Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/40-1.2.2.3-d+e-x^2-^m-a+b-x^2+c-x^4-^p

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [413]. This is test number [40].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.76 (412)	0.24 (1)
Mathematica	98.06 (405)	1.94 (8)
Maple	96.61 (399)	3.39 (14)
Fricas	80.15 (331)	19.85 (82)
Sympy	45.28 (187)	54.72 (226)
Mupad	44.55 (184)	55.45 (229)
Giac	44.07 (182)	55.93 (231)
Maxima	18.64 (77)	81.36 (336)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

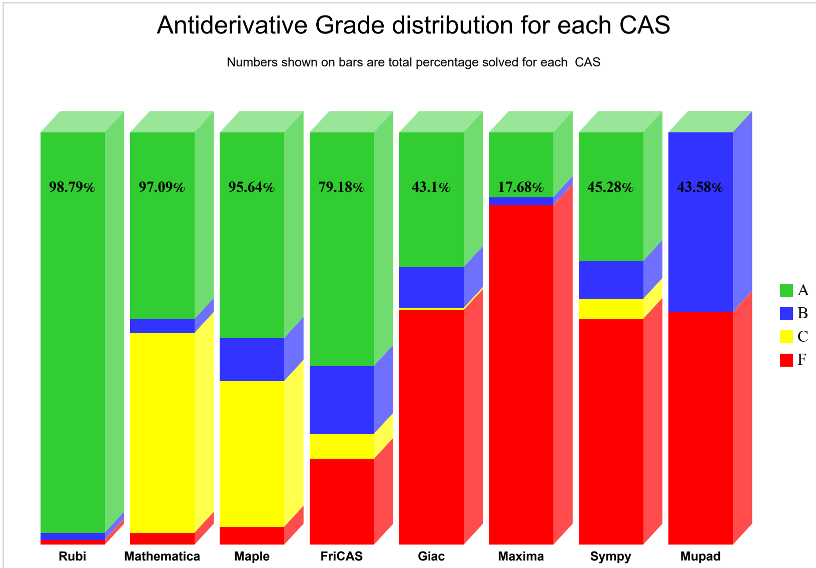
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

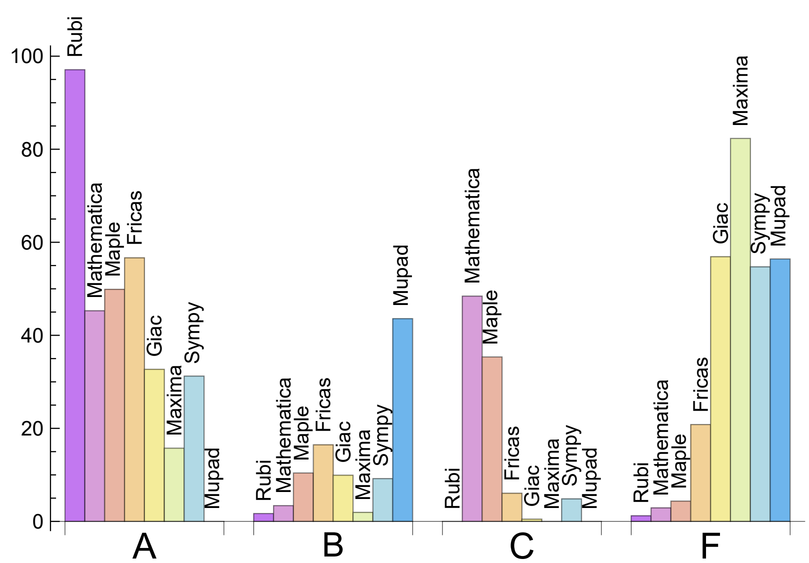
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.852	1.937	0.000	1.211
Fricas	56.659	16.465	6.053	20.823
Maple	49.879	10.412	35.351	4.358
Mathematica	45.278	3.390	48.426	2.906
Giac	32.688	9.927	0.484	56.901
Sympy	31.235	9.201	4.843	54.722
Maxima	15.738	1.937	0.000	82.324
Mupad	0.000	43.584	0.000	56.416

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	8	75.00	25.00	0.00
Maple	14	100.00	0.00	0.00
Fricas	82	79.27	20.73	0.00
Sympy	226	83.19	14.16	2.65
Mupad	229	0.00	100.00	0.00
Giac	231	96.54	0.00	3.46
Maxima	336	85.42	0.60	13.99

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Rubi	0.38
Giac	0.58
Maple	0.97
Fricas	2.03
Sympy	3.19
Mathematica	4.37
Mupad	7.64

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	97.43	1.14	45.00	0.93
Rubi	159.59	1.07	112.50	1.00
Maple	159.83	1.19	119.00	0.93
Mathematica	171.93	1.21	110.00	1.00
Sympy	233.94	2.07	82.00	1.08
Fricas	554.18	2.61	118.00	1.19
Giac	1126.08	3.80	83.00	1.01
Mupad	3780.98	8.37	67.00	0.96

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

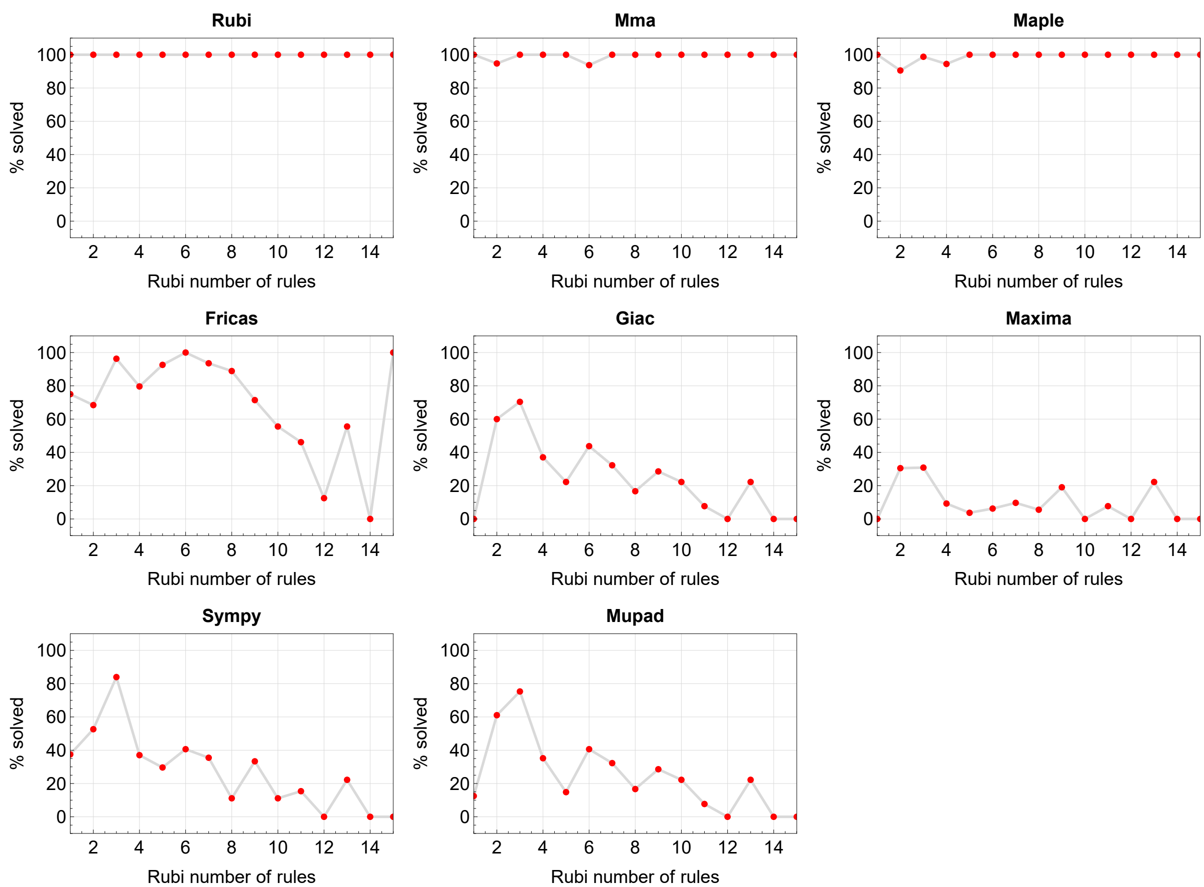


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

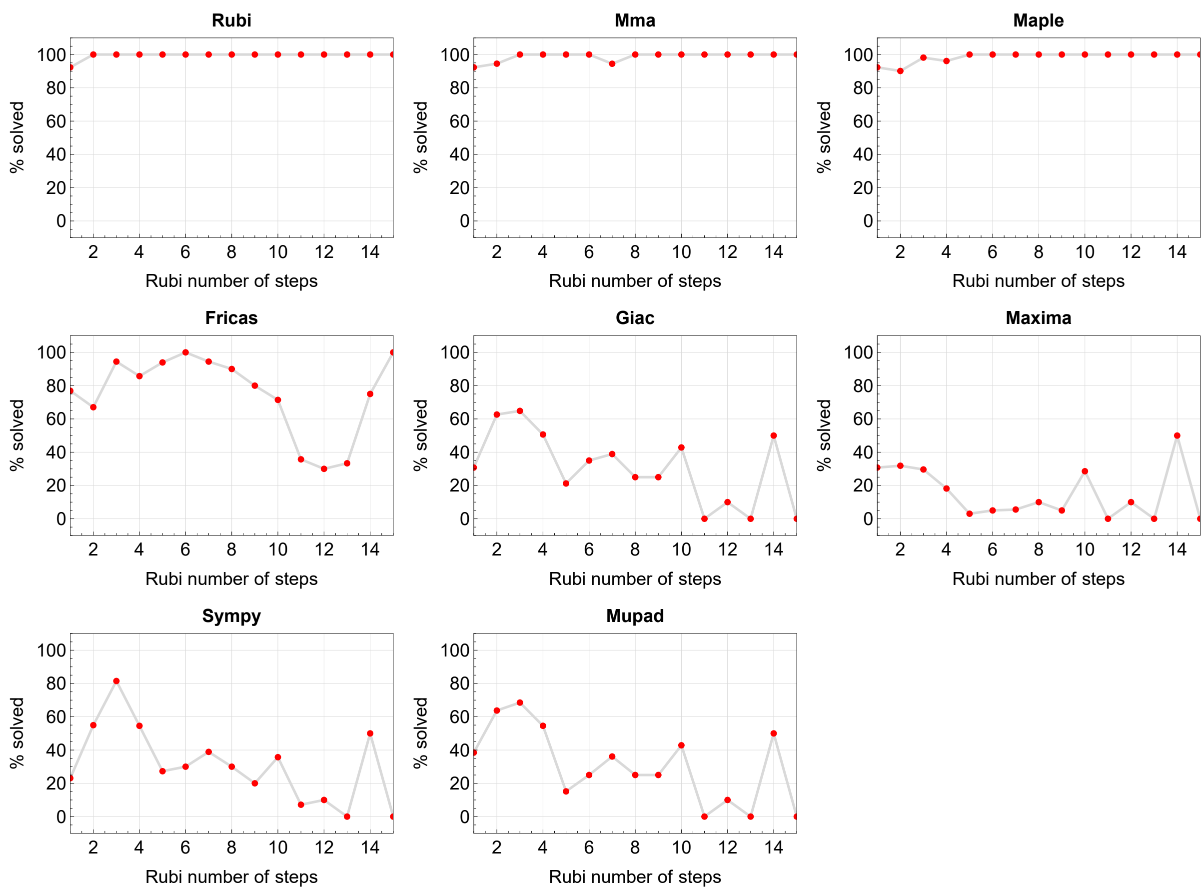


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

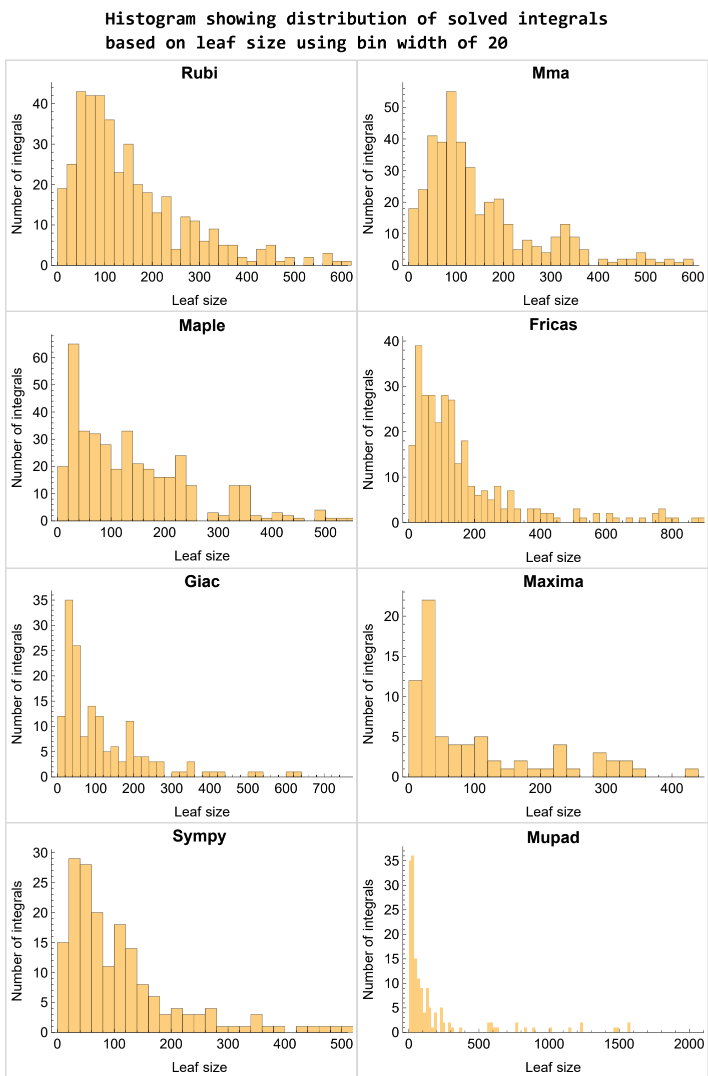


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

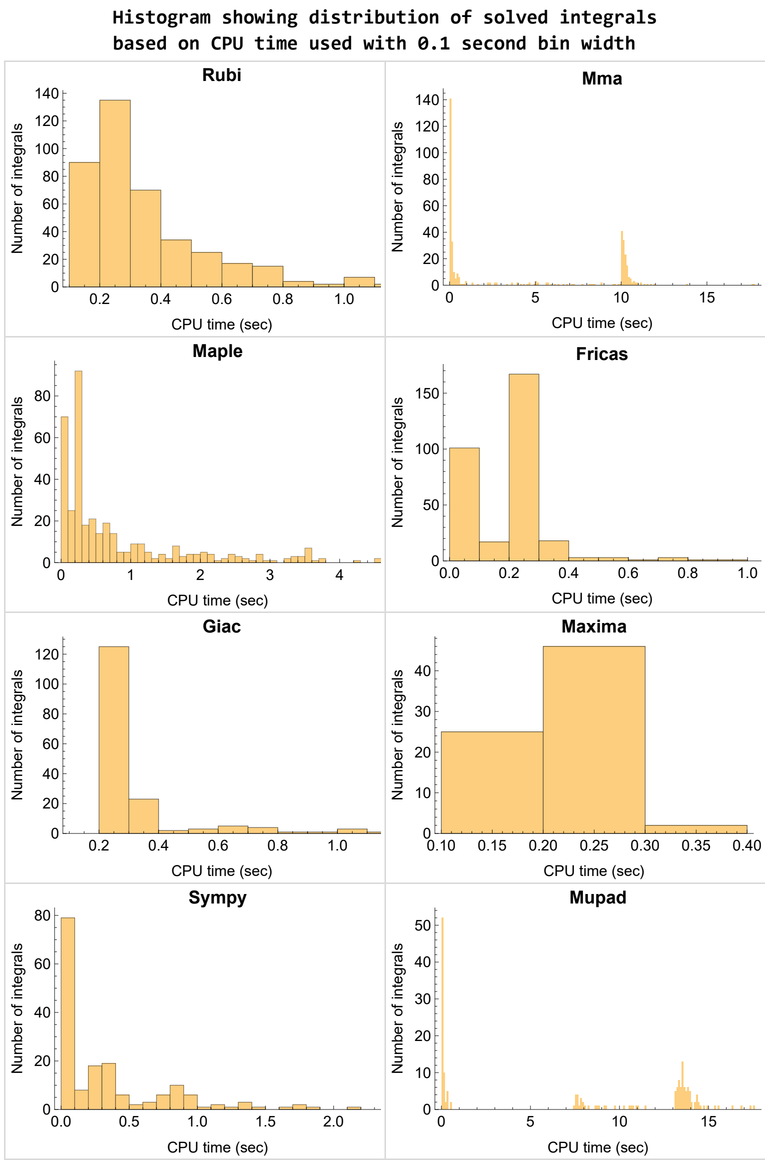


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

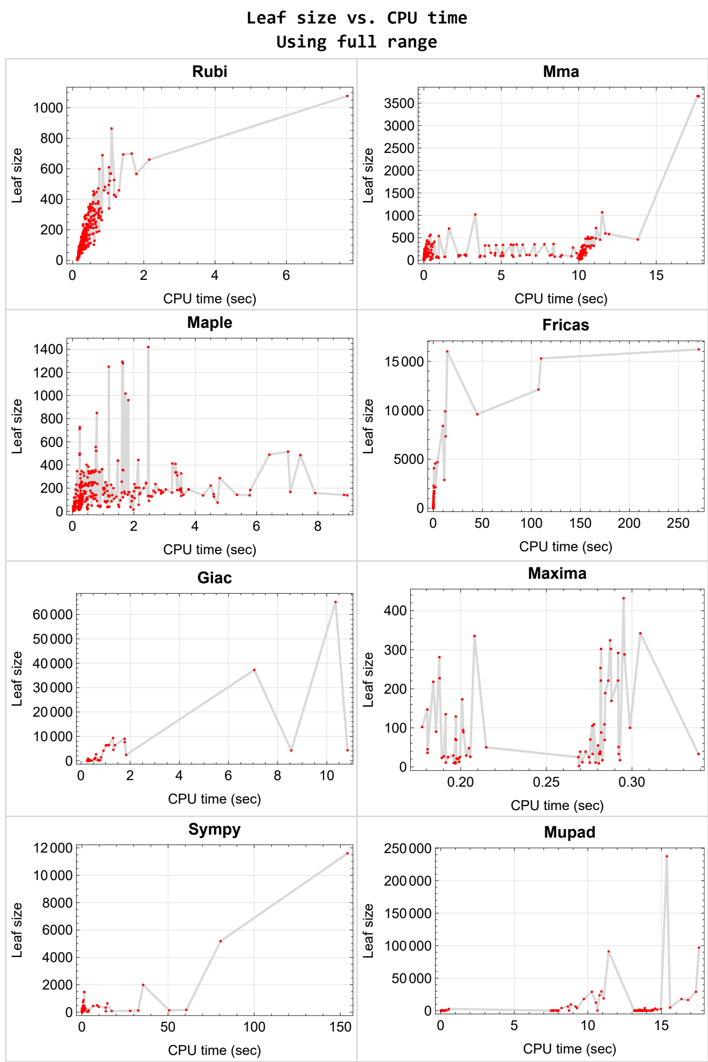


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{175, 399, 404, 405\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{113, 116, 118\}$

Mathematica $\{115, 402\}$

Maple $\{16, 17, 20, 21\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

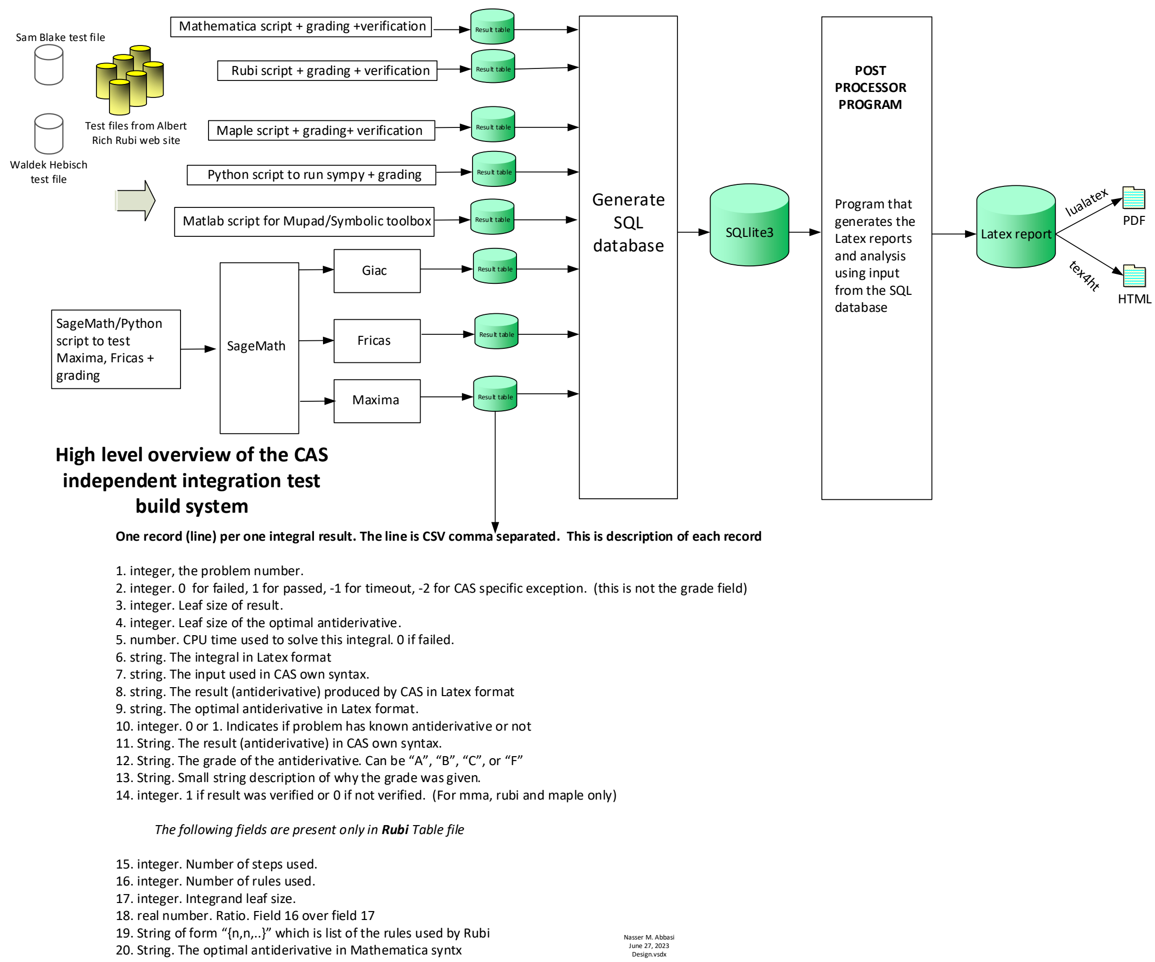
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 80, 81, 82, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

B grade { 53, 67, 78, 79, 83, 89, 90, 91 }

C grade { }

F normal fail { 174 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 22, 24, 34, 35, 36, 37, 41, 42, 43, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 94, 95, 96, 97, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 170, 176, 177, 178, 179, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 400, 401, 402, 403, 408, 409, 410, 411 }

B grade { 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 65, 80, 88 }

C grade { 14, 15, 16, 17, 18, 19, 20, 21, 23, 25, 44, 45, 46, 48, 49, 73, 92, 93, 98, 99, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 199, 200, 201, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 406, 407, 412, 413 }

F normal fail { 174, 180, 181, 186, 187, 188 }

F(-1) timedout fail { 104, 105 }

F(-2) exception fail { }

2.1.3 Maple

A grade { 5, 6, 7, 8, 11, 12, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 107, 108, 110, 111, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 142, 143, 148, 149, 157, 158, 159, 160, 164, 165, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 316, 317, 323, 324, 325, 326, 331, 338, 339, 340, 347, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 407, 412, 413 }

B grade { 9, 10, 113, 114, 115, 116, 117, 118, 161, 162, 163, 166, 167, 170, 172, 202, 203, 204, 208, 209, 210, 318, 319, 320, 321, 322, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 341, 342, 343, 344, 345, 346, 389 }

C grade { 1, 2, 3, 4, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 97, 100, 101, 102, 103, 104, 105, 106, 109, 112, 137, 138, 139, 140, 141, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 168, 169, 171, 173, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 266, 267, 270, 271, 272, 273, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 394, 395, 396, 397, 398, 406, 408, 409, 410, 411 }

F normal fail { 174, 176, 177, 178, 179, 180, 181, 186, 187, 188, 400, 401, 402, 403 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 98, 99, 108, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 150, 151, 152, 153, 157, 158, 159, 164, 168, 189, 190, 191, 192, 193, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 215, 216, 217, 218, 220, 221, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 332, 333, 334, 335, 339, 340, 341, 342, 343, 344, 348, 349, 350, 351, 352, 356, 357, 358, 359, 360, 364, 365, 366, 367, 371, 372, 373, 374, 375, 376, 380, 381, 382, 385, 386, 387, 390, 392 }

B grade { 1, 2, 3, 4, 7, 14, 15, 22, 23, 24, 25, 65, 88, 94, 95, 96, 97, 100, 101, 104, 105, 106, 107, 109, 110, 111, 112, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 166, 167, 194, 197, 198, 211, 212, 213, 214, 219, 222, 223, 224, 258, 259, 260, 263, 264, 265, 266, 267, 268, 270, 271, 272, 273, 408, 409, 410, 411 }

C grade { 102, 103, 141, 147, 286, 287, 288, 289, 293, 294, 295, 296, 300, 301, 302, 303, 307, 308, 309, 310, 311, 312, 394, 395, 396 }

F normal fail { 154, 160, 165, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 290, 291, 292, 297, 298, 299, 304, 305, 306, 313, 314, 315, 321, 322, 323, 329, 330, 331, 336, 337, 338, 345, 346, 347, 353, 354, 355, 361, 362, 363, 368, 369, 370, 377, 378, 379, 384, 397, 400, 401, 402, 403, 413 }

F(-1) timedout fail { 155, 156, 161, 162, 163, 269, 274, 275, 383, 388, 389, 391, 393, 398, 406, 407, 412 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 13, 37, 42, 43, 44, 47, 51, 52, 56, 57, 58, 61, 66, 72, 73, 74, 76, 77, 84, 85, 86, 89, 92, 93, 94, 96, 98, 99, 108, 120, 121, 122, 123, 128, 129, 130, 131, 137, 138, 139, 140, 141, 144, 145, 146, 147, 244, 245, 246, 247, 252, 253, 254, 255, 284, 285 }

B grade { 7, 11, 12, 65, 88, 95, 282, 283 }

C grade { }

F normal fail { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 45, 46, 48, 49, 50, 53, 54, 55, 59, 60, 62, 63, 64, 67, 68, 69, 70, 71, 75, 78, 79, 80, 81, 82, 83, 87, 90, 91, 97, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 266, 267, 270, 271, 272, 273, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-1) timedout fail { 214, 344 }

F(-2) exception fail { 38, 124, 125, 126, 127, 132, 133, 134, 135, 136, 142, 143, 148, 149, 189, 190, 191, 192, 193, 194, 215, 216, 217, 218, 219, 248, 249, 250, 251, 256, 257, 258, 259, 260, 261, 262, 268, 269, 274, 275, 276, 277, 278, 279, 280, 281, 397 }

2.1.6 Giac

A grade { 1, 2, 5, 6, 8, 11, 12, 37, 38, 41, 42, 43, 44, 45, 46, 48, 49, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 98, 99, 102, 103, 108, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 189, 190, 191, 192, 193, 194, 198, 214, 215, 216, 217, 218, 219, 222, 223, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285 }

B grade { 3, 4, 7, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 47, 50, 53, 65, 88, 95, 96, 100, 101, 195, 196, 197, 224, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275 }

C grade { 106, 109 }

F normal fail { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 39, 40, 54, 68, 80, 104, 105, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-1) timedout fail { }

F(-2) exception fail { 9, 10, 107, 110, 111, 112, 220, 221 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 179, 185, 189, 190, 191, 192, 193, 194, 214, 215, 216, 217, 218, 219, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 282, 283, 284, 285 }

C grade { }

F normal fail { }

F(-1) timedout fail { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 276, 277, 278, 279, 280, 281, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 100, 101, 106, 109, 120, 121, 122, 123, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 157, 158, 159, 164, 166, 189, 190, 191, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 258, 267, 277, 278, 279, 280 }

B grade { 7, 14, 15, 23, 25, 38, 42, 65, 88, 95, 96, 104, 105, 124, 125, 126, 132, 133, 167, 192, 193, 194, 214, 215, 216, 217, 248, 249, 256, 257, 261, 262, 276, 281, 282, 283, 284, 285 }

C grade { 18, 19, 20, 21, 98, 99, 108, 150, 151, 152, 153, 168, 176, 177, 178, 179, 182, 183, 184, 185 }

F normal fail { 22, 24, 113, 114, 115, 116, 117, 118, 119, 154, 155, 156, 160, 161, 162, 163, 165, 169, 170, 171, 172, 173, 174, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

F(-1) timedout fail { 142, 143, 148, 149, 175, 180, 181, 186, 187, 188, 218, 219, 220, 259, 260, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 399, 400, 401, 404, 405 }

F(-2) exception fail { 102, 103, 107, 110, 111, 112 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	230	183	34	221	767	109	241	599
N.S.	1	0.93	0.74	0.14	0.89	3.11	0.44	0.98	2.43
time (sec)	N/A	0.419	0.057	0.200	0.282	0.260	0.322	0.266	0.376

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	230	184	35	221	767	110	241	603
N.S.	1	0.93	0.74	0.14	0.89	3.11	0.45	0.98	2.44
time (sec)	N/A	0.394	0.033	0.193	0.286	0.256	0.323	0.284	13.696

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	85	95	36	109	755	110	230	579
N.S.	1	0.99	1.10	0.42	1.27	8.78	1.28	2.67	6.73
time (sec)	N/A	0.195	0.021	0.198	0.278	0.265	0.361	0.269	0.333

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	85	95	37	109	755	110	228	579
N.S.	1	0.99	1.10	0.43	1.27	8.78	1.28	2.65	6.73
time (sec)	N/A	0.195	0.020	0.191	0.284	0.281	0.334	0.267	13.680

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	33	35	39	33	41	52	29
N.S.	1	1.00	0.82	0.88	0.98	0.82	1.02	1.30	0.72
time (sec)	N/A	0.172	0.016	0.236	0.273	0.238	0.053	0.276	0.091

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	44	40	39	42	49	40	21
N.S.	1	1.00	0.86	0.78	0.76	0.82	0.96	0.78	0.41
time (sec)	N/A	0.193	0.011	0.200	0.270	0.258	0.046	0.260	13.563

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	32	13	25	29	32	29	12
N.S.	1	1.00	2.00	0.81	1.56	1.81	2.00	1.81	0.75
time (sec)	N/A	0.143	0.013	0.214	0.269	0.248	0.041	0.272	0.104

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	0.75	0.75
time (sec)	N/A	0.144	0.005	0.199	0.271	0.256	0.052	0.291	0.028

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	204	100	148	138	0	57
N.S.	1	1.00	0.80	2.72	1.33	1.97	1.84	0.00	0.76
time (sec)	N/A	0.202	0.017	0.247	0.299	0.283	0.172	0.000	13.896

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	91	204	70	151	131	0	43
N.S.	1	1.00	0.86	1.92	0.66	1.42	1.24	0.00	0.41
time (sec)	N/A	0.238	0.015	0.245	0.276	0.264	0.203	0.000	13.854

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	83	302	137	87	92	57
N.S.	1	1.00	0.80	1.11	4.03	1.83	1.16	1.23	0.76
time (sec)	N/A	0.202	0.025	0.272	0.282	0.263	0.096	0.287	0.070

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	75	75	302	140	80	100	41
N.S.	1	1.00	0.83	0.83	3.36	1.56	0.89	1.11	0.46
time (sec)	N/A	0.238	0.016	0.222	0.288	0.253	0.094	0.267	13.571

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	18	17	17	22	19	9
N.S.	1	1.00	1.92	1.38	1.31	1.31	1.69	1.46	0.69
time (sec)	N/A	0.145	0.006	0.224	0.293	0.262	0.070	0.263	13.492

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	45	36	0	71	70	0	0
N.S.	1	1.00	2.81	2.25	0.00	4.44	4.38	0.00	0.00
time (sec)	N/A	0.152	10.020	1.888	0.000	0.085	0.848	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	45	36	0	69	70	0	0
N.S.	1	1.00	1.29	1.03	0.00	1.97	2.00	0.00	0.00
time (sec)	N/A	0.184	10.020	1.130	0.000	0.074	0.881	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	43	43	74	88	0	56	61	0	0
N.S.	1	1.00	1.72	2.05	0.00	1.30	1.42	0.00	0.00
time (sec)	N/A	0.180	10.031	1.927	0.000	0.076	0.799	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	89	63	74	88	0	53	60	0	0
N.S.	1	0.71	0.83	0.99	0.00	0.60	0.67	0.00	0.00
time (sec)	N/A	0.213	10.026	1.139	0.000	0.074	0.822	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	47	38	0	73	66	0	0
N.S.	1	1.00	0.53	0.43	0.00	0.82	0.74	0.00	0.00
time (sec)	N/A	0.180	10.018	1.514	0.000	0.080	0.786	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	47	38	0	73	66	0	0
N.S.	1	1.00	0.31	0.25	0.00	0.48	0.43	0.00	0.00
time (sec)	N/A	0.241	10.015	0.471	0.000	0.078	0.780	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	90	90	76	90	0	87	70	0	0
N.S.	1	1.00	0.84	1.00	0.00	0.97	0.78	0.00	0.00
time (sec)	N/A	0.186	10.026	2.491	0.000	0.076	0.837	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	156	156	76	90	0	84	71	0	0
N.S.	1	1.00	0.49	0.58	0.00	0.54	0.46	0.00	0.00
time (sec)	N/A	0.253	10.019	0.795	0.000	0.080	0.854	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	15	0	76	0	0	0
N.S.	1	1.00	1.00	1.50	0.00	7.60	0.00	0.00	0.00
time (sec)	N/A	0.131	0.410	1.981	0.000	0.081	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	47	38	0	65	71	0	0
N.S.	1	1.00	4.70	3.80	0.00	6.50	7.10	0.00	0.00
time (sec)	N/A	0.144	10.021	1.200	0.000	0.076	0.900	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	73	0	0	0
N.S.	1	1.00	1.04	1.22	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.165	0.006	0.477	0.000	0.086	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	47	38	0	62	71	0	0
N.S.	1	1.00	2.04	1.65	0.00	2.70	3.09	0.00	0.00
time (sec)	N/A	0.179	10.020	1.146	0.000	0.080	0.862	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	90	181	71	0	162	122	189	94
N.S.	1	1.10	2.21	0.87	0.00	1.98	1.49	2.30	1.15
time (sec)	N/A	0.242	0.076	0.090	0.000	0.254	0.286	0.700	14.338

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	90	181	71	0	162	122	191	98
N.S.	1	1.10	2.21	0.87	0.00	1.98	1.49	2.33	1.20
time (sec)	N/A	0.248	0.075	0.088	0.000	0.259	0.286	0.688	0.100

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	88	189	75	0	176	110	195	30
N.S.	1	1.13	2.42	0.96	0.00	2.26	1.41	2.50	0.38
time (sec)	N/A	0.241	0.071	0.080	0.000	0.263	0.302	0.714	14.382

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	94	189	75	0	179	121	197	88
N.S.	1	1.09	2.20	0.87	0.00	2.08	1.41	2.29	1.02
time (sec)	N/A	0.244	0.067	0.079	0.000	0.254	0.293	0.711	0.105

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	182	88	0	172	121	189	99
N.S.	1	1.00	2.33	1.13	0.00	2.21	1.55	2.42	1.27
time (sec)	N/A	0.252	0.083	0.068	0.000	0.261	0.300	0.679	13.987

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	182	69	0	173	110	190	57
N.S.	1	1.00	2.33	0.88	0.00	2.22	1.41	2.44	0.73
time (sec)	N/A	0.245	0.079	0.066	0.000	0.270	0.303	0.699	13.569

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	168	112	199	29
N.S.	1	1.00	2.71	0.87	0.00	2.40	1.60	2.84	0.41
time (sec)	N/A	0.235	0.082	0.063	0.000	0.261	0.312	0.772	0.124

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	168	112	200	29
N.S.	1	1.00	2.71	0.87	0.00	2.40	1.60	2.86	0.41
time (sec)	N/A	0.236	0.084	0.062	0.000	0.258	0.324	0.687	13.198

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	250	121	0	244	158	6331	129
N.S.	1	1.00	1.87	0.90	0.00	1.82	1.18	47.25	0.96
time (sec)	N/A	0.330	0.123	0.224	0.000	0.255	0.362	1.009	0.192

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	152	248	119	0	232	160	6341	232
N.S.	1	1.17	1.91	0.92	0.00	1.78	1.23	48.78	1.78
time (sec)	N/A	0.295	0.077	0.093	0.000	0.263	0.393	1.041	13.935

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	152	248	119	0	232	160	6341	232
N.S.	1	1.17	1.91	0.92	0.00	1.78	1.23	48.78	1.78
time (sec)	N/A	0.309	0.030	0.052	0.000	0.262	0.382	1.097	13.832

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	25	25	26	25	12
N.S.	1	1.00	1.00	0.90	0.86	0.86	0.90	0.86	0.41
time (sec)	N/A	0.187	0.016	0.073	0.200	0.287	0.237	0.261	0.113

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	67	138	52	0	164	117	51	55
N.S.	1	1.12	2.30	0.87	0.00	2.73	1.95	0.85	0.92
time (sec)	N/A	0.224	0.133	0.071	0.000	0.259	0.254	0.269	13.774

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	71	126	74	0	110	95	0	66
N.S.	1	1.15	2.03	1.19	0.00	1.77	1.53	0.00	1.06
time (sec)	N/A	0.207	0.053	0.122	0.000	0.273	0.208	0.000	13.575

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	75	134	66	0	120	83	0	24
N.S.	1	1.14	2.03	1.00	0.00	1.82	1.26	0.00	0.36
time (sec)	N/A	0.206	0.047	0.086	0.000	0.253	0.217	0.000	13.198

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	81	83	35	0	31	42	39	29
N.S.	1	1.80	1.84	0.78	0.00	0.69	0.93	0.87	0.64
time (sec)	N/A	0.204	0.060	0.110	0.000	0.249	0.058	0.265	0.095

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	17	12	11	19	22	11	19
N.S.	1	1.00	1.13	0.80	0.73	1.27	1.47	0.73	1.27
time (sec)	N/A	0.152	0.008	0.064	0.279	0.257	0.062	0.272	13.611

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	11	11	14	11	11
N.S.	1	1.00	1.00	0.86	0.79	0.79	1.00	0.79	0.79
time (sec)	N/A	0.146	0.004	0.036	0.275	0.237	0.044	0.283	0.027

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	43	97	34	33	33	44	33	29
N.S.	1	1.13	2.55	0.89	0.87	0.87	1.16	0.87	0.76
time (sec)	N/A	0.181	0.153	0.067	0.292	0.237	0.059	0.271	13.745

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	51	99	35	0	29	42	45	29
N.S.	1	1.06	2.06	0.73	0.00	0.60	0.88	0.94	0.60
time (sec)	N/A	0.190	0.074	0.080	0.000	0.246	0.055	0.278	0.091

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	55	97	35	0	33	44	52	29
N.S.	1	1.20	2.11	0.76	0.00	0.72	0.96	1.13	0.63
time (sec)	N/A	0.195	0.181	0.101	0.000	0.249	0.061	0.294	13.719

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	16	17	15	14	46	15
N.S.	1	1.00	0.81	0.76	0.81	0.71	0.67	2.19	0.71
time (sec)	N/A	0.166	0.006	0.240	0.283	0.247	0.045	0.262	0.058

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	55	101	35	0	31	42	52	29
N.S.	1	1.20	2.20	0.76	0.00	0.67	0.91	1.13	0.63
time (sec)	N/A	0.186	0.210	0.079	0.000	0.255	0.062	0.306	13.650

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	51	99	27	0	26	29	46	21
N.S.	1	1.16	2.25	0.61	0.00	0.59	0.66	1.05	0.48
time (sec)	N/A	0.188	0.073	0.086	0.000	0.246	0.054	0.290	0.061

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	33	14	16	0	15	12	42	15
N.S.	1	1.43	0.61	0.70	0.00	0.65	0.52	1.83	0.65
time (sec)	N/A	0.184	0.010	0.090	0.000	0.251	0.054	0.300	0.081

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	12	11	12	12	8	12	12
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.73	1.09	1.09
time (sec)	N/A	0.144	0.005	0.027	0.197	0.262	0.040	0.266	0.044

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	41	29	26	29	25	26	33	14
N.S.	1	1.05	0.74	0.67	0.74	0.64	0.67	0.85	0.36
time (sec)	N/A	0.201	0.008	0.043	0.203	0.263	0.051	0.271	0.306

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	109	42	39	0	47	46	77	20
N.S.	1	2.48	0.95	0.89	0.00	1.07	1.05	1.75	0.45
time (sec)	N/A	0.274	0.010	0.087	0.000	0.247	0.049	0.310	13.329

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	127	78	0	109	94	0	63
N.S.	1	1.00	1.92	1.18	0.00	1.65	1.42	0.00	0.95
time (sec)	N/A	0.212	0.055	0.060	0.000	0.249	0.218	0.000	0.070

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	81	84	34	0	28	39	39	30
N.S.	1	1.76	1.83	0.74	0.00	0.61	0.85	0.85	0.65
time (sec)	N/A	0.189	0.045	0.047	0.000	0.241	0.061	0.276	13.492

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	12	10	9	17	14	9	17
N.S.	1	1.00	1.33	1.11	1.00	1.89	1.56	1.00	1.89
time (sec)	N/A	0.146	0.006	0.048	0.281	0.260	0.066	0.277	0.061

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	11	11	7	11	11
N.S.	1	1.00	1.00	1.00	1.00	1.00	0.64	1.00	1.00
time (sec)	N/A	0.147	0.006	0.025	0.191	0.254	0.041	0.264	0.027

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	25	25	26	25	12
N.S.	1	1.00	1.00	0.83	0.86	0.86	0.90	0.86	0.41
time (sec)	N/A	0.167	0.005	0.031	0.193	0.254	0.052	0.257	0.066

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	34	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.68	0.40
time (sec)	N/A	0.193	0.010	0.037	0.000	0.252	0.048	0.271	0.063

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	43	46	41	20
N.S.	1	1.00	0.84	0.78	0.00	0.86	0.92	0.82	0.40
time (sec)	N/A	0.187	0.010	0.043	0.000	0.251	0.051	0.298	13.207

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	27	27	22	34	15
N.S.	1	1.00	1.00	0.71	0.87	0.87	0.71	1.10	0.48
time (sec)	N/A	0.170	0.004	0.191	0.190	0.245	0.046	0.264	0.071

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	41	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.82	0.40
time (sec)	N/A	0.192	0.010	0.046	0.000	0.259	0.055	0.297	13.357

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	40	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.80	0.40
time (sec)	N/A	0.190	0.015	0.044	0.000	0.273	0.048	0.271	0.072

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	41	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.82	0.40
time (sec)	N/A	0.189	0.013	0.052	0.000	0.261	0.048	0.287	0.089

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	32	12	25	29	32	29	11
N.S.	1	1.00	2.29	0.86	1.79	2.07	2.29	2.07	0.79
time (sec)	N/A	0.147	0.012	0.030	0.275	0.239	0.046	0.276	13.368

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	49	31	26	29	27	29	33	15
N.S.	1	1.26	0.79	0.67	0.74	0.69	0.74	0.85	0.38
time (sec)	N/A	0.200	0.006	0.044	0.196	0.276	0.048	0.269	13.242

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	117	42	39	0	45	46	77	20
N.S.	1	2.44	0.88	0.81	0.00	0.94	0.96	1.60	0.42
time (sec)	N/A	0.260	0.014	0.050	0.000	0.245	0.050	0.327	0.132

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	63	124	74	0	101	88	0	73
N.S.	1	1.02	2.00	1.19	0.00	1.63	1.42	0.00	1.18
time (sec)	N/A	0.206	0.043	0.113	0.000	0.252	0.193	0.000	0.070

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	89	83	35	0	31	41	26	29
N.S.	1	1.82	1.69	0.71	0.00	0.63	0.84	0.53	0.59
time (sec)	N/A	0.229	0.102	0.103	0.000	0.248	0.053	0.271	13.277

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	79	81	35	0	31	41	26	29
N.S.	1	1.84	1.88	0.81	0.00	0.72	0.95	0.60	0.67
time (sec)	N/A	0.188	0.054	0.096	0.000	0.243	0.051	0.283	0.089

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	89	83	35	0	31	41	26	29
N.S.	1	1.82	1.69	0.71	0.00	0.63	0.84	0.53	0.59
time (sec)	N/A	0.206	0.065	0.092	0.000	0.278	0.057	0.265	0.086

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.135	0.003	0.032	0.269	0.240	0.059	0.290	13.386

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	37	99	34	33	31	41	26	29
N.S.	1	0.97	2.61	0.89	0.87	0.82	1.08	0.68	0.76
time (sec)	N/A	0.173	0.131	0.049	0.277	0.243	0.054	0.267	0.085

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	35	39	29	39	39	29
N.S.	1	1.00	0.86	1.00	1.11	0.83	1.11	1.11	0.83
time (sec)	N/A	0.172	0.011	0.217	0.281	0.248	0.049	0.274	13.347

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	12	8	0	7	7	30	7
N.S.	1	1.00	0.52	0.35	0.00	0.30	0.30	1.30	0.30
time (sec)	N/A	0.178	0.007	0.067	0.000	0.239	0.050	0.277	13.276

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	11	10	10	7	11	10
N.S.	1	1.00	0.91	1.00	0.91	0.91	0.64	1.00	0.91
time (sec)	N/A	0.141	0.004	0.027	0.196	0.235	0.036	0.298	13.361

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	63	29	22	21	21	19	43	12
N.S.	1	0.97	0.45	0.34	0.32	0.32	0.29	0.66	0.18
time (sec)	N/A	0.231	0.005	0.037	0.198	0.257	0.048	0.280	0.269

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	102	40	35	0	36	39	39	18
N.S.	1	2.37	0.93	0.81	0.00	0.84	0.91	0.91	0.42
time (sec)	N/A	0.263	0.010	0.076	0.000	0.240	0.048	0.288	13.528

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	105	40	35	0	39	39	39	18
N.S.	1	2.28	0.87	0.76	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.261	0.016	0.077	0.000	0.234	0.050	0.302	0.232

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	125	78	0	100	87	0	76
N.S.	1	1.00	2.02	1.26	0.00	1.61	1.40	0.00	1.23
time (sec)	N/A	0.205	0.060	0.063	0.000	0.236	0.204	0.000	0.062

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	89	87	35	0	31	42	26	31
N.S.	1	1.78	1.74	0.70	0.00	0.62	0.84	0.52	0.62
time (sec)	N/A	0.187	0.066	0.049	0.000	0.255	0.055	0.284	13.145

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	79	82	35	0	31	42	26	31
N.S.	1	1.80	1.86	0.80	0.00	0.70	0.95	0.59	0.70
time (sec)	N/A	0.182	0.041	0.050	0.000	0.251	0.067	0.271	0.085

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	87	10	14	0	13	10	26	13
N.S.	1	2.23	0.26	0.36	0.00	0.33	0.26	0.67	0.33
time (sec)	N/A	0.190	0.006	0.059	0.000	0.247	0.050	0.274	13.513

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	5	7	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.56	0.78	1.00
time (sec)	N/A	0.143	0.004	0.027	0.197	0.258	0.032	0.265	0.032

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	19	35	10
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.76	1.40	0.40
time (sec)	N/A	0.165	0.005	0.032	0.198	0.249	0.051	0.268	0.063

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	34	34	39	34	18
N.S.	1	1.00	0.87	0.76	0.74	0.74	0.85	0.74	0.39
time (sec)	N/A	0.183	0.008	0.218	0.281	0.265	0.042	0.268	13.246

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	0	39	39	39	18
N.S.	1	1.00	0.87	0.76	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.178	0.010	0.047	0.000	0.258	0.047	0.262	0.062

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	2
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00
time (sec)	N/A	0.134	0.002	0.023	0.199	0.258	0.045	0.266	0.065

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	85	40	34	55	39	39	39	18
N.S.	1	2.24	1.05	0.89	1.45	1.03	1.03	1.03	0.47
time (sec)	N/A	0.242	0.009	0.044	0.280	0.254	0.055	0.282	0.119

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	102	40	35	0	39	39	39	18
N.S.	1	2.17	0.85	0.74	0.00	0.83	0.83	0.83	0.38
time (sec)	N/A	0.259	0.012	0.042	0.000	0.265	0.050	0.296	0.074

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	105	40	35	0	39	39	39	18
N.S.	1	2.28	0.87	0.76	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.257	0.010	0.046	0.000	0.273	0.048	0.285	0.118

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	53	99	34	33	33	46	33	29
N.S.	1	1.23	2.30	0.79	0.77	0.77	1.07	0.77	0.67
time (sec)	N/A	0.181	0.076	0.061	0.281	0.275	0.062	0.289	13.395

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	53	99	34	33	33	46	33	29
N.S.	1	1.23	2.30	0.79	0.77	0.77	1.07	0.77	0.67
time (sec)	N/A	0.185	0.017	0.036	0.339	0.258	0.066	0.278	0.002

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	24	23	34	22	25	17
N.S.	1	1.00	1.29	1.14	1.10	1.62	1.05	1.19	0.81
time (sec)	N/A	0.149	0.010	0.036	0.189	0.251	0.044	0.271	13.453

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	53	26	38	49	53	44	17
N.S.	1	1.00	1.89	0.93	1.36	1.75	1.89	1.57	0.61
time (sec)	N/A	0.154	0.015	0.051	0.282	0.254	0.299	0.271	13.524

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	72	45	51	55	474	57	290
N.S.	1	1.00	2.00	1.25	1.42	1.53	13.17	1.58	8.06
time (sec)	N/A	0.175	0.034	0.067	0.292	0.266	0.793	0.287	13.410

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	91	73	40	0	147	46	41	117
N.S.	1	1.23	0.99	0.54	0.00	1.99	0.62	0.55	1.58
time (sec)	N/A	0.187	0.061	0.070	0.000	0.268	0.091	0.295	0.122

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	97	78	69	69	740	69	827
N.S.	1	1.05	1.17	0.94	0.83	0.83	8.92	0.83	9.96
time (sec)	N/A	0.258	0.089	0.122	0.284	0.263	0.630	0.281	13.439

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	128	147	136	105	185	874	109	897
N.S.	1	1.08	1.24	1.14	0.88	1.55	7.34	0.92	7.54
time (sec)	N/A	0.317	0.163	0.162	0.277	0.286	0.935	0.286	13.587

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	256	111	38	0	625	122	604	771
N.S.	1	1.09	0.47	0.16	0.00	2.67	0.52	2.58	3.29
time (sec)	N/A	0.455	0.081	0.159	0.000	0.283	0.640	0.540	13.521

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	343	166	82	0	953	165	1112	1491
N.S.	1	1.09	0.53	0.26	0.00	3.02	0.52	3.52	4.72
time (sec)	N/A	0.559	0.143	0.404	0.000	0.293	0.918	0.573	13.515

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	156	53	55	0	103	0	122	121
N.S.	1	0.98	0.33	0.34	0.00	0.64	0.00	0.76	0.76
time (sec)	N/A	0.389	0.043	0.238	0.000	0.263	0.000	0.368	13.946

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	168	53	52	0	103	0	126	121
N.S.	1	0.98	0.31	0.30	0.00	0.60	0.00	0.73	0.70
time (sec)	N/A	0.390	0.032	0.208	0.000	0.258	0.000	0.369	13.955

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	C	F	B	B	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	160	186	0	44	0	517	1469	0	1227
N.S.	1	1.16	0.00	0.28	0.00	3.23	9.18	0.00	7.67
time (sec)	N/A	0.375	0.000	0.117	0.000	0.265	1.339	0.000	14.434

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	C	F	B	B	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	160	184	0	42	0	513	1467	0	1227
N.S.	1	1.15	0.00	0.26	0.00	3.21	9.17	0.00	7.67
time (sec)	N/A	0.361	0.000	0.106	0.000	0.283	1.389	0.000	14.357

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	127	115	48	0	219	27	4217	133
N.S.	1	1.11	1.01	0.42	0.00	1.92	0.24	36.99	1.17
time (sec)	N/A	0.327	0.130	0.076	0.000	0.255	0.116	8.556	13.435

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	135	115	90	0	251	0	0	159
N.S.	1	1.11	0.94	0.74	0.00	2.06	0.00	0.00	1.30
time (sec)	N/A	0.329	0.103	0.121	0.000	0.267	0.000	0.000	14.209

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	111	115	87	88	264	143	92	133
N.S.	1	0.90	0.93	0.70	0.71	2.13	1.15	0.74	1.07
time (sec)	N/A	0.289	0.093	0.119	0.282	0.289	0.134	0.299	14.098

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	155	130	46	0	903	172	4293	1007
N.S.	1	1.14	0.96	0.34	0.00	6.64	1.26	31.57	7.40
time (sec)	N/A	0.335	0.104	0.053	0.000	0.286	0.934	10.843	13.843

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	179	138	188	0	1141	0	0	1155
N.S.	1	1.12	0.86	1.18	0.00	7.13	0.00	0.00	7.22
time (sec)	N/A	0.363	0.093	0.101	0.000	0.384	0.000	0.000	14.475

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	471	139	347	0	1457	0	0	3285
N.S.	1	1.14	0.34	0.84	0.00	3.52	0.00	0.00	7.93
time (sec)	N/A	0.722	0.105	0.148	0.000	0.600	0.000	0.000	14.576

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	261	163	75	0	1469	0	0	1575
N.S.	1	1.12	0.70	0.32	0.00	6.28	0.00	0.00	6.73
time (sec)	N/A	0.489	0.131	0.282	0.000	0.978	0.000	0.000	14.718

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	110	103	200	0	105	0	0	0
N.S.	1	1.15	1.07	2.08	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.281	10.134	2.321	0.000	0.100	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	19	113	0	51	0	0	0
N.S.	1	1.00	0.76	4.52	0.00	2.04	0.00	0.00	0.00
time (sec)	N/A	0.180	10.052	2.021	0.000	0.091	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	96	110	103	204	0	107	0	0	0
N.S.	1	1.15	1.07	2.12	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.303	10.139	2.087	0.000	0.096	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	106	107	204	0	107	0	0	0
N.S.	1	1.15	1.16	2.22	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.277	10.103	2.176	0.000	0.110	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	35	95	0	37	0	0	0
N.S.	1	1.00	1.30	3.52	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.191	10.090	1.559	0.000	0.089	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	106	107	204	0	107	0	0	0
N.S.	1	1.15	1.16	2.22	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.301	10.148	2.051	0.000	0.092	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	298	187	515	0	322	0	0	0
N.S.	1	1.01	0.63	1.74	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.387	10.233	7.023	0.000	0.115	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	96	94	94	110	98	95
N.S.	1	1.00	1.00	0.91	0.89	0.89	1.04	0.92	0.90
time (sec)	N/A	0.266	0.027	0.216	0.201	0.268	0.027	0.288	0.050

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	72	71	71	78	73	71
N.S.	1	1.00	1.00	0.91	0.90	0.90	0.99	0.92	0.90
time (sec)	N/A	0.241	0.013	0.207	0.197	0.271	0.030	0.348	13.896

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	49	48	48	56	50	49
N.S.	1	1.00	1.00	0.88	0.86	0.86	1.00	0.89	0.88
time (sec)	N/A	0.207	0.008	0.206	0.205	0.249	0.022	0.431	0.025

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.175	0.005	0.063	0.206	0.283	0.018	0.422	0.044

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	47	0	131	104	50	45
N.S.	1	1.00	1.00	0.85	0.00	2.38	1.89	0.91	0.82
time (sec)	N/A	0.198	0.028	0.224	0.000	0.284	0.144	0.346	14.265

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	85	78	70	0	222	138	72	68
N.S.	1	1.15	1.05	0.95	0.00	3.00	1.86	0.97	0.92
time (sec)	N/A	0.215	0.040	0.238	0.000	0.264	0.229	0.350	13.732

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	106	92	92	0	306	219	86	97
N.S.	1	1.14	0.99	0.99	0.00	3.29	2.35	0.92	1.04
time (sec)	N/A	0.234	0.043	0.248	0.000	0.283	0.352	0.327	13.730

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	131	113	113	0	424	204	111	129
N.S.	1	1.07	0.92	0.92	0.00	3.45	1.66	0.90	1.05
time (sec)	N/A	0.259	0.057	0.253	0.000	0.259	0.440	0.322	13.971

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	133	128	129	129	144	131	127
N.S.	1	1.00	1.00	0.96	0.97	0.97	1.08	0.98	0.95
time (sec)	N/A	0.313	0.021	0.240	0.197	0.243	0.036	0.316	0.060

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	90	89	89	104	91	89
N.S.	1	1.00	1.00	0.93	0.92	0.92	1.07	0.94	0.92
time (sec)	N/A	0.264	0.016	0.241	0.202	0.266	0.027	0.285	14.069

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	60	50	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.00	0.83	0.83
time (sec)	N/A	0.207	0.004	0.231	0.215	0.242	0.021	0.282	0.027

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.166	0.003	0.188	0.199	0.233	0.018	0.275	0.030

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	97	104	0	268	236	118	141
N.S.	1	1.00	0.90	0.96	0.00	2.48	2.19	1.09	1.31
time (sec)	N/A	0.255	0.052	0.284	0.000	0.279	0.233	0.275	13.691

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	142	134	129	0	394	314	144	183
N.S.	1	1.08	1.02	0.98	0.00	3.01	2.40	1.10	1.40
time (sec)	N/A	0.470	0.078	0.254	0.000	0.259	0.412	0.275	13.594

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	173	154	152	0	516	257	160	164
N.S.	1	1.12	0.99	0.98	0.00	3.33	1.66	1.03	1.06
time (sec)	N/A	0.563	0.077	0.268	0.000	0.267	0.751	0.283	13.184

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	212	174	179	0	662	292	183	199
N.S.	1	1.15	0.95	0.97	0.00	3.60	1.59	0.99	1.08
time (sec)	N/A	0.593	0.092	0.271	0.000	0.306	1.727	0.269	13.245

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	250	200	212	0	806	335	216	240
N.S.	1	1.12	0.90	0.95	0.00	3.61	1.50	0.97	1.08
time (sec)	N/A	0.620	0.127	0.285	0.000	0.280	13.765	0.296	13.305

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	444	118	432	2878	500	510	4022
N.S.	1	1.00	1.02	0.27	0.99	6.59	1.14	1.17	9.20
time (sec)	N/A	0.621	0.219	0.217	0.295	11.202	8.810	0.319	13.869

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	360	80	342	2133	350	411	2712
N.S.	1	1.00	0.97	0.22	0.92	5.76	0.95	1.11	7.33
time (sec)	N/A	0.578	0.174	0.230	0.305	2.391	1.440	0.276	0.540

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	269	56	288	1480	238	319	1479
N.S.	1	1.00	0.91	0.19	0.97	4.98	0.80	1.07	4.98
time (sec)	N/A	0.453	0.178	0.221	0.296	0.707	0.729	0.274	13.505

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	230	183	34	221	767	109	241	599
N.S.	1	0.93	0.74	0.14	0.89	3.11	0.44	0.98	2.43
time (sec)	N/A	0.430	0.035	0.216	0.292	0.296	0.341	0.277	0.354

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	200	134	27	169	112	20	179	33
N.S.	1	1.08	0.72	0.15	0.91	0.61	0.11	0.97	0.18
time (sec)	N/A	0.362	0.016	0.210	0.288	0.263	0.076	0.269	13.576

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	234	253	0	4084	0	344	4802
N.S.	1	1.00	0.70	0.75	0.00	12.15	0.00	1.02	14.29
time (sec)	N/A	0.459	0.106	0.310	0.000	0.697	0.000	0.286	15.587

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	362	307	0	8409	0	531	16369
N.S.	1	1.00	0.80	0.68	0.00	18.56	0.00	1.17	36.13
time (sec)	N/A	0.587	0.322	0.377	0.000	9.782	0.000	0.308	16.805

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	323	371	119	292	2116	352	430	2560
N.S.	1	0.89	1.02	0.33	0.80	5.83	0.97	1.18	7.05
time (sec)	N/A	0.626	0.175	0.479	0.292	0.716	1.639	0.290	14.968

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	325	295	97	324	1596	275	350	1565
N.S.	1	0.93	0.85	0.28	0.93	4.57	0.79	1.00	4.48
time (sec)	N/A	0.570	0.118	0.238	0.287	0.724	1.006	0.281	0.532

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	264	267	67	253	873	136	268	637
N.S.	1	0.96	0.97	0.24	0.92	3.17	0.49	0.97	2.32
time (sec)	N/A	0.460	0.196	0.237	0.282	0.287	0.450	0.279	14.343

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	225	183	46	189	183	39	194	58
N.S.	1	1.11	0.91	0.23	0.94	0.91	0.19	0.96	0.29
time (sec)	N/A	0.399	0.073	0.219	0.284	0.288	0.139	0.269	0.090

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	429	334	0	9892	0	621	17945
N.S.	1	1.00	0.62	0.48	0.00	14.36	0.00	0.90	26.04
time (sec)	N/A	0.824	0.198	0.361	0.000	12.224	0.000	0.285	16.364

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	864	864	540	402	0	15292	0	879	28923
N.S.	1	1.00	0.62	0.47	0.00	17.70	0.00	1.02	33.48
time (sec)	N/A	1.082	0.365	0.457	0.000	109.947	0.000	0.289	17.346

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	386	180	286	0	204	214	0	0
N.S.	1	0.99	0.46	0.74	0.00	0.53	0.55	0.00	0.00
time (sec)	N/A	0.724	10.180	4.797	0.000	0.094	2.113	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	322	140	235	0	166	173	0	0
N.S.	1	0.99	0.43	0.72	0.00	0.51	0.53	0.00	0.00
time (sec)	N/A	0.511	10.104	2.702	0.000	0.088	1.719	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	272	120	200	0	114	124	0	0
N.S.	1	1.03	0.45	0.76	0.00	0.43	0.47	0.00	0.00
time (sec)	N/A	0.352	10.067	1.333	0.000	0.086	1.297	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	228	77	169	0	89	78	0	0
N.S.	1	1.01	0.34	0.75	0.00	0.39	0.35	0.00	0.00
time (sec)	N/A	0.283	10.035	0.424	0.000	0.077	0.841	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	360	95	107	0	0	0	0	0
N.S.	1	1.08	0.28	0.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.494	10.159	0.474	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	569	522	556	0	0	0	0	0
N.S.	1	0.98	0.90	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.070	10.591	0.753	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	729	700	332	1018	0	0	0	0	0
N.S.	1	0.96	0.46	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.685	10.854	1.727	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	141	222	0	167	180	0	0
N.S.	1	1.00	0.66	1.04	0.00	0.78	0.85	0.00	0.00
time (sec)	N/A	0.521	10.115	4.503	0.000	0.102	1.881	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	167	121	186	0	115	129	0	0
N.S.	1	1.03	0.75	1.15	0.00	0.71	0.80	0.00	0.00
time (sec)	N/A	0.361	10.075	1.023	0.000	0.100	1.389	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	123	77	154	0	91	82	0	0
N.S.	1	0.99	0.62	1.24	0.00	0.73	0.66	0.00	0.00
time (sec)	N/A	0.289	10.043	0.418	0.000	0.097	0.947	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	91	97	0	0	0	0	0
N.S.	1	1.00	1.26	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	10.190	0.454	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	280	508	523	0	0	0	0	0
N.S.	1	0.94	1.70	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.720	10.810	0.768	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	417	321	961	0	0	0	0	0
N.S.	1	0.98	0.76	2.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.197	10.976	1.819	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	566	458	1420	0	0	0	0	0
N.S.	1	1.01	0.81	2.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.791	11.376	2.467	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	125	78	160	0	87	73	0	0
N.S.	1	0.99	0.62	1.27	0.00	0.69	0.58	0.00	0.00
time (sec)	N/A	0.287	10.040	1.051	0.000	0.090	0.900	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	92	99	0	0	0	0	0
N.S.	1	1.00	1.26	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	10.158	0.453	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	86	158	0	114	70	0	0
N.S.	1	1.00	1.59	2.93	0.00	2.11	1.30	0.00	0.00
time (sec)	N/A	0.197	10.038	0.622	0.000	0.084	0.916	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	85	165	0	131	76	0	0
N.S.	1	1.00	1.63	3.17	0.00	2.52	1.46	0.00	0.00
time (sec)	N/A	0.207	10.034	1.850	0.000	0.081	0.869	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	80	175	0	97	83	0	0
N.S.	1	1.00	0.34	0.74	0.00	0.41	0.35	0.00	0.00
time (sec)	N/A	0.294	10.039	1.132	0.000	0.081	0.867	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	369	98	110	0	0	0	0	0
N.S.	1	1.06	0.28	0.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.489	10.151	0.479	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	79	0	0	0	0	0
N.S.	1	1.00	1.00	1.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	10.213	1.084	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	338	50	86	0	0	0	0	0
N.S.	1	1.09	0.16	0.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	10.137	1.273	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	59	78	0	0	0	0	0
N.S.	1	1.00	1.48	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.165	10.133	1.084	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	318	65	86	0	0	0	0	0
N.S.	1	1.06	0.22	0.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	10.123	1.049	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	21	0	21	21
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.11
time (sec)	N/A	0.143	0.182	0.084	0.243	0.292	0.000	0.364	13.095

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	218	136	0	0	0	167	0	0
N.S.	1	1.07	0.67	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.455	0.586	0.000	0.000	0.000	60.588	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	158	106	0	0	0	119	0	0
N.S.	1	1.05	0.71	0.00	0.00	0.00	0.79	0.00	0.00
time (sec)	N/A	0.328	0.560	0.000	0.000	0.000	32.630	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	75	0	0	0	75	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.78	0.00	0.00
time (sec)	N/A	0.221	0.431	0.000	0.000	0.000	17.250	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	0.93
time (sec)	N/A	0.164	0.059	0.000	0.000	0.000	3.658	0.000	13.653

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.373	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	122	86	75	0	0	129	0	0
N.S.	1	1.13	0.80	0.69	0.00	0.00	1.19	0.00	0.00
time (sec)	N/A	0.341	0.868	4.728	0.000	0.000	50.668	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	94	65	56	0	0	94	0	0
N.S.	1	1.09	0.76	0.65	0.00	0.00	1.09	0.00	0.00
time (sec)	N/A	0.262	0.797	2.154	0.000	0.000	27.980	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	0	0	61	0	0
N.S.	1	1.00	1.00	0.88	0.00	0.00	1.45	0.00	0.00
time (sec)	N/A	0.181	0.549	0.891	0.000	0.000	13.868	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	0	0	29	0	15
N.S.	1	1.00	1.00	0.94	0.00	0.00	1.61	0.00	0.83
time (sec)	N/A	0.142	0.047	0.485	0.000	0.000	3.080	0.000	0.073

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	42	0	116	75	54	42
N.S.	1	1.00	1.00	0.82	0.00	2.27	1.47	1.06	0.82
time (sec)	N/A	0.199	0.018	0.249	0.000	0.256	0.101	0.265	0.098

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	0	90	58	42	28
N.S.	1	1.00	1.00	0.82	0.00	2.37	1.53	1.11	0.74
time (sec)	N/A	0.189	0.014	0.223	0.000	0.268	0.086	0.269	13.162

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	0	73	34	23	21
N.S.	1	1.00	1.00	0.76	0.00	2.52	1.17	0.79	0.72
time (sec)	N/A	0.163	0.008	0.496	0.000	0.280	0.072	0.266	10.652

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	68	46	18	16
N.S.	1	1.00	1.00	0.67	0.00	2.83	1.92	0.75	0.67
time (sec)	N/A	0.148	0.005	0.231	0.000	0.264	0.067	0.266	0.035

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	75	65	54	0	189	226	56	74
N.S.	1	1.04	0.90	0.75	0.00	2.62	3.14	0.78	1.03
time (sec)	N/A	0.205	0.026	0.296	0.000	0.270	0.211	0.275	0.095

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	101	76	64	0	278	257	66	96
N.S.	1	1.13	0.85	0.72	0.00	3.12	2.89	0.74	1.08
time (sec)	N/A	0.237	0.042	0.313	0.000	0.273	0.274	0.295	0.095

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	70	50	0	199	0	109	0
N.S.	1	1.00	1.13	0.81	0.00	3.21	0.00	1.76	0.00
time (sec)	N/A	0.213	0.087	0.623	0.000	0.264	0.000	0.332	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	50	33	0	138	0	82	0
N.S.	1	1.00	1.32	0.87	0.00	3.63	0.00	2.16	0.00
time (sec)	N/A	0.167	0.058	0.279	0.000	0.260	0.000	0.316	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	69	59	0	209	0	101	0
N.S.	1	1.00	1.13	0.97	0.00	3.43	0.00	1.66	0.00
time (sec)	N/A	0.189	0.112	0.285	0.000	0.270	0.000	0.335	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	88	80	69	0	279	0	113	0
N.S.	1	1.10	1.00	0.86	0.00	3.49	0.00	1.41	0.00
time (sec)	N/A	0.230	0.175	0.314	0.000	0.282	0.000	0.316	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	118	98	96	0	251	0	0	0
N.S.	1	0.77	0.64	0.63	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.236	2.724	0.284	0.000	0.271	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	88	86	75	0	223	0	0	0
N.S.	1	0.80	0.78	0.68	0.00	2.03	0.00	0.00	0.00
time (sec)	N/A	0.203	2.242	0.259	0.000	0.260	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	54	0	121	0	0	0
N.S.	1	1.00	0.77	0.83	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.180	0.959	0.230	0.000	0.269	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	249	0	152	0	0	0
N.S.	1	1.00	1.00	3.19	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.194	1.375	0.591	0.000	0.248	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	112	111	488	0	297	0	0	0
N.S.	1	0.90	0.89	3.90	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.222	2.332	0.231	0.000	0.253	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	149	123	711	0	365	0	0	0
N.S.	1	0.89	0.73	4.23	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.266	2.686	0.233	0.000	0.275	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	116	123	94	0	265	0	0	0
N.S.	1	0.76	0.81	0.62	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.239	2.707	0.233	0.000	0.256	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	86	110	74	0	236	0	0	0
N.S.	1	0.79	1.01	0.68	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.205	2.235	0.235	0.000	0.252	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	67	54	0	125	0	0	0
N.S.	1	1.00	1.05	0.84	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.186	0.934	0.231	0.000	0.261	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	256	0	155	0	0	0
N.S.	1	1.00	1.00	3.32	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.198	1.335	0.284	0.000	0.244	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	111	110	499	0	302	0	0	0
N.S.	1	0.90	0.89	4.02	0.00	2.44	0.00	0.00	0.00
time (sec)	N/A	0.221	2.351	0.234	0.000	0.254	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	148	122	728	0	376	0	0	0
N.S.	1	0.89	0.73	4.36	0.00	2.25	0.00	0.00	0.00
time (sec)	N/A	0.265	2.635	0.237	0.000	0.280	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	38	25	0	73	0	0	0
N.S.	1	1.00	1.27	0.83	0.00	2.43	0.00	0.00	0.00
time (sec)	N/A	0.158	0.510	0.231	0.000	0.238	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	40	34	33	0	65	0	0	0
N.S.	1	1.67	1.42	1.38	0.00	2.71	0.00	0.00	0.00
time (sec)	N/A	0.165	0.497	0.228	0.000	0.236	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	72	71	59	0	137	0	0	0
N.S.	1	0.99	0.97	0.81	0.00	1.88	0.00	0.00	0.00
time (sec)	N/A	0.312	3.608	0.252	0.000	0.255	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	121	134	0	446	345	158	182
N.S.	1	1.00	1.00	1.11	0.00	3.69	2.85	1.31	1.50
time (sec)	N/A	0.320	0.057	0.240	0.000	0.256	0.446	0.285	7.957

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	84	81	0	311	275	101	113
N.S.	1	1.00	0.98	0.94	0.00	3.62	3.20	1.17	1.31
time (sec)	N/A	0.262	0.035	0.221	0.000	0.269	0.350	0.290	8.007

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	51	0	210	212	56	52
N.S.	1	1.00	0.98	0.80	0.00	3.28	3.31	0.88	0.81
time (sec)	N/A	0.210	0.040	0.221	0.000	0.264	0.244	0.289	0.041

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	33	0	134	124	38	38
N.S.	1	1.00	0.98	0.67	0.00	2.73	2.53	0.78	0.78
time (sec)	N/A	0.178	0.011	0.051	0.000	0.258	0.113	0.279	8.720

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	153	133	109	0	895	0	144	3901
N.S.	1	1.12	0.98	0.80	0.00	6.58	0.00	1.06	28.68
time (sec)	N/A	0.311	0.137	0.479	0.000	0.352	0.000	0.291	9.265

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	225	177	174	0	1765	0	235	6267
N.S.	1	1.20	0.95	0.93	0.00	9.44	0.00	1.26	33.51
time (sec)	N/A	0.430	0.270	0.376	0.000	0.853	0.000	0.285	9.168

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	145	172	144	0	1079	0	0	0
N.S.	1	1.04	1.24	1.04	0.00	7.76	0.00	0.00	0.00
time (sec)	N/A	0.379	0.393	0.997	0.000	0.540	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	139	100	0	940	0	0	0
N.S.	1	1.00	1.29	0.93	0.00	8.70	0.00	0.00	0.00
time (sec)	N/A	0.267	0.176	0.333	0.000	0.334	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	94	64	0	432	0	88	0
N.S.	1	1.00	1.24	0.84	0.00	5.68	0.00	1.16	0.00
time (sec)	N/A	0.215	0.125	0.267	0.000	0.297	0.000	0.285	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	172	122	0	701	0	129	0
N.S.	1	1.00	1.62	1.15	0.00	6.61	0.00	1.22	0.00
time (sec)	N/A	0.253	0.264	0.303	0.000	0.343	0.000	0.286	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	166	174	152	0	1063	0	343	0
N.S.	1	1.11	1.17	1.02	0.00	7.13	0.00	2.30	0.00
time (sec)	N/A	0.323	0.435	0.381	0.000	0.513	0.000	0.301	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	191	169	235	0	131	0	0	0
N.S.	1	1.04	0.92	1.28	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.372	5.055	2.015	0.000	0.080	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	170	162	230	0	125	0	0	0
N.S.	1	1.04	0.99	1.40	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.313	4.611	0.494	0.000	0.081	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	142	168	223	0	110	0	0	0
N.S.	1	0.98	1.16	1.54	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.259	4.314	0.430	0.000	0.077	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	117	293	0	124	0	0	0
N.S.	1	1.00	0.85	2.14	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.421	8.896	0.490	0.000	0.102	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	164	224	0	113	0	0	0
N.S.	1	1.00	3.35	4.57	0.00	2.31	0.00	0.00	0.00
time (sec)	N/A	0.168	10.262	0.627	0.000	0.094	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	91	176	251	0	164	0	0	0
N.S.	1	0.98	1.89	2.70	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.446	10.217	1.072	0.000	0.102	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	215	240	341	0	216	0	0	0
N.S.	1	1.30	1.45	2.05	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.671	10.313	0.869	0.000	0.098	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	167	157	225	0	121	0	0	0
N.S.	1	1.05	0.99	1.42	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.333	10.150	1.115	0.000	0.080	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	143	143	218	0	113	0	0	0
N.S.	1	1.04	1.04	1.59	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.268	10.117	0.516	0.000	0.080	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	94	205	0	99	0	0	0
N.S.	1	1.00	0.82	1.78	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.223	10.063	0.271	0.000	0.089	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	72	104	0	55	0	0	0
N.S.	1	1.00	1.04	1.51	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.271	10.079	0.483	0.000	0.095	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	169	226	328	0	140	0	0	0
N.S.	1	1.43	1.92	2.78	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.476	10.284	0.600	0.000	0.102	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	185	235	336	0	181	0	0	0
N.S.	1	1.30	1.65	2.37	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.685	10.258	0.687	0.000	0.100	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	147	136	223	0	153	0	0	0
N.S.	1	1.02	0.94	1.55	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.273	10.137	1.131	0.000	0.080	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	158	225	0	131	0	0	0
N.S.	1	1.00	1.61	2.30	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.210	10.129	0.388	0.000	0.083	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	160	223	0	131	0	0	0
N.S.	1	1.00	1.67	2.32	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.201	9.860	0.292	0.000	0.086	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	218	204	328	0	168	0	0	0
N.S.	1	1.31	1.23	1.98	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.495	10.158	0.527	0.000	0.098	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	168	339	0	194	0	0	0
N.S.	1	1.00	1.51	3.05	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.451	10.286	0.621	0.000	0.103	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	192	346	0	251	0	0	0
N.S.	1	1.00	1.01	1.82	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.743	10.252	0.746	0.000	0.111	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	135	135	135	135	156	148	131
N.S.	1	1.00	1.00	1.00	1.00	1.00	1.16	1.10	0.97
time (sec)	N/A	0.323	0.035	0.258	0.191	0.260	0.030	0.292	0.036

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	104	102	102	102	112	111	101
N.S.	1	1.00	1.01	0.99	0.99	0.99	1.09	1.08	0.98
time (sec)	N/A	0.281	0.020	0.261	0.177	0.239	0.027	0.266	7.489

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	69	78	76	70
N.S.	1	1.00	1.00	0.96	0.95	0.95	1.07	1.04	0.96
time (sec)	N/A	0.232	0.014	0.262	0.197	0.241	0.026	0.274	7.514

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	36	39	40	38
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.93	0.95	0.90
time (sec)	N/A	0.190	0.007	0.064	0.181	0.235	0.018	0.294	0.026

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	57	0	159	117	62	57
N.S.	1	1.00	0.98	0.86	0.00	2.41	1.77	0.94	0.86
time (sec)	N/A	0.211	0.040	0.508	0.000	0.248	0.215	0.283	0.047

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	94	88	79	0	268	153	83	77
N.S.	1	1.13	1.06	0.95	0.00	3.23	1.84	1.00	0.93
time (sec)	N/A	0.240	0.039	0.255	0.000	0.262	0.387	0.274	7.559

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	125	110	107	0	391	196	109	112
N.S.	1	1.09	0.96	0.93	0.00	3.40	1.70	0.95	0.97
time (sec)	N/A	0.261	0.070	0.272	0.000	0.273	0.662	0.290	7.644

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	146	142	130	0	530	241	145	144
N.S.	1	0.97	0.95	0.87	0.00	3.53	1.61	0.97	0.96
time (sec)	N/A	0.286	0.094	0.566	0.000	0.252	1.180	0.282	7.585

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	223	219	218	218	272	261	220
N.S.	1	1.00	1.00	0.98	0.98	0.98	1.22	1.17	0.99
time (sec)	N/A	0.439	0.058	0.267	0.184	0.255	0.036	0.283	0.049

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	156	155	147	147	192	181	148
N.S.	1	1.00	1.01	1.00	0.95	0.95	1.24	1.17	0.95
time (sec)	N/A	0.354	0.038	0.260	0.181	0.259	0.030	0.274	7.638

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	91	90	90	107	100	90
N.S.	1	1.00	1.00	0.95	0.94	0.94	1.11	1.04	0.94
time (sec)	N/A	0.254	0.017	0.140	0.186	0.250	0.027	0.284	0.024

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	45	41	48	43	42
N.S.	1	1.00	1.00	0.86	0.92	0.84	0.98	0.88	0.86
time (sec)	N/A	0.193	0.005	0.013	0.181	0.238	0.019	0.287	0.013

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	144	178	0	406	371	205	229
N.S.	1	1.00	1.01	1.24	0.00	2.84	2.59	1.43	1.60
time (sec)	N/A	0.323	0.046	0.583	0.000	0.266	0.486	0.273	0.039

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	175	183	213	0	600	484	228	293
N.S.	1	1.05	1.10	1.28	0.00	3.61	2.92	1.37	1.77
time (sec)	N/A	0.566	0.069	0.284	0.000	0.254	1.171	0.283	7.678

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	220	217	237	0	794	398	265	257
N.S.	1	1.09	1.08	1.18	0.00	3.95	1.98	1.32	1.28
time (sec)	N/A	0.693	0.072	0.288	0.000	0.257	9.721	0.290	0.072

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	275	267	285	0	1016	0	320	308
N.S.	1	1.10	1.07	1.14	0.00	4.06	0.00	1.28	1.23
time (sec)	N/A	0.775	0.099	0.257	0.000	0.274	0.000	0.294	7.718

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	341	345	347	0	1266	0	393	375
N.S.	1	1.08	1.09	1.09	0.00	3.99	0.00	1.24	1.18
time (sec)	N/A	0.778	0.152	0.296	0.000	0.286	0.000	0.282	7.680

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	94	88	79	0	268	153	83	77
N.S.	1	1.13	1.06	0.95	0.00	3.23	1.84	1.00	0.93
time (sec)	N/A	0.244	0.031	0.215	0.000	0.251	0.386	0.286	0.001

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	94	88	79	0	268	153	83	77
N.S.	1	1.13	1.06	0.95	0.00	3.23	1.84	1.00	0.93
time (sec)	N/A	0.260	0.013	0.250	0.000	0.259	0.389	0.283	7.569

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	570	227	0	16218	0	9306	29551
N.S.	1	1.00	1.24	0.49	0.00	35.33	0.00	20.27	64.38
time (sec)	N/A	1.336	0.429	0.244	0.000	270.801	0.000	1.303	10.920

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	402	129	0	9584	0	6418	17954
N.S.	1	1.00	1.27	0.41	0.00	30.33	0.00	20.31	56.82
time (sec)	N/A	0.839	0.334	0.240	0.000	44.923	0.000	1.138	9.730

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	269	76	0	4690	0	4110	9600
N.S.	1	1.00	1.13	0.32	0.00	19.71	0.00	17.27	40.34
time (sec)	N/A	0.673	0.204	0.232	0.000	4.220	0.000	0.912	8.837

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	172	45	0	1525	0	1402	4109
N.S.	1	1.00	0.99	0.26	0.00	8.76	0.00	8.06	23.61
time (sec)	N/A	0.326	0.098	0.061	0.000	0.357	0.000	0.818	8.212

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	129	38	0	613	87	1026	763
N.S.	1	1.00	0.86	0.25	0.00	4.09	0.58	6.84	5.09
time (sec)	N/A	0.241	0.056	0.041	0.000	0.276	0.547	0.579	0.305

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	274	215	0	16013	0	7664	23640
N.S.	1	1.00	1.08	0.85	0.00	63.04	0.00	30.17	93.07
time (sec)	N/A	0.693	0.182	0.343	0.000	14.270	0.000	1.783	10.793

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	354	345	0	0	0	2394	91169
N.S.	1	1.00	0.83	0.80	0.00	0.00	0.00	5.58	212.52
time (sec)	N/A	1.193	0.495	0.518	0.000	0.000	0.000	1.833	11.414

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	521	540	320	0	12117	0	8992	29030
N.S.	1	0.93	0.96	0.57	0.00	21.52	0.00	15.97	51.56
time (sec)	N/A	1.062	0.966	0.262	0.000	107.283	0.000	1.768	10.269

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	372	415	224	0	7338	0	6390	18785
N.S.	1	0.96	1.08	0.58	0.00	19.01	0.00	16.55	48.67
time (sec)	N/A	0.681	0.679	0.218	0.000	12.700	0.000	1.381	11.076

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	279	310	178	0	4573	0	4425	12350
N.S.	1	0.95	1.06	0.61	0.00	15.61	0.00	15.10	42.15
time (sec)	N/A	0.531	0.465	0.163	0.000	1.979	0.000	1.317	10.551

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	236	243	151	0	2309	0	2682	6404
N.S.	1	0.94	0.96	0.60	0.00	9.16	0.00	10.64	25.41
time (sec)	N/A	0.417	0.266	0.105	0.000	0.407	0.000	0.611	8.642

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	660	660	708	850	0	0	0	37254	237586
N.S.	1	1.00	1.07	1.29	0.00	0.00	0.00	56.45	359.98
time (sec)	N/A	2.238	1.621	0.791	0.000	0.000	0.000	7.050	15.371

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1077	1077	1020	1250	0	0	0	65158	97073
N.S.	1	1.00	0.95	1.16	0.00	0.00	0.00	60.50	90.13
time (sec)	N/A	7.650	3.321	1.179	0.000	0.000	0.000	10.357	17.554

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	174	181	149	0	370	439	196	0
N.S.	1	0.81	0.84	0.69	0.00	1.72	2.04	0.91	0.00
time (sec)	N/A	0.282	0.320	0.350	0.000	0.351	0.521	0.290	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	152	148	124	0	304	274	155	0
N.S.	1	0.87	0.85	0.71	0.00	1.74	1.57	0.89	0.00
time (sec)	N/A	0.268	0.231	0.337	0.000	0.334	0.458	0.286	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	128	112	96	0	232	153	113	0
N.S.	1	0.97	0.85	0.73	0.00	1.76	1.16	0.86	0.00
time (sec)	N/A	0.244	0.173	0.319	0.000	0.340	0.386	0.287	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	105	93	72	0	174	114	81	0
N.S.	1	1.08	0.96	0.74	0.00	1.79	1.18	0.84	0.00
time (sec)	N/A	0.228	0.277	0.271	0.000	0.319	0.341	0.281	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	96	87	85	0	249	134	82	0
N.S.	1	1.08	0.98	0.96	0.00	2.80	1.51	0.92	0.00
time (sec)	N/A	0.234	0.156	0.303	0.000	0.315	3.904	0.282	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	111	92	95	0	289	450	93	0
N.S.	1	1.10	0.91	0.94	0.00	2.86	4.46	0.92	0.00
time (sec)	N/A	0.254	0.187	0.304	0.000	0.313	6.508	0.286	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	100	69	58	173	93	639	81	133
N.S.	1	1.16	0.80	0.67	2.01	1.08	7.43	0.94	1.55
time (sec)	N/A	0.291	0.157	0.285	0.201	0.338	14.752	0.308	7.871

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	130	103	83	227	136	1989	123	154
N.S.	1	1.03	0.82	0.66	1.80	1.08	15.79	0.98	1.22
time (sec)	N/A	0.319	0.215	0.291	0.188	0.336	35.551	0.323	7.881

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	159	132	108	281	177	5187	162	189
N.S.	1	0.96	0.80	0.65	1.70	1.07	31.44	0.98	1.15
time (sec)	N/A	0.342	0.295	0.309	0.188	0.401	80.517	0.304	7.875

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	185	167	133	335	224	11602	207	226
N.S.	1	0.88	0.80	0.63	1.60	1.07	55.25	0.99	1.08
time (sec)	N/A	0.362	0.463	0.314	0.208	0.442	154.155	0.299	7.908

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	196	119	138	0	63	0	0	0
N.S.	1	1.02	0.62	0.72	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.395	6.628	5.773	0.000	0.089	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	178	114	133	0	58	0	0	0
N.S.	1	1.06	0.68	0.79	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.330	5.689	1.932	0.000	0.101	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	154	109	128	0	53	0	0	0
N.S.	1	1.03	0.73	0.86	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.284	4.596	1.176	0.000	0.086	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	147	102	121	0	46	0	0	0
N.S.	1	1.04	0.72	0.86	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.264	3.650	0.614	0.000	0.091	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	90	138	0	0	0	0	0
N.S.	1	1.00	0.51	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	9.531	0.750	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	271	208	162	0	0	0	0	0
N.S.	1	1.30	1.00	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.591	10.255	3.243	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	304	174	183	0	0	0	0	0
N.S.	1	1.28	0.73	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.732	10.351	3.411	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	235	129	148	0	73	0	0	0
N.S.	1	1.07	0.59	0.68	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.439	10.088	4.597	0.000	0.088	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	209	124	143	0	68	0	0	0
N.S.	1	1.06	0.63	0.72	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.366	8.265	1.796	0.000	0.084	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	185	119	138	0	63	0	0	0
N.S.	1	1.03	0.66	0.77	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.324	6.819	1.169	0.000	0.088	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	178	114	133	0	58	0	0	0
N.S.	1	1.03	0.66	0.77	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.311	5.075	0.682	0.000	0.112	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	225	148	170	0	0	0	0	0
N.S.	1	1.09	0.71	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	10.236	1.688	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	333	213	177	0	0	0	0	0
N.S.	1	1.50	0.96	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.597	10.281	2.639	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	288	174	183	0	0	0	0	0
N.S.	1	1.25	0.75	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.787	10.387	3.534	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	159	106	126	0	53	0	0	0
N.S.	1	1.01	0.68	0.80	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.333	10.154	4.606	0.000	0.101	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	147	104	121	0	49	0	0	0
N.S.	1	1.04	0.73	0.85	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.279	10.076	1.642	0.000	0.081	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	123	69	106	0	41	0	0	0
N.S.	1	1.02	0.57	0.88	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.243	10.053	0.693	0.000	0.083	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	0	11	0	0	0
N.S.	1	1.00	1.04	0.96	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.160	10.020	0.213	0.000	0.079	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	55	47	0	0	0	0	0
N.S.	1	1.00	0.52	0.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	10.129	0.826	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	264	208	162	0	0	0	0	0
N.S.	1	1.26	1.00	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.588	10.303	2.698	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	297	186	183	0	0	0	0	0
N.S.	1	1.25	0.78	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.723	10.383	3.481	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	197	109	138	0	101	0	0	0
N.S.	1	1.04	0.58	0.73	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.419	10.094	8.956	0.000	0.094	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	178	104	133	0	96	0	0	0
N.S.	1	1.05	0.61	0.78	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.355	10.084	3.541	0.000	0.084	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	154	99	128	0	91	0	0	0
N.S.	1	1.03	0.66	0.86	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.285	10.085	2.590	0.000	0.085	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	154	99	128	0	84	0	0	0
N.S.	1	1.03	0.66	0.86	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.287	10.069	2.561	0.000	0.085	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	148	97	126	0	81	0	0	0
N.S.	1	1.02	0.67	0.87	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.277	10.058	1.423	0.000	0.088	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	154	99	128	0	84	0	0	0
N.S.	1	1.03	0.66	0.86	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.274	5.079	0.645	0.000	0.088	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	261	138	161	0	0	0	0	0
N.S.	1	1.51	0.80	0.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.535	10.192	1.602	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	208	185	0	0	0	0	0
N.S.	1	1.00	0.89	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.556	10.319	2.812	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	159	193	0	0	0	0	0
N.S.	1	1.00	0.60	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.830	10.486	3.512	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	127	112	157	0	78	0	0	0
N.S.	1	1.09	0.97	1.35	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.414	10.101	7.908	0.000	0.084	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	106	107	152	0	73	0	0	0
N.S.	1	1.12	1.13	1.60	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.316	8.137	2.888	0.000	0.082	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	80	102	147	0	68	0	0	0
N.S.	1	1.08	1.38	1.99	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.253	6.197	2.182	0.000	0.094	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	94	140	0	60	0	0	0
N.S.	1	1.00	2.04	3.04	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.209	4.952	1.292	0.000	0.086	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	47	90	125	0	56	0	0	0
N.S.	1	1.07	2.05	2.84	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.200	3.930	0.643	0.000	0.088	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	47	51	141	0	0	0	0	0
N.S.	1	1.02	1.11	3.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	9.930	0.762	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	80	196	165	0	0	0	0	0
N.S.	1	1.08	2.65	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.364	10.276	3.552	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	113	244	189	0	0	0	0	0
N.S.	1	1.11	2.39	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.498	10.352	3.779	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	163	122	167	0	89	0	0	0
N.S.	1	1.15	0.86	1.18	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.447	10.096	7.098	0.000	0.088	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	137	117	162	0	84	0	0	0
N.S.	1	1.13	0.97	1.34	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.368	10.119	2.895	0.000	0.083	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	111	112	157	0	79	0	0	0
N.S.	1	1.11	1.12	1.57	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.300	10.013	2.207	0.000	0.088	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	87	107	152	0	74	0	0	0
N.S.	1	1.07	1.32	1.88	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.258	7.229	1.707	0.000	0.092	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	80	102	147	0	69	0	0	0
N.S.	1	1.08	1.38	1.99	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.235	5.128	0.680	0.000	0.086	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	90	130	173	0	0	0	0	0
N.S.	1	1.25	1.81	2.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.398	10.269	2.001	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	201	180	0	0	0	0	0
N.S.	1	1.00	2.16	1.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.426	10.323	2.891	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	244	189	0	0	0	0	0
N.S.	1	1.00	2.39	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.593	10.392	3.786	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	68	97	142	0	64	0	0	0
N.S.	1	1.05	1.49	2.18	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.271	10.102	5.358	0.000	0.089	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	49	92	125	0	59	0	0	0
N.S.	1	1.07	2.00	2.72	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.210	10.091	1.950	0.000	0.084	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	34	110	0	52	0	0	0
N.S.	1	1.00	1.36	4.40	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.180	10.069	0.741	0.000	0.085	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	19	47	0	9	0	0	0
N.S.	1	1.00	1.90	4.70	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.146	0.011	0.214	0.000	0.074	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	24	48	0	0	0	0	0
N.S.	1	1.00	1.41	2.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.160	10.142	0.912	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	75	196	165	0	0	0	0	0
N.S.	1	1.01	2.65	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	10.316	2.951	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	108	108	189	0	0	0	0	0
N.S.	1	1.06	1.06	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	10.428	3.538	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	92	97	142	0	111	0	0	0
N.S.	1	0.99	1.04	1.53	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.355	10.112	8.850	0.000	0.082	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	73	92	137	0	106	0	0	0
N.S.	1	0.99	1.24	1.85	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.275	10.090	4.257	0.000	0.080	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	56	79	132	0	101	0	0	0
N.S.	1	1.02	1.44	2.40	0.00	1.84	0.00	0.00	0.00
time (sec)	N/A	0.216	10.090	2.520	0.000	0.086	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	56	79	132	0	80	0	0	0
N.S.	1	1.02	1.44	2.40	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.222	10.081	2.460	0.000	0.079	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	56	79	132	0	80	0	0	0
N.S.	1	1.02	1.44	2.40	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.212	10.070	1.204	0.000	0.081	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	54	79	130	0	77	0	0	0
N.S.	1	0.98	1.44	2.36	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.204	5.773	0.243	0.000	0.080	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	78	101	164	0	0	0	0	0
N.S.	1	1.08	1.40	2.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	10.225	1.823	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	196	188	0	0	0	0	0
N.S.	1	1.00	1.96	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.398	10.358	3.040	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	244	198	0	0	0	0	0
N.S.	1	1.00	1.91	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.637	10.438	3.533	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	243	354	246	0	138	0	0	0
N.S.	1	1.00	1.46	1.02	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.498	5.991	2.394	0.000	0.082	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	224	349	241	0	133	0	0	0
N.S.	1	1.01	1.58	1.09	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.403	5.617	0.664	0.000	0.084	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	343	236	0	128	0	0	0
N.S.	1	1.00	1.73	1.19	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.333	5.104	0.524	0.000	0.085	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	182	338	231	0	123	0	0	0
N.S.	1	1.03	1.91	1.31	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.307	4.658	0.421	0.000	0.089	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	175	331	224	0	115	0	0	0
N.S.	1	1.04	1.96	1.33	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.292	3.945	0.227	0.000	0.083	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	261	283	386	0	0	0	0	0
N.S.	1	0.81	0.88	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	9.630	0.514	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	299	481	339	0	0	0	0	0
N.S.	1	1.05	1.69	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	10.668	0.886	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	332	308	346	0	0	0	0	0
N.S.	1	1.06	0.99	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.748	10.593	0.738	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	287	364	256	0	148	0	0	0
N.S.	1	1.07	1.36	0.96	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.529	8.374	1.602	0.000	0.084	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	263	358	251	0	143	0	0	0
N.S.	1	1.06	1.45	1.02	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.450	7.786	0.630	0.000	0.085	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	237	354	246	0	138	0	0	0
N.S.	1	1.05	1.57	1.09	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.381	7.145	0.539	0.000	0.085	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	213	349	241	0	133	0	0	0
N.S.	1	1.03	1.69	1.16	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.355	6.379	0.423	0.000	0.084	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	206	343	236	0	128	0	0	0
N.S.	1	1.04	1.73	1.19	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.328	5.780	0.256	0.000	0.078	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	301	477	337	0	0	0	0	0
N.S.	1	1.06	1.68	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.705	10.562	0.558	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	372	309	346	0	0	0	0	0
N.S.	1	1.22	1.01	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.716	10.484	0.608	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	440	309	346	0	0	0	0	0
N.S.	1	1.00	0.70	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.009	10.546	0.750	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	189	337	229	0	123	0	0	0
N.S.	1	1.01	1.80	1.22	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.338	10.344	1.257	0.000	0.082	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	177	331	224	0	118	0	0	0
N.S.	1	1.04	1.95	1.32	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.298	10.309	0.520	0.000	0.080	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	153	214	209	0	111	0	0	0
N.S.	1	1.01	1.42	1.38	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.257	10.139	0.278	0.000	0.080	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	142	85	0	39	0	0	0
N.S.	1	1.00	2.22	1.33	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.163	10.054	0.117	0.000	0.076	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	171	159	107	0	0	0	0	0
N.S.	1	1.02	0.95	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	10.162	0.447	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	292	481	339	0	0	0	0	0
N.S.	1	1.02	1.68	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.616	10.752	0.608	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	321	308	346	0	0	0	0	0
N.S.	1	1.02	0.98	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.724	10.720	0.730	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	225	339	241	0	187	0	0	0
N.S.	1	1.03	1.55	1.10	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.437	10.362	2.401	0.000	0.091	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	206	333	236	0	182	0	0	0
N.S.	1	1.03	1.66	1.18	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.376	10.308	0.810	0.000	0.082	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	182	328	231	0	177	0	0	0
N.S.	1	1.01	1.81	1.28	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.304	10.295	0.661	0.000	0.090	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	182	329	231	0	160	0	0	0
N.S.	1	1.01	1.82	1.28	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.297	10.280	0.651	0.000	0.088	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	182	329	231	0	160	0	0	0
N.S.	1	1.01	1.82	1.28	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.312	10.263	0.524	0.000	0.085	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	182	328	231	0	158	0	0	0
N.S.	1	1.01	1.81	1.28	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.298	4.196	0.123	0.000	0.086	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	295	483	337	0	0	0	0	0
N.S.	1	1.04	1.70	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.588	10.435	0.546	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	311	351	0	0	0	0	0
N.S.	1	1.00	1.00	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.675	10.537	0.645	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	320	356	0	0	0	0	0
N.S.	1	1.00	0.94	1.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.034	10.677	0.747	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	467	448	584	485	0	566	0	0	0
N.S.	1	0.96	1.25	1.04	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.671	11.958	7.425	0.000	0.103	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	350	488	410	0	398	0	0	0
N.S.	1	0.98	1.37	1.15	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.434	11.105	3.355	0.000	0.096	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	285	302	362	0	298	0	0	0
N.S.	1	1.01	1.07	1.28	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.333	10.190	0.955	0.000	0.087	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	427	214	200	0	0	0	0	0
N.S.	1	1.06	0.53	0.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.590	10.224	0.997	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	694	1069	1279	0	0	0	0	0
N.S.	1	0.97	1.49	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.360	11.508	1.643	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	494	596	489	0	576	0	0	0
N.S.	1	0.89	1.08	0.88	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	1.010	11.719	6.414	0.000	0.117	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	382	503	413	0	408	0	0	0
N.S.	1	0.84	1.11	0.91	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.591	10.909	3.253	0.000	0.098	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	311	293	364	0	310	0	0	0
N.S.	1	0.81	0.76	0.95	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.460	10.192	0.964	0.000	0.105	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	205	201	0	0	0	0	0
N.S.	1	1.00	1.04	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	10.208	1.043	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	610	464	1293	0	0	0	0	0
N.S.	1	0.85	0.65	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.023	13.811	1.622	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	599	304	355	0	301	0	0	0
N.S.	1	1.25	0.63	0.74	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.775	10.211	1.653	0.000	0.097	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	216	198	0	0	0	0	0
N.S.	1	1.00	1.06	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	10.208	1.048	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	295	357	0	311	0	0	0
N.S.	1	1.00	1.01	1.22	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.362	10.240	1.636	0.000	0.099	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	434	207	199	0	0	0	0	0
N.S.	1	1.05	0.50	0.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.590	10.231	1.013	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	210	154	184	0	123	0	0	0
N.S.	1	0.92	0.67	0.80	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.427	10.164	5.791	0.000	0.099	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	166	127	143	0	82	0	0	0
N.S.	1	0.99	0.76	0.85	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.299	10.122	3.614	0.000	0.103	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	123	73	108	0	45	0	0	0
N.S.	1	1.01	0.60	0.89	0.00	0.37	0.00	0.00	0.00
time (sec)	N/A	0.235	10.065	1.354	0.000	0.093	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	59	55	0	0	0	0	0
N.S.	1	1.00	0.48	0.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	10.129	1.494	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	336	175	443	0	0	0	0	0
N.S.	1	1.06	0.55	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.755	10.465	2.142	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.160	0.882	0.078	0.237	0.549	0.000	1.438	8.074

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	498	481	373	0	0	0	0	0	0
N.S.	1	0.97	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.899	0.597	0.000	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	358	350	303	0	0	0	0	0	0
N.S.	1	0.98	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.519	0.551	0.000	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	232	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	0.476	0.000	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.147	0.000	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.162	0.896	0.051	0.224	0.267	0.000	0.302	7.809

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	37	0	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.54	0.00	1.08	1.08
time (sec)	N/A	0.163	0.862	0.065	0.213	0.280	0.000	0.292	8.059

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	446	459	270	251	0	0	0	0	0
N.S.	1	1.03	0.61	0.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.867	10.747	0.701	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	719	247	0	0	0	0	0
N.S.	1	1.00	3.30	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.671	11.115	0.741	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	73	77	327	0	323	0	0	0
N.S.	1	1.12	1.18	5.03	0.00	4.97	0.00	0.00	0.00
time (sec)	N/A	0.301	0.069	3.539	0.000	0.400	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	70	77	311	0	112	0	0	0
N.S.	1	1.11	1.22	4.94	0.00	1.78	0.00	0.00	0.00
time (sec)	N/A	0.296	0.054	3.415	0.000	0.388	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	80	81	336	0	328	0	0	0
N.S.	1	1.11	1.12	4.67	0.00	4.56	0.00	0.00	0.00
time (sec)	N/A	0.302	8.821	3.381	0.000	0.391	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	77	81	336	0	114	0	0	0
N.S.	1	1.10	1.16	4.80	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.302	8.440	3.349	0.000	0.395	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	568	3652	437	0	0	0	0	0
N.S.	1	1.01	6.52	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.063	17.740	1.477	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	527	527	3658	439	0	0	0	0	0
N.S.	1	1.00	6.94	0.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.153	17.676	1.478	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [147] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	9	0.93	17	0.529
2	A	10	9	0.93	18	0.500
3	A	3	3	0.99	18	0.167
4	A	3	3	0.99	19	0.158
5	A	4	3	1.00	17	0.176
6	A	3	3	1.00	17	0.176
7	A	2	2	1.00	17	0.118
8	A	2	2	1.00	17	0.118
9	A	4	3	1.00	27	0.111
10	A	4	4	1.00	28	0.143
11	A	4	3	1.00	21	0.143
12	A	4	4	1.00	22	0.182
13	A	3	3	1.00	15	0.200
14	A	2	2	1.00	22	0.091
15	A	5	5	1.00	23	0.217
16	A	3	3	1.00	21	0.143
17	A	6	6	0.71	22	0.273
18	A	1	1	1.00	22	0.045
19	A	3	3	1.00	21	0.143
20	A	1	1	1.00	23	0.043
21	A	3	3	1.00	22	0.136
22	A	1	1	1.00	28	0.036

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	2	2	1.00	24	0.083
24	A	4	4	1.00	28	0.143
25	A	5	5	1.00	25	0.200
26	A	4	3	1.10	26	0.115
27	A	4	3	1.10	26	0.115
28	A	4	3	1.13	27	0.111
29	A	4	3	1.09	27	0.111
30	A	4	4	1.00	27	0.148
31	A	4	4	1.00	27	0.148
32	A	4	4	1.00	28	0.143
33	A	4	4	1.00	28	0.143
34	A	4	4	1.00	30	0.133
35	A	4	3	1.17	29	0.103
36	A	5	4	1.17	29	0.138
37	A	4	4	1.00	32	0.125
38	A	4	3	1.12	31	0.097
39	A	4	3	1.15	22	0.136
40	A	4	3	1.14	23	0.130
41	A	2	2	1.80	22	0.091
42	A	2	2	1.00	22	0.091
43	A	3	3	1.00	22	0.136
44	A	4	3	1.13	22	0.136
45	A	4	3	1.06	22	0.136
46	A	4	3	1.20	20	0.150
47	A	4	3	1.00	17	0.176
48	A	4	3	1.20	22	0.136
49	A	4	3	1.16	22	0.136
50	A	4	3	1.43	22	0.136
51	A	3	3	1.00	22	0.136
52	A	3	3	1.05	22	0.136
53	B	3	3	2.48	22	0.136
54	A	3	3	1.00	22	0.136
55	A	2	2	1.76	22	0.091
56	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	3	3	1.00	22	0.136
58	A	3	3	1.00	22	0.136
59	A	3	3	1.00	22	0.136
60	A	3	3	1.00	20	0.150
61	A	3	3	1.00	17	0.176
62	A	3	3	1.00	22	0.136
63	A	3	3	1.00	22	0.136
64	A	3	3	1.00	22	0.136
65	A	3	3	1.00	22	0.136
66	A	3	3	1.26	22	0.136
67	B	3	3	2.44	22	0.136
68	A	4	3	1.02	18	0.167
69	A	2	2	1.82	18	0.111
70	A	2	2	1.84	18	0.111
71	A	2	2	1.82	18	0.111
72	A	2	2	1.00	18	0.111
73	A	4	3	0.97	16	0.188
74	A	4	3	1.00	13	0.231
75	A	4	3	1.00	18	0.167
76	A	2	2	1.00	18	0.111
77	A	3	3	0.97	18	0.167
78	B	3	3	2.37	18	0.167
79	B	3	3	2.28	18	0.167
80	A	3	3	1.00	20	0.150
81	A	2	2	1.78	20	0.100
82	A	2	2	1.80	20	0.100
83	B	2	2	2.23	20	0.100
84	A	2	2	1.00	20	0.100
85	A	3	3	1.00	18	0.167
86	A	4	4	1.00	15	0.267
87	A	3	3	1.00	20	0.150
88	A	2	2	1.00	20	0.100
89	B	3	3	2.24	20	0.150
90	B	3	3	2.17	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	B	3	3	2.28	20	0.150
92	A	4	3	1.23	22	0.136
93	A	4	3	1.23	22	0.136
94	A	3	3	1.00	20	0.150
95	A	2	2	1.00	22	0.091
96	A	2	2	1.00	22	0.091
97	A	2	2	1.23	18	0.111
98	A	7	6	1.05	18	0.333
99	A	8	7	1.08	18	0.389
100	A	7	6	1.09	18	0.333
101	A	8	7	1.09	18	0.389
102	A	7	6	0.98	29	0.207
103	A	7	6	0.98	26	0.231
104	A	7	6	1.16	24	0.250
105	A	7	6	1.15	22	0.273
106	A	8	7	1.11	25	0.280
107	A	8	7	1.11	31	0.226
108	A	8	7	0.90	32	0.219
109	A	7	6	1.14	23	0.261
110	A	7	6	1.12	25	0.240
111	A	9	8	1.14	29	0.276
112	A	9	8	1.12	32	0.250
113	A	4	4	1.15	22	0.182
114	A	4	4	1.00	24	0.167
115	A	4	4	1.15	24	0.167
116	A	4	4	1.15	24	0.167
117	A	5	5	1.00	24	0.208
118	A	4	4	1.15	24	0.167
119	A	4	4	1.01	39	0.103
120	A	2	2	1.00	17	0.118
121	A	2	2	1.00	17	0.118
122	A	2	2	1.00	17	0.118
123	A	2	2	1.00	15	0.133
124	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	5	5	1.15	17	0.294
126	A	6	6	1.14	17	0.353
127	A	7	7	1.07	17	0.412
128	A	2	2	1.00	19	0.105
129	A	2	2	1.00	19	0.105
130	A	2	2	1.00	17	0.118
131	A	2	2	1.00	9	0.222
132	A	2	2	1.00	19	0.105
133	A	4	4	1.08	19	0.211
134	A	6	6	1.12	19	0.316
135	A	10	10	1.15	19	0.526
136	A	9	9	1.12	19	0.474
137	A	2	2	1.00	19	0.105
138	A	2	2	1.00	19	0.105
139	A	2	2	1.00	19	0.105
140	A	10	9	0.93	17	0.529
141	A	9	8	1.08	9	0.889
142	A	2	2	1.00	19	0.105
143	A	2	2	1.00	19	0.105
144	A	14	13	0.89	19	0.684
145	A	14	13	0.93	19	0.684
146	A	12	11	0.96	17	0.647
147	A	10	9	1.11	9	1.000
148	A	2	2	1.00	19	0.105
149	A	2	2	1.00	19	0.105
150	A	8	8	0.99	21	0.381
151	A	7	7	0.99	21	0.333
152	A	5	5	1.03	21	0.238
153	A	4	4	1.01	19	0.211
154	A	4	4	1.08	21	0.190
155	A	9	9	0.98	21	0.429
156	A	11	11	0.96	21	0.524
157	A	11	11	1.00	22	0.500
158	A	9	9	1.03	22	0.409

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
159	A	7	7	0.99	20	0.350
160	A	2	2	1.00	22	0.091
161	A	12	12	0.94	22	0.545
162	A	13	13	0.98	22	0.591
163	A	14	14	1.01	22	0.636
164	A	7	7	0.99	21	0.333
165	A	2	2	1.00	23	0.087
166	A	3	3	1.00	29	0.103
167	A	3	3	1.00	29	0.103
168	A	4	4	1.00	22	0.182
169	A	4	4	1.06	24	0.167
170	A	2	2	1.00	21	0.095
171	A	4	4	1.09	21	0.190
172	A	1	1	1.00	22	0.045
173	A	4	4	1.06	21	0.190
174	F	0	0	N/A	0.000	N/A
175	N/A	1	0	1.00	19	0.000
176	A	3	3	1.07	19	0.158
177	A	4	4	1.05	19	0.211
178	A	2	2	1.00	17	0.118
179	A	2	2	1.00	9	0.222
180	A	2	2	1.00	19	0.105
181	A	2	2	1.00	19	0.105
182	A	3	3	1.13	19	0.158
183	A	4	4	1.09	19	0.211
184	A	2	2	1.00	17	0.118
185	A	1	1	1.00	9	0.111
186	A	2	2	1.00	19	0.105
187	A	2	2	1.00	19	0.105
188	A	2	2	1.00	19	0.105
189	A	3	3	1.00	24	0.125
190	A	3	3	1.00	24	0.125
191	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	2	2	1.00	22	0.091
193	A	7	7	1.04	24	0.292
194	A	9	9	1.13	24	0.375
195	A	7	6	1.00	26	0.231
196	A	4	3	1.00	26	0.115
197	A	5	4	1.00	26	0.154
198	A	9	8	1.10	26	0.308
199	A	8	7	0.77	28	0.250
200	A	5	4	0.80	28	0.143
201	A	4	3	1.00	28	0.107
202	A	4	3	1.00	28	0.107
203	A	5	4	0.90	28	0.143
204	A	9	8	0.89	28	0.286
205	A	7	6	0.76	29	0.207
206	A	5	4	0.79	29	0.138
207	A	4	3	1.00	29	0.103
208	A	4	3	1.00	29	0.103
209	A	5	4	0.90	29	0.138
210	A	8	7	0.89	29	0.241
211	A	2	2	1.00	19	0.105
212	A	4	3	1.67	19	0.158
213	A	2	2	0.99	31	0.065
214	A	3	3	1.00	39	0.077
215	A	3	3	1.00	39	0.077
216	A	3	3	1.00	39	0.077
217	A	2	2	1.00	37	0.054
218	A	7	7	1.12	39	0.179
219	A	10	10	1.20	39	0.256
220	A	10	9	1.04	41	0.220
221	A	8	7	1.00	41	0.171
222	A	4	3	1.00	41	0.073
223	A	6	5	1.00	41	0.122
224	A	9	8	1.11	41	0.195
225	A	8	8	1.04	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	A	7	7	1.04	20	0.350
227	A	5	5	0.98	18	0.278
228	A	7	6	1.00	20	0.300
229	A	1	1	1.00	20	0.050
230	A	9	8	0.98	20	0.400
231	A	14	13	1.30	20	0.650
232	A	6	6	1.05	20	0.300
233	A	5	5	1.04	20	0.250
234	A	3	3	1.00	18	0.167
235	A	5	4	1.00	20	0.200
236	A	10	9	1.43	20	0.450
237	A	12	11	1.30	20	0.550
238	A	5	5	1.02	20	0.250
239	A	3	3	1.00	20	0.150
240	A	2	2	1.00	18	0.111
241	A	11	10	1.31	20	0.500
242	A	2	2	1.00	20	0.100
243	A	2	2	1.00	20	0.100
244	A	2	2	1.00	22	0.091
245	A	2	2	1.00	22	0.091
246	A	2	2	1.00	22	0.091
247	A	2	2	1.00	20	0.100
248	A	2	2	1.00	22	0.091
249	A	4	4	1.13	22	0.182
250	A	5	5	1.09	22	0.227
251	A	6	6	0.97	22	0.273
252	A	2	2	1.00	24	0.083
253	A	2	2	1.00	24	0.083
254	A	2	2	1.00	22	0.091
255	A	2	2	1.00	14	0.143
256	A	2	2	1.00	24	0.083
257	A	3	3	1.05	24	0.125
258	A	4	4	1.09	24	0.167
259	A	7	7	1.10	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	6	6	1.08	24	0.250
261	A	4	4	1.13	22	0.182
262	A	5	5	1.13	23	0.217
263	A	2	2	1.00	24	0.083
264	A	2	2	1.00	24	0.083
265	A	2	2	1.00	24	0.083
266	A	2	2	1.00	22	0.091
267	A	2	2	1.00	14	0.143
268	A	2	2	1.00	24	0.083
269	A	2	2	1.00	24	0.083
270	A	4	4	0.93	24	0.167
271	A	4	4	0.96	24	0.167
272	A	4	4	0.95	22	0.182
273	A	4	4	0.94	14	0.286
274	A	2	2	1.00	24	0.083
275	A	2	2	1.00	24	0.083
276	A	8	7	0.81	24	0.292
277	A	7	6	0.87	24	0.250
278	A	7	6	0.97	24	0.250
279	A	5	4	1.08	24	0.167
280	A	6	5	1.08	24	0.208
281	A	7	6	1.10	24	0.250
282	A	4	4	1.16	24	0.167
283	A	5	5	1.03	24	0.208
284	A	6	6	0.96	24	0.250
285	A	7	7	0.88	24	0.292
286	A	8	8	1.02	24	0.333
287	A	6	6	1.06	24	0.250
288	A	5	5	1.03	22	0.227
289	A	4	4	1.04	14	0.286
290	A	7	7	1.00	24	0.292
291	A	12	12	1.30	24	0.500
292	A	13	13	1.28	24	0.542
293	A	9	9	1.07	24	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
294	A	7	7	1.06	24	0.292
295	A	6	6	1.03	22	0.273
296	A	6	6	1.03	14	0.429
297	A	11	11	1.09	24	0.458
298	A	2	2	1.50	24	0.083
299	A	2	2	1.25	24	0.083
300	A	7	7	1.01	24	0.292
301	A	4	4	1.04	24	0.167
302	A	3	3	1.02	22	0.136
303	A	1	1	1.00	14	0.071
304	A	5	5	1.00	24	0.208
305	A	11	11	1.26	24	0.458
306	A	13	13	1.25	24	0.542
307	A	8	8	1.04	24	0.333
308	A	6	6	1.05	24	0.250
309	A	5	5	1.03	24	0.208
310	A	5	5	1.03	24	0.208
311	A	5	5	1.02	22	0.227
312	A	4	4	1.03	14	0.286
313	A	12	12	1.51	24	0.500
314	A	2	2	1.00	24	0.083
315	A	2	2	1.00	24	0.083
316	A	13	13	1.09	24	0.542
317	A	11	11	1.12	24	0.458
318	A	9	9	1.08	24	0.375
319	A	7	7	1.00	22	0.318
320	A	6	6	1.07	14	0.429
321	A	10	10	1.02	24	0.417
322	A	11	11	1.08	24	0.458
323	A	12	12	1.11	24	0.500
324	A	15	15	1.15	24	0.625
325	A	13	13	1.13	24	0.542
326	A	11	11	1.11	24	0.458
327	A	9	9	1.07	22	0.409

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
328	A	8	8	1.08	14	0.571
329	A	13	13	1.25	24	0.542
330	A	2	2	1.00	24	0.083
331	A	2	2	1.00	24	0.083
332	A	9	9	1.05	24	0.375
333	A	7	7	1.07	24	0.292
334	A	5	5	1.00	22	0.227
335	A	3	3	1.00	14	0.214
336	A	3	3	1.00	24	0.125
337	A	11	11	1.01	24	0.458
338	A	12	12	1.06	24	0.500
339	A	10	10	0.99	24	0.417
340	A	8	8	0.99	24	0.333
341	A	6	6	1.02	24	0.250
342	A	7	7	1.02	24	0.292
343	A	7	7	1.02	22	0.318
344	A	7	7	0.98	14	0.500
345	A	12	12	1.08	24	0.500
346	A	2	2	1.00	24	0.083
347	A	2	2	1.00	24	0.083
348	A	11	11	1.00	24	0.458
349	A	9	9	1.01	24	0.375
350	A	7	7	1.00	24	0.292
351	A	6	6	1.03	22	0.273
352	A	5	5	1.04	14	0.357
353	A	7	7	0.81	24	0.292
354	A	8	8	1.05	24	0.333
355	A	11	11	1.06	24	0.458
356	A	12	12	1.07	24	0.500
357	A	10	10	1.06	24	0.417
358	A	8	8	1.05	24	0.333
359	A	7	7	1.03	22	0.318
360	A	7	7	1.04	14	0.500
361	A	11	11	1.06	24	0.458

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
362	A	2	2	1.22	24	0.083
363	A	2	2	1.00	24	0.083
364	A	8	8	1.01	24	0.333
365	A	5	5	1.04	24	0.208
366	A	4	4	1.01	22	0.182
367	A	1	1	1.00	14	0.071
368	A	4	4	1.02	24	0.167
369	A	8	8	1.02	24	0.333
370	A	10	10	1.02	24	0.417
371	A	9	9	1.03	24	0.375
372	A	7	7	1.03	24	0.292
373	A	5	5	1.01	24	0.208
374	A	5	5	1.01	24	0.208
375	A	6	6	1.01	22	0.273
376	A	5	5	1.01	14	0.357
377	A	9	9	1.04	24	0.375
378	A	2	2	1.00	24	0.083
379	A	2	2	1.00	24	0.083
380	A	6	6	0.96	26	0.231
381	A	5	5	0.98	26	0.192
382	A	4	4	1.01	24	0.167
383	A	4	4	1.06	26	0.154
384	A	9	9	0.97	26	0.346
385	A	8	8	0.89	27	0.296
386	A	6	6	0.84	27	0.222
387	A	4	4	0.81	25	0.160
388	A	2	2	1.00	27	0.074
389	A	9	9	0.85	27	0.333
390	A	5	5	1.25	26	0.192
391	A	2	2	1.00	28	0.071
392	A	4	4	1.00	27	0.148
393	A	4	4	1.05	29	0.138
394	A	6	6	0.92	24	0.250
395	A	4	4	0.99	24	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	3	3	1.01	22	0.136
397	A	5	5	1.00	24	0.208
398	A	12	12	1.06	24	0.500
399	N/A	1	0	1.00	24	0.000
400	A	4	4	0.97	24	0.167
401	A	4	4	0.98	24	0.167
402	A	2	2	1.00	22	0.091
403	A	2	2	1.00	14	0.143
404	N/A	1	0	1.00	24	0.000
405	N/A	1	0	1.00	24	0.000
406	A	10	9	1.03	24	0.375
407	A	11	10	1.00	26	0.385
408	A	3	2	1.12	40	0.050
409	A	3	2	1.11	40	0.050
410	A	3	2	1.11	46	0.043
411	A	3	2	1.10	46	0.043
412	A	10	9	1.01	29	0.310
413	A	11	10	1.00	31	0.323

CHAPTER 3

Listing of integrals

3.1 $\int \frac{c+dx^2}{a+bx^4}\,dx$

3.1.1 Optimal result

Integrand size = 17, antiderivative size = 247

$$
\begin{aligned}
\int \frac{c+dx^2}{a+bx^4}\,dx = & -\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\
& +\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\
& -\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\
& +\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}
\end{aligned}
$$

output
```
-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-d*a^(1/2)+c*b^(1
/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
b^(1/2))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(-1+b^(1
/4)*x*2^(1/2)/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*a
rctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2
^(1/2)
```

3.1.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^2}{a+bx^4}\,dx$$
$$= \frac{-2\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c-\sqrt{a}d\right)\left(\log\left(\sqrt{\right.\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$

input
```
Integrate[(c + d*x^2)/(a + b*x^4),x]
```

output
```
(-2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(S
qrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c
 - Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*
b^(3/4))
```

3.1.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{a+bx^4}\,dx$$

$$\Big\downarrow \text{1482}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{b}\left(\sqrt{a}-\sqrt{b}x^2\right)}{bx^4+a}dx}{2b}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{b}\left(\sqrt{b}x^2+\sqrt{a}\right)}{bx^4+a}dx}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{25}$$

3.1. $\int \frac{c+dx^2}{a+bx^4}\,dx$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt{b}}$$

$\downarrow$ 27

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt{b}}$$

$\downarrow$ 1103

$$\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}$$

input
```
Int[(c + d*x^2)/(a + b*x^4),x]
```

output
```
(((Sqrt[b]*c)/Sqrt[a] + d)*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqr
t[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*
a^(1/4)*b^(1/4))))/(2*Sqrt[b]) + (((Sqrt[b]*c)/Sqrt[a] - d)*(-1/2*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)
*b^(1/4))))/(2*Sqrt[b])
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

3.1.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.14

method	result
risch	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^4 b+a\right)} \frac{\left(\_R^2 d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\text{arct}\right.}{8b(}$

input
```
int((d*x^2+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum((_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.1.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(166) = 332$.

Time = 0.26 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.11

$$\int \frac{c+dx^2}{a+bx^4}\,dx = -\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x + \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right) + \frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x - \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right) + \frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x + \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right) - \frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x - \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$

input
```
integrate((d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c
*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*
c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-
(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt
(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b)
)*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 +
 a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4
- 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqr
t(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^
2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/
(a^3*b^3)) - a*b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt(-(b^2*c^4
 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2
*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
- a*b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b)))
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int \frac{c+dx^2}{a+bx^4}\,dx$$
$$= \operatorname{RootSum}\left(256t^4a^3b^3+64t^2a^2b^2cd+a^2d^4+2abc^2d^2+b^2c^4, \left(t\mapsto t\log\left(x+\frac{64t^3a^3b^2d+12ta^2bcd^2-4t\right.\right.\right.}{a^2d^4-b^2c^4}$$

input
```
integrate((d*x**2+c)/(b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c
**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d + 12*_t*
a**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

input
```
integrate((d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt
(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
*sqrt(b)) + 1/4*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*log(sqrt(b)*x^
2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(
sqrt(b)*c - sqrt(a)*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(3/4))
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)
*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3
)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a*b^3)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.43

$$\int \frac{c+dx^2}{a+bx^4}\,dx = -2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}} - \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{-\frac{b\,c^2\,\sqrt{-a^3\,b^3}-a\,d^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}} - 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}} - \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{-\frac{a\,d^2\,\sqrt{-a^3\,b^3}-b\,c^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$

input
```
int((c + d*x^2)/(a + b*x^4),x)
```

output
```
- 2*atanh((8*b^3*c^2*x*((d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3) - (c^2*(-a^3*b
^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 +
 (2*b*c^3*(-a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2
*d^2*x*((d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3) - (c^2*(-a^3*b^3)^(1/2))/(16*a
^3*b^2) - (c*d)/(8*a*b))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*
b^3)^(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a))*(-(b*c^2*(-a^3*b^3)^(1/2)
 - a*d^2*(-a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) - 2*atanh((
8*b^3*c^2*x*((c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) - (d^2*(-
a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(
-a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*x*((c^
2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) - (d^2*(-a^3*b^3)^(1/2))/
(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^3)^(1/2))
/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a))*(-(a*d^2*(-a^3*b^3)^(1/2) - b*c^2*(-
a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

3.2 $\int \frac{c-dx^2}{a+bx^4}\,dx$

3.2.1 Optimal result

Integrand size = 18, antiderivative size = 247

$$\begin{aligned}\int \frac{c-dx^2}{a+bx^4}\,dx = & -\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\ & +\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} \\ & -\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}} \\ & +\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}\end{aligned}$$

output
```
1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^
(3/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d*a^(1/2)+c*b^(1/2
))/a^(3/4)/b^(3/4)*2^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b
^(1/2))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^
(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.74

$$\int \frac{c-dx^2}{a+bx^4}\,dx$$
$$= \frac{\left(-2\sqrt{b}c+2\sqrt{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c+\sqrt{a}d\right)\left(\log\left(\right.\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$

input
```
Integrate[(c - d*x^2)/(a + b*x^4),x]
```

output
```
((-2*Sqrt[b]*c + 2*Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*
(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]
*c + Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] -
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4
)*b^(3/4))
```

3.2.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx^2}{a+bx^4}\,dx$$
$$\Big\downarrow \; 1482$$
$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{b}\left(\sqrt{a}-\sqrt{b}x^2\right)}{bx^4+a}dx}{2b}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{b}\left(\sqrt{b}x^2+\sqrt{a}\right)}{bx^4+a}dx}{2b}$$
$$\Big\downarrow \; 27$$
$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}$$
$$\Big\downarrow \; 1476$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 25}$$

3.2. $\int \frac{c-dx^2}{a+bx^4}\,dx$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}\right)}{2\sqrt{b}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt{b}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt{b}}+\frac{\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right)}{2\sqrt{b}}$$

input
```
Int[(c - d*x^2)/(a + b*x^4),x]
```

3.2. $\int \frac{c-dx^2}{a+bx^4}\,dx$

output
```
(((Sqrt[b]*c)/Sqrt[a] - d)*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqr
t[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*
a^(1/4)*b^(1/4))))/(2*Sqrt[b]) + (((Sqrt[b]*c)/Sqrt[a] + d)*(-1/2*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)
*b^(1/4))))/(2*Sqrt[b])
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

3.2.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.14

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(-\_R^2 d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{d\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arct}\right.}{8b(}$

input
```
int((-d*x^2+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum((-_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(166) = 332$.

Time = 0.26 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.11

$$\int \frac{c-dx^2}{a+bx^4}\,dx = -\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x + \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right) + \frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x - \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right) + \frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x + \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right) - \frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x - \left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$

input
```
integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*
d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c
^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b
^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((
a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*l
og(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^
2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*
a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt(-
(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c
^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^
3*b^3)) - a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d
^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4
- 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*
d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) -
 a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b)))
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.45

$$\int \frac{c-dx^2}{a+bx^4}\,dx =$$
$$-\operatorname{RootSum}\left(256t^4a^3b^3-64t^2a^2b^2cd+a^2d^4+2abc^2d^2+b^2c^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^3b^2d-12ta^2bcd^2+}{a^2d^4-b^2c^4}\right.\right.\right.$$

input
```
integrate((-d*x**2+c)/(b*x**4+a),x)
```

output
```
-RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*
c**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d - 12*_t
*a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.89

$$\int \frac{c-dx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

input
```
integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt
(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
*sqrt(b)) + 1/4*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*log(sqrt(b)*x^
2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(
sqrt(b)*c + sqrt(a)*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(3/4))
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

$$\int \frac{c-dx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)
*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3
)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a*b^3)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.44

$$\int \frac{c-dx^2}{a+bx^4}\,dx = 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}} - \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{\frac{a\,d^2\,\sqrt{-a^3\,b^3}-b\,c^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}} + 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}} - \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{-a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{-a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d-2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{-a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{-a^3\,b^3}}{a}}\right)\sqrt{\frac{b\,c^2\,\sqrt{-a^3\,b^3}-a\,d^2\,\sqrt{-a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$

input
```
int((c - d*x^2)/(a + b*x^4),x)
```

output
```
2*atanh((8*b^3*c^2*x*((c*d)/(8*a*b) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2)
+ (d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (
2*b*c^3*(-a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d
^2*x*((c*d)/(8*a*b) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(-a^3*b^3
)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^
3)^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a))*((a*d^2*(-a^3*b^3)^(1/2) -
b*c^2*(-a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) + 2*atanh((8*b
^3*c^2*x*((c*d)/(8*a*b) + (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(-a^3
*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^
3*b^3)^(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*x*((c*d)/
(8*a*b) + (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(-a^3*b^3)^(1/2))/(16
*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*b^3)^(1/2))/a^
2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a))*((b*c^2*(-a^3*b^3)^(1/2) - a*d^2*(-a^3*
b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

3.3 $\int \frac{c+dx^2}{a-bx^4}\,dx$

3.3.1 Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \frac{c+dx^2}{a-bx^4}\,dx=\frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c+\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

output
```
1/2*arctan(b^(1/4)*x/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*a
rctanh(b^(1/4)*x/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

3.3.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\int \frac{c+dx^2}{a-bx^4}\,dx =\frac{2\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c+\sqrt{a}d\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\right)}{4a^{3/4}b^{3/4}}$$

input
```
Integrate[(c + d*x^2)/(a - b*x^4),x]
```

output
```
(2*(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + Sqrt
[a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b
^(3/4))
```

3.3.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1481, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{a-bx^4} \, dx$$

$$\Big\downarrow \text{ 1481}$$

$$\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx-\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{-bx^2-\sqrt{a}\sqrt{b}}dx$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt[4]{a}b^{3/4}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt[4]{a}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt[4]{a}b^{3/4}}$$

input
```
Int[(c + d*x^2)/(a - b*x^4),x]
```

output
```
(((Sqrt[b]*c)/Sqrt[a] - d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(3/4)
) + (((Sqrt[b]*c)/Sqrt[a] + d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^
(3/4))
```

3.3.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1481
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Simp[(e/2 + c*(d/(2*q)))   Int[1/(-q + c*x^2), x], x] + Simp[(
e/2 - c*(d/(2*q)))   Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] &
& NeQ[c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

3.3.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4} b-a\right)}\frac{\left(\_R^{2} d+c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$	36
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}-\frac{d\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	104

input
```
int((d*x^2+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(58) = 116.

Time = 0.26 (sec) , antiderivative size = 755, normalized size of antiderivative = 8.78

$$\int \frac{c+dx^2}{a-bx^4}\,dx$$
$$= \frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x + \left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)$$
$$-\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x - \left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)$$
$$-\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x + \left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$
$$+\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x - \left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$

input
```
integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)
/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*
d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c
^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*
sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(
b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)
/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt((b^2*c^4
 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2
*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) +
 a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d
^4)/(a^3*b^3)) - 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a
^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 +
 a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3
)) - 2*c*d)/(a*b)))
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\int \frac{c+dx^2}{a-bx^4}\,dx =$$
$$-\operatorname{RootSum}\left(256t^4a^3b^3 - 64t^2a^2b^2cd - a^2d^4 + 2abc^2d^2 - b^2c^4, \left(t \mapsto t\log\left(x + \frac{-64t^3a^3b^2d + 12ta^2bcd^2}{a^2d^4 - b^2c^4}\right.\right.\right.$$

input
```
integrate((d*x**2+c)/(-b*x**4+a),x)
```

output
```
-RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*
c**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d + 12*_
t*a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27

$$\int \frac{c+dx^2}{a-bx^4}\,dx = \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*(sqrt(b)*c - sqrt(a)*d)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(b)*c + sqrt(a)*d)*log((sqrt(
b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(58) = 116$.

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.67

$$\begin{aligned}\int \frac{c+dx^2}{a-bx^4}\,dx = &-\frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} \\ &- \frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} \\ &- \frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}} \\ &+ \frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}\end{aligned}$$

input
```
integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c - sqrt(-a*b)
*b*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^
3)^(3/4) - 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*log(x^2 + sqrt(2)*x*(-a/b)
^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)
*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 579, normalized size of antiderivative = 6.73

$$\int \frac{c+dx^2}{a-bx^4}\,dx$$
$$= 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}} + \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{-\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}-2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$
$$+ 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}} + \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c\,d}{8\,a\,b}+\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$

input
```
int((c + d*x^2)/(a - b*x^4),x)
```

output
```
2*atanh((8*b^3*c^2*x*((c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) -
 (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*
b*c^3*(a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x
*((c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(a^3*b^3)^(1/2
))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2
))/a^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a))*(-(a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a
^3*b^3)^(1/2) - 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) + 2*atanh((8*b^3*c^2*x*
((c*d)/(8*a*b) + (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(a^3*b^3)^(1/2)
)/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/2)
)/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((c*d)/(8*a*b) + (c^
2*(a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2
))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(a^
3*b^3)^(1/2))/a))*((a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3)^(1/2) + 2*a^2*
b^2*c*d)/(16*a^3*b^3))^(1/2)
```

3.3. $\int \frac{c+dx^2}{a-bx^4}\,dx$

3.4 $\int \frac{c-dx^2}{a-bx^4} dx$

3.4.1 Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{c-dx^2}{a-bx^4} dx = \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

output
```
1/2*arctanh(b^(1/4)*x/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*
arctan(b^(1/4)*x/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

3.4.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\int \frac{c-dx^2}{a-bx^4} dx = \frac{2\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \left(\sqrt{b}c-\sqrt{a}d\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)\right)}{4a^{3/4}b^{3/4}}$$

input
```
Integrate[(c - d*x^2)/(a - b*x^4),x]
```

output
```
(2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c - Sqrt
[a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b
^(3/4))
```

3.4.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1481, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx^2}{a-bx^4}\,dx$$

$$\Big\downarrow \text{ 1481}$$

$$\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx-\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)\int \frac{1}{-bx^2-\sqrt{a}\sqrt{b}}dx$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}dx+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt[4]{a}b^{3/4}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)}{2\sqrt[4]{a}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)}{2\sqrt[4]{a}b^{3/4}}$$

input
```
Int[(c - d*x^2)/(a - b*x^4),x]
```

output
```
(((Sqrt[b]*c)/Sqrt[a] + d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^(3/4)
) + (((Sqrt[b]*c)/Sqrt[a] - d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*b^
(3/4))
```

3.4.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1481
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Simp[(e/2 + c*(d/(2*q)))   Int[1/(-q + c*x^2), x], x] + Simp[(
e/2 - c*(d/(2*q)))   Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] &
& NeQ[c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

3.4.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.43

method	result	size
risch	$-\frac{\sum_{\_R=\text{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(-\_R^2 d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	37
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	104

input
```
int((-d*x^2+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((-_R^2*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(58) = 116.

Time = 0.28 (sec) , antiderivative size = 755, normalized size of antiderivative = 8.78

$$\int \frac{c-dx^2}{a-bx^4}\,dx$$
$$= \frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x + \left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)$$
$$- \frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x - \left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)$$
$$- \frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x + \left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$
$$+ \frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-\left(b^2c^4-a^2d^4\right)x - \left(a^3b^2d\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$

input
```
integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d
)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2
*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a
*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log
(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d
^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b
*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt((b^2*
c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 -
a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)
) + a*b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2
*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^
2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (
a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3
+ a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3
)) - 2*c*d)/(a*b)))
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\int \frac{c-dx^2}{a-bx^4}\,dx$$
$$= \operatorname{RootSum}\left(256t^4a^3b^3 + 64t^2a^2b^2cd - a^2d^4 + 2abc^2d^2 - b^2c^4, \left(t \mapsto t\log\left(x + \frac{-64t^3a^3b^2d - 12ta^2bcd^2 - 4}{a^2d^4 - b^2c^4}\right.\right.\right.$$

input
```
integrate((-d*x**2+c)/(-b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c
**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d - 12*_t
*a**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27

$$\int \frac{c-dx^2}{a-bx^4}\,dx = \frac{\left(\sqrt{b}c+\sqrt{a}d\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{b}c-\sqrt{a}d\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*(sqrt(b)*c + sqrt(a)*d)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(b)*c - sqrt(a)*d)*log((sqrt(
b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(58) = 116$.

Time = 0.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.65

$$\begin{aligned}\int \frac{c-dx^2}{a-bx^4}\,dx = &-\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} \\ &- \frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}} \\ &- \frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}} \\ &+ \frac{\sqrt{2}\left(b^2c+\sqrt{-ab}bd\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}\end{aligned}$$

input
```
integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c + sqrt(-a*b)
*b*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^
3)^(3/4) - 1/8*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*log(x^2 + sqrt(2)*x*(-a/b)
^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)
*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 579, normalized size of antiderivative = 6.73

$$\int \frac{c-dx^2}{a-bx^4}\,dx$$

$$= -2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{-\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}} + \frac{8\,a\,b^2\,d^2\,x\,\sqrt{-\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{-\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$
$$- 2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}} + \frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\sqrt{\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}-2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$

input
```
int((c - d*x^2)/(a - b*x^4),x)
```

output
```
- 2*atanh((8*b^3*c^2*x*(- (c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^
2) - (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 +
 (2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d
^2*x*(- (c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(a^3*b^3
)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3
)^(1/2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a))*(-(a*d^2*(a^3*b^3)^(1/2) + b*
c^2*(a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) - 2*atanh((8*b^3*
c^2*x*((c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)
^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)
^(1/2))/a^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((c^2*(a^3*b^3
)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3)
)^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2))/a^2 - (2*c*d
^2*(a^3*b^3)^(1/2))/a))*((a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3)^(1/2) -
2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

3.4. $\int \frac{c-dx^2}{a-bx^4}\,dx$

3.5 $\int \frac{2+3x^2}{4+9x^4} \, dx$

3.5.1 Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{2+3x^2}{4+9x^4} \, dx = -\frac{\arctan\left(1-\sqrt{3}x\right)}{2\sqrt{3}} + \frac{\arctan\left(1+\sqrt{3}x\right)}{2\sqrt{3}}$$

output
```
1/6*arctan(-1+x*3^(1/2))*3^(1/2)+1/6*arctan(1+x*3^(1/2))*3^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{4+9x^4} \, dx = \frac{-\arctan\left(1-\sqrt{3}x\right)+\arctan\left(1+\sqrt{3}x\right)}{2\sqrt{3}}$$

input
```
Integrate[(2 + 3*x^2)/(4 + 9*x^4),x]
```

output
```
(-ArcTan[1 - Sqrt[3]*x] + ArcTan[1 + Sqrt[3]*x])/(2*Sqrt[3])
```

3.5.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1476, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{9x^4+4}dx$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{6}\int \frac{1}{x^2-\frac{2x}{\sqrt{3}}+\frac{2}{3}}dx+\frac{1}{6}\int \frac{1}{x^2+\frac{2x}{\sqrt{3}}+\frac{2}{3}}dx$$

$$\Big\downarrow \text{1082}$$

$$\frac{\int \frac{1}{-\left(1-\sqrt{3}x\right)^2-1}d\left(1-\sqrt{3}x\right)}{2\sqrt{3}}-\frac{\int \frac{1}{-\left(\sqrt{3}x+1\right)^2-1}d\left(\sqrt{3}x+1\right)}{2\sqrt{3}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\arctan\left(1-\sqrt{3}x\right)}{2\sqrt{3}}$$

input
```
Int[(2 + 3*x^2)/(4 + 9*x^4),x]
```

output
```
-1/2*ArcTan[1 - Sqrt[3]*x]/Sqrt[3] + ArcTan[1 + Sqrt[3]*x]/(2*Sqrt[3])
```

3.5.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.5.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{x\sqrt{3}}{2}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{3x^3\sqrt{3}}{4}+\frac{x\sqrt{3}}{2}\right)}{6}$
default	$\frac{\sqrt{6}\,\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^2-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}+1\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}-1\right)\right)}{48}+\frac{\sqrt{6}\,\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^2+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}\right.\right.}{48}$
meijerg	$\frac{\sqrt{6}\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{3}\left(x^4\right)^{\frac{1}{4}}+\frac{3\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{3}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{3}\left(x^4\right)^{\frac{1}{4}}+\frac{3\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\left(x^4\right)^{\frac{1}{4}}}{2+\sqrt{3}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}\right)}{24}+$

input
```
int((3*x^2+2)/(9*x^4+4),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*arctan(1/2*x*3^(1/2))+1/6*3^(1/2)*arctan(3/4*x^3*3^(1/2)+1/2*x
*3^(1/2))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{2+3x^2}{4+9x^4}\,dx=\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{4}\,\sqrt{3}(3\,x^3+2\,x)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{2}\,\sqrt{3}x\right)$$

input
```
integrate((3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/4*sqrt(3)*(3*x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/2*sqr
t(3)*x)
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{2}\right)+2\,\mathrm{atan}\left(\frac{3\sqrt{3}x^3}{4}+\frac{\sqrt{3}x}{2}\right)\right)}{12}$$

input
```
integrate((3*x**2+2)/(9*x**4+4),x)
```

output
```
sqrt(3)*(2*atan(sqrt(3)*x/2) + 2*atan(3*sqrt(3)*x**3/4 + sqrt(3)*x/2))/12
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(3\,x+\sqrt{3}\right)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(3\,x-\sqrt{3}\right)\right)$$

input
```
integrate((3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(3*x + sqrt(3))) + 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(3*x - sqrt(3)))
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{9}{8}\,\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{9}{8}\,\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)$$

input
```
integrate((3*x^2+2)/(9*x^4+4),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x + sqrt(2)*(4/9)^(1/4))) +
1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x - sqrt(2)*(4/9)^(1/4)))
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{3\sqrt{3}\,x^3}{4}+\frac{\sqrt{3}\,x}{2}\right)+\operatorname{atan}\left(\frac{\sqrt{3}\,x}{2}\right)\right)}{6}$$

input
```
int((3*x^2 + 2)/(9*x^4 + 4),x)
```

output
```
(3^(1/2)*(atan((3^(1/2)*x)/2 + (3*3^(1/2)*x^3)/4) + atan((3^(1/2)*x)/2)))/
6
```

3.6 $\int \frac{2-3x^2}{4+9x^4} dx$

3.6.1 Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{2-3x^2}{4+9x^4} dx = -\frac{\log \left(2-2\sqrt{3}x+3x^2\right)}{4\sqrt{3}}+\frac{\log \left(2+2\sqrt{3}x+3x^2\right)}{4\sqrt{3}}$$

output
```
-1/12*ln(2+3*x^2-2*x*3^(1/2))*3^(1/2)+1/12*ln(2+3*x^2+2*x*3^(1/2))*3^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{2-3x^2}{4+9x^4} dx = \frac{-\log \left(-2+2\sqrt{3}x-3x^2\right)+\log \left(2+2\sqrt{3}x+3x^2\right)}{4\sqrt{3}}$$

input
```
Integrate[(2 - 3*x^2)/(4 + 9*x^4),x]
```

output
```
(-Log[-2 + 2*Sqrt[3]*x - 3*x^2] + Log[2 + 2*Sqrt[3]*x + 3*x^2])/(4*Sqrt[3]
)
```

3.6.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1479, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-3x^2}{9x^4+4} \, dx$$

$$\Big\downarrow \text{1479}$$

$$-\frac{\int -\frac{2(\sqrt{3}-3x)}{3x^2-2\sqrt{3}x+2}dx}{4\sqrt{3}} - \frac{\int -\frac{2\sqrt{3}(\sqrt{3}x+1)}{3x^2+2\sqrt{3}x+2}dx}{4\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{3}-3x}{3x^2-2\sqrt{3}x+2}dx}{2\sqrt{3}} + \frac{1}{2}\int \frac{\sqrt{3}x+1}{3x^2+2\sqrt{3}x+2}dx$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log(3x^2+2\sqrt{3}x+2)}{4\sqrt{3}} - \frac{\log(3x^2-2\sqrt{3}x+2)}{4\sqrt{3}}$$

input
```
Int[(2 - 3*x^2)/(4 + 9*x^4),x]
```

output
```
-1/4*Log[2 - 2*Sqrt[3]*x + 3*x^2]/Sqrt[3] + Log[2 + 2*Sqrt[3]*x + 3*x^2]/(
4*Sqrt[3])
```

3.6.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.6.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\ln\left(2+3x^2-2x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(2+3x^2+2x\sqrt{3}\right)\sqrt{3}}{12}$
default	$\frac{\sqrt{6}\,\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^2-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}+1\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}-1\right)\right)}{48}-\frac{\sqrt{6}\,\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^2+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}\right.\right.}{48}$
meijerg	$\frac{\sqrt{6}\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{3}\,\left(x^4\right)^{\frac{1}{4}}+\frac{3\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{3}\,\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{3}\,\left(x^4\right)^{\frac{1}{4}}+\frac{3\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\left(x^4\right)^{\frac{1}{4}}}{2+\sqrt{3}\,\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}\right)}{24}-$

input
```
int((-3*x^2+2)/(9*x^4+4),x,method=_RETURNVERBOSE)
```

output
```
-1/12*ln(2+3*x^2-2*x*3^(1/2))*3^(1/2)+1/12*ln(2+3*x^2+2*x*3^(1/2))*3^(1/2)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{2-3x^2}{4+9x^4}\,dx=\frac{1}{12}\,\sqrt{3}\log\left(\frac{9\,x^4+24\,x^2+4\,\sqrt{3}(3\,x^3+2\,x)+4}{9\,x^4+4}\right)$$

input
```
integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((9*x^4 + 24*x^2 + 4*sqrt(3)*(3*x^3 + 2*x) + 4)/(9*x^4 + 4
))
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{2-3x^2}{4+9x^4}\,dx = -\frac{\sqrt{3}\log\left(x^2-\frac{2\sqrt{3}x}{3}+\frac{2}{3}\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\frac{2\sqrt{3}x}{3}+\frac{2}{3}\right)}{12}$$

input
```
integrate((-3*x**2+2)/(9*x**4+4),x)
```

output
```
-sqrt(3)*log(x**2 - 2*sqrt(3)*x/3 + 2/3)/12 + sqrt(3)*log(x**2 + 2*sqrt(3)
*x/3 + 2/3)/12
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{1}{12}\sqrt{3}\log\left(3x^2+2\sqrt{3}x+2\right)-\frac{1}{12}\sqrt{3}\log\left(3x^2-2\sqrt{3}x+2\right)$$

input
```
integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*log(3*x^2 + 2*sqrt(3)*x + 2) - 1/12*sqrt(3)*log(3*x^2 - 2*sqr
t(3)*x + 2)
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x+\frac{2}{3}\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x+\frac{2}{3}\right)$$

input
```
integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(x^2 + sqrt(2)*(4/9)^(1/4)*x + 2/3) - 1/12*sqrt(3)*log(x^2
 - sqrt(2)*(4/9)^(1/4)*x + 2/3)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.41

$$\int \frac{2-3x^2}{4+9x^4}\,dx = \frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{2\sqrt{3}\,x}{3\,x^2+2}\right)}{6}$$

input
```
int(-(3*x^2 - 2)/(9*x^4 + 4),x)
```

output
```
(3^(1/2)*atanh((2*3^(1/2)*x)/(3*x^2 + 2)))/6
```

3.7 $\int \frac{2+3x^2}{4-9x^4} dx$

3.7.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{2+3x^2}{4-9x^4} dx = \frac{\operatorname{arctanh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

output
```
1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{2+3x^2}{4-9x^4} dx = \frac{-\log\left(\sqrt{6}-3x\right)+\log\left(\sqrt{6}+3x\right)}{2\sqrt{6}}$$

input
```
Integrate[(2 + 3*x^2)/(4 - 9*x^4),x]
```

output
```
(-Log[Sqrt[6] - 3*x] + Log[Sqrt[6] + 3*x])/(2*Sqrt[6])
```

3.7.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1386, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{4-9x^4}\,dx$$

$$\Big\downarrow \text{1386}$$

$$\int \frac{1}{2-3x^2}dx$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

input
```
Int[(2 + 3*x^2)/(4 - 9*x^4),x]
```

output
```
ArcTanh[Sqrt[3/2]*x]/Sqrt[6]
```

3.7.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

3.7.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result
default	$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
risch	$\frac{\sqrt{6}\,\ln\left(3x+\sqrt{6}\right)}{12}-\frac{\sqrt{6}\,\ln\left(3x-\sqrt{6}\right)}{12}$
meijerg	$-\frac{\sqrt{6}\,x\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{\sqrt{3}\,\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}}{2}\right)-2\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}}{2}\right)\right)}{24\left(x^4\right)^{\frac{1}{4}}}-\frac{\sqrt{6}\,x^3\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{\sqrt{3}\right.\right.}{}\right.}{24(x}$

input
```
int((3*x^2+2)/(-9*x^4+4),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{2+3x^2}{4-9x^4}\,dx = \frac{1}{12}\,\sqrt{6}\log\left(\frac{3\,x^2+2\,\sqrt{6}x+2}{3\,x^2-2}\right)$$

input
```
integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
```

output
```
1/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2))
```

3.7.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{2+3x^2}{4-9x^4}\,dx = -\frac{\sqrt{6}\log\left(x-\frac{\sqrt{6}}{3}\right)}{12}+\frac{\sqrt{6}\log\left(x+\frac{\sqrt{6}}{3}\right)}{12}$$

input
```
integrate((3*x**2+2)/(-9*x**4+4),x)
```

output
```
-sqrt(6)*log(x - sqrt(6)/3)/12 + sqrt(6)*log(x + sqrt(6)/3)/12
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{2+3x^2}{4-9x^4}\,dx = -\frac{1}{12}\sqrt{6}\log\left(\frac{3\,x-\sqrt{6}}{3\,x+\sqrt{6}}\right)$$

input
```
integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
```

output
```
-1/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6)))
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{2+3x^2}{4-9x^4}\,dx = \frac{1}{12}\sqrt{6}\log\left(\left|x+\frac{1}{3}\sqrt{6}\right|\right) - \frac{1}{12}\sqrt{6}\log\left(\left|x-\frac{1}{3}\sqrt{6}\right|\right)$$

input
```
integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*log(abs(x + 1/3*sqrt(6))) - 1/12*sqrt(6)*log(abs(x - 1/3*sqrt
(6)))
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2+3x^2}{4-9x^4} \, dx = \frac{\sqrt{6} \operatorname{atanh}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

input
```
int(-(3*x^2 + 2)/(9*x^4 - 4),x)
```

output
```
(6^(1/2)*atanh((6^(1/2)*x)/2))/6
```

3.8 $\int \frac{2-3x^2}{4-9x^4} \, dx$

3.8.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{2-3x^2}{4-9x^4} \, dx = \frac{\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

output
```
1/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{2-3x^2}{4-9x^4} \, dx = \frac{\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

input
```
Integrate[(2 - 3*x^2)/(4 - 9*x^4),x]
```

output
```
ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

3.8.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1386, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-3x^2}{4-9x^4}\,dx$$

$$\Big\downarrow \text{1386}$$

$$\int \frac{1}{3x^2+2}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

input
```
Int[(2 - 3*x^2)/(4 - 9*x^4),x]
```

output
```
ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

3.8.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

3.8.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result
default	$\frac{\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
risch	$\frac{\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
meijerg	$-\frac{\sqrt{6}\,x\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{\sqrt{3}\,\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2}\right)-2\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2}\right)\right)}{24(x^4)^{\frac{1}{4}}}+\frac{\sqrt{6}\,x^3\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\sqrt{3}\right.\right.}{24(x}$

input
```
int((-3*x^2+2)/(-9*x^4+4),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}x\right)$$

input
```
integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
```

output
```
1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

input
```
integrate((-3*x**2+2)/(-9*x**4+4),x)
```

output
```
sqrt(6)*atan(sqrt(6)*x/2)/6
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
```

output
```
1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

output
```
1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{2-3x^2}{4-9x^4}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

input
```
int((3*x^2 - 2)/(9*x^4 - 4),x)
```

output
```
(6^(1/2)*atan((6^(1/2)*x)/2))/6
```

3.9 $\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx$

3.9.1 Optimal result

Integrand size = 27, antiderivative size = 75

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx=-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

output
```
1/2*b^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)*2^(1/2)+1/2*b^(1/
4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)*2^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx=\frac{\sqrt[4]{b}\left(-\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right)}{\sqrt{2}\sqrt[4]{a}}$$

input
```
Integrate[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

output
```
(b^(1/4)*(-ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)]))/(Sqrt[2]*a^(1/4))
```

3.9.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1476, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx+\frac{1}{2}\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx$$

$$\Big\downarrow \text{1082}$$

$$\frac{\sqrt[4]{b}\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

input
```
Int[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

output
```
-((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))) + (
b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))
```

3.9.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.9.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(51) = 102$.

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.72

method	result
default	$\frac{\sqrt{b}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8\sqrt{a}}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arc}\ldots}{8\ldots}$

input
```
int((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^
(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8/(a/b)^(1/4)*2^(1/2)*(ln(
(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx$$
$$=\left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4-4\sqrt{a}\sqrt{b}x^2+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3-ax\right)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right),\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{\right.\right.$$
$$\left.+\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\frac{\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3+ax\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{a}\right)\right]$$

input
```
integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*log((b*x^4 - 4*sqrt(a)*sqrt(b)*x^2 +
 4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a)) + a)/(b*x^
4 + a)), sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(sqrt(b)/s
qrt(a))) + sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(
b)*x^3 + a*x)*sqrt(sqrt(b)/sqrt(a))/a)]
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx=-\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$
$$+\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

input
```
integrate((b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

output
```
-sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(
a))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*l
og(sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a))/sqrt(b) - sqrt(a)/sqrt(b) + x*
*2)/4
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}$$

input
```
integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 1/2*sqrt(2)*sqrt(b)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/sqrt(sqrt(a)*sqrt(b))
```

3.9.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.9.9 Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\,b^{1/4}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{2\,a^{1/4}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,b^{3/4}\,x^3}{2\,a^{3/4}}+\frac{\sqrt{2}\,b^{1/4}\,x}{2\,a^{1/4}}\right)\right)}{4\,a^{1/4}}$$

input
```
int((b*x^2 + a^(1/2)*b^(1/2))/(a + b*x^4),x)
```

output
```
(2^(1/2)*b^(1/4)*(2*atan((2^(1/2)*b^(1/4)*x)/(2*a^(1/4))) + 2*atan((2^(1/2
)*b^(3/4)*x^3)/(2*a^(3/4)) + (2^(1/2)*b^(1/4)*x)/(2*a^(1/4)))))/(4*a^(1/4)
)
```

3.9. $\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx$

3.10 $\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx$

3.10.1 Optimal result

Integrand size = 28, antiderivative size = 106

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = -\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}} + \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

output
```
-1/4*b^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/4)*2^
(1/2)+1/4*b^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/4
)*2^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \frac{\sqrt[4]{b}\left(-\log\left(-\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x-\sqrt{b}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)\right)}{2\sqrt{2}\sqrt[4]{a}}$$

input
```
Integrate[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

output
```
(b^(1/4)*(-Log[-Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x - Sqrt[b]*x^2] + Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(2*Sqrt[2]*a^(1/4))
```

3.10.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}dx$$

$$\Big\downarrow \text{1479}$$

$$-\frac{\sqrt[4]{b}\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt[4]{b}\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

input
```
Int[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

output
```
-1/2*(b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(Sqr
t[2]*a^(1/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(2*Sqrt[2]*a^(1/4))
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(70) = 140$.

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.92

method	result
default	$\frac{\sqrt{b}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8\sqrt{a}}-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arc}\ldots}{8\ldots}$

input
```
int((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^
(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8/(a/b)^(1/4)*2^(1/2)*(ln(
(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4+4\sqrt{a}\sqrt{b}x^2+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3+ax\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right), -\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\right) + \sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\frac{\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3-ax\right)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{a}\right)\right]$$

input
```
integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*log((b*x^4 + 4*sqrt(a)*sqrt(b)*x^2 +
4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 + a*x)*sqrt(sqrt(b)/sqrt(a)) + a)/(b*x^4
+ a)), -sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(-sqrt(b)/
sqrt(a))) + sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqr
t(b)*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a))/a)]
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = -\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4} + \frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

input
```
integrate((-b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

output
```
-sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a)
)/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(
sqrt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/
4
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}} - \frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}}$$

input
```
integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*b^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))
/a^(1/4) - 1/4*sqrt(2)*b^(1/4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/a^(1/4)
```

3.10.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.10.9 Mupad [B] (verification not implemented)

Time = 13.85 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = \frac{\sqrt{2}\,b^{1/4}\,\mathrm{atanh}\left(\frac{2\sqrt{2}\,a^{1/4}\,b^{11/4}\,x}{2\sqrt{a}\,b^{5/2}+2\,b^3\,x^2}\right)}{2\,a^{1/4}}$$

input
```
int(-(b*x^2 - a^(1/2)*b^(1/2))/(a + b*x^4),x)
```

output
```
(2^(1/2)*b^(1/4)*atanh((2*2^(1/2)*a^(1/4)*b^(11/4)*x)/(2*a^(1/2)*b^(5/2) +
 2*b^3*x^2)))/(2*a^(1/4))
```

3.11 $\int \frac{d+ex^2}{d^2+e^2x^4} \, dx$

3.11.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{d+ex^2}{d^2+e^2x^4} \, dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

output
```
1/2*arctan(-1+x*2^(1/2)*e^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)+1/2*arcta
n(1+x*2^(1/2)*e^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{d+ex^2}{d^2+e^2x^4} \, dx = \frac{-\arctan\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)+\arctan\left(1+\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

output
```
(-ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]] + ArcTan[1 + (Sqrt[2]*Sqrt[e]*x)
/Sqrt[d]])/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

3.11.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1476, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{d^2+e^2x^4}dx$$

$$\Big\downarrow \text{1476}$$

$$\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}}dx}{2e}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}}dx}{2e}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

input
```
Int[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

output
```
-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])) + Arc
Tan[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

3.11.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.11.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{\sqrt{2}\,\ln\left(e\,x^{2}\sqrt{-ed}-dex\sqrt{2}-d\sqrt{-ed}\right)}{4\sqrt{-ed}}+\frac{\sqrt{2}\,\ln\left(e\,x^{2}\sqrt{-ed}+dex\sqrt{2}-d\sqrt{-ed}\right)}{4\sqrt{-ed}}$
default	$\frac{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}{x^{2}-\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8d}+\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}{x^{2}+\left(\frac{d^{2}}{e^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^{2}}{e^{2}}}}\right)\right.}{}$

input
```
int((e*x^2+d)/(e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)/(-e*d)^(1/2)*ln(e*x^2*(-e*d)^(1/2)-d*e*x*2^(1/2)-d*(-e*d)^(1/
2))+1/4*2^(1/2)/(-e*d)^(1/2)*ln(e*x^2*(-e*d)^(1/2)+d*e*x*2^(1/2)-d*(-e*d)^
(1/2))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.83

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx$$
$$=\left[-\frac{\sqrt{2}\sqrt{-de}\log\left(\frac{e^2x^4-4\,dex^2-2\,\sqrt{2}(ex^3-dx)\sqrt{-de}+d^2}{e^2x^4+d^2}\right)}{4\,de},\frac{\sqrt{2}\sqrt{de}\arctan\left(\frac{\sqrt{2}\sqrt{de}x}{2\,d}\right)+\sqrt{2}\sqrt{de}\arctan\left(\frac{\sqrt{2}(ex^3+dx)\sqrt{}}{2\,d^2}\right.}{2\,de}\right.$$

input
```
integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(2)*sqrt(-d*e)*log((e^2*x^4 - 4*d*e*x^2 - 2*sqrt(2)*(e*x^3 - d*x
)*sqrt(-d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), 1/2*(sqrt(2)*sqrt(d*e)*arctan(
1/2*sqrt(2)*sqrt(d*e)*x/d) + sqrt(2)*sqrt(d*e)*arctan(1/2*sqrt(2)*(e*x^3 +
 d*x)*sqrt(d*e)/d^2))/(d*e)]
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = -\frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(-\sqrt{2}dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4} + \frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(\sqrt{2}dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4}$$

input
```
integrate((e*x**2+d)/(e**2*x**4+d**2),x)
```

output
```
-sqrt(2)*sqrt(-1/(d*e))*log(-sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4 +
sqrt(2)*sqrt(-1/(d*e))*log(sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(51) = 102$.

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 4.03

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}} + \frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}} - \frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}} + \frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}}$$

input
```
integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(e + sqrt(e^2))*log((2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(1/
4) - sqrt(2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(1
/4) + sqrt(2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) + 1/8*sq
rt(2)*(e + sqrt(e^2))*log((2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) - s
qrt(2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) +
sqrt(2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) - 1/8*sqrt(2)*
(e - sqrt(e^2))*log(sqrt(e^2)*x^2 + sqrt(2)*sqrt(d)*(e^2)^(1/4)*x + d)/(sq
rt(d)*(e^2)^(3/4)) + 1/8*sqrt(2)*(e - sqrt(e^2))*log(sqrt(e^2)*x^2 - sqrt(
2)*sqrt(d)*(e^2)^(1/4)*x + d)/(sqrt(d)*(e^2)^(3/4))
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\sqrt{-de}\log\left(x^2+\sqrt{2}x\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{e^2}}\right)}{4\,de} - \frac{\sqrt{2}\sqrt{-de}\log\left(x^2-\sqrt{2}x\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}+\sqrt{\frac{d^2}{e^2}}\right)}{4\,de}$$

input
```
integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*sqrt(-d*e)*log(x^2 + sqrt(2)*x*(d^2/e^2)^(1/4) + sqrt(d^2/e^2)
)/(d*e) - 1/4*sqrt(2)*sqrt(-d*e)*log(x^2 - sqrt(2)*x*(d^2/e^2)^(1/4) + sqr
t(d^2/e^2))/(d*e)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{d+ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{3/2}\,x^3}{2\,d^{3/2}}+\frac{\sqrt{2}\,\sqrt{e}\,x}{2\,\sqrt{d}}\right)\right)}{4\,\sqrt{d}\,\sqrt{e}}$$

input
```
int((d + e*x^2)/(d^2 + e^2*x^4),x)
```

output
```
(2^(1/2)*(2*atan((2^(1/2)*e^(1/2)*x)/(2*d^(1/2))) + 2*atan((2^(1/2)*e^(3/2
)*x^3)/(2*d^(3/2)) + (2^(1/2)*e^(1/2)*x)/(2*d^(1/2)))))/(4*d^(1/2)*e^(1/2)
)
```

3.11. $\int \frac{d+ex^2}{d^2+e^2x^4}\,dx$

3.12 $\int \frac{d-ex^2}{d^2+e^2x^4}\,dx$

3.12.1 Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = -\frac{\log\left(d-\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\log\left(d+\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

output
```
-1/4*ln(d+e*x^2-x*2^(1/2)*d^(1/2)*e^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)+1/4*ln(
d+e*x^2+x*2^(1/2)*d^(1/2)*e^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = \frac{-\log\left(-d+\sqrt{2}\sqrt{d}\sqrt{e}x-ex^2\right)+\log\left(d+\sqrt{2}\sqrt{d}\sqrt{e}x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

input
```
Integrate[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

output
```
(-Log[-d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x - e*x^2] + Log[d + Sqrt[2]*Sqrt[d]*Sq
rt[e]*x + e*x^2])/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

3.12.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{d^2+e^2x^4} \, dx$$

$$\Big\downarrow \text{1479}$$

$$-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}x+\sqrt{d}\right)}{\sqrt{e}\left(x^2+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{e}x}{\sqrt{e}\left(x^2-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{e}x+\sqrt{d}\right)}{\sqrt{e}\left(x^2+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{e}x}{x^2-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}}dx}{2\sqrt{2}\sqrt{de}}+\frac{\int \frac{\sqrt{2}\sqrt{e}x+\sqrt{d}}{x^2+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{e}}+\frac{d}{e}}dx}{2\sqrt{de}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\log\left(-\sqrt{2}\sqrt{d}\sqrt{e}x+d+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

input
```
Int[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

output
```
-1/2*Log[d - Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[e])
+ Log[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.12.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\sqrt{2}\,\ln\left(e\,x^2\sqrt{ed}+dex\sqrt{2}+d\sqrt{ed}\right)}{4\sqrt{ed}}-\frac{\sqrt{2}\,\ln\left(e\,x^2\sqrt{ed}-dex\sqrt{2}+d\sqrt{ed}\right)}{4\sqrt{ed}}$
default	$\frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}}{x^2-\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8d}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}}{x^2+\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}}\right)\right.}{}$

input
```
int((-e*x^2+d)/(e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/(e*d)^(1/2)*ln(e*x^2*(e*d)^(1/2)+d*e*x*2^(1/2)+d*(e*d)^(1/2))-
1/4*2^(1/2)/(e*d)^(1/2)*ln(e*x^2*(e*d)^(1/2)-d*e*x*2^(1/2)+d*(e*d)^(1/2))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.56

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = \left[\frac{\sqrt{2}\sqrt{de}\log\left(\frac{e^2x^4+4\,dex^2+2\,\sqrt{2}(ex^3+dx)\sqrt{de}+d^2}{e^2x^4+d^2}\right)}{4\,de},\right.$$
$$\left.-\frac{\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\sqrt{-de}x}{2\,d}\right)-\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}(ex^3-dx)\sqrt{-de}}{2\,d^2}\right)}{2\,de}\right]$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(2)*sqrt(d*e)*log((e^2*x^4 + 4*d*e*x^2 + 2*sqrt(2)*(e*x^3 + d*x)*
sqrt(d*e) + d^2)/(e^2*x^4 + d^2))/(d*e), -1/2*(sqrt(2)*sqrt(-d*e)*arctan(1
/2*sqrt(2)*sqrt(-d*e)*x/d) - sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(e*x^3
- d*x)*sqrt(-d*e)/d^2))/(d*e)]
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = -\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(-\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{\frac{1}{de}}\log\left(\sqrt{2}dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}$$

input
```
integrate((-e*x**2+d)/(e**2*x**4+d**2),x)
```

output
```
-sqrt(2)*sqrt(1/(d*e))*log(-sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4 + sq
rt(2)*sqrt(1/(d*e))*log(sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4
```

3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(62) = 124$.

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.36

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = -\frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}} - \frac{\sqrt{2}\left(e-\sqrt{e^2}\right)\log\left(\frac{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}-\sqrt{2}\sqrt{-d\sqrt{e^2}}}{2\sqrt{e^2}x-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}+\sqrt{2}\sqrt{-d\sqrt{e^2}}}\right)}{8\sqrt{e^2}\sqrt{-d\sqrt{e^2}}} + \frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2+\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(e+\sqrt{e^2}\right)\log\left(\sqrt{e^2}x^2-\sqrt{2}\sqrt{d}(e^2)^{\frac{1}{4}}x+d\right)}{8\sqrt{d}(e^2)^{\frac{3}{4}}}$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*(e - sqrt(e^2))*log((2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(1
/4) - sqrt(2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x + sqrt(2)*sqrt(d)*(e^2)^(
1/4) + sqrt(2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) - 1/8*s
qrt(2)*(e - sqrt(e^2))*log((2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) -
sqrt(2)*sqrt(-d*sqrt(e^2)))/(2*sqrt(e^2)*x - sqrt(2)*sqrt(d)*(e^2)^(1/4) +
 sqrt(2)*sqrt(-d*sqrt(e^2))))/(sqrt(e^2)*sqrt(-d*sqrt(e^2))) + 1/8*sqrt(2)
*(e + sqrt(e^2))*log(sqrt(e^2)*x^2 + sqrt(2)*sqrt(d)*(e^2)^(1/4)*x + d)/(s
qrt(d)*(e^2)^(3/4)) - 1/8*sqrt(2)*(e + sqrt(e^2))*log(sqrt(e^2)*x^2 - sqrt
(2)*sqrt(d)*(e^2)^(1/4)*x + d)/(sqrt(d)*(e^2)^(3/4))
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\right)}{2\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}\right)}{2\,de} + \frac{\sqrt{2}\sqrt{-de}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\right)}{2\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}\right)}{2\,de}$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(d^2/e^2)^(1/4))/
(d^2/e^2)^(1/4))/(d*e) + 1/2*sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(d^2/e^2)^(1/4))/(d^2/e^2)^(1/4))/(d*e)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.46

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{2\sqrt{2}\sqrt{d}\,e^{7/2}\,x}{2\,e^4\,x^2+2\,d\,e^3}\right)}{2\sqrt{d}\sqrt{e}}$$

input
```
int((d - e*x^2)/(d^2 + e^2*x^4),x)
```

output
```
(2^(1/2)*atanh((2*2^(1/2)*d^(1/2)*e^(7/2)*x)/(2*d*e^3 + 2*e^4*x^2)))/(2*d^
(1/2)*e^(1/2))
```

3.13 $\int \frac{5+2x^2}{-1+x^4} \, dx$

3.13.1 Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{5+2x^2}{-1+x^4} \, dx = -\frac{3\arctan(x)}{2} - \frac{7\text{arctanh}(x)}{2}$$

output
```
-3/2*arctan(x)-7/2*arctanh(x)
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{5+2x^2}{-1+x^4} \, dx = -\frac{3\arctan(x)}{2} + \frac{7}{4}\log(1-x) - \frac{7}{4}\log(1+x)$$

input
```
Integrate[(5 + 2*x^2)/(-1 + x^4),x]
```

output
```
(-3*ArcTan[x])/2 + (7*Log[1 - x])/4 - (7*Log[1 + x])/4
```

3.13.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1481, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+5}{x^4-1}\,dx$$
$$\Big\downarrow \; 1481$$
$$\frac{7}{2}\int \frac{1}{x^2-1}dx-\frac{3}{2}\int \frac{1}{x^2+1}dx$$
$$\Big\downarrow \; 216$$
$$\frac{7}{2}\int \frac{1}{x^2-1}dx-\frac{3\arctan(x)}{2}$$
$$\Big\downarrow \; 220$$
$$-\frac{3\arctan(x)}{2}-\frac{7\text{arctanh}(x)}{2}$$

input
```
Int[(5 + 2*x^2)/(-1 + x^4),x]
```

output
```
(-3*ArcTan[x])/2 - (7*ArcTanh[x])/2
```

3.13.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1481
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Simp[(e/2 + c*(d/(2*q)))   Int[1/(-q + c*x^2), x], x] + Simp[(
e/2 - c*(d/(2*q)))   Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] &
& NeQ[c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

3.13.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

method	result	si
default	$\frac{7\ln(x-1)}{4}-\frac{7\ln(x+1)}{4}-\frac{3\arctan(x)}{2}$	18
risch	$\frac{7\ln(x-1)}{4}-\frac{7\ln(x+1)}{4}-\frac{3\arctan(x)}{2}$	18
parallelrisch	$\frac{7\ln(x-1)}{4}+\frac{3i\ln(x-i)}{4}-\frac{3i\ln(x+i)}{4}-\frac{7\ln(x+1)}{4}$	30
meijerg	$\frac{5x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{x^3\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{2(x^4)^{\frac{3}{4}}}$	78

input
```
int((2*x^2+5)/(x^4-1),x,method=_RETURNVERBOSE)
```

output
```
7/4*ln(x-1)-7/4*ln(x+1)-3/2*arctan(x)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{5+2x^2}{-1+x^4}\,dx=-\frac{3}{2}\arctan(x)-\frac{7}{4}\log(x+1)+\frac{7}{4}\log(x-1)$$

input
```
integrate((2*x^2+5)/(x^4-1),x, algorithm="fricas")
```

output
```
-3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{5+2x^2}{-1+x^4}\,dx = \frac{7\log(x-1)}{4} - \frac{7\log(x+1)}{4} - \frac{3\,\mathrm{atan}(x)}{2}$$

input
```
integrate((2*x**2+5)/(x**4-1),x)
```

output
```
7*log(x - 1)/4 - 7*log(x + 1)/4 - 3*atan(x)/2
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\frac{3}{2}\arctan(x) - \frac{7}{4}\log(x+1) + \frac{7}{4}\log(x-1)$$

input
```
integrate((2*x^2+5)/(x^4-1),x, algorithm="maxima")
```

output
```
-3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\frac{3}{2}\arctan(x) - \frac{7}{4}\log(|x+1|) + \frac{7}{4}\log(|x-1|)$$

input
```
integrate((2*x^2+5)/(x^4-1),x, algorithm="giac")
```

output
```
-3/2*arctan(x) - 7/4*log(abs(x + 1)) + 7/4*log(abs(x - 1))
```

3.13.9 Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{5+2x^2}{-1+x^4}\,dx = -\frac{3\,\mathrm{atan}(x)}{2} - \frac{7\,\mathrm{atanh}(x)}{2}$$

input
```
int((2*x^2 + 5)/(x^4 - 1),x)
```

output
```
- (3*atan(x))/2 - (7*atanh(x))/2
```

3.14 $\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx$

3.14.1 Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

output
```
EllipticE(x*b^(1/2),I)/b^(1/2)
```

3.14.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right) + \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)$$

input
```
Integrate[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + (b*x^3*Hypergeometric2F1[1/2
, 3/4, 7/4, b^2*x^4])/3
```

3.14.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\sqrt{1-b^2x^4}}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

input
```
Int[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

output
```
EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]
```

3.14.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.14.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

method	result	size
meijerg	$\frac{b\,x^{3}{}_{2}F_{1}\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^{2}x^{4}\right)}{3}+x{}_{2}F_{1}\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^{2}x^{4}\right)$	36
default	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,F\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}-\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(F\left(\sqrt{b}\,x,i\right)-E\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}$	100
elliptic	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,F\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}-\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(F\left(\sqrt{b}\,x,i\right)-E\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}$	100

input
```
int((b*x^2+1)/(-b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x^3*hypergeom([1/2,3/4],[7/4],b^2*x^4)+x*hypergeom([1/4,1/2],[5/4],b
^2*x^4)
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(11) = 22.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.44

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx=\frac{\frac{\sqrt{-b^2}(b+1)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}}-\frac{\sqrt{-b^2}xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}}-\sqrt{-b^2x^4+1}b}{b^2x}$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b^2)*(b + 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - sqrt
(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - sqrt(-b^2*x^4 + 1
)*b)/(b^2*x)
```

3.14.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(12) = 24.

Time = 0.85 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.38

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

output
```
b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(
4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar
(2*I*pi))/(4*gamma(5/4))
```

3.14.7 Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

3.14.8 Giac [F]

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{-b^2x^4+1}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx = \int \frac{b\,x^2+1}{\sqrt{1-b^2\,x^4}}\,dx$$

input
```
int((b*x^2 + 1)/(1 - b^2*x^4)^(1/2),x)
```

output
```
int((b*x^2 + 1)/(1 - b^2*x^4)^(1/2), x)
```

3.15 $\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx$

3.15.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx=-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}+\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}$$

output
```
-EllipticE(x*b^(1/2),I)/b^(1/2)+2*EllipticF(x*b^(1/2),I)/b^(1/2)
```

3.15.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)-\frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)$$

input
```
Integrate[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] - (b*x^3*Hypergeometric2F1[1/2
, 3/4, 7/4, b^2*x^4])/3
```

3.15.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1388, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}dx$$
$$\Big\downarrow \text{1388}$$
$$\int \frac{\sqrt{1-bx^2}}{\sqrt{bx^2+1}}dx$$
$$\Big\downarrow \text{326}$$
$$2\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx-\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx$$
$$\Big\downarrow \text{284}$$
$$2\int \frac{1}{\sqrt{1-b^2x^4}}dx-\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx$$
$$\Big\downarrow \text{327}$$
$$2\int \frac{1}{\sqrt{1-b^2x^4}}dx-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$
$$\Big\downarrow \text{762}$$
$$\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}-\frac{E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}$$

input
```
Int[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

output
```
-(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*EllipticF[ArcSin[Sqrt[b]*
x], -1])/Sqrt[b]
```

3.15.3.1 Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.15.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
meijerg	$-\frac{b\,x^{3}{}_{2}F_{1}\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^{2}x^{4}\right)}{3}+x{}_{2}F_{1}\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^{2}x^{4}\right)$	36
default	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,F\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}+\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(F\left(\sqrt{b}\,x,i\right)-E\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}$	99
elliptic	$\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,F\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}+\frac{\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(F\left(\sqrt{b}\,x,i\right)-E\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}$	99

input
```
int((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b*x^3*hypergeom([1/2,3/4],[7/4],b^2*x^4)+x*hypergeom([1/4,1/2],[5/4],
b^2*x^4)
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(25) = 50$.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = \frac{\frac{\sqrt{-b^2}(b-1)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}} + \frac{\sqrt{-b^2}xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1\right)}{\sqrt{b}} + \sqrt{-b^2x^4+1}b}{b^2x}$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b^2)*(b - 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + sqrt
(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + sqrt(-b^2*x^4 + 1
)*b)/(b^2*x)
```

3.15.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(27) = 54$.

Time = 0.88 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.00

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/
(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_pola
r(2*I*pi))/(4*gamma(5/4))
```

3.15.7 Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{-b^2x^4+1}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

3.15.8 Giac [F]

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{-b^2x^4+1}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx = -\int \frac{b\,x^2-1}{\sqrt{1-b^2\,x^4}}\,dx$$

input
```
int(-(b*x^2 - 1)/(1 - b^2*x^4)^(1/2),x)
```

output
```
-int((b*x^2 - 1)/(1 - b^2*x^4)^(1/2), x)
```

3.16 $\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx$

3.16.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = \frac{\sqrt{1-b^2x^4}\,E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}$$

output
```
EllipticE(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

3.16.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = \frac{\sqrt{1-b^2x^4}\left(3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\right)}{3\sqrt{-1+b^2x^4}}$$

input
```
Integrate[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

output
```
(Sqrt[1 - b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b*x^3*
Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[-1 + b^2*x^4])
```

3.16.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1390, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\sqrt{b^2x^4-1}}dx$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{\sqrt{1-b^2x^4}\int \frac{bx^2+1}{\sqrt{1-b^2x^4}}dx}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{ 1388}$$

$$\frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{1-b^2x^4}E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{b^2x^4-1}}$$

input
```
Int[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

output
```
(Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^
2*x^4])
```

3.16.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

3.16.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.93 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.05

method	result	size
meijerg	$\frac{b\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)}{3\sqrt{\operatorname{signum}(b^2x^4-1)}}+\frac{\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)}{\sqrt{\operatorname{signum}(b^2x^4-1)}}$	88
default	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,F(x\sqrt{-b},i)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,(F(x\sqrt{-b},i)-E(x\sqrt{-b},i))}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	107
elliptic	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,F(x\sqrt{-b},i)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,(F(x\sqrt{-b},i)-E(x\sqrt{-b},i))}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	107

input
```
int((b*x^2+1)/(b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2)*x^3*hypergeom([1/
2,3/4],[7/4],b^2*x^4)+1/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2)
*x*hypergeom([1/4,1/2],[5/4],b^2*x^4)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx=-\frac{\frac{(b+1)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\mid -1\right)}{\sqrt{b}}-\frac{xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\mid -1\right)}{\sqrt{b}}-\sqrt{b^2x^4-1}}{bx}$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-((b + 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - x*elliptic_e(a
rcsin(1/(sqrt(b)*x)), -1)/sqrt(b) - sqrt(b^2*x^4 - 1))/(b*x)
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

output
```
-I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) -
 I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

3.16.7 Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{b^2x^4-1}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

3.16.8 Giac [F]

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{b^2x^4-1}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int \frac{b\,x^2+1}{\sqrt{b^2\,x^4-1}}\,dx$$

input
```
int((b*x^2 + 1)/(b^2*x^4 - 1)^(1/2),x)
```

output
```
int((b*x^2 + 1)/(b^2*x^4 - 1)^(1/2), x)
```

3.17 $\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx$

3.17.1 Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{\sqrt{1-b^2x^4}E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}} + \frac{2\sqrt{1-b^2x^4}\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}\sqrt{-1+b^2x^4}}$$

output
```
-EllipticE(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)+2*Ell
ipticF(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

3.17.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{\sqrt{1-b^2x^4}\left(-3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},b^2x^4\right)\right)}{3\sqrt{-1+b^2x^4}}$$

input
```
Integrate[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

output
```
-1/3*(Sqrt[1 - b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] +
b*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/Sqrt[-1 + b^2*x^4]
```

3.17.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1390, 1388, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\sqrt{b^2x^4-1}}dx$$

$$\Big\downarrow \text{1390}$$

$$\frac{\sqrt{1-b^2x^4}\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}dx}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{1388}$$

$$\frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1-bx^2}}{\sqrt{bx^2+1}}dx}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{326}$$

$$\frac{\sqrt{1-b^2x^4}\left(2\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx-\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{284}$$

$$\frac{\sqrt{1-b^2x^4}\left(2\int \frac{1}{\sqrt{1-b^2x^4}}dx-\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx\right)}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{1-b^2x^4}\left(2\int \frac{1}{\sqrt{1-b^2x^4}}dx-\frac{E\left(\arcsin\left(\sqrt{b}x\right)|-1\right)}{\sqrt{b}}\right)}{\sqrt{b^2x^4-1}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{1-b^2x^4}\left(\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}-\frac{E\left(\arcsin\left(\sqrt{b}x\right)|-1\right)}{\sqrt{b}}\right)}{\sqrt{b^2x^4-1}}$$

input
```
Int[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

output
```
(Sqrt[1 - b^2*x^4]*(-(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*Ellip
ticF[ArcSin[Sqrt[b]*x], -1])/Sqrt[b]))/Sqrt[-1 + b^2*x^4]
```

3.17.3.1 Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

3.17.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

method	result	size
meijerg	$-\frac{b\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)}{3\sqrt{\operatorname{signum}(b^2x^4-1)}}+\frac{\sqrt{-\operatorname{signum}(b^2x^4-1)}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)}{\sqrt{\operatorname{signum}(b^2x^4-1)}}$	88
default	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,F(x\sqrt{-b},i)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}-\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,(F(x\sqrt{-b},i)-E(x\sqrt{-b},i))}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	108
elliptic	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,F(x\sqrt{-b},i)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}-\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,(F(x\sqrt{-b},i)-E(x\sqrt{-b},i))}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$	108

input
```
int((-b*x^2+1)/(b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2)*x^3*hypergeom([1
/2,3/4],[7/4],b^2*x^4)+1/signum(b^2*x^4-1)^(1/2)*(-signum(b^2*x^4-1))^(1/2
)*x*hypergeom([1/4,1/2],[5/4],b^2*x^4)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = -\frac{\frac{(b-1)xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}}+\frac{xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\,|\,-1\right)}{\sqrt{b}}+\sqrt{b^2x^4-1}}{bx}$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-((b - 1)*x*elliptic_f(arcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + x*elliptic_e(a
rcsin(1/(sqrt(b)*x)), -1)/sqrt(b) + sqrt(b^2*x^4 - 1))/(b*x)
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) -
I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

3.17.7 Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4-1}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

3.17.8 Giac [F]

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4-1}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx=-\int \frac{b\,x^2-1}{\sqrt{b^2\,x^4-1}}\,dx$$

input
```
int(-(b*x^2 - 1)/(b^2*x^4 - 1)^(1/2),x)
```

output
```
-int((b*x^2 - 1)/(b^2*x^4 - 1)^(1/2), x)
```

3.18 $\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx$

3.18.1 Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx=-\frac{x\sqrt{1+b^2x^4}}{1+bx^2}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

output
```
-x*(b^2*x^4+1)^(1/2)/(b*x^2+1)+(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2
)/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))
*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x^4+1)^(1/2)
```

3.18.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)-\frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)$$

input
```
Integrate[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] - (b*x^3*Hypergeometric2F1[
1/2, 3/4, 7/4, -(b^2*x^4)])/3
```

3.18.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}\,dx$$

$$\Big\downarrow \ 1510$$

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

input
```
Int[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

output
```
-((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1
+ b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4
])
```

3.18.3.1 Defintions of rubi rules used

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.18.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.43

method	result	size
meijerg	$-\frac{b\,x^{3}{}_{2}F_{1}\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^{2}x^{4}\right)}{3}+x\,{}_{2}F_{1}\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^{2}x^{4}\right)$	38
default	$\frac{\sqrt{-ib\,x^{2}+1}\,\sqrt{ib\,x^{2}+1}\,F\left(x\sqrt{ib},i\right)}{\sqrt{ib}\,\sqrt{b^{2}x^{4}+1}}-\frac{i\sqrt{-ib\,x^{2}+1}\,\sqrt{ib\,x^{2}+1}\,\left(F\left(x\sqrt{ib},i\right)-E\left(x\sqrt{ib},i\right)\right)}{\sqrt{ib}\,\sqrt{b^{2}x^{4}+1}}$	120
elliptic	$\frac{\sqrt{-ib\,x^{2}+1}\,\sqrt{ib\,x^{2}+1}\,F\left(x\sqrt{ib},i\right)}{\sqrt{ib}\,\sqrt{b^{2}x^{4}+1}}-\frac{i\sqrt{-ib\,x^{2}+1}\,\sqrt{ib\,x^{2}+1}\,\left(F\left(x\sqrt{ib},i\right)-E\left(x\sqrt{ib},i\right)\right)}{\sqrt{ib}\,\sqrt{b^{2}x^{4}+1}}$	120

input
```
int((-b*x^2+1)/(b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b*x^3*hypergeom([1/2,3/4],[7/4],-b^2*x^4)+x*hypergeom([1/4,1/2],[5/4]
,-b^2*x^4)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = -\frac{bx\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-(b^2+b)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+\sqrt{b^2x^4+1}}{bx}$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-(b*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1) - (b^2 + b)*
x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x), -1) + sqrt(b^2*x^4 +
 1))/(b*x)
```

3.18.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = -\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

output
```
-b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4
*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(
I*pi))/(4*gamma(5/4))
```

3.18.7 Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4+1}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

3.18.8 Giac [F]

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{b^2x^4+1}}\,dx$$

input
```
integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx = -\int \frac{b\,x^2-1}{\sqrt{b^2\,x^4+1}}\,dx$$

input
```
int(-(b*x^2 - 1)/(b^2*x^4 + 1)^(1/2),x)
```

output
```
-int((b*x^2 - 1)/(b^2*x^4 + 1)^(1/2), x)
```

3.19 $\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx$

3.19.1 Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \frac{x\sqrt{1+b^2x^4}}{1+bx^2} - \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{1+b^2x^4}}$$

output
```
x*(b^2*x^4+1)^(1/2)/(b*x^2+1)-(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2)
/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*
((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x^4+1)^(1/2)+(b*x^2+1)*(cos(2
*arctan(x*b^(1/2)))^2)^(1/2)/cos(2*arctan(x*b^(1/2)))*EllipticF(sin(2*arct
an(x*b^(1/2))),1/2*2^(1/2))*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x
^4+1)^(1/2)
```

3.19.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.31

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right) + \frac{1}{3}bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)$$

input
```
Integrate[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + (b*x^3*Hypergeometric2F1[
1/2, 3/4, 7/4, -(b^2*x^4)])/3
```

3.19.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\sqrt{b^2x^4+1}}dx$$

$$\Big\downarrow \ 1512$$

$$2\int \frac{1}{\sqrt{b^2x^4+1}}dx-\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx$$

$$\Big\downarrow \ 761$$

$$\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\int \frac{1-bx^2}{\sqrt{b^2x^4+1}}dx$$

$$\Big\downarrow \ 1510$$

$$\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}-\frac{\left(bx^2+1\right)\sqrt{\frac{b^2x^4+1}{\left(bx^2+1\right)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{b^2x^4+1}}+\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

input
```
Int[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

output
```
(x*Sqrt[1 + b^2*x^4])/(1 + b*x^2) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b
*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
+ ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b
]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

3.19.3.1 Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

3.19.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	size
meijerg	$\frac{b\,x^{3}{}_{2}F_{1}\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^{2}x^{4}\right)}{3}+x\,{}_{2}F_{1}\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^{2}x^{4}\right)$	38
default	$\frac{\sqrt{-ib\,x^{2}+1}\,\sqrt{ib\,x^{2}+1}\,F\left(x\sqrt{ib},i\right)}{\sqrt{ib}\,\sqrt{b^{2}x^{4}+1}}+\frac{i\sqrt{-ib\,x^{2}+1}\,\sqrt{ib\,x^{2}+1}\,\left(F\left(x\sqrt{ib},i\right)-E\left(x\sqrt{ib},i\right)\right)}{\sqrt{ib}\,\sqrt{b^{2}x^{4}+1}}$	120
elliptic	$\frac{\sqrt{-ib\,x^{2}+1}\,\sqrt{ib\,x^{2}+1}\,F\left(x\sqrt{ib},i\right)}{\sqrt{ib}\,\sqrt{b^{2}x^{4}+1}}+\frac{i\sqrt{-ib\,x^{2}+1}\,\sqrt{ib\,x^{2}+1}\,\left(F\left(x\sqrt{ib},i\right)-E\left(x\sqrt{ib},i\right)\right)}{\sqrt{ib}\,\sqrt{b^{2}x^{4}+1}}$	120

input
```
int((b*x^2+1)/(b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x^3*hypergeom([1/2,3/4],[7/4],-b^2*x^4)+x*hypergeom([1/4,1/2],[5/4],
-b^2*x^4)
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.48

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \frac{bx\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} E\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1\right) + (b^2-b)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right) \mid -1\right) + \sqrt{b^2x^4+1}}{bx}$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(b*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1) + (b^2 - b)*x
*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x), -1) + sqrt(b^2*x^4 +
1))/(b*x)
```

3.19.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \frac{bx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

output
```
b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I
*pi))/(4*gamma(5/4))
```

3.19.7 Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{b^2x^4+1}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

3.19.8 Giac [F]

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{b^2x^4+1}}\,dx$$

input
```
integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = \int \frac{b\,x^2+1}{\sqrt{b^2\,x^4+1}}\,dx$$

input
```
int((b*x^2 + 1)/(b^2*x^4 + 1)^(1/2),x)
```

output
```
int((b*x^2 + 1)/(b^2*x^4 + 1)^(1/2), x)
```

3.20 $\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx$

3.20.1 Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx=\frac{x\sqrt{-1-b^2x^4}}{1+bx^2}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

output
```
x*(-b^2*x^4-1)^(1/2)/(b*x^2+1)+(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2
)/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))
*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

3.20.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = -\frac{\sqrt{1+b^2x^4}\left(-3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)\right)}{3\sqrt{-1-b^2x^4}}$$

input
```
Integrate[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

output
```
-1/3*(Sqrt[1 + b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)]
 + b*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/Sqrt[-1 - b^2*x^4]
```

3.20.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}\,dx$$

$$\Big\downarrow \; 1510$$

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}+\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}$$

input
```
Int[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

output
```
(x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 +
b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4]
)
```

3.20.3.1 Defintions of rubi rules used

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.20.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

method	result	size
meijerg	$-\frac{b\sqrt{\operatorname{signum}(b^2x^4+1)}\,x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)}{3\sqrt{-\operatorname{signum}(b^2x^4+1)}}+\frac{\sqrt{\operatorname{signum}(b^2x^4+1)}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)}{\sqrt{-\operatorname{signum}(b^2x^4+1)}}$	90
default	$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,F\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(F\left(x\sqrt{-ib},i\right)-E\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$	122
elliptic	$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,F\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(F\left(x\sqrt{-ib},i\right)-E\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$	122

input
```
int((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/2)*x^3*hypergeom([1
/2,3/4],[7/4],-b^2*x^4)+1/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/
2)*x*hypergeom([1/4,1/2],[5/4],-b^2*x^4)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = -\frac{\sqrt{-b^2}(b+1)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-\sqrt{-b^2}x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-\sqrt{-b^2x^4}}{bx}$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b^2)*(b + 1)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x),
 -1) - sqrt(-b^2)*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1
) - sqrt(-b^2*x^4 - 1))/(b*x)
```

3.20.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = \frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

output
```
I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(
4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_pol
ar(I*pi))/(4*gamma(5/4))
```

3.20.7 Maxima [F]

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

3.20.8 Giac [F]

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = \int -\frac{bx^2-1}{\sqrt{-b^2x^4-1}}\,dx$$

input
```
integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx = -\int \frac{b\,x^2-1}{\sqrt{-b^2\,x^4-1}}\,dx$$

input
```
int(-(b*x^2 - 1)/(- b^2*x^4 - 1)^(1/2),x)
```

output
```
-int((b*x^2 - 1)/(- b^2*x^4 - 1)^(1/2), x)
```

3.21 $\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx$

3.21.1 Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = -\frac{x\sqrt{-1-b^2x^4}}{1+bx^2} - \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}}$$

output
```
-x*(-b^2*x^4-1)^(1/2)/(b*x^2+1)-(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/
2)/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2)
)*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b^2*x^4-1)^(1/2)+(b*x^2+1)*(co
s(2*arctan(x*b^(1/2)))^2)^(1/2)/cos(2*arctan(x*b^(1/2)))*EllipticF(sin(2*a
rctan(x*b^(1/2))),1/2*2^(1/2))*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b
^2*x^4-1)^(1/2)
```

3.21.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.49

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = \frac{\sqrt{1+b^2x^4}\left(3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-b^2x^4\right)+bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-b^2x^4\right)\right)}{3\sqrt{-1-b^2x^4}}$$

input
```
Integrate[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

output
```
(Sqrt[1 + b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + b*x
^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/(3*Sqrt[-1 - b^2*x^4])
```

3.21.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1512, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}dx$$

$$\Big\downarrow \text{1512}$$

$$2\int \frac{1}{\sqrt{-b^2x^4-1}}dx-\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-\int \frac{1-bx^2}{\sqrt{-b^2x^4-1}}dx$$

$$\Big\downarrow \text{1510}$$

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{b}x\right),\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}E\left(2\arctan\left(\sqrt{b}x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-b^2x^4-1}}-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}$$

input
```
Int[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

output
```
-((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2)) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1
 + b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x
^4]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[S
qrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```

3.21.3.1 Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

3.21.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.58

method	result	size
meijerg	$\frac{b\sqrt{\operatorname{signum}(b^2x^4+1)}\, x^3\, {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)}{3\sqrt{-\operatorname{signum}(b^2x^4+1)}}+\frac{\sqrt{\operatorname{signum}(b^2x^4+1)}\, x\, {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)}{\sqrt{-\operatorname{signum}(b^2x^4+1)}}$	90
default	$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,F\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}-\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(F\left(x\sqrt{-ib},i\right)-E\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$	122
elliptic	$\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,F\left(x\sqrt{-ib},i\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}-\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(F\left(x\sqrt{-ib},i\right)-E\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}$	122

input
```
int((b*x^2+1)/(-b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/2)*x^3*hypergeom([1/
2,3/4],[7/4],-b^2*x^4)+1/(-signum(b^2*x^4+1))^(1/2)*signum(b^2*x^4+1)^(1/2
)*x*hypergeom([1/4,1/2],[5/4],-b^2*x^4)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.54

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = -\frac{\sqrt{-b^2}(b-1)x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1\right) + \sqrt{-b^2}x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}} E\left(\arcsin\left(\frac{\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}}{x}\right)|-1\right) + \sqrt{-b^2x^4}}{bx}$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b^2)*(b - 1)*x*(-1/b^2)^(3/4)*elliptic_f(arcsin((-1/b^2)^(1/4)/x),
 -1) + sqrt(-b^2)*x*(-1/b^2)^(3/4)*elliptic_e(arcsin((-1/b^2)^(1/4)/x), -1
) + sqrt(-b^2*x^4 - 1))/(b*x)
```

3.21.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = -\frac{ibx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

output
```
-I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/
(4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_po
lar(I*pi))/(4*gamma(5/4))
```

3.21.7 Maxima [F]

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

3.21.8 Giac [F]

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = \int \frac{bx^2+1}{\sqrt{-b^2x^4-1}}\,dx$$

input
```
integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = \int \frac{b\,x^2+1}{\sqrt{-b^2\,x^4-1}}\,dx$$

input
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(1/2),x)
```

output
```
int((b*x^2 + 1)/(- b^2*x^4 - 1)^(1/2), x)
```

3.22 $\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx$

3.22.1 Optimal result

Integrand size = 28, antiderivative size = 10

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \frac{E(\arcsin(cx)|-1)}{c}$$

output
```
EllipticE(c*x,I)/c
```

3.22.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \frac{E(\arcsin(cx)|-1)}{c}$$

input
```
Integrate[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]
```

output
```
EllipticE[ArcSin[c*x], -1]/c
```

3.22.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}\,dx$$

$$\Big\downarrow \; 327$$

$$\frac{E(\arcsin(cx)|-1)}{c}$$

input
```
Int[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]
```

output
```
EllipticE[ArcSin[c*x], -1]/c
```

3.22.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.22.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.98 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{E(x\ \mathrm{csgn}(c)c,i)\,\mathrm{csgn}(c)}{c}$	15
elliptic	$\frac{\sqrt{-c^4x^4+1}\left(\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,F\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}-\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(F\left(x\sqrt{c^2},i\right)-E\left(x\sqrt{c^2},i\right)\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}\right)}{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}}$	154

input
```
int((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticE(x*csgn(c)*c,I)*csgn(c)/c
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(9) = 18$.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 7.60

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = -\frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}c^3-\sqrt{-c^4}\left((c^2+1)xF\left(\arcsin\left(\frac{1}{cx}\right)|-1\right)-xE\left(\arcsin\left(\frac{1}{cx}\right)|-1\right)\right)}{c^5x}$$

input
```
integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)*c^3 - sqrt(-c^4)*((c^2 + 1)*x*ellip
tic_f(arcsin(1/(c*x)), -1) - x*elliptic_e(arcsin(1/(c*x)), -1)))/(c^5*x)
```

3.22.6 Sympy [F]

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

input
```
integrate((c**2*x**2+1)**(1/2)/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(c**2*x**2 + 1)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.22.7 Maxima [F]

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

3.22.8 Giac [F]

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\sqrt{c^2x^2+1}}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\sqrt{c^2\,x^2+1}}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int((c^2*x^2 + 1)^(1/2)/(1 - c^2*x^2)^(1/2),x)
```

output
```
int((c^2*x^2 + 1)^(1/2)/(1 - c^2*x^2)^(1/2), x)
```

3.23 $\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx$

3.23.1 Optimal result

Integrand size = 24, antiderivative size = 10

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx=\frac{E(\arcsin(cx)|-1)}{c}$$

output
```
EllipticE(c*x,I)/c
```

3.23.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 4.70

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^4x^4\right)+\frac{1}{3}c^2x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^4x^4\right)$$

input
```
Integrate[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] + (c^2*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, c^4*x^4])/3
```

3.23.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c^2x^2+1}{\sqrt{1-c^4x^4}}\,dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$\frac{E(\arcsin(cx)|-1)}{c}$$

input
```
Int[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

output
```
EllipticE[ArcSin[c*x], -1]/c
```

3.23.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.23.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.80

method	result	size
meijerg	$\frac{c^{2} x^{3} {}_{2}F_{1}\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^{4} x^{4}\right)}{3}+x {}_{2}F_{1}\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^{4} x^{4}\right)$	38
default	$\frac{\sqrt{-c^{2} x^{2}+1}\, \sqrt{c^{2} x^{2}+1}\, F\left(x \sqrt{c^{2}}, i\right)}{\sqrt{c^{2}}\, \sqrt{-c^{4} x^{4}+1}}-\frac{\sqrt{-c^{2} x^{2}+1}\, \sqrt{c^{2} x^{2}+1}\, \left(F\left(x \sqrt{c^{2}}, i\right)-E\left(x \sqrt{c^{2}}, i\right)\right)}{\sqrt{c^{2}}\, \sqrt{-c^{4} x^{4}+1}}$	118
elliptic	$\frac{\sqrt{-c^{2} x^{2}+1}\, \sqrt{c^{2} x^{2}+1}\, F\left(x \sqrt{c^{2}}, i\right)}{\sqrt{c^{2}}\, \sqrt{-c^{4} x^{4}+1}}-\frac{\sqrt{-c^{2} x^{2}+1}\, \sqrt{c^{2} x^{2}+1}\, \left(F\left(x \sqrt{c^{2}}, i\right)-E\left(x \sqrt{c^{2}}, i\right)\right)}{\sqrt{c^{2}}\, \sqrt{-c^{4} x^{4}+1}}$	118

input
```
int((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*c^2*x^3*hypergeom([1/2,3/4],[7/4],c^4*x^4)+x*hypergeom([1/4,1/2],[5/4]
,c^4*x^4)
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(9) = 18$.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 6.50

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{-c^4x^4+1}c^3-\sqrt{-c^4}\left((c^2+1)xF\left(\arcsin\left(\frac{1}{cx}\right)\mid -1\right)-xE\left(\arcsin\left(\frac{1}{cx}\right)\mid -1\right)\right)}{c^5x}$$

input
```
integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-c^4*x^4 + 1)*c^3 - sqrt(-c^4)*((c^2 + 1)*x*elliptic_f(arcsin(1/(c*
x)), -1) - x*elliptic_e(arcsin(1/(c*x)), -1)))/(c^5*x)
```

3.23.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(5) = 10.

Time = 0.90 (sec) , antiderivative size = 71, normalized size of antiderivative = 7.10

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \frac{c^2x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

output
```
c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi)
)/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_po
lar(2*I*pi))/(4*gamma(5/4))
```

3.23.7 Maxima [F]

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \int \frac{c^2x^2+1}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

3.23.8 Giac [F]

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \int \frac{c^2x^2+1}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \int \frac{c^2\,x^2+1}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((c^2*x^2 + 1)/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((c^2*x^2 + 1)/(1 - c^4*x^4)^(1/2), x)
```

3.24 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$

3.24.1 Optimal result

Integrand size = 28, antiderivative size = 23

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = -\frac{E(\arcsin(cx)|-1)}{c} + \frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c}$$

output
```
-EllipticE(c*x,I)/c+2*EllipticF(c*x,I)/c
```

3.24.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{-c^2}x\right)\middle|-1\right)}{\sqrt{-c^2}}$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

output
```
EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]
```

3.24.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{326}$$

$$2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}dx-\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}dx$$

$$\Big\downarrow \text{284}$$

$$2\int \frac{1}{\sqrt{1-c^4x^4}}dx-\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$2\int \frac{1}{\sqrt{1-c^4x^4}}dx-\frac{E(\arcsin(cx)|-1)}{c}$$

$$\Big\downarrow \text{762}$$

$$\frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c}-\frac{E(\arcsin(cx)|-1)}{c}$$

input
```
Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

output
```
-(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

3.24.3.1 Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

3.24.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{(2F(x\ \mathrm{csgn}(c)c,i)-E(x\ \mathrm{csgn}(c)c,i))\,\mathrm{csgn}(c)}{c}$	28
elliptic	$\frac{\sqrt{-c^4x^4+1}\left(\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,F\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}+\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(F\left(x\sqrt{c^2},i\right)-E\left(x\sqrt{c^2},i\right)\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}\right)}{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}}$	153

input
```
int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2*EllipticF(x*csgn(c)*c,I)-EllipticE(x*csgn(c)*c,I))*csgn(c)/c
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$$
$$= \frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}c^3 + \sqrt{-c^4}\left((c^2-1)xF\left(\arcsin\left(\frac{1}{cx}\right)|-1\right) + xE\left(\arcsin\left(\frac{1}{cx}\right)|-1\right)\right)}{c^5x}$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)*c^3 + sqrt(-c^4)*((c^2 - 1)*x*ellipt
ic_f(arcsin(1/(c*x)), -1) + x*elliptic_e(arcsin(1/(c*x)), -1)))/(c^5*x)
```

3.24.6 Sympy [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)
```

3.24.7 Maxima [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

3.24.8 Giac [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{\sqrt{c^2\,x^2+1}}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2), x)
```

3.25 $\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx$

3.25.1 Optimal result

Integrand size = 25, antiderivative size = 23

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx=-\frac{E(\arcsin(cx)|-1)}{c}+\frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c}$$

output
```
-EllipticE(c*x,I)/c+2*EllipticF(c*x,I)/c
```

3.25.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx=x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^4x^4\right)-\frac{1}{3}c^2x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^4x^4\right)$$

input
```
Integrate[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

output
```
x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] - (c^2*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, c^4*x^4])/3
```

3.25.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1388, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{c^2x^2+1}}dx$$

$$\Big\downarrow \text{ 326}$$

$$2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}dx-\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}dx$$

$$\Big\downarrow \text{ 284}$$

$$2\int \frac{1}{\sqrt{1-c^4x^4}}dx-\int \frac{\sqrt{c^2x^2+1}}{\sqrt{1-c^2x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$2\int \frac{1}{\sqrt{1-c^4x^4}}dx-\frac{E(\arcsin(cx)|-1)}{c}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c}-\frac{E(\arcsin(cx)|-1)}{c}$$

input
```
Int[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

output
```
-(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

3.25.3.1 Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.25.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

method	result	size
meijerg	$-\frac{c^2x^3{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)}{3}+x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^4x^4\right)$	38
default	$\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,F\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}+\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(F\left(x\sqrt{c^2},i\right)-E\left(x\sqrt{c^2},i\right)\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}$	117
elliptic	$\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,F\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}+\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(F\left(x\sqrt{c^2},i\right)-E\left(x\sqrt{c^2},i\right)\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}$	117

input
```
int((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c^2*x^3*hypergeom([1/2,3/4],[7/4],c^4*x^4)+x*hypergeom([1/4,1/2],[5/4
],c^4*x^4)
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.70

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx = \frac{\sqrt{-c^4x^4+1}c^3+\sqrt{-c^4}\left((c^2-1)xF\left(\arcsin\left(\frac{1}{cx}\right)|-1\right)+xE\left(\arcsin\left(\frac{1}{cx}\right)|-1\right)\right)}{c^5x}$$

input
```
integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-c^4*x^4 + 1)*c^3 + sqrt(-c^4)*((c^2 - 1)*x*elliptic_f(arcsin(1/(c*x
)), -1) + x*elliptic_e(arcsin(1/(c*x)), -1)))/(c^5*x)
```

3.25.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(14) = 28$.

Time = 0.86 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx = -\frac{c^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

output
```
-c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi
))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_p
olar(2*I*pi))/(4*gamma(5/4))
```

3.25.7 Maxima [F]

$$\int \frac{1 - c^2x^2}{\sqrt{1 - c^4x^4}}\,dx = \int -\frac{c^2x^2 - 1}{\sqrt{-c^4x^4 + 1}}\,dx$$

input
```
integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

3.25.8 Giac [F]

$$\int \frac{1 - c^2x^2}{\sqrt{1 - c^4x^4}}\,dx = \int -\frac{c^2x^2 - 1}{\sqrt{-c^4x^4 + 1}}\,dx$$

input
```
integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1 - c^2x^2}{\sqrt{1 - c^4x^4}}\,dx = -\int \frac{c^2\,x^2 - 1}{\sqrt{1 - c^4\,x^4}}\,dx$$

input
```
int(-(c^2*x^2 - 1)/(1 - c^4*x^4)^(1/2),x)
```

output
```
-int((c^2*x^2 - 1)/(1 - c^4*x^4)^(1/2), x)
```

3.26 $\int \frac{d+ex^2}{d^2+bx^2+e^2x^4} \, dx$

3.26.1 Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4} \, dx = -\frac{\arctan\left(\frac{\sqrt{-b+2de}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}+\frac{\arctan\left(\frac{\sqrt{-b+2de}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

output
```
-arctan((-2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+arctan((
2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)
```

3.26.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(82) = 164.

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.21

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4} \, dx = \frac{\frac{\left(-b+2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}+\frac{\left(b-2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

input
```
Integrate[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

output
```
(((-b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[
b^2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] + ((b - 2*d*e + Sqrt[b
^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sq
rt[b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

3.26.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{bx^2+d^2+e^2x^4}dx$$
$$\Big\downarrow \text{ 1475}$$
$$\frac{\int \frac{1}{x^2-\frac{\sqrt{2de-b}x}{e}+\frac{d}{e}}dx}{2e}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2de-b}x}{e}+\frac{d}{e}}dx}{2e}$$
$$\Big\downarrow \text{ 1083}$$
$$-\frac{\int \frac{1}{-\left(2x-\frac{\sqrt{2de-b}}{e}\right)^2-\frac{b+2de}{e^2}}d\left(2x-\frac{\sqrt{2de-b}}{e}\right)}{e}-\frac{\int \frac{1}{-\left(2x+\frac{\sqrt{2de-b}}{e}\right)^2-\frac{b+2de}{e^2}}d\left(2x+\frac{\sqrt{2de-b}}{e}\right)}{e}$$
$$\Big\downarrow \text{ 217}$$
$$\frac{\arctan\left(\frac{e\left(2x-\frac{\sqrt{2de-b}}{e}\right)}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}+\frac{\arctan\left(\frac{e\left(\frac{\sqrt{2de-b}}{e}+2x\right)}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

input
```
Int[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

output
```
ArcTan[(e*(-(Sqrt[-b + 2*d*e]/e) + 2*x))/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]
+ ArcTan[(e*(Sqrt[-b + 2*d*e]/e + 2*x))/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]
```

3.26.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.26.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\arctan\left(\frac{-2ex+\sqrt{2ed-b}}{\sqrt{2ed+b}}\right)}{\sqrt{2ed+b}}+\frac{\arctan\left(\frac{2ex+\sqrt{2ed-b}}{\sqrt{2ed+b}}\right)}{\sqrt{2ed+b}}$	71
risch	$-\frac{\ln\left(-e\,x^{2}\sqrt{-2ed-b}+(2ed+b)x+d\sqrt{-2ed-b}\right)}{2\sqrt{-2ed-b}}+\frac{\ln\left(-e\,x^{2}\sqrt{-2ed-b}+(-2ed-b)x+d\sqrt{-2ed-b}\right)}{2\sqrt{-2ed-b}}$	104

input
```
int((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
-arctan((-2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+arctan((
2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.98

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx$$
$$= \left[-\frac{\sqrt{-2\,de-b}\log\left(\frac{e^2x^4-(4\,de+b)x^2+d^2-2\,(ex^3-dx)\sqrt{-2\,de-b}}{e^2x^4+bx^2+d^2}\right)}{2\,(2\,de+b)}, \frac{\sqrt{2\,de+b}\arctan\left(\frac{ex}{\sqrt{2\,de+b}}\right)+\sqrt{2\,de+b}\arctan\left(\right.}{2\,de+b}\right.$$

input
```
integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-2*d*e - b)*log((e^2*x^4 - (4*d*e + b)*x^2 + d^2 - 2*(e*x^3 - d
*x)*sqrt(-2*d*e - b))/(e^2*x^4 + b*x^2 + d^2))/(2*d*e + b), (sqrt(2*d*e +
b)*arctan(e*x/sqrt(2*d*e + b)) + sqrt(2*d*e + b)*arctan((e^2*x^3 + (d*e +
b)*x)*sqrt(2*d*e + b)/(2*d^2*e + b*d)))/(2*d*e + b)]
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.49

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx = -\frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b+2de}}-2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2} + \frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b+2de}}+2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

input
```
integrate((e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

output
```
-sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(-1/(b + 2*d*e)) - 2*d*e
*sqrt(-1/(b + 2*d*e)))/e)/2 + sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(b*
sqrt(-1/(b + 2*d*e)) + 2*d*e*sqrt(-1/(b + 2*d*e)))/e)/2
```

3.26.7 Maxima [F]

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx = \int \frac{ex^2+d}{e^2x^4+bx^2+d^2}\,dx$$

input
```
integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(e^2*x^4 + b*x^2 + d^2), x)
```

3.26.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(73) = 146$.

Time = 0.70 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.30

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx = \frac{(2\,d^2e^3+de^4-bde^2)\sqrt{2\,de+b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b+\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5+bde^4-b^2de^2} + \frac{(2\,d^2e^3+de^4-bde^2)\sqrt{2\,de+b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b-\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5+bde^4-b^2de^2}$$

input
```
integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")
```

output
```
(2*d^2*e^3 + d*e^4 - b*d*e^2)*sqrt(2*d*e + b)*arctan(2*sqrt(1/2)*x/sqrt((b
 + sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 + b*d*e^4 - b^2*d*
e^2) + (2*d^2*e^3 + d*e^4 - b*d*e^2)*sqrt(2*d*e + b)*arctan(2*sqrt(1/2)*x/
sqrt((b - sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 + b*d*e^4 -
 b^2*d*e^2)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx=\frac{\operatorname{atan}\left(\frac{e\,x}{\sqrt{b+2\,d\,e}}\right)+\operatorname{atan}\left(\frac{b^2\,x-\frac{x\,(b+2\,d\,e)^2}{2}+\frac{b\,x\,(b+2\,d\,e)}{2}+2\,b\,e^2\,x^3-e^2\,x^3\,(b+2\,d\,e)}{(b\,d-2\,d^2\,e)\,\sqrt{b+2\,d\,e}}\right)}{\sqrt{b+2\,d\,e}}$$

input
```
int((d + e*x^2)/(b*x^2 + d^2 + e^2*x^4),x)
```

output
```
(atan((e*x)/(b + 2*d*e)^(1/2)) + atan((b^2*x - (x*(b + 2*d*e)^2)/2 + (b*x*
(b + 2*d*e))/2 + 2*b*e^2*x^3 - e^2*x^3*(b + 2*d*e))/((b*d - 2*d^2*e)*(b +
2*d*e)^(1/2))))/(b + 2*d*e)^(1/2)
```

3.27 $\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx$

3.27.1 Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx=-\frac{\arctan\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}+\frac{\arctan\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

output
```
-arctan((-2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+arctan((
2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)
```

3.27.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(82) = 164.

Time = 0.07 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.21

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx = \frac{\frac{\left(2de-f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}}+\frac{\left(-2de+f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\sqrt{-4d^2e^2+f^2}}$$

input
```
Integrate[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

output
```
(((2*d*e - f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[
-4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e + f + Sqr
t[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]
])/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

3.27.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{d^2+e^2x^4+fx^2}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{\int \frac{1}{x^2-\frac{\sqrt{2de-f}x}{e}+\frac{d}{e}}dx}{2e}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2de-f}x}{e}+\frac{d}{e}}dx}{2e}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\int \frac{1}{-\left(2x-\frac{\sqrt{2de-f}}{e}\right)^2-\frac{2de+f}{e^2}}d\left(2x-\frac{\sqrt{2de-f}}{e}\right)}{e}-\frac{\int \frac{1}{-\left(2x+\frac{\sqrt{2de-f}}{e}\right)^2-\frac{2de+f}{e^2}}d\left(2x+\frac{\sqrt{2de-f}}{e}\right)}{e}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{e\left(2x-\frac{\sqrt{2de-f}}{e}\right)}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}+\frac{\arctan\left(\frac{e\left(\frac{\sqrt{2de-f}}{e}+2x\right)}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

input
```
Int[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

output
```
ArcTan[(e*(-(Sqrt[2*d*e - f]/e) + 2*x))/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f] +
 ArcTan[(e*(Sqrt[2*d*e - f]/e + 2*x))/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]
```

3.27.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.27.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\arctan\left(\frac{-2ex+\sqrt{2ed-f}}{\sqrt{2ed+f}}\right)}{\sqrt{2ed+f}}+\frac{\arctan\left(\frac{2ex+\sqrt{2ed-f}}{\sqrt{2ed+f}}\right)}{\sqrt{2ed+f}}$	71
risch	$-\frac{\ln\left(e\,x^{2}\sqrt{-2ed-f}+(-2ed-f)x-d\sqrt{-2ed-f}\right)}{2\sqrt{-2ed-f}}+\frac{\ln\left(e\,x^{2}\sqrt{-2ed-f}+(2ed+f)x-d\sqrt{-2ed-f}\right)}{2\sqrt{-2ed-f}}$	104

input
```
int((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
-arctan((-2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+arctan((
2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.98

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx$$
$$= \left[-\frac{\sqrt{-2\,de-f}\log\left(\frac{e^2x^4-(4\,de+f)x^2+d^2-2\,(ex^3-dx)\sqrt{-2\,de-f}}{e^2x^4+fx^2+d^2}\right)}{2\,(2\,de+f)},\ \frac{\sqrt{2\,de+f}\arctan\left(\frac{ex}{\sqrt{2\,de+f}}\right)+\sqrt{2\,de+f}\,\mathrm{arctan}}{2\,de+f}\right.$$

input
```
integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-2*d*e - f)*log((e^2*x^4 - (4*d*e + f)*x^2 + d^2 - 2*(e*x^3 - d
*x)*sqrt(-2*d*e - f))/(e^2*x^4 + f*x^2 + d^2))/(2*d*e + f), (sqrt(2*d*e +
f)*arctan(e*x/sqrt(2*d*e + f)) + sqrt(2*d*e + f)*arctan((e^2*x^3 + (d*e +
f)*x)*sqrt(2*d*e + f)/(2*d^2*e + d*f)))/(2*d*e + f)]
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.49

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx = -\frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{-\frac{1}{2de+f}}-f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2} + \frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{-\frac{1}{2de+f}}+f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

input
```
integrate((e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

output
```
-sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e + f)) - f
*sqrt(-1/(2*d*e + f)))/e)/2 + sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(2*
d*e*sqrt(-1/(2*d*e + f)) + f*sqrt(-1/(2*d*e + f)))/e)/2
```

3.27.7 Maxima [F]

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx=\int \frac{ex^2+d}{e^2x^4+fx^2+d^2}\,dx$$

input
```
integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(e^2*x^4 + f*x^2 + d^2), x)
```

3.27.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(73) = 146.

Time = 0.69 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.33

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx=-\frac{(2\,d^2e^3+de^4-de^2f)\sqrt{2\,de+f}\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{\frac{f+\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5+de^4f-de^2f^2}-\frac{(2\,d^2e^3+de^4-de^2f)\sqrt{2\,de+f}\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{\frac{f-\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5+de^4f-de^2f^2}$$

input
```
integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")
```

output
```
-(2*d^2*e^3 + d*e^4 - d*e^2*f)*sqrt(2*d*e + f)*arctan(2*sqrt(1/2)*x/sqrt((
f + sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 + d*e^4*f - d*e^2
*f^2) - (2*d^2*e^3 + d*e^4 - d*e^2*f)*sqrt(2*d*e + f)*arctan(2*sqrt(1/2)*x
/sqrt((f - sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 + d*e^4*f
- d*e^2*f^2)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx = \frac{\operatorname{atan}\left(\frac{f^2\,x-\frac{x\,{\left(f+2\,d\,e\right)}^2}{2}+\frac{f\,x\,\left(f+2\,d\,e\right)}{2}+2\,e^2\,f\,x^3-e^2\,x^3\,\left(f+2\,d\,e\right)}{\left(2\,d\,f-d\,\left(f+2\,d\,e\right)\right)\,\sqrt{f+2\,d\,e}}\right)+\operatorname{atan}\left(\frac{e\,x}{\sqrt{f+2\,d\,e}}\right)}{\sqrt{f+2\,d\,e}}$$

input
```
int((d + e*x^2)/(f*x^2 + d^2 + e^2*x^4),x)
```

output
```
(atan((f^2*x - (x*(f + 2*d*e)^2)/2 + (f*x*(f + 2*d*e))/2 + 2*e^2*f*x^3 - e
^2*x^3*(f + 2*d*e))/((2*d*f - d*(f + 2*d*e))*(f + 2*d*e)^(1/2))) + atan((e
*x)/(f + 2*d*e)^(1/2)))/(f + 2*d*e)^(1/2)
```

3.28 $\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx$

3.28.1 Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

output
```
arctanh((-2*e*x+(2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2)-arctan
h((2*e*x+(2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2)
```

3.28.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(78) = 156.

Time = 0.07 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.42

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{\frac{\left(b+2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}}+\frac{\left(-b-2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

input
```
Integrate[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

output
```
(((b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b - Sqrt[
b^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]] + ((-b - 2*d*e + Sqrt
[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]]])
/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

3.28.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1475, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{-bx^2+d^2+e^2x^4}\,dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{\int \frac{1}{x^2-\frac{\sqrt{b+2de}x}{e}+\frac{d}{e}}dx}{2e}+\frac{\int \frac{1}{x^2+\frac{\sqrt{b+2de}x}{e}+\frac{d}{e}}dx}{2e}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\int \frac{1}{\frac{b-2de}{e^2}-\left(2x-\frac{\sqrt{b+2de}}{e}\right)^2}d\left(2x-\frac{\sqrt{b+2de}}{e}\right)}{e}-\frac{\int \frac{1}{\frac{b-2de}{e^2}-\left(2x+\frac{\sqrt{b+2de}}{e}\right)^2}d\left(2x+\frac{\sqrt{b+2de}}{e}\right)}{e}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{e\left(2x-\frac{\sqrt{b+2de}}{e}\right)}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\text{arctanh}\left(\frac{e\left(\frac{\sqrt{b+2de}}{e}+2x\right)}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

input
```
Int[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

output
```
-(ArcTanh[(e*(-(Sqrt[b + 2*d*e]/e) + 2*x))/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e
]) - ArcTanh[(e*(Sqrt[b + 2*d*e]/e + 2*x))/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e
]
```

3.28.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.28.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\arctan\left(\frac{2ex+\sqrt{2ed+b}}{\sqrt{2ed-b}}\right)}{\sqrt{2ed-b}}-\frac{\arctan\left(\frac{-2ex+\sqrt{2ed+b}}{\sqrt{2ed-b}}\right)}{\sqrt{2ed-b}}$	75
risch	$\frac{\ln\left(e\,x^2\sqrt{-2ed+b}+(2ed-b)x-d\sqrt{-2ed+b}\right)}{2\sqrt{-2ed+b}}-\frac{\ln\left(e\,x^2\sqrt{-2ed+b}+(-2ed+b)x-d\sqrt{-2ed+b}\right)}{2\sqrt{-2ed+b}}$	92

input
```
int((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/(2*d*e-b)^(1/2)*arctan((2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))-1/(2*d*e
-b)^(1/2)*arctan((-2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.26

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx = \left[-\frac{\sqrt{-2\,de+b}\log\left(\frac{e^2x^4-(4\,de-b)x^2+d^2-2\,(ex^3-dx)\sqrt{-2\,de+b}}{e^2x^4-bx^2+d^2}\right)}{2\,(2\,de-b)}, \frac{\sqrt{2\,de-b}\arctan\left(\frac{ex}{\sqrt{2\,de-b}}\right)+\sqrt{2\,de-b}\arctan\left(\right.}{2\,de-b}\right.$$

input
```
integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-2*d*e + b)*log((e^2*x^4 - (4*d*e - b)*x^2 + d^2 - 2*(e*x^3 - d
*x)*sqrt(-2*d*e + b))/(e^2*x^4 - b*x^2 + d^2))/(2*d*e - b), (sqrt(2*d*e -
b)*arctan(e*x/sqrt(2*d*e - b)) + sqrt(2*d*e - b)*arctan((e^2*x^3 + (d*e -
b)*x)*sqrt(2*d*e - b)/(2*d^2*e - b*d)))/(2*d*e - b)]
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx = \frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{\frac{1}{b-2de}}+2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2} - \frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{\frac{1}{b-2de}}-2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

input
```
integrate((e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

output
```
sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(1/(b - 2*d*e)) + 2*d*e*sq
rt(1/(b - 2*d*e)))/e)/2 - sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(b*sqrt(
1/(b - 2*d*e)) - 2*d*e*sqrt(1/(b - 2*d*e)))/e)/2
```

3.28.7 Maxima [F]

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx=\int \frac{ex^2+d}{e^2x^4-bx^2+d^2}\,dx$$

input
```
integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(e^2*x^4 - b*x^2 + d^2), x)
```

3.28.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(69) = 138.

Time = 0.71 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.50

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{(2\,d^2e^3+de^4+bde^2)\sqrt{2\,de-b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{b+\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5-bde^4-b^2de^2}+\frac{(2\,d^2e^3+de^4+bde^2)\sqrt{2\,de-b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{b-\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5-bde^4-b^2de^2}$$

input
```
integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")
```

output
```
(2*d^2*e^3 + d*e^4 + b*d*e^2)*sqrt(2*d*e - b)*arctan(2*sqrt(1/2)*x/sqrt(-(
b + sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 - b*d*e^4 - b^2*d
*e^2) + (2*d^2*e^3 + d*e^4 + b*d*e^2)*sqrt(2*d*e - b)*arctan(2*sqrt(1/2)*x
/sqrt(-(b - sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 - b*d*e^4
 - b^2*d*e^2)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.38

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{b-2\,d\,e}}{d-e\,x^2}\right)}{\sqrt{b-2\,d\,e}}$$

input
```
int((d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x)
```

output
```
atanh((x*(b - 2*d*e)^(1/2))/(d - e*x^2))/(b - 2*d*e)^(1/2)
```

3.29 $\int \frac{d+ex^2}{d^2-fx^2+e^2x^4} \, dx$

3.29.1 Optimal result

Integrand size = 27, antiderivative size = 86

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4} \, dx = -\frac{\arctan\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}} + \frac{\arctan\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

output
```
-arctan((-2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+arctan((
2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
```

3.29.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(86) = 172.

Time = 0.07 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.20

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4} \, dx = \frac{\frac{\left(2de+f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}} + \frac{\left(-2de-f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\sqrt{-4d^2e^2+f^2}}$$

input
```
Integrate[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

output
```
(((2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f - Sqrt
[-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e - f + S
qrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^
2]]])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

3.29.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{d^2+e^2x^4-fx^2}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{\int \frac{1}{x^2-\frac{\sqrt{2de+f}x}{e}+\frac{d}{e}}dx}{2e}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2de+f}x}{e}+\frac{d}{e}}dx}{2e}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\int \frac{1}{-\left(2x-\frac{\sqrt{2de+f}}{e}\right)^2-\frac{2de-f}{e^2}}d\left(2x-\frac{\sqrt{2de+f}}{e}\right)}{e}-\frac{\int \frac{1}{-\left(2x+\frac{\sqrt{2de+f}}{e}\right)^2-\frac{2de-f}{e^2}}d\left(2x+\frac{\sqrt{2de+f}}{e}\right)}{e}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{e\left(2x-\frac{\sqrt{2de+f}}{e}\right)}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}+\frac{\arctan\left(\frac{e\left(\frac{\sqrt{2de+f}}{e}+2x\right)}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

input
```
Int[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

output
```
ArcTan[(e*(-(Sqrt[2*d*e + f]/e) + 2*x))/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f] +
 ArcTan[(e*(Sqrt[2*d*e + f]/e + 2*x))/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]
```

3.29.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.29.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\arctan\left(\frac{-2ex+\sqrt{2ed+f}}{\sqrt{2ed-f}}\right)}{\sqrt{2ed-f}}+\frac{\arctan\left(\frac{2ex+\sqrt{2ed+f}}{\sqrt{2ed-f}}\right)}{\sqrt{2ed-f}}$	75
risch	$-\frac{\ln\left(e\,x^{2}\sqrt{-2ed+f}+(-2ed+f)x-d\sqrt{-2ed+f}\right)}{2\sqrt{-2ed+f}}+\frac{\ln\left(e\,x^{2}\sqrt{-2ed+f}+(2ed-f)x-d\sqrt{-2ed+f}\right)}{2\sqrt{-2ed+f}}$	92

input
```
int((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
-arctan((-2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+arctan((
2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.08

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx = \left[-\frac{\sqrt{-2\,de+f}\log\left(\frac{e^2x^4-(4\,de-f)x^2+d^2-2\,(ex^3-dx)\sqrt{-2\,de+f}}{e^2x^4-fx^2+d^2}\right)}{2\,(2\,de-f)},\right.$$
$$\left.-\frac{\sqrt{2\,de-f}\arctan\left(-\frac{ex}{\sqrt{2\,de-f}}\right)+\sqrt{2\,de-f}\arctan\left(-\frac{(e^2x^3+(de-f)x)\sqrt{2\,de-f}}{2\,d^2e-df}\right)}{2\,de-f}\right]$$

input
```
integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-2*d*e + f)*log((e^2*x^4 - (4*d*e - f)*x^2 + d^2 - 2*(e*x^3 - d
*x)*sqrt(-2*d*e + f))/(e^2*x^4 - f*x^2 + d^2))/(2*d*e - f), -(sqrt(2*d*e -
 f)*arctan(-e*x/sqrt(2*d*e - f)) + sqrt(2*d*e - f)*arctan(-(e^2*x^3 + (d*e
 - f)*x)*sqrt(2*d*e - f)/(2*d^2*e - d*f)))/(2*d*e - f)]
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.41

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx = -\frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{-\frac{1}{2de-f}}+f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}$$
$$+\frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{-\frac{1}{2de-f}}-f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

input
```
integrate((e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

output
```
-sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e - f)) + f
*sqrt(-1/(2*d*e - f)))/e)/2 + sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(2*
d*e*sqrt(-1/(2*d*e - f)) - f*sqrt(-1/(2*d*e - f)))/e)/2
```

3.29.7 Maxima [F]

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx = \int \frac{ex^2+d}{e^2x^4-fx^2+d^2}\,dx$$

input
```
integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(e^2*x^4 - f*x^2 + d^2), x)
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(77) = 154.

Time = 0.71 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.29

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx = -\frac{(2\,d^2e^3+de^4+de^2f)\sqrt{2\,de-f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{f+\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5-de^4f-de^2f^2} - \frac{(2\,d^2e^3+de^4+de^2f)\sqrt{2\,de-f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{f-\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4+2\,d^2e^5-de^4f-de^2f^2}$$

input
```
integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")
```

output
```
-(2*d^2*e^3 + d*e^4 + d*e^2*f)*sqrt(2*d*e - f)*arctan(2*sqrt(1/2)*x/sqrt(-
(f + sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 - d*e^4*f - d*e^
2*f^2) - (2*d^2*e^3 + d*e^4 + d*e^2*f)*sqrt(2*d*e - f)*arctan(2*sqrt(1/2)*
x/sqrt(-(f - sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 + 2*d^2*e^5 - d*e^4*
f - d*e^2*f^2)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx = -\frac{\operatorname{atan}\left(\frac{e^2\,x^3\,\sqrt{2\,d\,e-f}-f\,x\,\sqrt{2\,d\,e-f}+d\,e\,x\,\sqrt{2\,d\,e-f}}{d\,(f-2\,d\,e)}\right)-\operatorname{atan}\left(\frac{e\,x}{\sqrt{2\,d\,e-f}}\right)}{\sqrt{2\,d\,e-f}}$$

input
```
int((d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x)
```

output
```
-(atan((e^2*x^3*(2*d*e - f)^(1/2) - f*x*(2*d*e - f)^(1/2) + d*e*x*(2*d*e -
 f)^(1/2))/(d*(f - 2*d*e))) - atan((e*x)/(2*d*e - f)^(1/2)))/(2*d*e - f)^(
1/2)
```

3.30 $\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx$

3.30.1 Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx=-\frac{\log\left(d-\sqrt{-b+2de}x+ex^2\right)}{2\sqrt{-b+2de}}+\frac{\log\left(d+\sqrt{-b+2de}x+ex^2\right)}{2\sqrt{-b+2de}}$$

output
```
-1/2*ln(d+e*x^2-x*(2*d*e-b)^(1/2))/(2*d*e-b)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e
-b)^(1/2))/(2*d*e-b)^(1/2)
```

3.30.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(78) = 156.

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.33

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx=\frac{\frac{\left(b+2de-\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}-\frac{\left(b+2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

input
```
Integrate[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

output
```
(((b + 2*d*e - Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b
^2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] - ((b + 2*d*e + Sqrt[b^
2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqr
t[b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

3.30.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1478, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{bx^2+d^2+e^2x^4}dx$$

$$\Big\downarrow \text{1478}$$

$$-\frac{\int -\frac{\sqrt{2de-b}-2ex}{e\left(x^2-\frac{\sqrt{2de-b}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de-b}}-\frac{\int -\frac{2ex+\sqrt{2de-b}}{e\left(x^2+\frac{\sqrt{2de-b}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de-b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{2de-b}-2ex}{e\left(x^2-\frac{\sqrt{2de-b}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de-b}}+\frac{\int \frac{2ex+\sqrt{2de-b}}{e\left(x^2+\frac{\sqrt{2de-b}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de-b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{2de-b}-2ex}{x^2-\frac{\sqrt{2de-b}x}{e}+\frac{d}{e}}dx}{2e\sqrt{2de-b}}+\frac{\int \frac{2ex+\sqrt{2de-b}}{x^2+\frac{\sqrt{2de-b}x}{e}+\frac{d}{e}}dx}{2e\sqrt{2de-b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}-\frac{\log\left(-x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}$$

input
```
Int[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

output
```
-1/2*Log[d - Sqrt[-b + 2*d*e]*x + e*x^2]/Sqrt[-b + 2*d*e] + Log[d + Sqrt[-
b + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e])
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.30.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\sqrt{2ed-b}\,\ln\left(-e\,x^{2}+x\sqrt{2ed-b}-d\right)}{-4ed+2b}-\frac{\sqrt{2ed-b}\,\ln\left(d+e\,x^{2}+x\sqrt{2ed-b}\right)}{-4ed+2b}$	88
risch	$-\frac{\ln\left(-e\,x^{2}\sqrt{2ed-b}+(2ed-b)x-d\sqrt{2ed-b}\right)}{2\sqrt{2ed-b}}+\frac{\ln\left(-e\,x^{2}\sqrt{2ed-b}+(-2ed+b)x-d\sqrt{2ed-b}\right)}{2\sqrt{2ed-b}}$	106

input
```
int((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/(-4*d*e+2*b)*(2*d*e-b)^(1/2)*ln(-e*x^2+x*(2*d*e-b)^(1/2)-d)-1/(-4*d*e+2*
b)*(2*d*e-b)^(1/2)*ln(d+e*x^2+x*(2*d*e-b)^(1/2))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.21

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx = \left[\frac{\log\left(\frac{e^2x^4+(4\,de-b)x^2+d^2+2\,(ex^3+dx)\sqrt{2\,de-b}}{e^2x^4+bx^2+d^2}\right)}{2\sqrt{2\,de-b}}, -\frac{\sqrt{-2\,de+b}\arctan\left(\frac{\sqrt{-2\,de+b}ex}{2\,de-b}\right)-\sqrt{-2\,de+b}\arctan\left(\frac{(e^2x^3-(de-b)x)\sqrt{-2\,de+b}}{2\,d^2e-bd}\right)}{2\,de-b}\right]$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")
```

output
```
[1/2*log((e^2*x^4 + (4*d*e - b)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - b
))/(e^2*x^4 + b*x^2 + d^2))/sqrt(2*d*e - b), -(sqrt(-2*d*e + b)*arctan(sqr
t(-2*d*e + b)*e*x/(2*d*e - b)) - sqrt(-2*d*e + b)*arctan((e^2*x^3 - (d*e -
 b)*x)*sqrt(-2*d*e + b)/(2*d^2*e - b*d)))/(2*d*e - b)]
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.55

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx = \frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b-2de}}+2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2} - \frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b-2de}}-2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

input
```
integrate((-e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

output
```
sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(-1/(b - 2*d*e)) + 2*d*e*s
qrt(-1/(b - 2*d*e)))/e)/2 - sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(b*sqr
t(-1/(b - 2*d*e)) - 2*d*e*sqrt(-1/(b - 2*d*e)))/e)/2
```

3.30.7 Maxima [F]

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx = \int -\frac{ex^2-d}{e^2x^4+bx^2+d^2}\,dx$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)/(e^2*x^4 + b*x^2 + d^2), x)
```

3.30.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(66) = 132.

Time = 0.68 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.42

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx = \frac{(2\,d^2e^3-de^4+bde^2)\sqrt{-2\,de+b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b+\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5+bde^4-b^2de^2} + \frac{(2\,d^2e^3-de^4+bde^2)\sqrt{-2\,de+b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b-\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5+bde^4-b^2de^2}$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")
```

output
```
(2*d^2*e^3 - d*e^4 + b*d*e^2)*sqrt(-2*d*e + b)*arctan(2*sqrt(1/2)*x/sqrt((
b + sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 + b*d*e^4 - b^2*d
*e^2) + (2*d^2*e^3 - d*e^4 + b*d*e^2)*sqrt(-2*d*e + b)*arctan(2*sqrt(1/2)*
x/sqrt((b - sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 + b*d*e^4
 - b^2*d*e^2)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{b\,x\,(b-2\,d\,e)+2\,b\,e^2\,x^3+4\,d^2\,e^2\,x-e^2\,x^3\,(b-2\,d\,e)+3\,d\,e\,x\,(b-2\,d\,e)}{(2\,e\,d^2+b\,d)\,\sqrt{b-2\,d\,e}}\right)-\operatorname{atan}\left(\frac{e\,x}{\sqrt{b-2\,d\,e}}\right)}{\sqrt{b-2\,d\,e}}$$

input
```
int((d - e*x^2)/(b*x^2 + d^2 + e^2*x^4),x)
```

output
```
(atan((b*x*(b - 2*d*e) + 2*b*e^2*x^3 + 4*d^2*e^2*x - e^2*x^3*(b - 2*d*e) +
 3*d*e*x*(b - 2*d*e))/((b*d + 2*d^2*e)*(b - 2*d*e)^(1/2))) - atan((e*x)/(b
 - 2*d*e)^(1/2)))/(b - 2*d*e)^(1/2)
```

3.31 $\int \frac{d-ex^2}{d^2+fx^2+e^2x^4} \, dx$

3.31.1 Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4} \, dx = -\frac{\log\left(d-\sqrt{2de-f}x+ex^2\right)}{2\sqrt{2de-f}}+\frac{\log\left(d+\sqrt{2de-f}x+ex^2\right)}{2\sqrt{2de-f}}$$

output
```
-1/2*ln(d+e*x^2-x*(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e
-f)^(1/2))/(2*d*e-f)^(1/2)
```

3.31.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(78) = 156.

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.33

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4} \, dx = \frac{\frac{\left(2de+f-\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}}-\frac{\left(2de+f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\sqrt{-4d^2e^2+f^2}}$$

input
```
Integrate[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

output
```
(((2*d*e + f - Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[
-4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] - ((2*d*e + f + Sqrt
[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]]
)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

3.31.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1478, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{d^2+e^2x^4+fx^2}dx$$

$$\Big\downarrow \text{1478}$$

$$-\frac{\int -\frac{\sqrt{2de-f}-2ex}{e\left(x^2-\frac{\sqrt{2de-f}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de-f}}-\frac{\int -\frac{2ex+\sqrt{2de-f}}{e\left(x^2+\frac{\sqrt{2de-f}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de-f}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{2de-f}-2ex}{e\left(x^2-\frac{\sqrt{2de-f}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de-f}}+\frac{\int \frac{2ex+\sqrt{2de-f}}{e\left(x^2+\frac{\sqrt{2de-f}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de-f}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{2de-f}-2ex}{x^2-\frac{\sqrt{2de-f}x}{e}+\frac{d}{e}}dx}{2e\sqrt{2de-f}}+\frac{\int \frac{2ex+\sqrt{2de-f}}{x^2+\frac{\sqrt{2de-f}x}{e}+\frac{d}{e}}dx}{2e\sqrt{2de-f}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}-\frac{\log\left(-x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}$$

input
```
Int[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

output
```
-1/2*Log[d - Sqrt[2*d*e - f]*x + e*x^2]/Sqrt[2*d*e - f] + Log[d + Sqrt[2*d
*e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f])
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.31.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\ln\left(d+e\,x^{2}+x\sqrt{2ed-f}\right)}{2\sqrt{2ed-f}}-\frac{\ln\left(-e\,x^{2}+x\sqrt{2ed-f}-d\right)}{2\sqrt{2ed-f}}$	69
risch	$\frac{\ln\left(\sqrt{2ed-f}\,e\,x^{2}+(2ed-f)x+\sqrt{2ed-f}\,d\right)}{2\sqrt{2ed-f}}-\frac{\ln\left(\sqrt{2ed-f}\,e\,x^{2}+(-2ed+f)x+\sqrt{2ed-f}\,d\right)}{2\sqrt{2ed-f}}$	102

input
```
int((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(d+e*x^2+x*(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)-1/2/(2*d*e-f)^(1/2)*ln(-
e*x^2+x*(2*d*e-f)^(1/2)-d)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.22

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx = \left[\frac{\log\left(\frac{e^2x^4+(4\,de-f)x^2+d^2+2\,(ex^3+dx)\sqrt{2\,de-f}}{e^2x^4+fx^2+d^2}\right)}{2\sqrt{2\,de-f}}, \frac{\sqrt{-2\,de+f}\arctan\left(-\frac{\sqrt{-2\,de+f}ex}{2\,de-f}\right)-\sqrt{-2\,de+f}\arctan\left(-\frac{(e^2}{}\right.}{2\,de-f}\right.$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")
```

output
```
[1/2*log((e^2*x^4 + (4*d*e - f)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e - f
))/(e^2*x^4 + f*x^2 + d^2))/sqrt(2*d*e - f), (sqrt(-2*d*e + f)*arctan(-sqr
t(-2*d*e + f)*e*x/(2*d*e - f)) - sqrt(-2*d*e + f)*arctan(-(e^2*x^3 - (d*e
- f)*x)*sqrt(-2*d*e + f)/(2*d^2*e - d*f)))/(2*d*e - f)]
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx = -\frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{\frac{1}{2de-f}}+f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2} + \frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{\frac{1}{2de-f}}-f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

input
```
integrate((-e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

output
```
-sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e - f)) + f*sq
rt(1/(2*d*e - f)))/e)/2 + sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(2*d*e*sq
rt(1/(2*d*e - f)) - f*sqrt(1/(2*d*e - f)))/e)/2
```

3.31.7 Maxima [F]

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx = \int -\frac{ex^2-d}{e^2x^4+fx^2+d^2}\,dx$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)/(e^2*x^4 + f*x^2 + d^2), x)
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(66) = 132.

Time = 0.70 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.44

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx = -\frac{(2\,d^2e^3-de^4+de^2f)\sqrt{-2\,de+f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{f+\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5+de^4f-de^2f^2} + \frac{(2\,d^2e^3-de^4+de^2f)\sqrt{-2\,de+f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{f-\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5+de^4f-de^2f^2}$$

input
```
integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")
```

output
```
-(2*d^2*e^3 - d*e^4 + d*e^2*f)*sqrt(-2*d*e + f)*arctan(2*sqrt(1/2)*x/sqrt(
(f + sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 + d*e^4*f - d*e^
2*f^2) + (2*d^2*e^3 - d*e^4 + d*e^2*f)*sqrt(-2*d*e + f)*arctan(2*sqrt(1/2)
*x/sqrt((f - sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 + d*e^4*
f - d*e^2*f^2)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx = \frac{\operatorname{atan}\left(\frac{f\,x\,1\mathrm{i}-d\,e\,x\,2\mathrm{i}}{d\,\sqrt{2\,d\,e-f}+e\,x^2\,\sqrt{2\,d\,e-f}}\right)\,1\mathrm{i}}{\sqrt{2\,d\,e-f}}$$

input
```
int((d - e*x^2)/(f*x^2 + d^2 + e^2*x^4),x)
```

output
```
(atan((f*x*1i - d*e*x*2i)/(d*(2*d*e - f)^(1/2) + e*x^2*(2*d*e - f)^(1/2)))
*1i)/(2*d*e - f)^(1/2)
```

3.32 $\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx$

3.32.1 Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx=-\frac{\log\left(d-\sqrt{b+2de}\,x+ex^2\right)}{2\sqrt{b+2de}}+\frac{\log\left(d+\sqrt{b+2de}\,x+ex^2\right)}{2\sqrt{b+2de}}$$

output
```
-1/2*ln(d+e*x^2-x*(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e
+b)^(1/2))/(2*d*e+b)^(1/2)
```

3.32.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(70) = 140.

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.71

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{-\frac{\left(b-2de+\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}}+\frac{\left(b-2de-\sqrt{b^2-4d^2e^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

input
```
Integrate[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

output
```
(-(((b - 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b - Sqr
t[b^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]]) + ((b - 2*d*e - Sq
rt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]]
])/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

3.32.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1478, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{-bx^2+d^2+e^2x^4}dx$$

$$\Big\downarrow \text{1478}$$

$$-\frac{\int -\frac{\sqrt{b+2de}-2ex}{e\left(x^2-\frac{\sqrt{b+2de}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{b+2de}}-\frac{\int -\frac{2ex+\sqrt{b+2de}}{e\left(x^2+\frac{\sqrt{b+2de}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{b+2de}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{b+2de}-2ex}{e\left(x^2-\frac{\sqrt{b+2de}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{b+2de}}+\frac{\int \frac{2ex+\sqrt{b+2de}}{e\left(x^2+\frac{\sqrt{b+2de}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{b+2de}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{b+2de}-2ex}{x^2-\frac{\sqrt{b+2de}x}{e}+\frac{d}{e}}dx}{2e\sqrt{b+2de}}+\frac{\int \frac{2ex+\sqrt{b+2de}}{x^2+\frac{\sqrt{b+2de}x}{e}+\frac{d}{e}}dx}{2e\sqrt{b+2de}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}-\frac{\log\left(-x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}$$

input
```
Int[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

output
```
-1/2*Log[d - Sqrt[b + 2*d*e]*x + e*x^2]/Sqrt[b + 2*d*e] + Log[d + Sqrt[b +
 2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e])
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.32.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\ln\left(d+e\,x^{2}+x\sqrt{2ed+b}\right)}{2\sqrt{2ed+b}}-\frac{\ln\left(-e\,x^{2}+x\sqrt{2ed+b}-d\right)}{2\sqrt{2ed+b}}$	61
risch	$\frac{\ln\left(\sqrt{2ed+b}\,e\,x^{2}+(2ed+b)x+\sqrt{2ed+b}\,d\right)}{2\sqrt{2ed+b}}-\frac{\ln\left(\sqrt{2ed+b}\,e\,x^{2}+(-2ed-b)x+\sqrt{2ed+b}\,d\right)}{2\sqrt{2ed+b}}$	90

input
```
int((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(d+e*x^2+x*(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)-1/2/(2*d*e+b)^(1/2)*ln(-
e*x^2+x*(2*d*e+b)^(1/2)-d)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.40

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx = \left[\frac{\log\left(\frac{e^2x^4+(4\,de+b)x^2+d^2+2\,(ex^3+dx)\sqrt{2\,de+b}}{e^2x^4-bx^2+d^2}\right)}{2\,\sqrt{2\,de+b}}, -\frac{\sqrt{-2\,de-b}\arctan\left(\frac{\sqrt{-2\,de-b}ex}{2\,de+b}\right)-\sqrt{-2\,de-b}\arctan\left(\frac{(e^2x^3-(de+b)x)\sqrt{-2\,de-b}}{2\,d^2e+bd}\right)}{2\,de+b}\right]$$

input
```
integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")
```

output
```
[1/2*log((e^2*x^4 + (4*d*e + b)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e + b
))/(e^2*x^4 - b*x^2 + d^2))/sqrt(2*d*e + b), -(sqrt(-2*d*e - b)*arctan(sqr
t(-2*d*e - b)*e*x/(2*d*e + b)) - sqrt(-2*d*e - b)*arctan((e^2*x^3 - (d*e +
 b)*x)*sqrt(-2*d*e - b)/(2*d^2*e + b*d)))/(2*d*e + b)]
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.60

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx = -\frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{\frac{1}{b+2de}}-2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2} + \frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(b\sqrt{\frac{1}{b+2de}}+2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

input
```
integrate((-e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

output
```
-sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(1/(b + 2*d*e)) - 2*d*e*sq
rt(1/(b + 2*d*e)))/e)/2 + sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(b*sqrt(1
/(b + 2*d*e)) + 2*d*e*sqrt(1/(b + 2*d*e)))/e)/2
```

3.32.7 Maxima [F]

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx=\int -\frac{ex^2-d}{e^2x^4-bx^2+d^2}\,dx$$

input
```
integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)/(e^2*x^4 - b*x^2 + d^2), x)
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(58) = 116.

Time = 0.77 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.84

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx=\frac{(2\,d^2e^3-de^4-bde^2)\sqrt{-2\,de-b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{b+\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5-bde^4-b^2de^2}+\frac{(2\,d^2e^3-de^4-bde^2)\sqrt{-2\,de-b}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{b-\sqrt{-4\,d^2e^2+b^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5-bde^4-b^2de^2}$$

input
```
integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")
```

output
```
(2*d^2*e^3 - d*e^4 - b*d*e^2)*sqrt(-2*d*e - b)*arctan(2*sqrt(1/2)*x/sqrt(-
(b + sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 - b*d*e^4 - b^2*
d*e^2) + (2*d^2*e^3 - d*e^4 - b*d*e^2)*sqrt(-2*d*e - b)*arctan(2*sqrt(1/2)
*x/sqrt(-(b - sqrt(-4*d^2*e^2 + b^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 - b*d*e
^4 - b^2*d*e^2)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx = \frac{\operatorname{atanh}\left(\frac{x\,\sqrt{b+2\,d\,e}}{e\,x^2+d}\right)}{\sqrt{b+2\,d\,e}}$$

input
```
int((d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x)
```

output
```
atanh((x*(b + 2*d*e)^(1/2))/(d + e*x^2))/(b + 2*d*e)^(1/2)
```

3.33 $\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx$

3.33.1 Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = -\frac{\log\left(d-\sqrt{2de+f}\,x+ex^2\right)}{2\sqrt{2de+f}}+\frac{\log\left(d+\sqrt{2de+f}\,x+ex^2\right)}{2\sqrt{2de+f}}$$

output
```
-1/2*ln(d+e*x^2-x*(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e
+f)^(1/2))/(2*d*e+f)^(1/2)
```

3.33.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(70) = 140.

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.71

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = \frac{-\frac{\left(-2de+f+\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}}+\frac{\left(-2de+f-\sqrt{-4d^2e^2+f^2}\right)\arctan\left(\frac{\sqrt{2}ex}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\sqrt{-4d^2e^2+f^2}}$$

input
```
Integrate[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

output
```
(-(((-2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f - S
qrt[-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]]) + ((-2*d*e + f
 - Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f + Sqrt[-4*d^2*e^2
+ f^2]]])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^
2])
```

3.33.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1478, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{d^2+e^2x^4-fx^2}dx$$

$$\Big\downarrow \text{1478}$$

$$-\frac{\int -\frac{\sqrt{2de+f}-2ex}{e\left(x^2-\frac{\sqrt{2de+f}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de+f}}-\frac{\int -\frac{2ex+\sqrt{2de+f}}{e\left(x^2+\frac{\sqrt{2de+f}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de+f}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{2de+f}-2ex}{e\left(x^2-\frac{\sqrt{2de+f}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de+f}}+\frac{\int \frac{2ex+\sqrt{2de+f}}{e\left(x^2+\frac{\sqrt{2de+f}x}{e}+\frac{d}{e}\right)}dx}{2\sqrt{2de+f}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{2de+f}-2ex}{x^2-\frac{\sqrt{2de+f}x}{e}+\frac{d}{e}}dx}{2e\sqrt{2de+f}}+\frac{\int \frac{2ex+\sqrt{2de+f}}{x^2+\frac{\sqrt{2de+f}x}{e}+\frac{d}{e}}dx}{2e\sqrt{2de+f}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}-\frac{\log\left(-x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}$$

input
```
Int[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

output
```
-1/2*Log[d - Sqrt[2*d*e + f]*x + e*x^2]/Sqrt[2*d*e + f] + Log[d + Sqrt[2*d
*e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f])
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.33.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\ln\left(d+e\,x^{2}+x\sqrt{2ed+f}\right)}{2\sqrt{2ed+f}}-\frac{\ln\left(-e\,x^{2}+x\sqrt{2ed+f}-d\right)}{2\sqrt{2ed+f}}$	61
risch	$\frac{\ln\left(\sqrt{2ed+f}\,e\,x^{2}+(2ed+f)x+\sqrt{2ed+f}\,d\right)}{2\sqrt{2ed+f}}-\frac{\ln\left(\sqrt{2ed+f}\,e\,x^{2}+(-2ed-f)x+\sqrt{2ed+f}\,d\right)}{2\sqrt{2ed+f}}$	90

input
```
int((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(d+e*x^2+x*(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)-1/2/(2*d*e+f)^(1/2)*ln(-
e*x^2+x*(2*d*e+f)^(1/2)-d)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.40

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx$$
$$=\left[\frac{\log\left(\frac{e^2x^4+(4\,de+f)x^2+d^2+2\,(ex^3+dx)\sqrt{2\,de+f}}{e^2x^4-fx^2+d^2}\right)}{2\sqrt{2\,de+f}},\right.$$
$$\left.-\frac{\sqrt{-2\,de-f}\arctan\left(\frac{\sqrt{-2\,de-f}ex}{2\,de+f}\right)-\sqrt{-2\,de-f}\arctan\left(\frac{(e^2x^3-(de+f)x)\sqrt{-2\,de-f}}{2\,d^2e+df}\right)}{2\,de+f}\right]$$

input
```
integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")
```

output
```
[1/2*log((e^2*x^4 + (4*d*e + f)*x^2 + d^2 + 2*(e*x^3 + d*x)*sqrt(2*d*e + f
))/(e^2*x^4 - f*x^2 + d^2))/sqrt(2*d*e + f), -(sqrt(-2*d*e - f)*arctan(sqr
t(-2*d*e - f)*e*x/(2*d*e + f)) - sqrt(-2*d*e - f)*arctan((e^2*x^3 - (d*e +
 f)*x)*sqrt(-2*d*e - f)/(2*d^2*e + d*f)))/(2*d*e + f)]
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.60

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx=-\frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{\frac{1}{2de+f}}-f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}$$
$$+\frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{\frac{1}{2de+f}}+f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

input
```
integrate((-e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

output
```
-sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e + f)) - f*sq
rt(1/(2*d*e + f)))/e)/2 + sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(2*d*e*sq
rt(1/(2*d*e + f)) + f*sqrt(1/(2*d*e + f)))/e)/2
```

3.33.7 Maxima [F]

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = \int -\frac{ex^2-d}{e^2x^4-fx^2+d^2}\,dx$$

input
```
integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)/(e^2*x^4 - f*x^2 + d^2), x)
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(58) = 116$.

Time = 0.69 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.86

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = \frac{(2\,d^2e^3-de^4-de^2f)\sqrt{-2\,de-f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{f+\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5-de^4f-de^2f^2} - \frac{(2\,d^2e^3-de^4-de^2f)\sqrt{-2\,de-f}\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{f-\sqrt{-4\,d^2e^2+f^2}}{e^2}}}\right)}{4\,d^3e^4-2\,d^2e^5-de^4f-de^2f^2}$$

input
```
integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")
```

output
```
(2*d^2*e^3 - d*e^4 - d*e^2*f)*sqrt(-2*d*e - f)*arctan(2*sqrt(1/2)*x/sqrt(-
(f + sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 - d*e^4*f - d*e^
2*f^2) - (2*d^2*e^3 - d*e^4 - d*e^2*f)*sqrt(-2*d*e - f)*arctan(2*sqrt(1/2)
*x/sqrt(-(f - sqrt(-4*d^2*e^2 + f^2))/e^2))/(4*d^3*e^4 - 2*d^2*e^5 - d*e^4
*f - d*e^2*f^2)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx=\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{f+2\,d\,e}}{e\,x^2+d}\right)}{\sqrt{f+2\,d\,e}}$$

input
```
int((d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x)
```

output
```
atanh((x*(f + 2*d*e)^(1/2))/(d + e*x^2))/(f + 2*d*e)^(1/2)
```

3.34 $\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx$

3.34.1 Optimal result

Integrand size = 30, antiderivative size = 134

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx = -\frac{e^{3/2}\log\left(\sqrt{c}d-\sqrt{e}\sqrt{2cd-be}x+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}} + \frac{e^{3/2}\log\left(\sqrt{c}d+\sqrt{e}\sqrt{2cd-be}x+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}$$

output
```
-1/2*e^(3/2)*ln(d*c^(1/2)+e*x^2*c^(1/2)-x*e^(1/2)*(-b*e+2*c*d)^(1/2))/c^(1
/2)/(-b*e+2*c*d)^(1/2)+1/2*e^(3/2)*ln(d*c^(1/2)+e*x^2*c^(1/2)+x*e^(1/2)*(-
b*e+2*c*d)^(1/2))/c^(1/2)/(-b*e+2*c*d)^(1/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.87

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4} dx = \frac{e^{3/2}\left(-\frac{\left(-2cd-be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}-\frac{\left(2cd+be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-4c^2d^2+b^2e^2}}$$

input
```
Integrate[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

output
```
(e^(3/2)*(-(((-2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*S
qrt[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt
[-4*c^2*d^2 + b^2*e^2]]) - ((2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*Arc
Tan[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/S
qrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 +
 b^2*e^2])
```

3.34.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1478, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-ex^2}{bx^2+\frac{cd^2}{e^2}+cx^4}dx$$

$$\Big\downarrow \text{1478}$$

$$-\frac{e^{3/2}\int -\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{c}\sqrt{e}\left(x^2-\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{c}\sqrt{2cd-be}}-\frac{e^{3/2}\int -\frac{2\sqrt{c}\sqrt{e}x+\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}\left(x^2+\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{c}\sqrt{2cd-be}}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^{3/2}\int \frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{c}\sqrt{e}\left(x^2-\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{c}\sqrt{2cd-be}}+\frac{e^{3/2}\int \frac{2\sqrt{c}\sqrt{e}x+\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}\left(x^2+\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+\frac{d}{e}\right)}dx}{2\sqrt{c}\sqrt{2cd-be}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{x^2-\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+\frac{d}{e}}dx}{2c\sqrt{2cd-be}}+\frac{e\int \frac{2\sqrt{c}\sqrt{e}x+\sqrt{2cd-be}}{x^2+\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+\frac{d}{e}}dx}{2c\sqrt{2cd-be}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{e^{3/2}\log\left(\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}-\frac{e^{3/2}\log\left(-\sqrt{e}x\sqrt{2cd-be}+\sqrt{c}d+\sqrt{c}ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}}$$

input
```
Int[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

output
```
-1/2*(e^(3/2)*Log[Sqrt[c]*d - Sqrt[e]*Sqrt[2*c*d - b*e]*x + Sqrt[c]*e*x^2]
)/(Sqrt[c]*Sqrt[2*c*d - b*e]) + (e^(3/2)*Log[Sqrt[c]*d + Sqrt[e]*Sqrt[2*c*
d - b*e]*x + Sqrt[c]*e*x^2])/(2*Sqrt[c]*Sqrt[2*c*d - b*e])
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.34.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\sqrt{-c(be-2cd)e}\,e\ln\left(-c\,x^2e+\sqrt{-c(be-2cd)e}\,x-cd\right)}{2c(be-2cd)}-\frac{\sqrt{-c(be-2cd)e}\,e\ln\left(-c\,x^2e-\sqrt{-c(be-2cd)e}\,x-cd\right)}{2c(be-2cd)}$
default	$4e^4c\left(\frac{\left(-b\,e^2-2dce-\sqrt{e^2(be-2cd)(be+2cd)}\right)\sqrt{2}\,\arctan\left(\frac{cxe\sqrt{2}}{\sqrt{c\left(b\,e^2+\sqrt{e^2(be-2cd)(be+2cd)}\right)}}\right)}{8\sqrt{e^2(be-2cd)(be+2cd)}\,c\,e^2\sqrt{c\left(b\,e^2+\sqrt{e^2(be-2cd)(be+2cd)}\right)}}-\frac{\left(b\,e^2+2dce-\sqrt{e^2(be-2cd)(be+2\right.}{8\sqrt{e^2(be-2cd)(be+2c}}\right.$

input
```
int((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-c*(b*e-2*c*d)*e)^(1/2)/c/(b*e-2*c*d)*e*ln(-c*x^2*e+(-c*(b*e-2*c*d)*e
)^(1/2)*x-c*d)-1/2*(-c*(b*e-2*c*d)*e)^(1/2)/c/(b*e-2*c*d)*e*ln(-c*x^2*e-(-
c*(b*e-2*c*d)*e)^(1/2)*x-c*d)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.82

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \left[\frac{1}{2}e\sqrt{\frac{e}{2\,c^2d-bce}}\log\left(\frac{ce^2x^4+cd^2+(4\,cde-be^2)x^2+2\left((2\,c^2de-bce^2)x^3+(2\,c^2d^2-bcde)x\right)\sqrt{\frac{e}{2\,c^2d-bce}}}{ce^2x^4+be^2x^2+cd^2}\right),\; -e\sqrt{-\frac{e}{2\,c^2d-bce}}\arctan\left(cx\sqrt{-\frac{e}{2\,c^2d-bce}}\right) + e\sqrt{-\frac{e}{2\,c^2d-bce}}\arctan\left(\frac{(cex^3-(cd-be)x)\sqrt{-\frac{e}{2\,c^2d-bce}}}{d}\right)\right]$$

input
```
integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")
```

output
```
[1/2*e*sqrt(e/(2*c^2*d - b*c*e))*log((c*e^2*x^4 + c*d^2 + (4*c*d*e - b*e^2
)*x^2 + 2*((2*c^2*d*e - b*c*e^2)*x^3 + (2*c^2*d^2 - b*c*d*e)*x)*sqrt(e/(2*
c^2*d - b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), -e*sqrt(-e/(2*c^2*d - b
*c*e))*arctan(c*x*sqrt(-e/(2*c^2*d - b*c*e))) + e*sqrt(-e/(2*c^2*d - b*c*e
))*arctan((c*e*x^3 - (c*d - b*e)*x)*sqrt(-e/(2*c^2*d - b*c*e))/d)]
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.18

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be-2cd)}}+2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2} - \frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be-2cd)}}-2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2}$$

input
```
integrate((-e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

output
```
sqrt(-e**3/(c*(b*e - 2*c*d)))*log(d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e
- 2*c*d))) + 2*c*d*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2 - sqrt(-e**3/(c*
(b*e - 2*c*d)))*log(d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e - 2*c*d))) - 2*
c*d*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2
```

3.34.7 Maxima [F]

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \int -\frac{ex^2-d}{cx^4+bx^2+\frac{cd^2}{e^2}}\,dx$$

input
```
integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 - d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6331 vs. 2(102) = 204.

Time = 1.01 (sec) , antiderivative size = 6331, normalized size of antiderivative = 47.25

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")
```

output
```
1/8*(2*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*
e + b*c*e^2)*b^2*c^2*e^7*sgn(c)*sgn(e) - 12*b*c^4*d^2*e^6 + 3*b^3*c^2*e^8
+ 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4*sgn
(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*
d^2*e^4*sgn(c)*sgn(e) - sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^
2)*b^3*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c
*e^2)*b^3*e^6*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e
+ b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c
^2*d*e + b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b
*c^3*d^2*e^5 - 2*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6 + sqrt(-4*c^2*d^2
+ b^2*e^2)*b^3*c*e^7 - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^7 - sqrt(-2*
c^2*d*e + b*c*e^2)*b^2*c^2*e^7 - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d
*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^5*sgn(c)*sgn(e) + 4*sqrt(-4
*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4 - sqrt(-4*c^2
*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^6 + 2*sqrt(-4*c^2*d^2 + b
^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^6 + (4*sqrt(2*c^2*d*e + b*c*e^2
)*c^4*d^2*e^3*sgn(c)*sgn(e) - 4*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^4*sgn(
c)*sgn(e) + sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*e^5*sgn(c)*sgn(e) - 24*c^5*d
^3*e^3 + 12*b*c^4*d^2*e^4 + 6*b^2*c^3*d*e^5 - 3*b^3*c^2*e^6 + 8*sqrt(-4*c^
2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn(c)*sgn(e) + 8*...
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.96

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = -\frac{e^{3/2}\left(\operatorname{atan}\left(\frac{\sqrt{e}\,x\,\sqrt{b\,c\,e-2\,c^2\,d}}{b\,e-2\,c\,d}\right)+\operatorname{atan}\left(\frac{c\,e^{3/2}\,x^3\,\sqrt{b\,c\,e-2\,c^2\,d}+b\,e^{3/2}\,x\,\sqrt{b\,c\,e-2\,c^2\,d}-c\,d\,\sqrt{e}\,x\,\sqrt{b\,c\,e-2\,c^2\,d}}{d\,(2\,c^2\,d-b\,c\,e)}\right)\right)}{\sqrt{b\,c\,e-2\,c^2\,d}}$$

input
```
int((d - e*x^2)/(b*x^2 + c*x^4 + (c*d^2)/e^2),x)
```

output
```
-(e^(3/2)*(atan((e^(1/2)*x*(b*c*e - 2*c^2*d)^(1/2))/(b*e - 2*c*d)) + atan(
(c*e^(3/2)*x^3*(b*c*e - 2*c^2*d)^(1/2) + b*e^(3/2)*x*(b*c*e - 2*c^2*d)^(1/
2) - c*d*e^(1/2)*x*(b*c*e - 2*c^2*d)^(1/2))/(d*(2*c^2*d - b*c*e)))))/(b*c*
e - 2*c^2*d)^(1/2)
```

3.35 $\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$

3.35.1 Optimal result

Integrand size = 29, antiderivative size = 130

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = -\frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}$$

output
```
-e^(3/2)*arctan((-2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d)^(1/2))/(b*e+2*c*d)^(1/2
))/c^(1/2)/(b*e+2*c*d)^(1/2)+e^(3/2)*arctan((2*x*c^(1/2)*e^(1/2)+(-b*e+2*c
*d)^(1/2))/(b*e+2*c*d)^(1/2))/c^(1/2)/(b*e+2*c*d)^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.91

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \frac{e^{3/2}\left(\frac{\left(2cd-be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}+\frac{\left(-2cd+be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-4c^2d^2+b^2e^2}}$$

input
```
Integrate[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

output
```
(e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt
[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4
*c^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt
[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^
2*e^2])
```

3.35.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{bx^2+\frac{cd^2}{e^2}+cx^4} \, dx$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{e\int \frac{1}{x^2-\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+\frac{d}{e}}dx}{2c}+\frac{e\int \frac{1}{x^2+\frac{\sqrt{2cd-be}x}{\sqrt{c}\sqrt{e}}+\frac{d}{e}}dx}{2c}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{e\int \frac{1}{-\left(2x-\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}\right)^2-\frac{b}{c}-\frac{2d}{e}}d\left(2x-\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}\right)}{c}-\frac{e\int \frac{1}{-\left(2x+\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}\right)^2-\frac{b}{c}-\frac{2d}{e}}d\left(2x+\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}\right)}{c}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{e^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{e}\left(2x-\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}\right)}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{e}\left(\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

input
```
Int[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

output
```
(e^(3/2)*ArcTan[(Sqrt[c]*Sqrt[e]*(-(Sqrt[2*c*d - b*e]/(Sqrt[c]*Sqrt[e])) +
 2*x))/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e]) + (e^(3/2)*ArcTan[(
Sqrt[c]*Sqrt[e]*(Sqrt[2*c*d - b*e]/(Sqrt[c]*Sqrt[e]) + 2*x))/Sqrt[2*c*d +
b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

3.35.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.35.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\sqrt{-c(be+2cd)e}\, e \ln\left(-c\,x^{2} e-\sqrt{-c(be+2cd)e}\, x+cd\right)}{2c(be+2cd)}-\frac{\sqrt{-c(be+2cd)e}\, e \ln\left(-c\,x^{2} e+\sqrt{-c(be+2cd)e}\, x+cd\right)}{2c(be+2cd)}$
default	$4e^{4}c\left(\frac{\left(b\,e^{2}-2dce+\sqrt{e^{2}(be-2cd)(be+2cd)}\right)\sqrt{2}\,\arctan\left(\frac{cxe\sqrt{2}}{\sqrt{c\left(b\,e^{2}+\sqrt{e^{2}(be-2cd)(be+2cd)}\right)}}\right)}{8\sqrt{e^{2}(be-2cd)(be+2cd)}\,c\,e^{2}\sqrt{c\left(b\,e^{2}+\sqrt{e^{2}(be-2cd)(be+2cd)}\right)}}-\frac{\left(-b\,e^{2}+2dce+\sqrt{e^{2}(be-2cd)(be+2\right.}{8\sqrt{e^{2}(be-2cd)(be+2cd}}\right.$

input
```
int((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-c*(b*e+2*c*d)*e)^(1/2)/c/(b*e+2*c*d)*e*ln(-c*x^2*e-(-c*(b*e+2*c*d)*e
)^(1/2)*x+c*d)-1/2*(-c*(b*e+2*c*d)*e)^(1/2)/c/(b*e+2*c*d)*e*ln(-c*x^2*e+(-
c*(b*e+2*c*d)*e)^(1/2)*x+c*d)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$$
$$=\left[\frac{1}{2}\,e\sqrt{-\frac{e}{2\,c^2d+bce}}\log\left(\frac{ce^2x^4+cd^2-(4\,cde+be^2)x^2+2\,((2\,c^2de+bce^2)x^3-(2\,c^2d^2+bcde)x)\sqrt{-\frac{e}{2\,c^2}}}{ce^2x^4+be^2x^2+cd^2}\right.\right.$$
$$\left.+e\sqrt{\frac{e}{2\,c^2d+bce}}\arctan\left(\frac{(cex^3+(cd+be)x)\sqrt{\frac{e}{2\,c^2d+bce}}}{d}\right)\right]$$

input
```
integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")
```

output
```
[1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d*e + b*e^
2)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c*d*e)*x)*sqrt(-e/(
2*c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), e*sqrt(e/(2*c^2*d + b
*c*e))*arctan(c*x*sqrt(e/(2*c^2*d + b*c*e))) + e*sqrt(e/(2*c^2*d + b*c*e))
*arctan((c*e*x^3 + (c*d + b*e)*x)*sqrt(e/(2*c^2*d + b*c*e))/d)]
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.23

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx=-\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be+2cd)}}-2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$
$$+\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be+2cd)}}+2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$

input
```
integrate((e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

output
```
-sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*
e + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(
c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) +
 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2
```

3.35.7 Maxima [F]

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx=\int \frac{ex^2+d}{cx^4+bx^2+\frac{cd^2}{e^2}}\,dx$$

input
```
integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(c*x^4 + b*x^2 + c*d^2/e^2), x)
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6341 vs. 2(105) = 210.

Time = 1.04 (sec) , antiderivative size = 6341, normalized size of antiderivative = 48.78

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")
```

output
```
1/8*(2*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*
e + b*c*e^2)*b^2*c^2*e^7*sgn(c)*sgn(e) - 12*b*c^4*d^2*e^6 + 3*b^3*c^2*e^8
+ 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4*sgn
(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*
d^2*e^4*sgn(c)*sgn(e) - sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^
2)*b^3*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c
*e^2)*b^3*e^6*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e
+ b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c
^2*d*e + b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b
*c^3*d^2*e^5 - 2*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6 + sqrt(-4*c^2*d^2
+ b^2*e^2)*b^3*c*e^7 - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^7 - sqrt(-2*
c^2*d*e + b*c*e^2)*b^2*c^2*e^7 - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d
*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^5*sgn(c)*sgn(e) + 4*sqrt(-4
*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4 - sqrt(-4*c^2
*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^6 + 2*sqrt(-4*c^2*d^2 + b
^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^6 - (4*sqrt(2*c^2*d*e + b*c*e^2
)*c^4*d^2*e^3*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*e^5*sgn(c)
*sgn(e) - 24*c^5*d^3*e^3 - 12*b*c^4*d^2*e^4 + 6*b^2*c^3*d*e^5 + 3*b^3*c^2*
e^6 + 8*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn
(c)*sgn(e) + 8*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3...
```

3.35.9 Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$$
$$= \frac{e^{3/2}\left(\operatorname{atan}\left(\frac{c\sqrt{e}\,x}{\sqrt{c\,(b\,e+2\,c\,d)}}\right)-\operatorname{atan}\left(\frac{(2\,d\,c^2+b\,e\,c)\left(x\left(\frac{\sqrt{e}\left(c\,d\,e^7-\frac{4\,c^3\,d^2\,e^7}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}+\frac{e^{3/2}\left(2\,c^2\,d\,e^6-b\,c\,e^7\right)}{c\,d\,\sqrt{2\,d\,c^2+b\,e\,c}\,(b\,e-2\,c\,d)}\right)+\frac{\sqrt{e}\,x^3\left(c\,e^8-\frac{2\,b\,c^2\,e^9}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}\right)}{c\,e^7}\right)\right)}{\sqrt{2\,d\,c^2+b\,e\,c}}$$

input
```
int((d + e*x^2)/(b*x^2 + c*x^4 + (c*d^2)/e^2),x)
```

output
```
(e^(3/2)*(atan((c*e^(1/2)*x)/(c*(b*e + 2*c*d))^(1/2)) - atan(((2*c^2*d + b
*c*e)*(x*((e^(1/2)*(c*d*e^7 - (4*c^3*d^2*e^7)/(2*c^2*d + b*c*e)))/(d*(c*(b
*e + 2*c*d))^(1/2)*(b*e - 2*c*d)) + (e^(3/2)*(2*c^2*d*e^6 - b*c*e^7))/(c*d
*(2*c^2*d + b*c*e)^(1/2)*(b*e - 2*c*d))) + (e^(1/2)*x^3*(c*e^8 - (2*b*c^2*
e^9)/(2*c^2*d + b*c*e)))/(d*(c*(b*e + 2*c*d))^(1/2)*(b*e - 2*c*d))))/(c*e^
7))))/(2*c^2*d + b*c*e)^(1/2)
```

3.36 $\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx$

3.36.1 Optimal result

Integrand size = 29, antiderivative size = 130

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx=-\frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}$$

output
```
-e^(3/2)*arctan((-2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d)^(1/2))/(b*e+2*c*d)^(1/2
))/c^(1/2)/(b*e+2*c*d)^(1/2)+e^(3/2)*arctan((2*x*c^(1/2)*e^(1/2)+(-b*e+2*c
*d)^(1/2))/(b*e+2*c*d)^(1/2))/c^(1/2)/(b*e+2*c*d)^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.91

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx = \frac{e^{3/2}\left(\frac{\left(2cd-be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}+\frac{\left(-2cd+be+\sqrt{-4c^2d^2+b^2e^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}x}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-4c^2d^2+b^2e^2}}$$

input
```
Integrate[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
```

output
```
(e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt
[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4
*c^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt
[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^
2*e^2])
```

3.36.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2087, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}dx$$

$$\Big\downarrow \text{ 2087}$$

$$\int \frac{d+ex^2}{bx^2+\frac{cd^2}{e^2}+cx^4}dx$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{e\int \frac{1}{x^2-\frac{\sqrt{2cd-bex}}{\sqrt{c}\sqrt{e}}+\frac{d}{e}}dx}{2c}+\frac{e\int \frac{1}{x^2+\frac{\sqrt{2cd-bex}}{\sqrt{c}\sqrt{e}}+\frac{d}{e}}dx}{2c}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{e\int \frac{1}{-\left(2x-\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}\right)^2-\frac{b}{c}-\frac{2d}{e}}d\left(2x-\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}\right)}{c}-\frac{e\int \frac{1}{-\left(2x+\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}\right)^2-\frac{b}{c}-\frac{2d}{e}}d\left(2x+\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}\right)}{c}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{e^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{e}\left(2x-\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}\right)}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{e}\left(\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

input
```
Int[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
```

output
```
(e^(3/2)*ArcTan[(Sqrt[c]*Sqrt[e]*(-(Sqrt[2*c*d - b*e]/(Sqrt[c]*Sqrt[e])) +
 2*x))/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e]) + (e^(3/2)*ArcTan[(
Sqrt[c]*Sqrt[e]*(Sqrt[2*c*d - b*e]/(Sqrt[c]*Sqrt[e]) + 2*x))/Sqrt[2*c*d +
b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

3.36.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2087
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

3.36.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

method	result
risch	$\frac{\sqrt{-c(be+2cd)e}\,e\ln\left(-c\,x^2e-\sqrt{-c(be+2cd)e}\,x+cd\right)}{2c(be+2cd)}-\frac{\sqrt{-c(be+2cd)e}\,e\ln\left(-c\,x^2e+\sqrt{-c(be+2cd)e}\,x+cd\right)}{2c(be+2cd)}$
default	$4e^4c\left(\frac{\left(b\,e^2-2dce+\sqrt{e^2(be-2cd)(be+2cd)}\right)\sqrt{2}\arctan\left(\frac{cxe\sqrt{2}}{\sqrt{c\left(b\,e^2+\sqrt{e^2(be-2cd)(be+2cd)}\right)}}\right)}{8\sqrt{e^2(be-2cd)(be+2cd)}\,c\,e^2\sqrt{c\left(b\,e^2+\sqrt{e^2(be-2cd)(be+2cd)}\right)}}-\frac{\left(-b\,e^2+2dce+\sqrt{e^2(be-2cd)(be+2\right.}{8\sqrt{e^2(be-2cd)(be+2cd}}\right.$

input
```
int((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-c*(b*e+2*c*d)*e)^(1/2)/c/(b*e+2*c*d)*e*ln(-c*x^2*e-(-c*(b*e+2*c*d)*e
)^(1/2)*x+c*d)-1/2*(-c*(b*e+2*c*d)*e)^(1/2)/c/(b*e+2*c*d)*e*ln(-c*x^2*e+(-
c*(b*e+2*c*d)*e)^(1/2)*x+c*d)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx$$
$$=\left[\frac{1}{2}e\sqrt{-\frac{e}{2\,c^2d+bce}}\log\left(\frac{ce^2x^4+cd^2-(4\,cde+be^2)x^2+2\left((2\,c^2de+bce^2)x^3-(2\,c^2d^2+bcde)x\right)\sqrt{-\frac{e}{2\,c^2}}}{ce^2x^4+be^2x^2+cd^2}\right.\right.$$
$$\left.+e\sqrt{\frac{e}{2\,c^2d+bce}}\arctan\left(\frac{(cex^3+(cd+be)x)\sqrt{\frac{e}{2\,c^2d+bce}}}{d}\right)\right]$$

input
```
integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="fricas")
```

output
```
[1/2*e*sqrt(-e/(2*c^2*d + b*c*e))*log((c*e^2*x^4 + c*d^2 - (4*c*d*e + b*e^
2)*x^2 + 2*((2*c^2*d*e + b*c*e^2)*x^3 - (2*c^2*d^2 + b*c*d*e)*x)*sqrt(-e/(
2*c^2*d + b*c*e)))/(c*e^2*x^4 + b*e^2*x^2 + c*d^2)), e*sqrt(e/(2*c^2*d + b
*c*e))*arctan(c*x*sqrt(e/(2*c^2*d + b*c*e))) + e*sqrt(e/(2*c^2*d + b*c*e))
*arctan((c*e*x^3 + (c*d + b*e)*x)*sqrt(e/(2*c^2*d + b*c*e))/d)]
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.23

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx = -\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be+2cd)}}-2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2} + \frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be+2cd)}}+2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$

input
```
integrate((e*x**2+d)/(b*x**2+c*(d**2/e**2+x**4)),x)
```

output
```
-sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*
e + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(
c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) +
 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2
```

3.36.7 Maxima [F]

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx = \int \frac{ex^2+d}{bx^2+\left(x^4+\frac{d^2}{e^2}\right)c}\,dx$$

input
```
integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(b*x^2 + (x^4 + d^2/e^2)*c), x)
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6341 vs. $2(105) = 210$.

Time = 1.10 (sec) , antiderivative size = 6341, normalized size of antiderivative = 48.78

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="giac")
```

output
```
1/8*(2*sqrt(2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*
e + b*c*e^2)*b^2*c^2*e^7*sgn(c)*sgn(e) - 12*b*c^4*d^2*e^6 + 3*b^3*c^2*e^8
+ 4*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4*sgn
(c)*sgn(e) + 4*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*
d^2*e^4*sgn(c)*sgn(e) - sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^
2)*b^3*e^6*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c
*e^2)*b^3*e^6*sgn(c)*sgn(e) + 2*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e
+ b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) + 2*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c
^2*d*e + b*c*e^2)*b^2*c*e^6*sgn(c)*sgn(e) - 4*sqrt(-4*c^2*d^2 + b^2*e^2)*b
*c^3*d^2*e^5 - 2*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^3*d*e^6 + sqrt(-4*c^2*d^2
+ b^2*e^2)*b^3*c*e^7 - 2*sqrt(-4*c^2*d^2 + b^2*e^2)*b^2*c^2*e^7 - sqrt(-2*
c^2*d*e + b*c*e^2)*b^2*c^2*e^7 - 3*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d
*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c*e^5*sgn(c)*sgn(e) + 4*sqrt(-4
*c^2*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b*c^2*d^2*e^4 - sqrt(-4*c^2
*d^2 + b^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^3*e^6 + 2*sqrt(-4*c^2*d^2 + b
^2*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*b^2*c*e^6 - (4*sqrt(2*c^2*d*e + b*c*e^2
)*c^4*d^2*e^3*sgn(c)*sgn(e) - sqrt(2*c^2*d*e + b*c*e^2)*b^2*c^2*e^5*sgn(c)
*sgn(e) - 24*c^5*d^3*e^3 - 12*b*c^4*d^2*e^4 + 6*b^2*c^3*d*e^5 + 3*b^3*c^2*
e^6 + 8*sqrt(-4*c^2*d^2 + b^2*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*c^3*d^3*e*sgn
(c)*sgn(e) + 8*sqrt(2*c^2*d*e + b*c*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*c^3...
```

3.36.9 Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx = \frac{e^{3/2}\left(\mathrm{atan}\left(\frac{c\sqrt{e}\,x}{\sqrt{c\,(b\,e+2\,c\,d)}}\right)-\mathrm{atan}\left(\frac{(2\,d\,c^2+b\,e\,c)\left(x\left(\frac{\sqrt{e}\left(c\,d\,e^7-\frac{4\,c^3\,d^2\,e^7}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}+\frac{e^{3/2}\left(2\,c^2\,d\,e^6-b\,c\,e^7\right)}{c\,d\,\sqrt{2\,d\,c^2+b\,e\,c}\,(b\,e-2\,c\,d)}\right)+\frac{\sqrt{e}\,x^3\left(c\,e^8-\frac{2\,b\,c^2\,e^9}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}\cdots}{c\,e^7}\right.\right.}{\sqrt{2\,d\,c^2+b\,e\,c}}$$

input
```
int((d + e*x^2)/(b*x^2 + c*(x^4 + d^2/e^2)),x)
```

output
```
(e^(3/2)*(atan((c*e^(1/2)*x)/(c*(b*e + 2*c*d))^(1/2)) - atan(((2*c^2*d + b
*c*e)*(x*((e^(1/2)*(c*d*e^7 - (4*c^3*d^2*e^7)/(2*c^2*d + b*c*e)))/(d*(c*(b
*e + 2*c*d))^(1/2)*(b*e - 2*c*d)) + (e^(3/2)*(2*c^2*d*e^6 - b*c*e^7))/(c*d
*(2*c^2*d + b*c*e)^(1/2)*(b*e - 2*c*d))) + (e^(1/2)*x^3*(c*e^8 - (2*b*c^2*
e^9)/(2*c^2*d + b*c*e)))/(d*(c*(b*e + 2*c*d))^(1/2)*(b*e - 2*c*d))))/(c*e^
7))))/(2*c^2*d + b*c*e)^(1/2)
```

3.36. $\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx$

3.37 $\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx$

3.37.1 Optimal result

Integrand size = 32, antiderivative size = 29

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=-\frac{1}{2}\log\left(a-x+bx^2\right)+\frac{1}{2}\log\left(a+x+bx^2\right)$$

output
```
-1/2*ln(b*x^2+a-x)+1/2*ln(b*x^2+a+x)
```

3.37.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=-\frac{1}{2}\log\left(a-x+bx^2\right)+\frac{1}{2}\log\left(a+x+bx^2\right)$$

input
```
Integrate[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

output
```
-1/2*Log[a - x + b*x^2] + Log[a + x + b*x^2]/2
```

3.37.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1478, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^2}{a^2+x^2(2ab-1)+b^2x^4}dx$$

$$\Big\downarrow \text{1478}$$

$$-\frac{1}{2}\int -\frac{1-2bx}{b\left(x^2-\frac{x}{b}+\frac{a}{b}\right)}dx-\frac{1}{2}\int -\frac{2bx+1}{b\left(x^2+\frac{x}{b}+\frac{a}{b}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{1-2bx}{b\left(x^2-\frac{x}{b}+\frac{a}{b}\right)}dx+\frac{1}{2}\int \frac{2bx+1}{b\left(x^2+\frac{x}{b}+\frac{a}{b}\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1-2bx}{x^2-\frac{x}{b}+\frac{a}{b}}dx}{2b}+\frac{\int \frac{2bx+1}{x^2+\frac{x}{b}+\frac{a}{b}}dx}{2b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

input
```
Int[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

output
```
-1/2*Log[a - x + b*x^2] + Log[a + x + b*x^2]/2
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.37.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{\ln\left(b\,x^{2}+a-x\right)}{2}+\frac{\ln\left(b\,x^{2}+a+x\right)}{2}$	26
norman	$-\frac{\ln\left(b\,x^{2}+a-x\right)}{2}+\frac{\ln\left(b\,x^{2}+a+x\right)}{2}$	26
risch	$-\frac{\ln\left(b\,x^{2}+a-x\right)}{2}+\frac{\ln\left(b\,x^{2}+a+x\right)}{2}$	26
parallelrisch	$-\frac{\ln\left(b\,x^{2}+a-x\right)}{2}+\frac{\ln\left(b\,x^{2}+a+x\right)}{2}$	26

input
```
int((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(b*x^2+a-x)+1/2*ln(b*x^2+a+x)
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

input
```
integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")
```

output
```
1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=-\frac{\log\left(\frac{a}{b}+x^2-\frac{x}{b}\right)}{2}+\frac{\log\left(\frac{a}{b}+x^2+\frac{x}{b}\right)}{2}$$

input
```
integrate((-b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
```

output
```
-log(a/b + x**2 - x/b)/2 + log(a/b + x**2 + x/b)/2
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

input
```
integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")
```

output
```
1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

input
```
integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")
```

output
```
1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.41

$$\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = \operatorname{atanh}\left(\frac{x}{b\,x^2+a}\right)$$

input
```
int((a - b*x^2)/(x^2*(2*a*b - 1) + a^2 + b^2*x^4),x)
```

output
```
atanh(x/(a + b*x^2))
```

3.38 $\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx$

3.38.1 Optimal result

Integrand size = 31, antiderivative size = 60

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\operatorname{arctanh}\left(\frac{1+2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

output
```
arctanh((-2*b*x+1)/(-4*a*b+1)^(1/2))/(-4*a*b+1)^(1/2)-arctanh((2*b*x+1)/(-
4*a*b+1)^(1/2))/(-4*a*b+1)^(1/2)
```

3.38.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 138 vs. 2(60) = 120.

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.30

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx=\frac{\frac{\left(1+\sqrt{1-4ab}\right)\arctan\left(\frac{bx}{\sqrt{-\frac{1}{2}+ab-\frac{1}{2}\sqrt{1-4ab}}}\right)}{\sqrt{-1+2ab-\sqrt{1-4ab}}}+\frac{\left(-1+\sqrt{1-4ab}\right)\arctan\left(\frac{\sqrt{2}bx}{\sqrt{-1+2ab+\sqrt{1-4ab}}}\right)}{\sqrt{-1+2ab+\sqrt{1-4ab}}}}{\sqrt{2-8ab}}$$

input
```
Integrate[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

output
```
(((1 + Sqrt[1 - 4*a*b])*ArcTan[(b*x)/Sqrt[-1/2 + a*b - Sqrt[1 - 4*a*b]/2]]
)/Sqrt[-1 + 2*a*b - Sqrt[1 - 4*a*b]] + ((-1 + Sqrt[1 - 4*a*b])*ArcTan[(Sqr
t[2]*b*x)/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a*b]]])/Sqrt[-1 + 2*a*b + Sqrt[1 -
4*a*b]])/Sqrt[2 - 8*a*b]
```

3.38.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1475, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{a^2+x^2(2ab-1)+b^2x^4}dx$$

$$\Big\downarrow \; 1475$$

$$\frac{\int \frac{1}{x^2-\frac{x}{b}+\frac{a}{b}}dx}{2b}+\frac{\int \frac{1}{x^2+\frac{x}{b}+\frac{a}{b}}dx}{2b}$$

$$\Big\downarrow \; 1083$$

$$-\frac{\int \frac{1}{\frac{1-4ab}{b^2}-\left(2x-\frac{1}{b}\right)^2}d\left(2x-\frac{1}{b}\right)}{b}-\frac{\int \frac{1}{\frac{1-4ab}{b^2}-\left(2x+\frac{1}{b}\right)^2}d\left(2x+\frac{1}{b}\right)}{b}$$

$$\Big\downarrow \; 219$$

$$-\frac{\text{arctanh}\left(\frac{b\left(2x-\frac{1}{b}\right)}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\text{arctanh}\left(\frac{b\left(\frac{1}{b}+2x\right)}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

input
```
Int[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

output
```
-(ArcTanh[(b*(-b^(-1) + 2*x))/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b]) - ArcTanh[
(b*(b^(-1) + 2*x))/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b]
```

3.38.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.38.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\arctan\left(\frac{2bx+1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}+\frac{\arctan\left(\frac{2bx-1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}$	52
risch	$-\frac{\ln\left(b\,x^{2}\sqrt{-4ab+1}+(-4ab+1)x-a\sqrt{-4ab+1}\right)}{2\sqrt{-4ab+1}}+\frac{\ln\left(b\,x^{2}\sqrt{-4ab+1}+x(4ab-1)-a\sqrt{-4ab+1}\right)}{2\sqrt{-4ab+1}}$	90

input
```
int((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x,method=_RETURNVERBOSE)
```

output
```
1/(4*a*b-1)^(1/2)*arctan((2*b*x+1)/(4*a*b-1)^(1/2))+1/(4*a*b-1)^(1/2)*arct
an((2*b*x-1)/(4*a*b-1)^(1/2))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.73

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx$$
$$= \left[-\frac{\sqrt{-4\,ab+1}\log\left(\frac{b^2x^4-(6\,ab-1)x^2+a^2-2\,(bx^3-ax)\sqrt{-4\,ab+1}}{b^2x^4+(2\,ab-1)x^2+a^2}\right)}{2\,(4\,ab-1)}, \frac{\sqrt{4\,ab-1}\arctan\left(\frac{bx}{\sqrt{4\,ab-1}}\right)+\sqrt{4\,ab-1}\arctan}{4\,ab-1}\right.$$

input
```
integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-4*a*b + 1)*log((b^2*x^4 - (6*a*b - 1)*x^2 + a^2 - 2*(b*x^3 - a
*x)*sqrt(-4*a*b + 1))/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2))/(4*a*b - 1), (sqr
t(4*a*b - 1)*arctan(b*x/sqrt(4*a*b - 1)) + sqrt(4*a*b - 1)*arctan((b^2*x^3
 + (3*a*b - 1)*x)*sqrt(4*a*b - 1)/(4*a^2*b - a)))/(4*a*b - 1)]
```

3.38.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(56) = 112$.

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.95

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = -\frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^2+\frac{x\left(-4ab\sqrt{-\frac{1}{4ab-1}}+\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2} + \frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^2+\frac{x\left(4ab\sqrt{-\frac{1}{4ab-1}}-\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}$$

input
```
integrate((b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
```

output
```
-sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(-4*a*b*sqrt(-1/(4*a*b - 1)) + s
qrt(-1/(4*a*b - 1)))/b)/2 + sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(4*a*
b*sqrt(-1/(4*a*b - 1)) - sqrt(-1/(4*a*b - 1)))/b)/2
```

3.38.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*b-0.25>0)', see `assume?` for
more detai
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = \frac{\arctan\left(\frac{2\,bx+1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}} + \frac{\arctan\left(\frac{2\,bx-1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}}$$

input
```
integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")
```

output
```
arctan((2*b*x + 1)/sqrt(4*a*b - 1))/sqrt(4*a*b - 1) + arctan((2*b*x - 1)/s
qrt(4*a*b - 1))/sqrt(4*a*b - 1)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = \frac{\mathrm{atan}\left(\frac{b\,x}{\sqrt{4\,a\,b-1}}\right)+\mathrm{atan}\left(\frac{\frac{3\,x\,(4\,a\,b-1)}{4}-\frac{x}{4}+b^2\,x^3}{a\,\sqrt{4\,a\,b-1}}\right)}{\sqrt{4\,a\,b-1}}$$

input
```
int((a + b*x^2)/(x^2*(2*a*b - 1) + a^2 + b^2*x^4),x)
```

output
```
(atan((b*x)/(4*a*b - 1)^(1/2)) + atan(((3*x*(4*a*b - 1))/4 - x/4 + b^2*x^3
)/(a*(4*a*b - 1)^(1/2))))/(4*a*b - 1)^(1/2)
```

3.39 $\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx$

3.39.1 Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx=-\frac{\arctan\left(\frac{\sqrt{4-b}-4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}+\frac{\arctan\left(\frac{\sqrt{4-b}+4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}$$

output
```
-arctan((-4*x+(4-b)^(1/2))/(4+b)^(1/2))/(4+b)^(1/2)+arctan((4*x+(4-b)^(1/2
))/(4+b)^(1/2))/(4+b)^(1/2)
```

3.39.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 126 vs. 2(62) = 124.

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.03

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx=\frac{\frac{\left(4-b+\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{-16+b^2}}}\right)}{\sqrt{b-\sqrt{-16+b^2}}}+\frac{\left(-4+b+\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{b+\sqrt{-16+b^2}}}\right)}{\sqrt{b+\sqrt{-16+b^2}}}}{\sqrt{2}\sqrt{-16+b^2}}$$

input
```
Integrate[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

output
```
(((4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]
])/Sqrt[b - Sqrt[-16 + b^2]] + ((-4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[
2]*x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt
[-16 + b^2])
```

3.39.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{bx^2+4x^4+1}\,dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{4}\int \frac{1}{x^2-\frac{1}{2}\sqrt{4-b}x+\frac{1}{2}}dx+\frac{1}{4}\int \frac{1}{x^2+\frac{1}{2}\sqrt{4-b}x+\frac{1}{2}}dx$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2}\int \frac{1}{\frac{1}{4}(-b-4)-\left(2x-\frac{\sqrt{4-b}}{2}\right)^2}d\left(2x-\frac{\sqrt{4-b}}{2}\right)-\frac{1}{2}\int \frac{1}{\frac{1}{4}(-b-4)-\left(2x+\frac{\sqrt{4-b}}{2}\right)^2}d\left(2x+\frac{\sqrt{4-b}}{2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2\left(2x-\frac{\sqrt{4-b}}{2}\right)}{\sqrt{b+4}}\right)}{\sqrt{b+4}}+\frac{\arctan\left(\frac{2\left(\frac{\sqrt{4-b}}{2}+2x\right)}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

input
```
Int[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

output
```
ArcTan[(2*(-1/2*Sqrt[4 - b] + 2*x))/Sqrt[4 + b]]/Sqrt[4 + b] + ArcTan[(2*(
Sqrt[4 - b]/2 + 2*x))/Sqrt[4 + b]]/Sqrt[4 + b]
```

3.39.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.39.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{\ln\left(-2x^2\sqrt{-4-b}+x(4+b)+\sqrt{-4-b}\right)}{2\sqrt{-4-b}}+\frac{\ln\left(-2x^2\sqrt{-4-b}+(-4-b)x+\sqrt{-4-b}\right)}{2\sqrt{-4-b}}$	74
default	$\frac{\left(-4+\sqrt{(b-4)(4+b)}+b\right)\arctan\left(\frac{4x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)}{\sqrt{(b-4)(4+b)}\,\sqrt{2\sqrt{(b-4)(4+b)}+2b}}+\frac{\left(4+\sqrt{(b-4)(4+b)}-b\right)\arctan\left(\frac{4x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)}{\sqrt{(b-4)(4+b)}\,\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}$	124

input
```
int((2*x^2+1)/(4*x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-4-b)^(1/2)*ln(-2*x^2*(-4-b)^(1/2)+x*(4+b)+(-4-b)^(1/2))+1/2/(-4-b)^
(1/2)*ln(-2*x^2*(-4-b)^(1/2)+(-4-b)*x+(-4-b)^(1/2))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx = \left[-\frac{\sqrt{-b-4}\log\left(\frac{4x^4-(b+8)x^2-2(2x^3-x)\sqrt{-b-4}+1}{4x^4+bx^2+1}\right)}{2(b+4)}, \frac{\sqrt{b+4}\arctan\left(\frac{4x^3+(b+2)x}{\sqrt{b+4}}\right)+\sqrt{b+4}\arctan\left(\frac{2x}{\sqrt{b+4}}\right)}{b+4}\right]$$

input
```
integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b - 4)*log((4*x^4 - (b + 8)*x^2 - 2*(2*x^3 - x)*sqrt(-b - 4) +
 1)/(4*x^4 + b*x^2 + 1))/(b + 4), (sqrt(b + 4)*arctan((4*x^3 + (b + 2)*x)/
sqrt(b + 4)) + sqrt(b + 4)*arctan(2*x/sqrt(b + 4)))/(b + 4)]
```

3.39.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.53

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx = -\frac{\sqrt{-\frac{1}{b+4}}\log\left(x^2+x\left(-\frac{b\sqrt{-\frac{1}{b+4}}}{2}-2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2} + \frac{\sqrt{-\frac{1}{b+4}}\log\left(x^2+x\left(\frac{b\sqrt{-\frac{1}{b+4}}}{2}+2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2}$$

input
```
integrate((2*x**2+1)/(4*x**4+b*x**2+1),x)
```

output
```
-sqrt(-1/(b + 4))*log(x**2 + x*(-b*sqrt(-1/(b + 4))/2 - 2*sqrt(-1/(b + 4))
) - 1/2)/2 + sqrt(-1/(b + 4))*log(x**2 + x*(b*sqrt(-1/(b + 4))/2 + 2*sqrt(
-1/(b + 4))) - 1/2)/2
```

3.39.7 Maxima [F]

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4+bx^2+1}\,dx$$

input
```
integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 1)/(4*x^4 + b*x^2 + 1), x)
```

3.39.8 Giac [F]

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4+bx^2+1}\,dx$$

input
```
integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")
```

output
```
sage0*x
```

3.39.9 Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx = -\frac{\operatorname{atan}\left(\frac{-b^3\,x-4\,b^2\,x^3-2\,b^2\,x+16\,b\,x+64\,x^3+32\,x}{(b^2-16)\,\sqrt{b+4}}\right)-\operatorname{atan}\left(\frac{2\,x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

input
```
int((2*x^2 + 1)/(b*x^2 + 4*x^4 + 1),x)
```

output
```
-(atan((32*x + 16*b*x - 2*b^2*x - b^3*x + 64*x^3 - 4*b^2*x^3)/((b^2 - 16)*
(b + 4)^(1/2))) - atan((2*x)/(b + 4)^(1/2)))/(b + 4)^(1/2)
```

3.40 $\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx$

3.40.1 Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx=-\frac{\arctan\left(\frac{\sqrt{4+b}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}+\frac{\arctan\left(\frac{\sqrt{4+b}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

output
```
-arctan((-4*x+(4+b)^(1/2))/(4-b)^(1/2))/(4-b)^(1/2)+arctan((4*x+(4+b)^(1/2
))/(4-b)^(1/2))/(4-b)^(1/2)
```

3.40.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 134 vs. 2(66) = 132.

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.03

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx=\frac{\frac{\left(4+b+\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{-b-\sqrt{-16+b^2}}}\right)}{\sqrt{-b-\sqrt{-16+b^2}}}+\frac{\left(-4-b+\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{-b+\sqrt{-16+b^2}}}\right)}{\sqrt{-b+\sqrt{-16+b^2}}}}{\sqrt{2}\sqrt{-16+b^2}}$$

input
```
Integrate[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

output
```
(((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[-b - Sqrt[-16 + b^2]
]])/Sqrt[-b - Sqrt[-16 + b^2]] + ((-4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqr
t[2]*x)/Sqrt[-b + Sqrt[-16 + b^2]]])/Sqrt[-b + Sqrt[-16 + b^2]])/(Sqrt[2]*
Sqrt[-16 + b^2])
```

3.40.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{-bx^2+4x^4+1}dx$$
$$\Big\downarrow \ 1475$$
$$\frac{1}{4}\int \frac{1}{x^2-\frac{1}{2}\sqrt{b+4}x+\frac{1}{2}}dx+\frac{1}{4}\int \frac{1}{x^2+\frac{1}{2}\sqrt{b+4}x+\frac{1}{2}}dx$$
$$\Big\downarrow \ 1083$$
$$-\frac{1}{2}\int \frac{1}{\frac{b-4}{4}-\left(2x-\frac{\sqrt{b+4}}{2}\right)^2}d\left(2x-\frac{\sqrt{b+4}}{2}\right)-\frac{1}{2}\int \frac{1}{\frac{b-4}{4}-\left(2x+\frac{\sqrt{b+4}}{2}\right)^2}d\left(2x+\frac{\sqrt{b+4}}{2}\right)$$
$$\Big\downarrow \ 217$$
$$\frac{\arctan\left(\frac{2\left(2x-\frac{\sqrt{b+4}}{2}\right)}{\sqrt{4-b}}\right)}{\sqrt{4-b}}+\frac{\arctan\left(\frac{2\left(\frac{\sqrt{b+4}}{2}+2x\right)}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

input
```
Int[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

output
```
ArcTan[(2*(-1/2*Sqrt[4 + b] + 2*x))/Sqrt[4 - b]]/Sqrt[4 - b] + ArcTan[(2*(
Sqrt[4 + b]/2 + 2*x))/Sqrt[4 - b]]/Sqrt[4 - b]
```

3.40.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.40.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\ln\left(2x^{2}\sqrt{b-4}+\left(4-b\right)x-\sqrt{b-4}\right)}{2\sqrt{b-4}}-\frac{\ln\left(2x^{2}\sqrt{b-4}+x\left(b-4\right)-\sqrt{b-4}\right)}{2\sqrt{b-4}}$	66
default	$\frac{\left(4+\sqrt{\left(b-4\right)\left(4+b\right)}+b\right)\arctan\left(\frac{4x}{\sqrt{-2\sqrt{\left(b-4\right)\left(4+b\right)}-2b}}\right)}{\sqrt{\left(b-4\right)\left(4+b\right)}\,\sqrt{-2\sqrt{\left(b-4\right)\left(4+b\right)}-2b}}+\frac{\left(-4+\sqrt{\left(b-4\right)\left(4+b\right)}-b\right)\arctan\left(\frac{4x}{\sqrt{2\sqrt{\left(b-4\right)\left(4+b\right)}-2b}}\right)}{\sqrt{\left(b-4\right)\left(4+b\right)}\,\sqrt{2\sqrt{\left(b-4\right)\left(4+b\right)}-2b}}$	124

input
```
int((2*x^2+1)/(4*x^4-b*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2/(b-4)^(1/2)*ln(2*x^2*(b-4)^(1/2)+(4-b)*x-(b-4)^(1/2))-1/2/(b-4)^(1/2)*
ln(2*x^2*(b-4)^(1/2)+x*(b-4)-(b-4)^(1/2))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.82

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = \left[\frac{\log\left(\frac{4\,x^4+(b-8)x^2-2\,(2\,x^3-x)\sqrt{b-4}+1}{4\,x^4-bx^2+1}\right)}{2\,\sqrt{b-4}}, \frac{\sqrt{-b+4}\arctan\left(\frac{(4\,x^3-(b-2)x)\sqrt{-b+4}}{b-4}\right)+\sqrt{-b+4}\arctan\left(\frac{2\,\sqrt{-b+4}x}{b-4}\right)}{b-4}\right]$$

input
```
integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="fricas")
```

output
```
[1/2*log((4*x^4 + (b - 8)*x^2 - 2*(2*x^3 - x)*sqrt(b - 4) + 1)/(4*x^4 - b*
x^2 + 1))/sqrt(b - 4), (sqrt(-b + 4)*arctan((4*x^3 - (b - 2)*x)*sqrt(-b +
4)/(b - 4)) + sqrt(-b + 4)*arctan(2*sqrt(-b + 4)*x/(b - 4)))/(b - 4)]
```

3.40.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = \frac{\sqrt{\frac{1}{b-4}}\log\left(x^2+x\left(-\frac{b\sqrt{\frac{1}{b-4}}}{2}+2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2} - \frac{\sqrt{\frac{1}{b-4}}\log\left(x^2+x\left(\frac{b\sqrt{\frac{1}{b-4}}}{2}-2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2}$$

input
```
integrate((2*x**2+1)/(4*x**4-b*x**2+1),x)
```

output
```
sqrt(1/(b - 4))*log(x**2 + x*(-b*sqrt(1/(b - 4))/2 + 2*sqrt(1/(b - 4))) -
1/2)/2 - sqrt(1/(b - 4))*log(x**2 + x*(b*sqrt(1/(b - 4))/2 - 2*sqrt(1/(b -
 4))) - 1/2)/2
```

3.40.7 Maxima [F]

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4-bx^2+1}\,dx$$

input
```
integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 1)/(4*x^4 - b*x^2 + 1), x)
```

3.40.8 Giac [F]

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4-bx^2+1}\,dx$$

input
```
integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="giac")
```

output
```
sage0*x
```

3.40.9 Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.36

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = -\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{b-4}}{2\,x^2-1}\right)}{\sqrt{b-4}}$$

input
```
int((2*x^2 + 1)/(4*x^4 - b*x^2 + 1),x)
```

output
```
-atanh((x*(b - 4)^(1/2))/(2*x^2 - 1))/(b - 4)^(1/2)
```

3.41 $\int \frac{1+2x^2}{1+6x^2+4x^4} dx$

3.41.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{1+2x^2}{1+6x^2+4x^4} dx = \frac{\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}} + \frac{\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

output
```
1/10*arctan(2*x/(1/2*10^(1/2)-1/2*2^(1/2)))*10^(1/2)+1/10*arctan(2*x/(1/2*
10^(1/2)+1/2*2^(1/2)))*10^(1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.84

$$\int \frac{1+2x^2}{1+6x^2+4x^4} dx = \frac{\left(-1+\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{2\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\left(1+\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{2\sqrt{5\left(3+\sqrt{5}\right)}}$$

input
```
Integrate[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

output
```
((-1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]])/(2*Sqrt[5*(3 - Sqrt[5])])
 + ((1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(2*Sqrt[5*(3 + Sqrt[5])
])
```

3.41.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4+6x^2+1}dx$$
$$\Big\downarrow \text{1477}$$
$$\frac{1}{5}\left(5-\sqrt{5}\right)\int \frac{1}{4x^2-\sqrt{5}+3}dx+\frac{1}{5}\left(5+\sqrt{5}\right)\int \frac{1}{4x^2+\sqrt{5}+3}dx$$
$$\Big\downarrow \text{216}$$
$$\frac{\left(5-\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{10\sqrt{3-\sqrt{5}}}+\frac{\left(5+\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{10\sqrt{3+\sqrt{5}}}$$

input
```
Int[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

output
```
((5 - Sqrt[5])*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]])/(10*Sqrt[3 - Sqrt[5]]) + (
(5 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(10*Sqrt[3 + Sqrt[5]])
```

3.41.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

3.41.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{\sqrt{10}\,\arctan\left(\frac{\sqrt{10}\,x}{5}\right)}{10}+\frac{\sqrt{10}\,\arctan\left(\frac{2\sqrt{10}\,x^3}{5}+\frac{4\sqrt{10}\,x}{5}\right)}{10}$	35
default	$\frac{2\left(\sqrt{5}+1\right)\sqrt{5}\,\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}+\frac{2\left(\sqrt{5}-1\right)\sqrt{5}\,\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}$	82

input
```
int((2*x^2+1)/(4*x^4+6*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/10*10^(1/2)*arctan(1/5*10^(1/2)*x)+1/10*10^(1/2)*arctan(2/5*10^(1/2)*x^3
+4/5*10^(1/2)*x)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \frac{1}{10}\sqrt{10}\arctan\left(\frac{2}{5}\sqrt{10}(x^3+2x)\right)+\frac{1}{10}\sqrt{10}\arctan\left(\frac{1}{5}\sqrt{10}x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="fricas")
```

output
```
1/10*sqrt(10)*arctan(2/5*sqrt(10)*(x^3 + 2*x)) + 1/10*sqrt(10)*arctan(1/5*
sqrt(10)*x)
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \frac{\sqrt{10}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{10}x}{5}\right)+2\operatorname{atan}\left(\frac{2\sqrt{10}x^3}{5}+\frac{4\sqrt{10}x}{5}\right)\right)}{20}$$

input
```
integrate((2*x**2+1)/(4*x**4+6*x**2+1),x)
```

output
```
sqrt(10)*(2*atan(sqrt(10)*x/5) + 2*atan(2*sqrt(10)*x**3/5 + 4*sqrt(10)*x/5
))/20
```

3.41.7 Maxima [F]

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4+6\,x^2+1}\,dx$$

input
```
integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 1)/(4*x^4 + 6*x^2 + 1), x)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \frac{1}{10}\,\sqrt{10}\arctan\left(\frac{4\,x}{\sqrt{10}+\sqrt{2}}\right) + \frac{1}{10}\,\sqrt{10}\arctan\left(\frac{4\,x}{\sqrt{10}-\sqrt{2}}\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")
```

output
```
1/10*sqrt(10)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/10*sqrt(10)*arctan(4*x/
(sqrt(10) - sqrt(2)))
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \frac{\sqrt{10}\,\left(\mathrm{atan}\left(\frac{2\,\sqrt{10}\,x^3}{5}+\frac{4\,\sqrt{10}\,x}{5}\right)+\mathrm{atan}\left(\frac{\sqrt{10}\,x}{5}\right)\right)}{10}$$

input
```
int((2*x^2 + 1)/(6*x^2 + 4*x^4 + 1),x)
```

output
```
(10^(1/2)*(atan((4*10^(1/2)*x)/5 + (2*10^(1/2)*x^3)/5) + atan((10^(1/2)*x)
/5)))/10
```

3.42 $\int \frac{1+2x^2}{1+5x^2+4x^4} \, dx$

3.42.1 Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{1+2x^2}{1+5x^2+4x^4} \, dx = \frac{\arctan(x)}{3} + \frac{1}{3}\arctan(2x)$$

output
```
1/3*arctan(x)+1/3*arctan(2*x)
```

3.42.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1+2x^2}{1+5x^2+4x^4} \, dx = -\frac{1}{3}\arctan\left(\frac{3x}{-1+2x^2}\right)$$

input
```
Integrate[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

output
```
-1/3*ArcTan[(3*x)/(-1 + 2*x^2)]
```

3.42.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4+5x^2+1}dx$$

$$\Big\downarrow \text{1477}$$

$$\frac{2}{3}\int \frac{1}{4x^2+1}dx+\frac{4}{3}\int \frac{1}{4x^2+4}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan(x)}{3}+\frac{1}{3}\arctan(2x)$$

input
```
Int[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

output
```
ArcTan[x]/3 + ArcTan[2*x]/3
```

3.42.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

3.42.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(x)}{3}+\frac{\arctan(2x)}{3}$	12
risch	$\frac{\arctan\left(\frac{2x}{3}\right)}{3}+\frac{\arctan\left(\frac{4}{3}x^3+\frac{7}{3}x\right)}{3}$	20
parallelrisch	$-\frac{i\ln(x-i)}{6}+\frac{i\ln(x+i)}{6}-\frac{i\ln\left(x-\frac{i}{2}\right)}{6}+\frac{i\ln\left(x+\frac{i}{2}\right)}{6}$	34

input
```
int((2*x^2+1)/(4*x^4+5*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctan(x)+1/3*arctan(2*x)
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = \frac{1}{3}\arctan\left(\frac{4}{3}x^3+\frac{7}{3}x\right)+\frac{1}{3}\arctan\left(\frac{2}{3}x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")
```

output
```
1/3*arctan(4/3*x^3 + 7/3*x) + 1/3*arctan(2/3*x)
```

3.42.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = \frac{\operatorname{atan}\left(\frac{2x}{3}\right)}{3}+\frac{\operatorname{atan}\left(\frac{4x^3}{3}+\frac{7x}{3}\right)}{3}$$

input
```
integrate((2*x**2+1)/(4*x**4+5*x**2+1),x)
```

output
```
atan(2*x/3)/3 + atan(4*x**3/3 + 7*x/3)/3
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = \frac{1}{3}\arctan(2x) + \frac{1}{3}\arctan(x)$$

input
```
integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")
```

output
```
1/3*arctan(2*x) + 1/3*arctan(x)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = \frac{1}{3}\arctan(2x) + \frac{1}{3}\arctan(x)$$

input
```
integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")
```

output
```
1/3*arctan(2*x) + 1/3*arctan(x)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{1+2x^2}{1+5x^2+4x^4}\,dx = \frac{\operatorname{atan}\left(\frac{2x}{3}\right)}{3} + \frac{\operatorname{atan}\left(\frac{4x^3}{3}+\frac{7x}{3}\right)}{3}$$

input
```
int((2*x^2 + 1)/(5*x^2 + 4*x^4 + 1),x)
```

output
```
atan((2*x)/3)/3 + atan((7*x)/3 + (4*x^3)/3)/3
```

3.43 $\int \frac{1+2x^2}{1+4x^2+4x^4} \, dx$

3.43.1 Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{1+2x^2}{1+4x^2+4x^4} \, dx = \frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

output
```
1/2*arctan(x*2^(1/2))*2^(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+2x^2}{1+4x^2+4x^4} \, dx = \frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

input
```
Integrate[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

output
```
ArcTan[Sqrt[2]*x]/Sqrt[2]
```

3.43.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1380, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4+4x^2+1}dx$$

$$\Big\downarrow \ 1380$$

$$4\int \frac{1}{4(2x^2+1)}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{1}{2x^2+1}dx$$

$$\Big\downarrow \ 216$$

$$\frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

input
```
Int[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

output
```
ArcTan[Sqrt[2]*x]/Sqrt[2]
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.43.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$	12
risch	$\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$	12

input
```
int((2*x^2+1)/(4*x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x*2^(1/2))*2^(1/2)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*x)
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x\right)}{2}$$

input
```
integrate((2*x**2+1)/(4*x**4+4*x**2+1),x)
```

output
```
sqrt(2)*atan(sqrt(2)*x)/2
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*x)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*x)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\right)}{2}$$

input
```
int((2*x^2 + 1)/(4*x^2 + 4*x^4 + 1),x)
```

output
```
(2^(1/2)*atan(2^(1/2)*x))/2
```

3.44 $\int \frac{1+2x^2}{1+3x^2+4x^4} \, dx$

3.44.1 Optimal result

Integrand size = 22, antiderivative size = 38

$$\int \frac{1+2x^2}{1+3x^2+4x^4} \, dx = -\frac{\arctan\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{\arctan\left(\frac{1+4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

output
```
-1/7*arctan(1/7*(1-4*x)*7^(1/2))*7^(1/2)+1/7*arctan(1/7*(1+4*x)*7^(1/2))*7
^(1/2)
```

3.44.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.55

$$\int \frac{1+2x^2}{1+3x^2+4x^4} \, dx = \frac{(-i+\sqrt{7}) \arctan\left(\frac{2x}{\sqrt{\frac{1}{2}(3-i\sqrt{7})}}\right)}{\sqrt{42-14i\sqrt{7}}} + \frac{(i+\sqrt{7}) \arctan\left(\frac{2x}{\sqrt{\frac{1}{2}(3+i\sqrt{7})}}\right)}{\sqrt{42+14i\sqrt{7}}}$$

input
```
Integrate[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

output
```
((-I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 - I*Sqrt[7])/2]])/Sqrt[42 - (14*I)*Sq
rt[7]] + ((I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 + I*Sqrt[7])/2]])/Sqrt[42 + (
14*I)*Sqrt[7]]
```

3.44.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4+3x^2+1}dx$$
$$\Big\downarrow \text{1475}$$
$$\frac{1}{4}\int \frac{1}{x^2-\frac{x}{2}+\frac{1}{2}}dx+\frac{1}{4}\int \frac{1}{x^2+\frac{x}{2}+\frac{1}{2}}dx$$
$$\Big\downarrow \text{1083}$$
$$-\frac{1}{2}\int \frac{1}{-\left(2x-\frac{1}{2}\right)^2-\frac{7}{4}}d\left(2x-\frac{1}{2}\right)-\frac{1}{2}\int \frac{1}{-\left(2x+\frac{1}{2}\right)^2-\frac{7}{4}}d\left(2x+\frac{1}{2}\right)$$
$$\Big\downarrow \text{217}$$
$$\frac{\arctan\left(\frac{2\left(2x-\frac{1}{2}\right)}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\arctan\left(\frac{2\left(2x+\frac{1}{2}\right)}{\sqrt{7}}\right)}{\sqrt{7}}$$

input
```
Int[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

output
```
ArcTan[(2*(-1/2 + 2*x))/Sqrt[7]]/Sqrt[7] + ArcTan[(2*(1/2 + 2*x))/Sqrt[7]]
/Sqrt[7]
```

3.44.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.44.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\sqrt{7}\,\arctan\left(\frac{(4x-1)\sqrt{7}}{7}\right)}{7}+\frac{\arctan\left(\frac{(1+4x)\sqrt{7}}{7}\right)\sqrt{7}}{7}$	34
risch	$\frac{\sqrt{7}\,\arctan\left(\frac{2x\sqrt{7}}{7}\right)}{7}+\frac{\sqrt{7}\,\arctan\left(\frac{4x^3\sqrt{7}}{7}+\frac{5x\sqrt{7}}{7}\right)}{7}$	35

input
```
int((2*x^2+1)/(4*x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/7*7^(1/2)*arctan(1/7*(4*x-1)*7^(1/2))+1/7*arctan(1/7*(1+4*x)*7^(1/2))*7^
(1/2)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx=\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4\,x^3+5\,x)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{2}{7}\sqrt{7}x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="fricas")
```

output
```
1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x^3 + 5*x)) + 1/7*sqrt(7)*arctan(2/7*sqr
t(7)*x)
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx = \frac{\sqrt{7}\cdot\left(2\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}\right)+2\,\mathrm{atan}\left(\frac{4\sqrt{7}x^3}{7}+\frac{5\sqrt{7}x}{7}\right)\right)}{14}$$

input
```
integrate((2*x**2+1)/(4*x**4+3*x**2+1),x)
```

output
```
sqrt(7)*(2*atan(2*sqrt(7)*x/7) + 2*atan(4*sqrt(7)*x**3/7 + 5*sqrt(7)*x/7))
/14
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx = \frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+1)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="maxima")
```

output
```
1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)
*(4*x - 1))
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx = \frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+1)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")
```

output
```
1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)
*(4*x - 1))
```

3.44.9 Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx=\frac{\sqrt{7}\left(\operatorname{atan}\left(\frac{4\sqrt{7}\,x^3}{7}+\frac{5\sqrt{7}\,x}{7}\right)+\operatorname{atan}\left(\frac{2\sqrt{7}\,x}{7}\right)\right)}{7}$$

input
```
int((2*x^2 + 1)/(3*x^2 + 4*x^4 + 1),x)
```

output
```
(7^(1/2)*(atan((5*7^(1/2)*x)/7 + (4*7^(1/2)*x^3)/7) + atan((2*7^(1/2)*x)/7
)))/7
```

3.45 $\int \frac{1+2x^2}{1+2x^2+4x^4} \, dx$

3.45.1 Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{1+2x^2}{1+2x^2+4x^4} \, dx = -\frac{\arctan\left(\frac{1-2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}} + \frac{\arctan\left(\frac{1+2\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

output
```
-1/6*arctan(1/3*(1-2*x*2^(1/2))*3^(1/2))*6^(1/2)+1/6*arctan(1/3*(1+2*x*2^(
1/2))*3^(1/2))*6^(1/2)
```

3.45.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.06

$$\int \frac{1+2x^2}{1+2x^2+4x^4} \, dx = \frac{(-i+\sqrt{3}) \arctan\left(\frac{2x}{\sqrt{1-i\sqrt{3}}}\right)}{2\sqrt{3(1-i\sqrt{3})}} + \frac{(i+\sqrt{3}) \arctan\left(\frac{2x}{\sqrt{1+i\sqrt{3}}}\right)}{2\sqrt{3(1+i\sqrt{3})}}$$

input
```
Integrate[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

output
```
((-I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 - I*Sqrt[3]]])/(2*Sqrt[3*(1 - I*Sqrt[3
])]) + ((I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 + I*Sqrt[3]]])/(2*Sqrt[3*(1 + I*
Sqrt[3])])
```

3.45.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4+2x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{4}\int \frac{1}{x^2-\frac{x}{\sqrt{2}}+\frac{1}{2}}dx+\frac{1}{4}\int \frac{1}{x^2+\frac{x}{\sqrt{2}}+\frac{1}{2}}dx$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2}\int \frac{1}{-\left(2x-\frac{1}{\sqrt{2}}\right)^2-\frac{3}{2}}d\left(2x-\frac{1}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1}{-\left(2x+\frac{1}{\sqrt{2}}\right)^2-\frac{3}{2}}d\left(2x+\frac{1}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\sqrt{\frac{2}{3}}\left(2x-\frac{1}{\sqrt{2}}\right)\right)}{\sqrt{6}}+\frac{\arctan\left(\sqrt{\frac{2}{3}}\left(2x+\frac{1}{\sqrt{2}}\right)\right)}{\sqrt{6}}$$

input
```
Int[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

output
```
ArcTan[Sqrt[2/3]*(-(1/Sqrt[2]) + 2*x)]/Sqrt[6] + ArcTan[Sqrt[2/3]*(1/Sqrt[
2] + 2*x)]/Sqrt[6]
```

3.45.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.45.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{\sqrt{6}\,\arctan\left(\frac{x\sqrt{6}}{3}\right)}{6}+\frac{\sqrt{6}\,\arctan\left(\frac{2x^3\sqrt{6}}{3}+\frac{2x\sqrt{6}}{3}\right)}{6}$	35
default	$\frac{\sqrt{6}\,\arctan\left(\frac{\left(4x-\sqrt{2}\right)\sqrt{6}}{6}\right)}{6}+\frac{\sqrt{6}\,\arctan\left(\frac{\left(4x+\sqrt{2}\right)\sqrt{6}}{6}\right)}{6}$	40

input
```
int((2*x^2+1)/(4*x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*6^(1/2)*arctan(1/3*x*6^(1/2))+1/6*6^(1/2)*arctan(2/3*x^3*6^(1/2)+2/3*x
*6^(1/2))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{2}{3}\,\sqrt{6}\left(x^3+x\right)\right)+\frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{3}\,\sqrt{6}x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(6)*arctan(2/3*sqrt(6)*(x^3 + x)) + 1/6*sqrt(6)*arctan(1/3*sqrt(6)
*x)
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx = \frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}\right)+2\,\mathrm{atan}\left(\frac{2\sqrt{6}x^3}{3}+\frac{2\sqrt{6}x}{3}\right)\right)}{12}$$

input
```
integrate((2*x**2+1)/(4*x**4+2*x**2+1),x)
```

output
```
sqrt(6)*(2*atan(sqrt(6)*x/3) + 2*atan(2*sqrt(6)*x**3/3 + 2*sqrt(6)*x/3))/1
2
```

3.45.7 Maxima [F]

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4+2\,x^2+1}\,dx$$

input
```
integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1), x)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{4}{3}\,\sqrt{3}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x+\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{4}{3}\,\sqrt{3}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x-\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(6)*arctan(4/3*sqrt(3)*(1/4)^(3/4)*(2*x + (1/4)^(1/4))) + 1/6*sqrt
(6)*arctan(4/3*sqrt(3)*(1/4)^(3/4)*(2*x - (1/4)^(1/4)))
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx = \frac{\sqrt{6}\left(\operatorname{atan}\left(\frac{2\sqrt{6}\,x^3}{3}+\frac{2\sqrt{6}\,x}{3}\right)+\operatorname{atan}\left(\frac{\sqrt{6}\,x}{3}\right)\right)}{6}$$

input
```
int((2*x^2 + 1)/(2*x^2 + 4*x^4 + 1),x)
```

output
```
(6^(1/2)*(atan((2*6^(1/2)*x)/3 + (2*6^(1/2)*x^3)/3) + atan((6^(1/2)*x)/3))
)/6
```

3.46 $\int \frac{1+2x^2}{1+x^2+4x^4}\,dx$

3.46.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx=-\frac{\arctan\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\arctan\left(\frac{\sqrt{3}+4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

output
```
-1/5*arctan(1/5*(-4*x+3^(1/2))*5^(1/2))*5^(1/2)+1/5*arctan(1/5*(4*x+3^(1/2
))*5^(1/2))*5^(1/2)
```

3.46.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx=\frac{\left(-3i+\sqrt{15}\right)\arctan\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{15}\right)}}\right)}{\sqrt{30-30i\sqrt{15}}}+\frac{\left(3i+\sqrt{15}\right)\arctan\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{15}\right)}}\right)}{\sqrt{30+30i\sqrt{15}}}$$

input
```
Integrate[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

output
```
((-3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 - I*Sqrt[15])/2]])/Sqrt[30 - (30*I
)*Sqrt[15]] + ((3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 + I*Sqrt[15])/2]])/Sq
rt[30 + (30*I)*Sqrt[15]]
```

3.46.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4+x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{4}\int \frac{1}{x^2-\frac{\sqrt{3}x}{2}+\frac{1}{2}}dx+\frac{1}{4}\int \frac{1}{x^2+\frac{\sqrt{3}x}{2}+\frac{1}{2}}dx$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2}\int \frac{1}{-\left(2x-\frac{\sqrt{3}}{2}\right)^2-\frac{5}{4}}d\left(2x-\frac{\sqrt{3}}{2}\right)-\frac{1}{2}\int \frac{1}{-\left(2x+\frac{\sqrt{3}}{2}\right)^2-\frac{5}{4}}d\left(2x+\frac{\sqrt{3}}{2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2\left(2x-\frac{\sqrt{3}}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\arctan\left(\frac{2\left(2x+\frac{\sqrt{3}}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

input
```
Int[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

output
```
ArcTan[(2*(-1/2*Sqrt[3] + 2*x))/Sqrt[5]]/Sqrt[5] + ArcTan[(2*(Sqrt[3]/2 +
2*x))/Sqrt[5]]/Sqrt[5]
```

3.46.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.46.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\sqrt{5}\,\arctan\left(\frac{2x\sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5}\,\arctan\left(\frac{4x^{3}\sqrt{5}}{5}+\frac{3x\sqrt{5}}{5}\right)}{5}$	35
default	$\frac{\arctan\left(\frac{\left(4x+\sqrt{3}\right)\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\sqrt{5}\,\arctan\left(\frac{\left(4x-\sqrt{3}\right)\sqrt{5}}{5}\right)}{5}$	40

input
```
int((2*x^2+1)/(4*x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/5*5^(1/2)*arctan(2/5*x*5^(1/2))+1/5*5^(1/2)*arctan(4/5*x^3*5^(1/2)+3/5*x
*5^(1/2))
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx = \frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}(4x^3+3x)\right)+\frac{1}{5}\sqrt{5}\arctan\left(\frac{2}{5}\sqrt{5}x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/5*sqrt(5)*arctan(1/5*sqrt(5)*(4*x^3 + 3*x)) + 1/5*sqrt(5)*arctan(2/5*sqr
t(5)*x)
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx = \frac{\sqrt{5}\cdot\left(2\operatorname{atan}\left(\frac{2\sqrt{5}x}{5}\right)+2\operatorname{atan}\left(\frac{4\sqrt{5}x^3}{5}+\frac{3\sqrt{5}x}{5}\right)\right)}{10}$$

input
```
integrate((2*x**2+1)/(4*x**4+x**2+1),x)
```

output
```
sqrt(5)*(2*atan(2*sqrt(5)*x/5) + 2*atan(4*sqrt(5)*x**3/5 + 3*sqrt(5)*x/5))
/10
```

3.46.7 Maxima [F]

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx = \int \frac{2x^2+1}{4x^4+x^2+1}\,dx$$

input
```
integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1), x)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx = \frac{1}{5}\sqrt{5}\arctan\left(\frac{2}{5}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x+\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{5}\sqrt{5}\arctan\left(\frac{2}{5}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x-\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")
```

output
```
1/5*sqrt(5)*arctan(2/5*sqrt(10)*(1/4)^(3/4)*(4*x + sqrt(6)*(1/4)^(1/4))) +
 1/5*sqrt(5)*arctan(2/5*sqrt(10)*(1/4)^(3/4)*(4*x - sqrt(6)*(1/4)^(1/4)))
```

3.46.9 Mupad [B] (verification not implemented)

Time = 13.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx = \frac{\sqrt{5}\left(\operatorname{atan}\left(\frac{4\sqrt{5}\,x^3}{5}+\frac{3\sqrt{5}\,x}{5}\right)+\operatorname{atan}\left(\frac{2\sqrt{5}\,x}{5}\right)\right)}{5}$$

input
```
int((2*x^2 + 1)/(x^2 + 4*x^4 + 1),x)
```

output
```
(5^(1/2)*(atan((3*5^(1/2)*x)/5 + (4*5^(1/2)*x^3)/5) + atan((2*5^(1/2)*x)/5
)))/5
```

3.47 $\int \frac{1+2x^2}{1+4x^4} dx$

3.47.1 Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1+2x^2}{1+4x^4} dx = -\frac{1}{2}\arctan(1-2x)+\frac{1}{2}\arctan(1+2x)$$

output
```
1/2*arctan(-1+2*x)+1/2*arctan(1+2*x)
```

3.47.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+2x^2}{1+4x^4} dx = -\frac{1}{2}\arctan\left(\frac{2x}{-1+2x^2}\right)$$

input
```
Integrate[(1 + 2*x^2)/(1 + 4*x^4),x]
```

output
```
-1/2*ArcTan[(2*x)/(-1 + 2*x^2)]
```

3.47.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1476, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4+1}dx$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{4}\int \frac{1}{x^2-x+\frac{1}{2}}dx+\frac{1}{4}\int \frac{1}{x^2+x+\frac{1}{2}}dx$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\int \frac{1}{-(1-2x)^2-1}d(1-2x)-\frac{1}{2}\int \frac{1}{-(2x+1)^2-1}d(2x+1)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\arctan(2x+1)-\frac{1}{2}\arctan(1-2x)$$

input
```
Int[(1 + 2*x^2)/(1 + 4*x^4),x]
```

output
```
-1/2*ArcTan[1 - 2*x] + ArcTan[1 + 2*x]/2
```

3.47.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.47.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result
risch	$\frac{\arctan(x)}{2}+\frac{\arctan(2x^3+x)}{2}$
default	$\frac{\arctan(2x-1)}{2}+\frac{\arctan(1+2x)}{2}$
parallelrisch	$-\frac{i\ln(x-\frac{1}{2}-\frac{i}{2})}{4}+\frac{i\ln(x-\frac{1}{2}+\frac{i}{2})}{4}-\frac{i\ln(x+\frac{1}{2}-\frac{i}{2})}{4}+\frac{i\ln(x+\frac{1}{2}+\frac{i}{2})}{4}$
meijerg	$\frac{\sqrt{2}\left(\frac{x^3\sqrt{2}\,\ln\left(1-2\left(x^4\right)^{\frac{1}{4}}+2\sqrt{x^4}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\left(x^4\right)^{\frac{1}{4}}}{1-\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+2\left(x^4\right)^{\frac{1}{4}}+2\sqrt{x^4}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\left(x^4\right)^{\frac{1}{4}}}{1+\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}\right)}{8}+ -$ $\sqrt{}$

input
```
int((2*x^2+1)/(4*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)+1/2*arctan(2*x^3+x)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+2x^2}{1+4x^4}\,dx=\frac{1}{2}\arctan\left(2\,x^3+x\right)+\frac{1}{2}\arctan\left(x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+1),x, algorithm="fricas")
```

output
```
1/2*arctan(2*x^3 + x) + 1/2*arctan(x)
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1+2x^2}{1+4x^4}\,dx = \frac{\operatorname{atan}(x)}{2} + \frac{\operatorname{atan}(2x^3+x)}{2}$$

input
```
integrate((2*x**2+1)/(4*x**4+1),x)
```

output
```
atan(x)/2 + atan(2*x**3 + x)/2
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+2x^2}{1+4x^4}\,dx = \frac{1}{2}\arctan(2\,x+1) + \frac{1}{2}\arctan(2\,x-1)$$

input
```
integrate((2*x^2+1)/(4*x^4+1),x, algorithm="maxima")
```

output
```
1/2*arctan(2*x + 1) + 1/2*arctan(2*x - 1)
```

3.47.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{1+2x^2}{1+4x^4}\,dx = \frac{1}{2}\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{2}\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

input
```
integrate((2*x^2+1)/(4*x^4+1),x, algorithm="giac")
```

output
```
1/2*arctan(2*sqrt(2)*(1/4)^(3/4)*(2*x + sqrt(2)*(1/4)^(1/4))) + 1/2*arctan
(2*sqrt(2)*(1/4)^(3/4)*(2*x - sqrt(2)*(1/4)^(1/4)))
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+2x^2}{1+4x^4}\,dx = \frac{\operatorname{atan}(2\,x^3+x)}{2} + \frac{\operatorname{atan}(x)}{2}$$

input
```
int((2*x^2 + 1)/(4*x^4 + 1),x)
```

output
```
atan(x + 2*x^3)/2 + atan(x)/2
```

3.48 $\int \frac{1+2x^2}{1-x^2+4x^4} \, dx$

3.48.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{1+2x^2}{1-x^2+4x^4} \, dx = -\frac{\arctan\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{5}+4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/3*arctan(1/3*(-4*x+5^(1/2))*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(4*x+5^(1/2
))*3^(1/2))*3^(1/2)
```

3.48.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.20

$$\int \frac{1+2x^2}{1-x^2+4x^4} \, dx = \frac{\left(-5i+\sqrt{15}\right)\arctan\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1-i\sqrt{15}\right)}} + \frac{\left(5i+\sqrt{15}\right)\arctan\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1+i\sqrt{15}\right)}}$$

input
```
Integrate[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

output
```
((-5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 - I*Sqrt[15])/2]])/Sqrt[30*(-1 -
I*Sqrt[15])] + ((5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 + I*Sqrt[15])/2]])/
Sqrt[30*(-1 + I*Sqrt[15])]
```

3.48.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{1}{4}\int \frac{1}{x^2-\frac{\sqrt{5}x}{2}+\frac{1}{2}}dx+\frac{1}{4}\int \frac{1}{x^2+\frac{\sqrt{5}x}{2}+\frac{1}{2}}dx$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{1}{2}\int \frac{1}{-\left(2x-\frac{\sqrt{5}}{2}\right)^2-\frac{3}{4}}d\left(2x-\frac{\sqrt{5}}{2}\right)-\frac{1}{2}\int \frac{1}{-\left(2x+\frac{\sqrt{5}}{2}\right)^2-\frac{3}{4}}d\left(2x+\frac{\sqrt{5}}{2}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\arctan\left(\frac{2\left(2x-\frac{\sqrt{5}}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2\left(2x+\frac{\sqrt{5}}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

output
```
ArcTan[(2*(-1/2*Sqrt[5] + 2*x))/Sqrt[3]]/Sqrt[3] + ArcTan[(2*(Sqrt[5]/2 +
2*x))/Sqrt[3]]/Sqrt[3]
```

3.48.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.48.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{2x\sqrt{3}}{3}\right)}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{4x^{3}\sqrt{3}}{3}+\frac{x\sqrt{3}}{3}\right)}{3}$	35
default	$\frac{\arctan\left(\frac{\left(4x+\sqrt{5}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(4x-\sqrt{5}\right)\sqrt{3}}{3}\right)}{3}$	40

input
```
int((2*x^2+1)/(4*x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arctan(2/3*x*3^(1/2))+1/3*3^(1/2)*arctan(4/3*x^3*3^(1/2)+1/3*x
*3^(1/2))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(4x^3+x)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(4*x^3 + x)) + 1/3*sqrt(3)*arctan(2/3*sqrt(
3)*x)
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \frac{\sqrt{3}\cdot\left(2\,\text{atan}\left(\frac{2\sqrt{3}x}{3}\right) + 2\,\text{atan}\left(\frac{4\sqrt{3}x^3}{3} + \frac{\sqrt{3}x}{3}\right)\right)}{6}$$

input
```
integrate((2*x**2+1)/(4*x**4-x**2+1),x)
```

output
```
sqrt(3)*(2*atan(2*sqrt(3)*x/3) + 2*atan(4*sqrt(3)*x**3/3 + sqrt(3)*x/3))/6
```

3.48.7 Maxima [F]

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \int \frac{2x^2+1}{4x^4-x^2+1}\,dx$$

input
```
integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1), x)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{6}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x+\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{6}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x-\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

input
```
integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(2/3*sqrt(6)*(1/4)^(3/4)*(4*x + sqrt(10)*(1/4)^(1/4))) +
 1/3*sqrt(3)*arctan(2/3*sqrt(6)*(1/4)^(3/4)*(4*x - sqrt(10)*(1/4)^(1/4)))
```

3.48.9 Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{4\sqrt{3}\,x^3}{3}+\frac{\sqrt{3}\,x}{3}\right)+\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}\right)\right)}{3}$$

input
```
int((2*x^2 + 1)/(4*x^4 - x^2 + 1),x)
```

output
```
(3^(1/2)*(atan((3^(1/2)*x)/3 + (4*3^(1/2)*x^3)/3) + atan((2*3^(1/2)*x)/3))
)/3
```

3.49 $\int \frac{1+2x^2}{1-2x^2+4x^4} dx$

3.49.1 Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \frac{1+2x^2}{1-2x^2+4x^4} dx = -\frac{\arctan\left(\sqrt{3}-2\sqrt{2}x\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{3}+2\sqrt{2}x\right)}{\sqrt{2}}$$

output
```
1/2*arctan(2*x*2^(1/2)-3^(1/2))*2^(1/2)+1/2*arctan(2*x*2^(1/2)+3^(1/2))*2^
(1/2)
```

3.49.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.25

$$\int \frac{1+2x^2}{1-2x^2+4x^4} dx = \frac{\left(-3i+\sqrt{3}\right)\arctan\left(\frac{2x}{\sqrt{-1-i\sqrt{3}}}\right)}{2\sqrt{3\left(-1-i\sqrt{3}\right)}}+\frac{\left(3i+\sqrt{3}\right)\arctan\left(\frac{2x}{\sqrt{-1+i\sqrt{3}}}\right)}{2\sqrt{3\left(-1+i\sqrt{3}\right)}}$$

input
```
Integrate[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

output
```
((-3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 - I*Sqrt[3]]])/(2*Sqrt[3*(-1 - I*Sq
rt[3])]) + ((3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 + I*Sqrt[3]]])/(2*Sqrt[3*
(-1 + I*Sqrt[3])])
```

3.49.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4-2x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{4}\int \frac{1}{x^2-\sqrt{\frac{3}{2}}x+\frac{1}{2}}dx+\frac{1}{4}\int \frac{1}{x^2+\sqrt{\frac{3}{2}}x+\frac{1}{2}}dx$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2}\int \frac{1}{-\left(2x-\sqrt{\frac{3}{2}}\right)^2-\frac{1}{2}}d\left(2x-\sqrt{\frac{3}{2}}\right)-\frac{1}{2}\int \frac{1}{-\left(2x+\sqrt{\frac{3}{2}}\right)^2-\frac{1}{2}}d\left(2x+\sqrt{\frac{3}{2}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\sqrt{2}\left(2x-\sqrt{\frac{3}{2}}\right)\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}\left(2x+\sqrt{\frac{3}{2}}\right)\right)}{\sqrt{2}}$$

input
```
Int[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

output
```
ArcTan[Sqrt[2]*(-Sqrt[3/2] + 2*x)]/Sqrt[2] + ArcTan[Sqrt[2]*(Sqrt[3/2] + 2
*x)]/Sqrt[2]
```

3.49.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.49.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}+\frac{\sqrt{2}\,\arctan\left(2x^{3}\sqrt{2}\right)}{2}$	27
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\left(4x-\sqrt{6}\right)\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\frac{\left(4x+\sqrt{6}\right)\sqrt{2}}{2}\right)}{2}$	40

input
```
int((2*x^2+1)/(4*x^4-2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x*2^(1/2))*2^(1/2)+1/2*2^(1/2)*arctan(2*x^3*2^(1/2))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx=\frac{1}{2}\,\sqrt{2}\arctan\left(2\,\sqrt{2}x^3\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(2*sqrt(2)*x^3) + 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx = \frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(\sqrt{2}x\right)+2\,\mathrm{atan}\left(2\sqrt{2}x^3\right)\right)}{4}$$

input
```
integrate((2*x**2+1)/(4*x**4-2*x**2+1),x)
```

output
```
sqrt(2)*(2*atan(sqrt(2)*x) + 2*atan(2*sqrt(2)*x**3))/4
```

3.49.7 Maxima [F]

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4-2\,x^2+1}\,dx$$

input
```
integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1), x)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

input
```
integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(4*(1/4)^(3/4)*(2*x + sqrt(3)*(1/4)^(1/4))) + 1/2*sqrt(2
)*arctan(4*(1/4)^(3/4)*(2*x - sqrt(3)*(1/4)^(1/4)))
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx=\frac{\sqrt{2}\,\left(\mathrm{atan}\left(\sqrt{2}\,x\right)+\mathrm{atan}\left(2\,\sqrt{2}\,x^3\right)\right)}{2}$$

input
```
int((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1),x)
```

output
```
(2^(1/2)*(atan(2^(1/2)*x) + atan(2*2^(1/2)*x^3)))/2
```

3.50 $\int \frac{1+2x^2}{1-3x^2+4x^4} \, dx$

3.50.1 Optimal result

Integrand size = 22, antiderivative size = 23

$$\int \frac{1+2x^2}{1-3x^2+4x^4} \, dx = -\arctan\left(\sqrt{7}-4x\right)+\arctan\left(\sqrt{7}+4x\right)$$

output
```
arctan(4*x-7^(1/2))+arctan(4*x+7^(1/2))
```

3.50.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1+2x^2}{1-3x^2+4x^4} \, dx = -\arctan\left(\frac{x}{-1+2x^2}\right)$$

input
```
Integrate[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

output
```
-ArcTan[x/(-1 + 2*x^2)]
```

3.50.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4-3x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{4}\int \frac{1}{x^2-\frac{\sqrt{7}x}{2}+\frac{1}{2}}dx+\frac{1}{4}\int \frac{1}{x^2+\frac{\sqrt{7}x}{2}+\frac{1}{2}}dx$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2}\int \frac{1}{-\left(2x-\frac{\sqrt{7}}{2}\right)^2-\frac{1}{4}}d\left(2x-\frac{\sqrt{7}}{2}\right)-\frac{1}{2}\int \frac{1}{-\left(2x+\frac{\sqrt{7}}{2}\right)^2-\frac{1}{4}}d\left(2x+\frac{\sqrt{7}}{2}\right)$$

$$\Big\downarrow \text{217}$$

$$\arctan\left(2\left(2x-\frac{\sqrt{7}}{2}\right)\right)+\arctan\left(2\left(2x+\frac{\sqrt{7}}{2}\right)\right)$$

input
```
Int[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

output
```
ArcTan[2*(-1/2*Sqrt[7] + 2*x)] + ArcTan[2*(Sqrt[7]/2 + 2*x)]
```

3.50.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.50.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
risch	$\arctan\left(4x^{3}-x\right)+\arctan\left(2x\right)$	16
default	$\arctan\left(4x-\sqrt{7}\right)+\arctan\left(4x+\sqrt{7}\right)$	20
parallelrisch	$-\frac{i\ln\left(x^{2}-\frac{1}{2}ix-\frac{1}{2}\right)}{2}+\frac{i\ln\left(x^{2}+\frac{1}{2}ix-\frac{1}{2}\right)}{2}$	28

input
```
int((2*x^2+1)/(4*x^4-3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctan(4*x^3-x)+arctan(2*x)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = \arctan\left(4\,x^{3}-x\right)+\arctan\left(2\,x\right)$$

input
```
integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="fricas")
```

output
```
arctan(4*x^3 - x) + arctan(2*x)
```

3.50.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = \operatorname{atan}(2x) + \operatorname{atan}(4x^3-x)$$

input
```
integrate((2*x**2+1)/(4*x**4-3*x**2+1),x)
```

output
```
atan(2*x) + atan(4*x**3 - x)
```

3.50.7 Maxima [F]

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4-3\,x^2+1}\,dx$$

input
```
integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1), x)
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = \arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x+\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x-\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

input
```
integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")
```

output
```
arctan(2*sqrt(2)*(1/4)^(3/4)*(4*x + sqrt(14)*(1/4)^(1/4))) + arctan(2*sqrt
(2)*(1/4)^(3/4)*(4*x - sqrt(14)*(1/4)^(1/4)))
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = \operatorname{atan}(2\,x) - \operatorname{atan}\left(x-4\,x^3\right)$$

input
```
int((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1),x)
```

output
```
atan(2*x) - atan(x - 4*x^3)
```

3.51 $\int \frac{1+2x^2}{1-4x^2+4x^4} \, dx$

3.51.1 Optimal result

Integrand size = 22, antiderivative size = 11

$$\int \frac{1+2x^2}{1-4x^2+4x^4} \, dx = \frac{x}{1-2x^2}$$

output
```
x/(-2*x^2+1)
```

3.51.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1+2x^2}{1-4x^2+4x^4} \, dx = -\frac{x}{-1+2x^2}$$

input
```
Integrate[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

output
```
-(x/(-1 + 2*x^2))
```

3.51.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1380, 27, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4-4x^2+1}\,dx$$

$$\Big\downarrow \text{1380}$$

$$4\int \frac{2x^2+1}{4(1-2x^2)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{2x^2+1}{(1-2x^2)^2}dx$$

$$\Big\downarrow \text{297}$$

$$\frac{x}{1-2x^2}$$

input
```
Int[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

output
```
x/(1 - 2*x^2)
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.51.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{x}{2\left(x^{2}-\frac{1}{2}\right)}$	11
risch	$-\frac{x}{2\left(x^{2}-\frac{1}{2}\right)}$	11
gosper	$-\frac{x}{2x^{2}-1}$	13
norman	$-\frac{x}{2x^{2}-1}$	13
parallelrisch	$-\frac{x}{2x^{2}-1}$	13

input
```
int((2*x^2+1)/(4*x^4-4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^2-1/2)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx = -\frac{x}{2\,x^2-1}$$

input
```
integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="fricas")
```

output
```
-x/(2*x^2 - 1)
```

3.51.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx = -\frac{x}{2x^2-1}$$

input
```
integrate((2*x**2+1)/(4*x**4-4*x**2+1),x)
```

output
```
-x/(2*x**2 - 1)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx = -\frac{x}{2\,x^2-1}$$

input
```
integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="maxima")
```

output
```
-x/(2*x^2 - 1)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx = -\frac{x}{2\,x^2-1}$$

input
```
integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")
```

output
```
-x/(2*x^2 - 1)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx = -\frac{x}{2\,\left(x^2-\frac{1}{2}\right)}$$

input
```
int((2*x^2 + 1)/(4*x^4 - 4*x^2 + 1),x)
```

output
```
-x/(2*(x^2 - 1/2))
```

3.52 $\int \frac{1+2x^2}{1-5x^2+4x^4} \, dx$

3.52.1 Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{1+2x^2}{1-5x^2+4x^4} \, dx = -\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(1+x)+\frac{1}{2}\log(1+2x)$$

output
```
-1/2*ln(1-2*x)+1/2*ln(1-x)-1/2*ln(1+x)+1/2*ln(1+2*x)
```

3.52.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1+2x^2}{1-5x^2+4x^4} \, dx = -\frac{1}{2}\log\left(1-x-2x^2\right)+\frac{1}{2}\log\left(1+x-2x^2\right)$$

input
```
Integrate[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

output
```
-1/2*Log[1 - x - 2*x^2] + Log[1 + x - 2*x^2]/2
```

3.52.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4-5x^2+1}dx$$
$$\Big\downarrow \text{1475}$$
$$\frac{1}{4}\int \frac{1}{x^2-\frac{3x}{2}+\frac{1}{2}}dx+\frac{1}{4}\int \frac{1}{x^2+\frac{3x}{2}+\frac{1}{2}}dx$$
$$\Big\downarrow \text{1081}$$
$$\frac{1}{4}\int \left(\frac{4}{1-2x}-\frac{2}{1-x}\right)dx+\frac{1}{4}\int \left(\frac{4}{2x+1}-\frac{2}{x+1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}(2\log(1-x)-2\log(1-2x))+\frac{1}{4}(2\log(2x+1)-2\log(x+1))$$

input
```
Int[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

output
```
(-2*Log[1 - 2*x] + 2*Log[1 - x])/4 + (-2*Log[1 + x] + 2*Log[1 + 2*x])/4
```

3.52.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.52.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{\ln(2x^2-x-1)}{2}-\frac{\ln(2x^2+x-1)}{2}$	26
parallelrisch	$\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}-\frac{\ln(x-\frac{1}{2})}{2}+\frac{\ln(x+\frac{1}{2})}{2}$	26
default	$\frac{\ln(1+2x)}{2}-\frac{\ln(2x-1)}{2}-\frac{\ln(x+1)}{2}+\frac{\ln(x-1)}{2}$	30
norman	$\frac{\ln(1+2x)}{2}-\frac{\ln(2x-1)}{2}-\frac{\ln(x+1)}{2}+\frac{\ln(x-1)}{2}$	30

input
```
int((2*x^2+1)/(4*x^4-5*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(2*x^2-x-1)-1/2*ln(2*x^2+x-1)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = -\frac{1}{2}\log\left(2\,x^2+x-1\right)+\frac{1}{2}\log\left(2\,x^2-x-1\right)$$

input
```
integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="fricas")
```

output
```
-1/2*log(2*x^2 + x - 1) + 1/2*log(2*x^2 - x - 1)
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = \frac{\log\left(x^2-\frac{x}{2}-\frac{1}{2}\right)}{2} - \frac{\log\left(x^2+\frac{x}{2}-\frac{1}{2}\right)}{2}$$

input
```
integrate((2*x**2+1)/(4*x**4-5*x**2+1),x)
```

output
```
log(x**2 - x/2 - 1/2)/2 - log(x**2 + x/2 - 1/2)/2
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = \frac{1}{2}\log(2x+1) - \frac{1}{2}\log(2x-1) - \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)$$

input
```
integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="maxima")
```

output
```
1/2*log(2*x + 1) - 1/2*log(2*x - 1) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = \frac{1}{2}\log(|2x+1|) - \frac{1}{2}\log(|2x-1|) - \frac{1}{2}\log(|x+1|) + \frac{1}{2}\log(|x-1|)$$

input
```
integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
```

output
```
1/2*log(abs(2*x + 1)) - 1/2*log(abs(2*x - 1)) - 1/2*log(abs(x + 1)) + 1/2*
log(abs(x - 1))
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.36

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = -\mathrm{atanh}\left(\frac{x}{2\,x^2-1}\right)$$

input
```
int((2*x^2 + 1)/(4*x^4 - 5*x^2 + 1),x)
```

output
```
-atanh(x/(2*x^2 - 1))
```

3.53 $\int \frac{1+2x^2}{1-6x^2+4x^4} \, dx$

3.53.1 Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \frac{1+2x^2}{1-6x^2+4x^4} \, dx = \frac{\operatorname{arctanh}\left(\sqrt{5}-2\sqrt{2}x\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\sqrt{5}+2\sqrt{2}x\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(2*x*2^(1/2)-5^(1/2))*2^(1/2)-1/2*arctanh(2*x*2^(1/2)+5^(1/2))
*2^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1+2x^2}{1-6x^2+4x^4} \, dx = \frac{\log\left(1+\sqrt{2}x-2x^2\right) - \log\left(-1+\sqrt{2}x+2x^2\right)}{2\sqrt{2}}$$

input
```
Integrate[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

output
```
(Log[1 + Sqrt[2]*x - 2*x^2] - Log[-1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])
```

3.53.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(44) = 88.

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.48, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{4x^4-6x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{4}\int \frac{1}{x^2-\sqrt{\frac{5}{2}}x+\frac{1}{2}}dx+\frac{1}{4}\int \frac{1}{x^2+\sqrt{\frac{5}{2}}x+\frac{1}{2}}dx$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{4}\int \left(-\frac{4\sqrt{2}}{4x+\sqrt{10}+\sqrt{2}}-\frac{4\sqrt{2}}{\sqrt{2}\left(1-\sqrt{5}\right)-4x}\right)dx+\frac{1}{4}\int \left(-\frac{4\sqrt{2}}{4x+\sqrt{2}\left(1-\sqrt{5}\right)}-\frac{4\sqrt{2}}{-4x+\sqrt{10}+\sqrt{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\sqrt{2}\log\left(-4x+\sqrt{10}+\sqrt{2}\right)-\sqrt{2}\log\left(-4x-\sqrt{2}\left(1-\sqrt{5}\right)\right)\right)+\frac{1}{4}\left(\sqrt{2}\log\left(4x-\sqrt{2}\left(1-\sqrt{5}\right)\right)-\sqrt{2}\log\left(4x+\sqrt{10}+\sqrt{2}\right)\right)$$

input
```
Int[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

output
```
(Sqrt[2]*Log[Sqrt[2] + Sqrt[10] - 4*x] - Sqrt[2]*Log[-(Sqrt[2]*(1 - Sqrt[5
])) - 4*x]])/4 + (-(Sqrt[2]*Log[Sqrt[2] + Sqrt[10] + 4*x]) + Sqrt[2]*Log[-(
Sqrt[2]*(1 - Sqrt[5])) + 4*x]])/4
```

3.53.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.53.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{\sqrt{2}\,\ln\left(-x\sqrt{2}+2x^2-1\right)}{4}-\frac{\sqrt{2}\,\ln\left(x\sqrt{2}+2x^2-1\right)}{4}$	39
default	$-\frac{2\left(-5+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}-\frac{2\sqrt{5}\,\left(5+\sqrt{5}\right)\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}$	82

input
```
int((2*x^2+1)/(4*x^4-6*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*ln(-x*2^(1/2)+2*x^2-1)-1/4*2^(1/2)*ln(x*2^(1/2)+2*x^2-1)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{4\,x^4-2\,x^2-2\,\sqrt{2}(2\,x^3-x)+1}{4\,x^4-6\,x^2+1}\right)$$

input
```
integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((4*x^4 - 2*x^2 - 2*sqrt(2)*(2*x^3 - x) + 1)/(4*x^4 - 6*x^2
 + 1))
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx = \frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x}{2}-\frac{1}{2}\right)}{4} - \frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{2}-\frac{1}{2}\right)}{4}$$

input
```
integrate((2*x**2+1)/(4*x**4-6*x**2+1),x)
```

output
```
sqrt(2)*log(x**2 - sqrt(2)*x/2 - 1/2)/4 - sqrt(2)*log(x**2 + sqrt(2)*x/2 -
 1/2)/4
```

3.53.7 Maxima [F]

$$\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx = \int \frac{2\,x^2+1}{4\,x^4-6\,x^2+1}\,dx$$

input
```
integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1), x)
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(35) = 70.

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.75

$$\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx = -\frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)+\frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right) -\frac{1}{4}\sqrt{2}\log\left(\left|x-\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)+\frac{1}{4}\sqrt{2}\log\left(\left|x-\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right)$$

input
```
integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/4*sqrt(2)*log(ab
s(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10)
+ 1/4*sqrt(2))) + 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2)))
```

3.53.9 Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2\,x^2-1}\right)}{2}$$

input
```
int((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*x)/(2*x^2 - 1)))/2
```

3.54 $\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx$

3.54.1 Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx=-\frac{\log\left(1-\sqrt{4-b}x+2x^2\right)}{2\sqrt{4-b}}+\frac{\log\left(1+\sqrt{4-b}x+2x^2\right)}{2\sqrt{4-b}}$$

output
```
-1/2*ln(1+2*x^2-x*(4-b)^(1/2))/(4-b)^(1/2)+1/2*ln(1+2*x^2+x*(4-b)^(1/2))/(
4-b)^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.92

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx=\frac{\frac{\left(4+b-\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{b-\sqrt{-16+b^2}}}\right)}{\sqrt{b-\sqrt{-16+b^2}}}-\frac{\left(4+b+\sqrt{-16+b^2}\right)\arctan\left(\frac{2\sqrt{2}x}{\sqrt{b+\sqrt{-16+b^2}}}\right)}{\sqrt{b+\sqrt{-16+b^2}}}}{\sqrt{2}\sqrt{-16+b^2}}$$

input
```
Integrate[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

output
```
(((4 + b - Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]
])/Sqrt[b - Sqrt[-16 + b^2]] - ((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2
]*x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[
-16 + b^2])
```

3.54.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{bx^2+4x^4+1} \, dx$$

$$\Big\downarrow \text{1478}$$

$$-\frac{\int -\frac{\sqrt{4-b}-4x}{2x^2-\sqrt{4-b}x+1}dx}{2\sqrt{4-b}}-\frac{\int -\frac{4x+\sqrt{4-b}}{2x^2+\sqrt{4-b}x+1}dx}{2\sqrt{4-b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{4-b}-4x}{2x^2-\sqrt{4-b}x+1}dx}{2\sqrt{4-b}}+\frac{\int \frac{4x+\sqrt{4-b}}{2x^2+\sqrt{4-b}x+1}dx}{2\sqrt{4-b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(\sqrt{4-b}x+2x^2+1\right)}{2\sqrt{4-b}}-\frac{\log\left(-\sqrt{4-b}x+2x^2+1\right)}{2\sqrt{4-b}}$$

input
```
Int[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

output
```
-1/2*Log[1 - Sqrt[4 - b]*x + 2*x^2]/Sqrt[4 - b] + Log[1 + Sqrt[4 - b]*x +
2*x^2]/(2*Sqrt[4 - b])
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.54.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

method	result	size
risch	$-\frac{\ln\left(-2x^2\sqrt{4-b}+(4-b)x-\sqrt{4-b}\right)}{2\sqrt{4-b}}+\frac{\ln\left(-2x^2\sqrt{4-b}+x(b-4)-\sqrt{4-b}\right)}{2\sqrt{4-b}}$	78
default	$\frac{\left(-4-\sqrt{(b-4)(4+b)}-b\right)\arctan\left(\frac{4x}{\sqrt{2\sqrt{(b-4)(4+b)}+2b}}\right)}{\sqrt{(b-4)(4+b)}\sqrt{2\sqrt{(b-4)(4+b)}+2b}}+\frac{\left(4-\sqrt{(b-4)(4+b)}+b\right)\arctan\left(\frac{4x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)}{\sqrt{(b-4)(4+b)}\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}$	128

input
```
int((-2*x^2+1)/(4*x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(4-b)^(1/2)*ln(-2*x^2*(4-b)^(1/2)+(4-b)*x-(4-b)^(1/2))+1/2/(4-b)^(1/2
)*ln(-2*x^2*(4-b)^(1/2)+x*(b-4)-(4-b)^(1/2))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.65

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = \left[-\frac{\sqrt{-b+4}\log\left(\frac{4x^4-(b-8)x^2+2(2x^3+x)\sqrt{-b+4}+1}{4x^4+bx^2+1}\right)}{2(b-4)}, \frac{\sqrt{b-4}\arctan\left(\frac{4x^3+(b-2)x}{\sqrt{b-4}}\right)-\sqrt{b-4}\arctan\left(\frac{2x}{\sqrt{b-4}}\right)}{b-4}\right]$$

input
```
integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b + 4)*log((4*x^4 - (b - 8)*x^2 + 2*(2*x^3 + x)*sqrt(-b + 4) +
 1)/(4*x^4 + b*x^2 + 1))/(b - 4), (sqrt(b - 4)*arctan((4*x^3 + (b - 2)*x)/
sqrt(b - 4)) - sqrt(b - 4)*arctan(2*x/sqrt(b - 4)))/(b - 4)]
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = \frac{\sqrt{-\frac{1}{b-4}}\log\left(x^2+x\left(-\frac{b\sqrt{-\frac{1}{b-4}}}{2}+2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2} - \frac{\sqrt{-\frac{1}{b-4}}\log\left(x^2+x\left(\frac{b\sqrt{-\frac{1}{b-4}}}{2}-2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2}$$

input
```
integrate((-2*x**2+1)/(4*x**4+b*x**2+1),x)
```

output
```
sqrt(-1/(b - 4))*log(x**2 + x*(-b*sqrt(-1/(b - 4))/2 + 2*sqrt(-1/(b - 4)))
 + 1/2)/2 - sqrt(-1/(b - 4))*log(x**2 + x*(b*sqrt(-1/(b - 4))/2 - 2*sqrt(-
1/(b - 4))) + 1/2)/2
```

3.54.7 Maxima [F]

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4+bx^2+1}\,dx$$

input
```
integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((2*x^2 - 1)/(4*x^4 + b*x^2 + 1), x)
```

3.54.8 Giac [F]

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4+bx^2+1}\,dx$$

input
```
integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")
```

output
```
sage0*x
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = -\frac{\operatorname{atan}\left(\frac{2\,x}{\sqrt{b-4}}\right)-\operatorname{atan}\left(\frac{b^3\,x+4\,b^2\,x^3-2\,b^2\,x-16\,b\,x-64\,x^3+32\,x}{(b-4)^{3/2}\,(b+4)}\right)}{\sqrt{b-4}}$$

input
```
int(-(2*x^2 - 1)/(b*x^2 + 4*x^4 + 1),x)
```

output
```
-(atan((2*x)/(b - 4)^(1/2)) - atan((32*x - 16*b*x - 2*b^2*x + b^3*x - 64*x
^3 + 4*b^2*x^3)/((b - 4)^(3/2)*(b + 4))))/(b - 4)^(1/2)
```

3.55 $\int \frac{1-2x^2}{1+6x^2+4x^4} dx$

3.55.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx=\frac{\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(2*x/(1/2*10^(1/2)-1/2*2^(1/2)))*2^(1/2)-1/2*arctan(2*x/(1/2*10^
(1/2)+1/2*2^(1/2)))*2^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = \frac{-\left(\left(-5+\sqrt{5}\right)\sqrt{3+\sqrt{5}}\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)\right)-\sqrt{3-\sqrt{5}}\left(5+\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{4\sqrt{5}}$$

input
```
Integrate[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

output
```
(-((-5 + Sqrt[5])*Sqrt[3 + Sqrt[5]]*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]) - Sqr
t[3 - Sqrt[5]]*(5 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(4*Sqrt[5])
```

3.55.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4+6x^2+1}\,dx$$

$$\Big\downarrow \text{1477}$$

$$-\left(\left(1-\sqrt{5}\right)\int \frac{1}{4x^2-\sqrt{5}+3}dx\right)-\left(1+\sqrt{5}\right)\int \frac{1}{4x^2+\sqrt{5}+3}dx$$

$$\Big\downarrow \text{216}$$

$$-\frac{\left(1-\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{2\sqrt{3-\sqrt{5}}}-\frac{\left(1+\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{2\sqrt{3+\sqrt{5}}}$$

input
```
Int[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

output
```
-1/2*((1 - Sqrt[5])*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]])/Sqrt[3 - Sqrt[5]] - (
(1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(2*Sqrt[3 + Sqrt[5]])
```

3.55.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

3.55.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}+\frac{\sqrt{2}\,\arctan\left(2x^{3}\sqrt{2}+2x\sqrt{2}\right)}{2}$	34
default	$-\frac{2\sqrt{5}\left(5+\sqrt{5}\right)\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}-\frac{2\left(-5+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}$	82

input
```
int((-2*x^2+1)/(4*x^4+6*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(x*2^(1/2))*2^(1/2)+1/2*2^(1/2)*arctan(2*x^3*2^(1/2)+2*x*2^(1/2
))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(2\sqrt{2}(x^3+x)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(2*sqrt(2)*(x^3 + x)) - 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = -\frac{\sqrt{2}\cdot\left(2\operatorname{atan}\left(\sqrt{2}x\right)-2\operatorname{atan}\left(2\sqrt{2}x^3+2\sqrt{2}x\right)\right)}{4}$$

input
```
integrate((-2*x**2+1)/(4*x**4+6*x**2+1),x)
```

output
```
-sqrt(2)*(2*atan(sqrt(2)*x) - 2*atan(2*sqrt(2)*x**3 + 2*sqrt(2)*x))/4
```

3.55.7 Maxima [F]

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4+6\,x^2+1}\,dx$$

input
```
integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((2*x^2 - 1)/(4*x^4 + 6*x^2 + 1), x)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = -\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{4\,x}{\sqrt{10}+\sqrt{2}}\right) + \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{4\,x}{\sqrt{10}-\sqrt{2}}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/2*sqrt(2)*arctan(4*x/(sq
rt(10) - sqrt(2)))
```

3.55.9 Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = \frac{\sqrt{2}\,\left(\mathrm{atan}\left(2\,\sqrt{2}\,x^3+2\,\sqrt{2}\,x\right)-\mathrm{atan}\left(\sqrt{2}\,x\right)\right)}{2}$$

input
```
int(-(2*x^2 - 1)/(6*x^2 + 4*x^4 + 1),x)
```

output
```
(2^(1/2)*(atan(2*2^(1/2)*x + 2*2^(1/2)*x^3) - atan(2^(1/2)*x)))/2
```

3.56 $\int \frac{1-2x^2}{1+5x^2+4x^4} dx$

3.56.1 Optimal result

Integrand size = 22, antiderivative size = 9

$$\int \frac{1-2x^2}{1+5x^2+4x^4} dx = -\arctan(x) + \arctan(2x)$$

output
```
-arctan(x)+arctan(2*x)
```

3.56.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{1-2x^2}{1+5x^2+4x^4} dx = \arctan\left(\frac{x}{1+2x^2}\right)$$

input
```
Integrate[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

output
```
ArcTan[x/(1 + 2*x^2)]
```

3.56.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4+5x^2+1} \, dx$$

$$\Big\downarrow \text{ 1477}$$

$$2\int \frac{1}{4x^2+1}dx-4\int \frac{1}{4x^2+4}dx$$

$$\Big\downarrow \text{ 216}$$

$$\arctan(2x)-\arctan(x)$$

input
```
Int[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

output
```
-ArcTan[x] + ArcTan[2*x]
```

3.56.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

3.56.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$-\arctan(x)+\arctan(2x)$	10
risch	$-\arctan(2x)+\arctan(4x^3+3x)$	18
parallelrisch	$\frac{i\ln(x-i)}{2}-\frac{i\ln(x+i)}{2}-\frac{i\ln(x-\frac{i}{2})}{2}+\frac{i\ln(x+\frac{i}{2})}{2}$	34

input
```
int((-2*x^2+1)/(4*x^4+5*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-arctan(x)+arctan(2*x)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = \arctan\left(4\,x^3+3\,x\right)-\arctan\left(2\,x\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")
```

output
```
arctan(4*x^3 + 3*x) - arctan(2*x)
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = -\operatorname{atan}(2x)+\operatorname{atan}\left(4x^3+3x\right)$$

input
```
integrate((-2*x**2+1)/(4*x**4+5*x**2+1),x)
```

output
```
-atan(2*x) + atan(4*x**3 + 3*x)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = \arctan(2\,x) - \arctan(x)$$

input
```
integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")
```

output
```
arctan(2*x) - arctan(x)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = \arctan(2\,x) - \arctan(x)$$

input
```
integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")
```

output
```
arctan(2*x) - arctan(x)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx = \mathrm{atan}\left(4\,x^3+3\,x\right) - \mathrm{atan}(2\,x)$$

input
```
int(-(2*x^2 - 1)/(5*x^2 + 4*x^4 + 1),x)
```

output
```
atan(3*x + 4*x^3) - atan(2*x)
```

3.57 $\int \frac{1-2x^2}{1+4x^2+4x^4} \, dx$

3.57.1 Optimal result

Integrand size = 22, antiderivative size = 11

$$\int \frac{1-2x^2}{1+4x^2+4x^4} \, dx = \frac{x}{1+2x^2}$$

output
```
x/(2*x^2+1)
```

3.57.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+4x^2+4x^4} \, dx = \frac{x}{1+2x^2}$$

input
```
Integrate[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

output
```
x/(1 + 2*x^2)
```

3.57.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1380, 27, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4+4x^2+1}dx$$

$$\Big\downarrow \text{1380}$$

$$4\int \frac{1-2x^2}{4\left(2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-2x^2}{\left(2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{297}$$

$$\frac{x}{2x^2+1}$$

input
```
Int[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

output
```
x/(1 + 2*x^2)
```

3.57.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.57.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{x}{2x^2+1}$	11
risch	$\frac{x}{2x^2+1}$	11
gosper	$\frac{x}{2x^2+1}$	12
norman	$\frac{x}{2x^2+1}$	12
parallelrisch	$\frac{x}{2x^2+1}$	12

input
```
int((-2*x^2+1)/(4*x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(x^2+1/2)
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{2\,x^2+1}$$

input
```
integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")
```

output
```
x/(2*x^2 + 1)
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{2x^2+1}$$

input
```
integrate((-2*x**2+1)/(4*x**4+4*x**2+1),x)
```

output
```
x/(2*x**2 + 1)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{2\,x^2+1}$$

input
```
integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")
```

output
```
x/(2*x^2 + 1)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{2\,x^2+1}$$

input
```
integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")
```

output
```
x/(2*x^2 + 1)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx = \frac{x}{2\,\left(x^2+\frac{1}{2}\right)}$$

input
```
int(-(2*x^2 - 1)/(4*x^2 + 4*x^4 + 1),x)
```

output
```
x/(2*(x^2 + 1/2))
```

3.58 $\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx$

3.58.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = -\frac{1}{2}\log\left(1-x+2x^2\right)+\frac{1}{2}\log\left(1+x+2x^2\right)$$

output
```
-1/2*ln(2*x^2-x+1)+1/2*ln(2*x^2+x+1)
```

3.58.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = -\frac{1}{2}\log\left(1-x+2x^2\right)+\frac{1}{2}\log\left(1+x+2x^2\right)$$

input
```
Integrate[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

output
```
-1/2*Log[1 - x + 2*x^2] + Log[1 + x + 2*x^2]/2
```

3.58.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4+3x^2+1}\,dx$$
$$\Big\downarrow \; 1478$$
$$-\frac{1}{2}\int -\frac{1-4x}{2x^2-x+1}dx-\frac{1}{2}\int -\frac{4x+1}{2x^2+x+1}dx$$
$$\Big\downarrow \; 25$$
$$\frac{1}{2}\int \frac{1-4x}{2x^2-x+1}dx+\frac{1}{2}\int \frac{4x+1}{2x^2+x+1}dx$$
$$\Big\downarrow \; 1103$$
$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

input
```
Int[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

output
```
-1/2*Log[1 - x + 2*x^2] + Log[1 + x + 2*x^2]/2
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.58.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$-\frac{\ln\left(x^2-\frac{1}{2}x+\frac{1}{2}\right)}{2}+\frac{\ln\left(x^2+\frac{1}{2}x+\frac{1}{2}\right)}{2}$	24
default	$-\frac{\ln\left(2x^2-x+1\right)}{2}+\frac{\ln\left(2x^2+x+1\right)}{2}$	26
norman	$-\frac{\ln\left(2x^2-x+1\right)}{2}+\frac{\ln\left(2x^2+x+1\right)}{2}$	26
risch	$-\frac{\ln\left(2x^2-x+1\right)}{2}+\frac{\ln\left(2x^2+x+1\right)}{2}$	26

input
```
int((-2*x^2+1)/(4*x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x^2-1/2*x+1/2)+1/2*ln(x^2+1/2*x+1/2)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = \frac{1}{2}\,\log\left(2\,x^2+x+1\right)-\frac{1}{2}\,\log\left(2\,x^2-x+1\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="fricas")
```

output
```
1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = -\frac{\log\left(x^2-\frac{x}{2}+\frac{1}{2}\right)}{2}+\frac{\log\left(x^2+\frac{x}{2}+\frac{1}{2}\right)}{2}$$

input
```
integrate((-2*x**2+1)/(4*x**4+3*x**2+1),x)
```

output
```
-log(x**2 - x/2 + 1/2)/2 + log(x**2 + x/2 + 1/2)/2
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = \frac{1}{2}\log\left(2\,x^2+x+1\right)-\frac{1}{2}\log\left(2\,x^2-x+1\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="maxima")
```

output
```
1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = \frac{1}{2}\log\left(2\,x^2+x+1\right)-\frac{1}{2}\log\left(2\,x^2-x+1\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")
```

output
```
1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.41

$$\int \frac{1-2x^2}{1+3x^2+4x^4}\,dx = \operatorname{atanh}\left(\frac{x}{2\,x^2+1}\right)$$

input
```
int(-(2*x^2 - 1)/(3*x^2 + 4*x^4 + 1),x)
```

output
```
atanh(x/(2*x^2 + 1))
```

3.59 $\int \frac{1-2x^2}{1+2x^2+4x^4} \, dx$

3.59.1 Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{1-2x^2}{1+2x^2+4x^4} \, dx = -\frac{\log\left(1-\sqrt{2}x+2x^2\right)}{2\sqrt{2}} + \frac{\log\left(1+\sqrt{2}x+2x^2\right)}{2\sqrt{2}}$$

output
```
-1/4*ln(1+2*x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+2*x^2+x*2^(1/2))*2^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-2x^2}{1+2x^2+4x^4} \, dx = \frac{-\log\left(-1+\sqrt{2}x-2x^2\right)+\log\left(1+\sqrt{2}x+2x^2\right)}{2\sqrt{2}}$$

input
```
Integrate[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

output
```
(-Log[-1 + Sqrt[2]*x - 2*x^2] + Log[1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])
```

3.59.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4+2x^2+1}\,dx$$

$$\Big\downarrow \text{ 1478}$$

$$-\frac{\int -\frac{\sqrt{2}-4x}{2x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{4x+\sqrt{2}}{2x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sqrt{2}-4x}{2x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{4x+\sqrt{2}}{2x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\log\left(2x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(2x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

input
```
Int[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

output
```
-1/2*Log[1 - Sqrt[2]*x + 2*x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x + 2*x^2]/(2*Sq
rt[2])
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.59.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+2x^2+x\sqrt{2}\right)\sqrt{2}}{4}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+2x^2+x\sqrt{2}\right)\sqrt{2}}{4}$	39

input
```
int((-2*x^2+1)/(4*x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(1+2*x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+2*x^2+x*2^(1/2))*2^(1/2)
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx=\frac{1}{4}\sqrt{2}\log\left(\frac{4\,x^4+6\,x^2+2\,\sqrt{2}(2\,x^3+x)+1}{4\,x^4+2\,x^2+1}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((4*x^4 + 6*x^2 + 2*sqrt(2)*(2*x^3 + x) + 1)/(4*x^4 + 2*x^2
 + 1))
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx = -\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{2}+\frac{1}{2}\right)}{4}$$

input
```
integrate((-2*x**2+1)/(4*x**4+2*x**2+1),x)
```

output
```
-sqrt(2)*log(x**2 - sqrt(2)*x/2 + 1/2)/4 + sqrt(2)*log(x**2 + sqrt(2)*x/2
+ 1/2)/4
```

3.59.7 Maxima [F]

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4+2\,x^2+1}\,dx$$

input
```
integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((2*x^2 - 1)/(4*x^4 + 2*x^2 + 1), x)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(x^2+\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(x^2 + (1/4)^(1/4)*x + 1/2) - 1/4*sqrt(2)*log(x^2 - (1/4)^(
1/4)*x + 1/2)
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx=\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2\,x^2+1}\right)}{2}$$

input
```
int(-(2*x^2 - 1)/(2*x^2 + 4*x^4 + 1),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*x)/(2*x^2 + 1)))/2
```

3.60 $\int \frac{1-2x^2}{1+x^2+4x^4}\,dx$

3.60.1 Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx=-\frac{\log\left(1-\sqrt{3}x+2x^2\right)}{2\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+2x^2\right)}{2\sqrt{3}}$$

output
```
-1/6*ln(1+2*x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+2*x^2+x*3^(1/2))*3^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx=\frac{-\log\left(-1+\sqrt{3}x-2x^2\right)+\log\left(1+\sqrt{3}x+2x^2\right)}{2\sqrt{3}}$$

input
```
Integrate[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
```

output
```
(-Log[-1 + Sqrt[3]*x - 2*x^2] + Log[1 + Sqrt[3]*x + 2*x^2])/(2*Sqrt[3])
```

3.60.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4+x^2+1} \, dx$$

$$\Big\downarrow \text{ 1478}$$

$$-\frac{\int -\frac{\sqrt{3}-4x}{2x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int -\frac{4x+\sqrt{3}}{2x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sqrt{3}-4x}{2x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{4x+\sqrt{3}}{2x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\log\left(2x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(2x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

input
```
Int[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
```

output
```
-1/2*Log[1 - Sqrt[3]*x + 2*x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + 2*x^2]/(2*Sq
rt[3])
```

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.60.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+2x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+2x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	39

input
```
int((-2*x^2+1)/(4*x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(1+2*x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+2*x^2+x*3^(1/2))*3^(1/2)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{4x^4+7x^2+2\sqrt{3}(2x^3+x)+1}{4x^4+x^2+1}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log((4*x^4 + 7*x^2 + 2*sqrt(3)*(2*x^3 + x) + 1)/(4*x^4 + x^2 +
 1))
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = -\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}$$

input
```
integrate((-2*x**2+1)/(4*x**4+x**2+1),x)
```

output
```
-sqrt(3)*log(x**2 - sqrt(3)*x/2 + 1/2)/6 + sqrt(3)*log(x**2 + sqrt(3)*x/2
+ 1/2)/6
```

3.60.7 Maxima [F]

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4+x^2+1}\,dx$$

input
```
integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="maxima")
```

output
```
-integrate((2*x^2 - 1)/(4*x^4 + x^2 + 1), x)
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(x^2+\frac{1}{2}\,\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{6}\,\sqrt{3}\log\left(x^2-\frac{1}{2}\,\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(x^2 + 1/2*sqrt(6)*(1/4)^(1/4)*x + 1/2) - 1/6*sqrt(3)*log(x
^2 - 1/2*sqrt(6)*(1/4)^(1/4)*x + 1/2)
```

3.60.9 Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{1-2x^2}{1+x^2+4x^4}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{2\,x^2+1}\right)}{3}$$

input
```
int(-(2*x^2 - 1)/(x^2 + 4*x^4 + 1),x)
```

output
```
(3^(1/2)*atanh((3^(1/2)*x)/(2*x^2 + 1)))/3
```

3.61 $\int \frac{1-2x^2}{1+4x^4} \, dx$

3.61.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{1-2x^2}{1+4x^4} \, dx = -\frac{1}{4} \log \left(1-2x+2x^2\right)+\frac{1}{4} \log \left(1+2x+2x^2\right)$$

output
```
-1/4*ln(2*x^2-2*x+1)+1/4*ln(2*x^2+2*x+1)
```

3.61.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1-2x^2}{1+4x^4} \, dx = -\frac{1}{4} \log \left(1-2x+2x^2\right)+\frac{1}{4} \log \left(1+2x+2x^2\right)$$

input
```
Integrate[(1 - 2*x^2)/(1 + 4*x^4),x]
```

output
```
-1/4*Log[1 - 2*x + 2*x^2] + Log[1 + 2*x + 2*x^2]/4
```

3.61.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1479, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4+1}\,dx$$

$$\Big\downarrow \text{1479}$$

$$-\frac{1}{4}\int -\frac{2(1-2x)}{2x^2-2x+1}dx-\frac{1}{4}\int -\frac{2(2x+1)}{2x^2+2x+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1-2x}{2x^2-2x+1}dx+\frac{1}{2}\int \frac{2x+1}{2x^2+2x+1}dx$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

input
```
Int[(1 - 2*x^2)/(1 + 4*x^4),x]
```

output
```
-1/4*Log[1 - 2*x + 2*x^2] + Log[1 + 2*x + 2*x^2]/4
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.61.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result
parallelrisch	$-\frac{\ln\left(x^{2}-x+\frac{1}{2}\right)}{4}+\frac{\ln\left(x^{2}+x+\frac{1}{2}\right)}{4}$
default	$-\frac{\ln\left(2x^{2}-2x+1\right)}{4}+\frac{\ln\left(2x^{2}+2x+1\right)}{4}$
norman	$-\frac{\ln\left(2x^{2}-2x+1\right)}{4}+\frac{\ln\left(2x^{2}+2x+1\right)}{4}$
risch	$-\frac{\ln\left(2x^{2}-2x+1\right)}{4}+\frac{\ln\left(2x^{2}+2x+1\right)}{4}$
meijerg	$-\frac{\sqrt{2}\left(\frac{x^{3}\sqrt{2}\,\ln\left(1-2\left(x^{4}\right)^{\frac{1}{4}}+2\sqrt{x^{4}}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{\left(x^{4}\right)^{\frac{1}{4}}}{1-\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\,\ln\left(1+2\left(x^{4}\right)^{\frac{1}{4}}+2\sqrt{x^{4}}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{\left(x^{4}\right)^{\frac{1}{4}}}{1+\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}\right)}{8}+$

input
```
int((-2*x^2+1)/(4*x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(x^2-x+1/2)+1/4*ln(x^2+x+1/2)
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1-2x^2}{1+4x^4}\,dx = \frac{1}{4}\,\log\left(2\,x^{2}+2\,x+1\right)-\frac{1}{4}\,\log\left(2\,x^{2}-2\,x+1\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="fricas")
```

output
```
1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1-2x^2}{1+4x^4}\,dx = -\frac{\log\left(x^2-x+\frac{1}{2}\right)}{4} + \frac{\log\left(x^2+x+\frac{1}{2}\right)}{4}$$

input
```
integrate((-2*x**2+1)/(4*x**4+1),x)
```

output
```
-log(x**2 - x + 1/2)/4 + log(x**2 + x + 1/2)/4
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1-2x^2}{1+4x^4}\,dx = \frac{1}{4}\log\left(2x^2+2x+1\right) - \frac{1}{4}\log\left(2x^2-2x+1\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="maxima")
```

output
```
1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1-2x^2}{1+4x^4}\,dx = \frac{1}{4}\log\left(x^2+\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right) - \frac{1}{4}\log\left(x^2-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="giac")
```

output
```
1/4*log(x^2 + sqrt(2)*(1/4)^(1/4)*x + 1/2) - 1/4*log(x^2 - sqrt(2)*(1/4)^(
1/4)*x + 1/2)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.48

$$\int \frac{1-2x^2}{1+4x^4}\,dx = \frac{\operatorname{atanh}\left(\frac{2\,x}{2\,x^2+1}\right)}{2}$$

input
```
int(-(2*x^2 - 1)/(4*x^4 + 1),x)
```

output
```
atanh((2*x)/(2*x^2 + 1))/2
```

3.62 $\int \frac{1-2x^2}{1-x^2+4x^4} \, dx$

3.62.1 Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{1-2x^2}{1-x^2+4x^4} \, dx = -\frac{\log \left(1-\sqrt{5}x+2x^2\right)}{2\sqrt{5}}+\frac{\log \left(1+\sqrt{5}x+2x^2\right)}{2\sqrt{5}}$$

output
```
-1/10*ln(1+2*x^2-x*5^(1/2))*5^(1/2)+1/10*ln(1+2*x^2+x*5^(1/2))*5^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-2x^2}{1-x^2+4x^4} \, dx = \frac{-\log \left(-1+\sqrt{5}x-2x^2\right)+\log \left(1+\sqrt{5}x+2x^2\right)}{2\sqrt{5}}$$

input
```
Integrate[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
```

output
```
(-Log[-1 + Sqrt[5]*x - 2*x^2] + Log[1 + Sqrt[5]*x + 2*x^2])/(2*Sqrt[5])
```

3.62.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4-x^2+1} \, dx$$

$$\Big\downarrow \text{1478}$$

$$-\frac{\int -\frac{\sqrt{5}-4x}{2x^2-\sqrt{5}x+1}dx}{2\sqrt{5}}-\frac{\int -\frac{4x+\sqrt{5}}{2x^2+\sqrt{5}x+1}dx}{2\sqrt{5}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{5}-4x}{2x^2-\sqrt{5}x+1}dx}{2\sqrt{5}}+\frac{\int \frac{4x+\sqrt{5}}{2x^2+\sqrt{5}x+1}dx}{2\sqrt{5}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(2x^2+\sqrt{5}x+1\right)}{2\sqrt{5}}-\frac{\log\left(2x^2-\sqrt{5}x+1\right)}{2\sqrt{5}}$$

input
```
Int[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
```

output
```
-1/2*Log[1 - Sqrt[5]*x + 2*x^2]/Sqrt[5] + Log[1 + Sqrt[5]*x + 2*x^2]/(2*Sq
rt[5])
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.62.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(1+2x^2+x\sqrt{5}\right)\sqrt{5}}{10}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(1+2x^2+x\sqrt{5}\right)\sqrt{5}}{10}$	39

input
```
int((-2*x^2+1)/(4*x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/10*ln(1+2*x^2-x*5^(1/2))*5^(1/2)+1/10*ln(1+2*x^2+x*5^(1/2))*5^(1/2)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = \frac{1}{10}\sqrt{5}\log\left(\frac{4\,x^4+9\,x^2+2\,\sqrt{5}(2\,x^3+x)+1}{4\,x^4-x^2+1}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/10*sqrt(5)*log((4*x^4 + 9*x^2 + 2*sqrt(5)*(2*x^3 + x) + 1)/(4*x^4 - x^2
+ 1))
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = -\frac{\sqrt{5}\log\left(x^2-\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\frac{\sqrt{5}x}{2}+\frac{1}{2}\right)}{10}$$

input
```
integrate((-2*x**2+1)/(4*x**4-x**2+1),x)
```

output
```
-sqrt(5)*log(x**2 - sqrt(5)*x/2 + 1/2)/10 + sqrt(5)*log(x**2 + sqrt(5)*x/2
 + 1/2)/10
```

3.62.7 Maxima [F]

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4-x^2+1}\,dx$$

input
```
integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="maxima")
```

output
```
-integrate((2*x^2 - 1)/(4*x^4 - x^2 + 1), x)
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = \frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right) - \frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")
```

output
```
1/10*sqrt(5)*log(x^2 + 1/2*sqrt(10)*(1/4)^(1/4)*x + 1/2) - 1/10*sqrt(5)*lo
g(x^2 - 1/2*sqrt(10)*(1/4)^(1/4)*x + 1/2)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{1-2x^2}{1-x^2+4x^4}\,dx = \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,x}{2\,x^2+1}\right)}{5}$$

input
```
int(-(2*x^2 - 1)/(4*x^4 - x^2 + 1),x)
```

output
```
(5^(1/2)*atanh((5^(1/2)*x)/(2*x^2 + 1)))/5
```

3.63 $\int \frac{1-2x^2}{1-2x^2+4x^4} \, dx$

3.63.1 Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{1-2x^2}{1-2x^2+4x^4} \, dx = -\frac{\log\left(1-\sqrt{6}x+2x^2\right)}{2\sqrt{6}} + \frac{\log\left(1+\sqrt{6}x+2x^2\right)}{2\sqrt{6}}$$

output
```
-1/12*ln(1+2*x^2-x*6^(1/2))*6^(1/2)+1/12*ln(1+2*x^2+x*6^(1/2))*6^(1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-2x^2}{1-2x^2+4x^4} \, dx = \frac{-\log\left(-1+\sqrt{6}x-2x^2\right)+\log\left(1+\sqrt{6}x+2x^2\right)}{2\sqrt{6}}$$

input
```
Integrate[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

output
```
(-Log[-1 + Sqrt[6]*x - 2*x^2] + Log[1 + Sqrt[6]*x + 2*x^2])/(2*Sqrt[6])
```

3.63.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4-2x^2+1}\,dx$$

$$\Big\downarrow \text{ 1478}$$

$$-\frac{\int -\frac{\sqrt{6}-4x}{2x^2-\sqrt{6}x+1}dx}{2\sqrt{6}}-\frac{\int -\frac{4x+\sqrt{6}}{2x^2+\sqrt{6}x+1}dx}{2\sqrt{6}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sqrt{6}-4x}{2x^2-\sqrt{6}x+1}dx}{2\sqrt{6}}+\frac{\int \frac{4x+\sqrt{6}}{2x^2+\sqrt{6}x+1}dx}{2\sqrt{6}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\log\left(2x^2+\sqrt{6}x+1\right)}{2\sqrt{6}}-\frac{\log\left(2x^2-\sqrt{6}x+1\right)}{2\sqrt{6}}$$

input
```
Int[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

output
```
-1/2*Log[1 - Sqrt[6]*x + 2*x^2]/Sqrt[6] + Log[1 + Sqrt[6]*x + 2*x^2]/(2*Sq
rt[6])
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.63.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{6}\right)\sqrt{6}}{12}+\frac{\ln\left(1+2x^2+x\sqrt{6}\right)\sqrt{6}}{12}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{6}\right)\sqrt{6}}{12}+\frac{\ln\left(1+2x^2+x\sqrt{6}\right)\sqrt{6}}{12}$	39

input
```
int((-2*x^2+1)/(4*x^4-2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/12*ln(1+2*x^2-x*6^(1/2))*6^(1/2)+1/12*ln(1+2*x^2+x*6^(1/2))*6^(1/2)
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx=\frac{1}{12}\sqrt{6}\log\left(\frac{4\,x^4+10\,x^2+2\,\sqrt{6}(2\,x^3+x)+1}{4\,x^4-2\,x^2+1}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(6)*log((4*x^4 + 10*x^2 + 2*sqrt(6)*(2*x^3 + x) + 1)/(4*x^4 - 2*x
^2 + 1))
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx = -\frac{\sqrt{6}\log\left(x^2-\frac{\sqrt{6}x}{2}+\frac{1}{2}\right)}{12}+\frac{\sqrt{6}\log\left(x^2+\frac{\sqrt{6}x}{2}+\frac{1}{2}\right)}{12}$$

input
```
integrate((-2*x**2+1)/(4*x**4-2*x**2+1),x)
```

output
```
-sqrt(6)*log(x**2 - sqrt(6)*x/2 + 1/2)/12 + sqrt(6)*log(x**2 + sqrt(6)*x/2
 + 1/2)/12
```

3.63.7 Maxima [F]

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4-2\,x^2+1}\,dx$$

input
```
integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((2*x^2 - 1)/(4*x^4 - 2*x^2 + 1), x)
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx = \frac{1}{12}\sqrt{6}\log\left(x^2+\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{12}\sqrt{6}\log\left(x^2-\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*log(x^2 + sqrt(3)*(1/4)^(1/4)*x + 1/2) - 1/12*sqrt(6)*log(x^2
 - sqrt(3)*(1/4)^(1/4)*x + 1/2)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx = \frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{2\,x^2+1}\right)}{6}$$

input
```
int(-(2*x^2 - 1)/(4*x^4 - 2*x^2 + 1),x)
```

output
```
(6^(1/2)*atanh((6^(1/2)*x)/(2*x^2 + 1)))/6
```

3.64 $\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx$

3.64.1 Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx=-\frac{\log\left(1-\sqrt{7}x+2x^2\right)}{2\sqrt{7}}+\frac{\log\left(1+\sqrt{7}x+2x^2\right)}{2\sqrt{7}}$$

output
```
-1/14*ln(1+2*x^2-x*7^(1/2))*7^(1/2)+1/14*ln(1+2*x^2+x*7^(1/2))*7^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx=\frac{-\log\left(-1+\sqrt{7}x-2x^2\right)+\log\left(1+\sqrt{7}x+2x^2\right)}{2\sqrt{7}}$$

input
```
Integrate[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

output
```
(-Log[-1 + Sqrt[7]*x - 2*x^2] + Log[1 + Sqrt[7]*x + 2*x^2])/(2*Sqrt[7])
```

3.64.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4-3x^2+1}dx$$

$$\Big\downarrow \ 1478$$

$$-\frac{\int -\frac{\sqrt{7}-4x}{2x^2-\sqrt{7}x+1}dx}{2\sqrt{7}}-\frac{\int -\frac{4x+\sqrt{7}}{2x^2+\sqrt{7}x+1}dx}{2\sqrt{7}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{7}-4x}{2x^2-\sqrt{7}x+1}dx}{2\sqrt{7}}+\frac{\int \frac{4x+\sqrt{7}}{2x^2+\sqrt{7}x+1}dx}{2\sqrt{7}}$$

$$\Big\downarrow \ 1103$$

$$\frac{\log\left(2x^2+\sqrt{7}x+1\right)}{2\sqrt{7}}-\frac{\log\left(2x^2-\sqrt{7}x+1\right)}{2\sqrt{7}}$$

input
```
Int[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

output
```
-1/2*Log[1 - Sqrt[7]*x + 2*x^2]/Sqrt[7] + Log[1 + Sqrt[7]*x + 2*x^2]/(2*Sq
rt[7])
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.64.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{7}\right)\sqrt{7}}{14}+\frac{\ln\left(1+2x^2+x\sqrt{7}\right)\sqrt{7}}{14}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{7}\right)\sqrt{7}}{14}+\frac{\ln\left(1+2x^2+x\sqrt{7}\right)\sqrt{7}}{14}$	39

input
```
int((-2*x^2+1)/(4*x^4-3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/14*ln(1+2*x^2-x*7^(1/2))*7^(1/2)+1/14*ln(1+2*x^2+x*7^(1/2))*7^(1/2)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx=\frac{1}{14}\sqrt{7}\log\left(\frac{4\,x^4+11\,x^2+2\,\sqrt{7}(2\,x^3+x)+1}{4\,x^4-3\,x^2+1}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="fricas")
```

output
```
1/14*sqrt(7)*log((4*x^4 + 11*x^2 + 2*sqrt(7)*(2*x^3 + x) + 1)/(4*x^4 - 3*x
^2 + 1))
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx = -\frac{\sqrt{7}\log\left(x^2-\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\frac{\sqrt{7}x}{2}+\frac{1}{2}\right)}{14}$$

input
```
integrate((-2*x**2+1)/(4*x**4-3*x**2+1),x)
```

output
```
-sqrt(7)*log(x**2 - sqrt(7)*x/2 + 1/2)/14 + sqrt(7)*log(x**2 + sqrt(7)*x/2
 + 1/2)/14
```

3.64.7 Maxima [F]

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4-3\,x^2+1}\,dx$$

input
```
integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((2*x^2 - 1)/(4*x^4 - 3*x^2 + 1), x)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx = \frac{1}{14}\sqrt{7}\log\left(x^2+\frac{1}{2}\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right) - \frac{1}{14}\sqrt{7}\log\left(x^2-\frac{1}{2}\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")
```

output
```
1/14*sqrt(7)*log(x^2 + 1/2*sqrt(14)*(1/4)^(1/4)*x + 1/2) - 1/14*sqrt(7)*lo
g(x^2 - 1/2*sqrt(14)*(1/4)^(1/4)*x + 1/2)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx = \frac{\sqrt{7}\,\mathrm{atanh}\left(\frac{\sqrt{7}\,x}{2\,x^2+1}\right)}{7}$$

input
```
int(-(2*x^2 - 1)/(4*x^4 - 3*x^2 + 1),x)
```

output
```
(7^(1/2)*atanh((7^(1/2)*x)/(2*x^2 + 1)))/7
```

3.65 $\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx$

3.65.1 Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx=\frac{\operatorname{arctanh}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(x*2^(1/2))*2^(1/2)
```

3.65.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx=\frac{-\log\left(\sqrt{2}-2x\right)+\log\left(\sqrt{2}+2x\right)}{2\sqrt{2}}$$

input
```
Integrate[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

output
```
(-Log[Sqrt[2] - 2*x] + Log[Sqrt[2] + 2*x])/(2*Sqrt[2])
```

3.65.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1380, 27, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4-4x^2+1} \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$4 \int \frac{1}{4 (1-2x^2)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{1-2x^2}dx$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

input
```
Int[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

output
```
ArcTanh[Sqrt[2]*x]/Sqrt[2]
```

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.65.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\operatorname{arctanh}\left(x\sqrt{2}\right)\sqrt{2}}{2}$	12
risch	$\frac{\sqrt{2}\,\ln\left(2x+\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(2x-\sqrt{2}\right)}{4}$	30

input
```
int((-2*x^2+1)/(4*x^4-4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(x*2^(1/2))*2^(1/2)
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{2\,x^2+2\,\sqrt{2}x+1}{2\,x^2-1}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((2*x^2 + 2*sqrt(2)*x + 1)/(2*x^2 - 1))
```

3.65.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = -\frac{\sqrt{2}\log\left(x-\frac{\sqrt{2}}{2}\right)}{4} + \frac{\sqrt{2}\log\left(x+\frac{\sqrt{2}}{2}\right)}{4}$$

input
```
integrate((-2*x**2+1)/(4*x**4-4*x**2+1),x)
```

output
```
-sqrt(2)*log(x - sqrt(2)/2)/4 + sqrt(2)*log(x + sqrt(2)/2)/4
```

3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = -\frac{1}{4}\sqrt{2}\log\left(\frac{2x-\sqrt{2}}{2x+\sqrt{2}}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*log((2*x - sqrt(2))/(2*x + sqrt(2)))
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right) - \frac{1}{4}\sqrt{2}\log\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(x + 1/2*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/2*sqrt(2
)))
```

3.65.9 Mupad [B] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,x\right)}{2}$$

input
```
int(-(2*x^2 - 1)/(4*x^4 - 4*x^2 + 1),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*x))/2
```

3.66 $\int \frac{1-2x^2}{1-5x^2+4x^4} \, dx$

3.66.1 Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{1-2x^2}{1-5x^2+4x^4} \, dx = -\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(1+x)+\frac{1}{6}\log(1+2x)$$

output
```
-1/6*ln(1-2*x)-1/6*ln(1-x)+1/6*ln(1+x)+1/6*ln(1+2*x)
```

3.66.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1-2x^2}{1-5x^2+4x^4} \, dx = -\frac{1}{6}\log\left(1-3x+2x^2\right)+\frac{1}{6}\log\left(1+3x+2x^2\right)$$

input
```
Integrate[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

output
```
-1/6*Log[1 - 3*x + 2*x^2] + Log[1 + 3*x + 2*x^2]/6
```

3.66.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4-5x^2+1} \, dx$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{4} \int \frac{1}{x^2-\frac{x}{2}-\frac{1}{2}}dx-\frac{1}{4} \int \frac{1}{x^2+\frac{x}{2}-\frac{1}{2}}dx$$

$$\Big\downarrow \text{1081}$$

$$-\frac{1}{4} \int \left(-\frac{2}{3(x+1)}-\frac{4}{3(1-2x)}\right)dx-\frac{1}{4} \int \left(-\frac{4}{3(2x+1)}-\frac{2}{3(1-x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(\frac{2}{3} \log (x+1)-\frac{2}{3} \log (1-2x)\right)+\frac{1}{4} \left(\frac{2}{3} \log (2x+1)-\frac{2}{3} \log (1-x)\right)$$

input
```
Int[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

output
```
((-2*Log[1 - 2*x])/3 + (2*Log[1 + x])/3)/4 + ((-2*Log[1 - x])/3 + (2*Log[1
 + 2*x])/3)/4
```

3.66.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.66.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln(x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln\left(x-\frac{1}{2}\right)}{6}+\frac{\ln\left(x+\frac{1}{2}\right)}{6}$	26
risch	$-\frac{\ln(2x^2-3x+1)}{6}+\frac{\ln(2x^2+3x+1)}{6}$	28
default	$\frac{\ln(1+2x)}{6}-\frac{\ln(2x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln(x-1)}{6}$	30
norman	$\frac{\ln(1+2x)}{6}-\frac{\ln(2x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln(x-1)}{6}$	30

input
```
int((-2*x^2+1)/(4*x^4-5*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(x-1)+1/6*ln(x+1)-1/6*ln(x-1/2)+1/6*ln(x+1/2)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx = \frac{1}{6}\,\log\left(2\,x^2+3\,x+1\right)-\frac{1}{6}\,\log\left(2\,x^2-3\,x+1\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="fricas")
```

output
```
1/6*log(2*x^2 + 3*x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx = -\frac{\log\left(x^2-\frac{3x}{2}+\frac{1}{2}\right)}{6}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

input
```
integrate((-2*x**2+1)/(4*x**4-5*x**2+1),x)
```

output
```
-log(x**2 - 3*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx = \frac{1}{6}\,\log\left(2\,x+1\right)-\frac{1}{6}\,\log\left(2\,x-1\right)+\frac{1}{6}\,\log\left(x+1\right)-\frac{1}{6}\,\log\left(x-1\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="maxima")
```

output
```
1/6*log(2*x + 1) - 1/6*log(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx = \frac{1}{6}\,\log\left(|2\,x+1|\right) - \frac{1}{6}\,\log\left(|2\,x-1|\right) + \frac{1}{6}\,\log\left(|x+1|\right) - \frac{1}{6}\,\log\left(|x-1|\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
```

output
```
1/6*log(abs(2*x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*
log(abs(x - 1))
```

3.66.9 Mupad [B] (verification not implemented)

Time = 13.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx = \frac{\operatorname{atanh}\left(\frac{3\,x}{2\,x^2+1}\right)}{3}$$

input
```
int(-(2*x^2 - 1)/(4*x^4 - 5*x^2 + 1),x)
```

output
```
atanh((3*x)/(2*x^2 + 1))/3
```

3.67 $\int \frac{1-2x^2}{1-6x^2+4x^4} \, dx$

3.67.1 Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{1-2x^2}{1-6x^2+4x^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{1-2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}+\frac{\operatorname{arctanh}\left(\frac{1+2\sqrt{2}x}{\sqrt{5}}\right)}{\sqrt{10}}$$

output
```
-1/10*arctanh(1/5*(1-2*x*2^(1/2))*5^(1/2))*10^(1/2)+1/10*arctanh(1/5*(1+2*
x*2^(1/2))*5^(1/2))*10^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{1-2x^2}{1-6x^2+4x^4} \, dx = \frac{-\log\left(-1+\sqrt{10}x-2x^2\right)+\log\left(1+\sqrt{10}x+2x^2\right)}{2\sqrt{10}}$$

input
```
Integrate[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

output
```
(-Log[-1 + Sqrt[10]*x - 2*x^2] + Log[1 + Sqrt[10]*x + 2*x^2])/(2*Sqrt[10])
```

3.67.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 117 vs. $2(48) = 96$.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.44, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-2x^2}{4x^4-6x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{4}\int \frac{1}{x^2-\frac{x}{\sqrt{2}}-\frac{1}{2}}dx-\frac{1}{4}\int \frac{1}{x^2+\frac{x}{\sqrt{2}}-\frac{1}{2}}dx$$

$$\Big\downarrow \text{1081}$$

$$-\frac{1}{4}\int \left(\frac{4\sqrt{\frac{2}{5}}}{\sqrt{2}\left(1-\sqrt{5}\right)-4x}-\frac{4\sqrt{\frac{2}{5}}}{-4x+\sqrt{10}+\sqrt{2}}\right)dx-\frac{1}{4}\int \left(\frac{4\sqrt{\frac{2}{5}}}{4x+\sqrt{2}\left(1-\sqrt{5}\right)}-\frac{4\sqrt{\frac{2}{5}}}{4x+\sqrt{10}+\sqrt{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\sqrt{\frac{2}{5}}\log\left(4x+\sqrt{10}+\sqrt{2}\right)-\sqrt{\frac{2}{5}}\log\left(-4x-\sqrt{2}\left(1-\sqrt{5}\right)\right)\right)+\frac{1}{4}\left(\sqrt{\frac{2}{5}}\log\left(4x-\sqrt{2}\left(1-\sqrt{5}\right)\right)-\sqrt{\frac{2}{5}}\log\left(-4x+\sqrt{10}+\sqrt{2}\right)\right)$$

input
```
Int[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

output
```
(-(Sqrt[2/5]*Log[-(Sqrt[2]*(1 - Sqrt[5])) - 4*x]) + Sqrt[2/5]*Log[Sqrt[2]
+ Sqrt[10] + 4*x])/4 + (-(Sqrt[2/5]*Log[Sqrt[2] + Sqrt[10] - 4*x]) + Sqrt[
2/5]*Log[-(Sqrt[2]*(1 - Sqrt[5])) + 4*x])/4
```

3.67.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.67.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{\sqrt{10}\,\ln\left(\sqrt{10}\,x+2x^2+1\right)}{20}-\frac{\sqrt{10}\,\ln\left(-\sqrt{10}\,x+2x^2+1\right)}{20}$	39
default	$\frac{2\left(\sqrt{5}-1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}+\frac{2\left(\sqrt{5}+1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}$	82

input
```
int((-2*x^2+1)/(4*x^4-6*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/20*10^(1/2)*ln(10^(1/2)*x+2*x^2+1)-1/20*10^(1/2)*ln(-10^(1/2)*x+2*x^2+1)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx = \frac{1}{20}\sqrt{10}\log\left(\frac{4\,x^4+14\,x^2+2\,\sqrt{10}(2\,x^3+x)+1}{4\,x^4-6\,x^2+1}\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")
```

output
```
1/20*sqrt(10)*log((4*x^4 + 14*x^2 + 2*sqrt(10)*(2*x^3 + x) + 1)/(4*x^4 - 6
*x^2 + 1))
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx = -\frac{\sqrt{10}\log\left(x^2-\frac{\sqrt{10}x}{2}+\frac{1}{2}\right)}{20}+\frac{\sqrt{10}\log\left(x^2+\frac{\sqrt{10}x}{2}+\frac{1}{2}\right)}{20}$$

input
```
integrate((-2*x**2+1)/(4*x**4-6*x**2+1),x)
```

output
```
-sqrt(10)*log(x**2 - sqrt(10)*x/2 + 1/2)/20 + sqrt(10)*log(x**2 + sqrt(10)
*x/2 + 1/2)/20
```

3.67.7 Maxima [F]

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx = \int -\frac{2\,x^2-1}{4\,x^4-6\,x^2+1}\,dx$$

input
```
integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((2*x^2 - 1)/(4*x^4 - 6*x^2 + 1), x)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx = \frac{1}{20}\sqrt{10}\log\left(\left|x+\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right) + \frac{1}{20}\sqrt{10}\log\left(\left|x+\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right) - \frac{1}{20}\sqrt{10}\log\left(\left|x-\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right) - \frac{1}{20}\sqrt{10}\log\left(\left|x-\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right)$$

input
```
integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")
```

output
```
1/20*sqrt(10)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/20*sqrt(10)*log
(abs(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt
(10) + 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2
)))
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.42

$$\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx = \frac{\sqrt{10}\,\mathrm{atanh}\left(\frac{\sqrt{10}\,x}{2\,x^2+1}\right)}{10}$$

input
```
int(-(2*x^2 - 1)/(4*x^4 - 6*x^2 + 1),x)
```

output
```
(10^(1/2)*atanh((10^(1/2)*x)/(2*x^2 + 1)))/10
```

3.68 $\int \frac{1+x^2}{1+bx^2+x^4} \, dx$

3.68.1 Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{1+x^2}{1+bx^2+x^4} \, dx = -\frac{\arctan\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}+\frac{\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}$$

output
```
-arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))/(2+b)^(1/2)+arctan((2*x+(2-b)^(1/2
))/(2+b)^(1/2))/(2+b)^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.00

$$\int \frac{1+x^2}{1+bx^2+x^4} \, dx = \frac{\frac{\left(2-b+\sqrt{-4+b^2}\right)\arctan\left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{-4+b^2}}}\right)}{\sqrt{b-\sqrt{-4+b^2}}}+\frac{\left(-2+b+\sqrt{-4+b^2}\right)\arctan\left(\frac{\sqrt{2}x}{\sqrt{b+\sqrt{-4+b^2}}}\right)}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2}\sqrt{-4+b^2}}$$

input
```
Integrate[(1 + x^2)/(1 + b*x^2 + x^4),x]
```

output
```
(((2 - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/S
qrt[b - Sqrt[-4 + b^2]] + ((-2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sq
rt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2]
)
```

3.68.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{bx^2+x^4+1} \, dx$$

$$\Big\downarrow \ 1475$$

$$\frac{1}{2}\int \frac{1}{x^2-\sqrt{2-b}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2-b}x+1}dx$$

$$\Big\downarrow \ 1083$$

$$-\int \frac{1}{-\left(2x-\sqrt{2-b}\right)^2-b-2}d\left(2x-\sqrt{2-b}\right)-\int \frac{1}{-\left(2x+\sqrt{2-b}\right)^2-b-2}d\left(2x+\sqrt{2-b}\right)$$

$$\Big\downarrow \ 217$$

$$\frac{\arctan\left(\frac{2x-\sqrt{2-b}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}+\frac{\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

input
```
Int[(1 + x^2)/(1 + b*x^2 + x^4),x]
```

output
```
ArcTan[(-Sqrt[2 - b] + 2*x)/Sqrt[2 + b]]/Sqrt[2 + b] + ArcTan[(Sqrt[2 - b]
 + 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]
```

3.68.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.68.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{\ln\left(-x^2\sqrt{-2-b}+x(2+b)+\sqrt{-2-b}\right)}{2\sqrt{-2-b}}+\frac{\ln\left(-x^2\sqrt{-2-b}+(-2-b)x+\sqrt{-2-b}\right)}{2\sqrt{-2-b}}$	74
default	$\frac{\left(-2+\sqrt{(b-2)(2+b)}+b\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\sqrt{2\sqrt{(b-2)(2+b)}+2b}}+\frac{\left(2+\sqrt{(b-2)(2+b)}-b\right)\arctan\left(\frac{2x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$	124

input
```
int((x^2+1)/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-2-b)^(1/2)*ln(-x^2*(-2-b)^(1/2)+x*(2+b)+(-2-b)^(1/2))+1/2/(-2-b)^(1
/2)*ln(-x^2*(-2-b)^(1/2)+(-2-b)*x+(-2-b)^(1/2))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

$$\int\frac{1+x^2}{1+bx^2+x^4}\,dx = \left[-\frac{\sqrt{-b-2}\log\left(\frac{x^4-(b+4)x^2-2\left(x^3-x\right)\sqrt{-b-2}+1}{x^4+bx^2+1}\right)}{2\,(b+2)},\frac{\sqrt{b+2}\arctan\left(\frac{x^3+(b+1)x}{\sqrt{b+2}}\right)+\sqrt{b+2}\arctan\left(\frac{x}{\sqrt{b+2}}\right)}{b+2}\right]$$

input
```
integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b - 2)*log((x^4 - (b + 4)*x^2 - 2*(x^3 - x)*sqrt(-b - 2) + 1)/
(x^4 + b*x^2 + 1))/(b + 2), (sqrt(b + 2)*arctan((x^3 + (b + 1)*x)/sqrt(b +
 2)) + sqrt(b + 2)*arctan(x/sqrt(b + 2)))/(b + 2)]
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.42

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx = -\frac{\sqrt{-\frac{1}{b+2}}\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b+2}}-2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2} + \frac{\sqrt{-\frac{1}{b+2}}\log\left(x^2+x\left(b\sqrt{-\frac{1}{b+2}}+2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2}$$

input
```
integrate((x**2+1)/(x**4+b*x**2+1),x)
```

output
```
-sqrt(-1/(b + 2))*log(x**2 + x*(-b*sqrt(-1/(b + 2)) - 2*sqrt(-1/(b + 2)))
- 1)/2 + sqrt(-1/(b + 2))*log(x**2 + x*(b*sqrt(-1/(b + 2)) + 2*sqrt(-1/(b
+ 2))) - 1)/2
```

3.68.7 Maxima [F]

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx = \int \frac{x^2+1}{x^4+bx^2+1}\,dx$$

input
```
integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/(x^4 + b*x^2 + 1), x)
```

3.68.8 Giac [F]

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx = \int \frac{x^2+1}{x^4+bx^2+1}\,dx$$

input
```
integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")
```

output
```
sage0*x
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx = \frac{\operatorname{atan}\left(\frac{x}{\sqrt{b+2}}\right)+\operatorname{atan}\left((b+2)\,\left(x\,\left(\frac{1}{\sqrt{b+2}}+\frac{\frac{4}{b+2}-1}{(b-2)\,\sqrt{b+2}}\right)+\frac{x^3\,\left(\frac{2\,b}{b+2}-1\right)}{(b-2)\,\sqrt{b+2}}\right)\right)}{\sqrt{b+2}}$$

input
```
int((x^2 + 1)/(b*x^2 + x^4 + 1),x)
```

output
```
(atan(x/(b + 2)^(1/2)) + atan((b + 2)*(x*(1/(b + 2)^(1/2) + (4/(b + 2) - 1
)/((b - 2)*(b + 2)^(1/2))) + (x^3*((2*b)/(b + 2) - 1))/((b - 2)*(b + 2)^(1
/2)))))/(b + 2)^(1/2)
```

3.69 $\int \frac{1+x^2}{1+5x^2+x^4}\,dx$

3.69.1 Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx=\frac{\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{7}}+\frac{\arctan\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{7}}$$

output
```
1/7*arctan(x*2^(1/2)/(5+21^(1/2))^(1/2))*7^(1/2)+1/7*arctan(x*(1/2*7^(1/2)
+1/2*3^(1/2)))*7^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx=\frac{\left(-3+\sqrt{21}\right)\arctan\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}}+\frac{\left(3+\sqrt{21}\right)\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

input
```
Integrate[(1 + x^2)/(1 + 5*x^2 + x^4),x]
```

output
```
((-3 + Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sqrt[21])]
 + ((3 + Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[42*(5 + Sqrt[21]
)]
```

3.69.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.82, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4+5x^2+1}dx$$

$$\Big\downarrow \ 1477$$

$$\frac{1}{14}\left(7-\sqrt{21}\right)\int \frac{1}{x^2+\frac{1}{2}\left(5-\sqrt{21}\right)}dx+\frac{1}{14}\left(7+\sqrt{21}\right)\int \frac{1}{x^2+\frac{1}{2}\left(5+\sqrt{21}\right)}dx$$

$$\Big\downarrow \ 216$$

$$\frac{\left(7+\sqrt{21}\right)\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{7\sqrt{2\left(5+\sqrt{21}\right)}}+\frac{1}{14}\left(7-\sqrt{21}\right)\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)$$

input
```
Int[(1 + x^2)/(1 + 5*x^2 + x^4),x]
```

output
```
((7 + Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/(7*Sqrt[2*(5 + Sqrt[21])
]) + ((7 - Sqrt[21])*Sqrt[(5 + Sqrt[21])/2]*ArcTan[Sqrt[(5 + Sqrt[21])/2]*
x])/14
```

3.69.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

3.69.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{\sqrt{7}\,\arctan\left(\frac{x\sqrt{7}}{7}\right)}{7}+\frac{\sqrt{7}\,\arctan\left(\frac{x^{3}\sqrt{7}}{7}+\frac{6x\sqrt{7}}{7}\right)}{7}$	35
default	$\frac{2\left(3+\sqrt{21}\right)\sqrt{21}\,\arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}+\frac{2\left(-3+\sqrt{21}\right)\sqrt{21}\,\arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}$	82

input
```
int((x^2+1)/(x^4+5*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/7*7^(1/2)*arctan(1/7*x*7^(1/2))+1/7*7^(1/2)*arctan(1/7*x^3*7^(1/2)+6/7*x
*7^(1/2))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx=\frac{1}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}\left(x^3+6\,x\right)\right)+\frac{1}{7}\,\sqrt{7}\arctan\left(\frac{1}{7}\,\sqrt{7}x\right)$$

input
```
integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="fricas")
```

output
```
1/7*sqrt(7)*arctan(1/7*sqrt(7)*(x^3 + 6*x)) + 1/7*sqrt(7)*arctan(1/7*sqrt(
7)*x)
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx=\frac{\sqrt{7}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{7}x}{7}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{7}x^3}{7}+\frac{6\sqrt{7}x}{7}\right)\right)}{14}$$

input
```
integrate((x**2+1)/(x**4+5*x**2+1),x)
```

output
```
sqrt(7)*(2*atan(sqrt(7)*x/7) + 2*atan(sqrt(7)*x**3/7 + 6*sqrt(7)*x/7))/14
```

3.69.7 Maxima [F]

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx = \int \frac{x^2+1}{x^4+5\,x^2+1}\,dx$$

input
```
integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/(x^4 + 5*x^2 + 1), x)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx = \frac{1}{14}\,\sqrt{7}\left(\pi\mathrm{sgn}(x) + 2\,\arctan\left(\frac{\sqrt{7}(x^2-1)}{7\,x}\right)\right)$$

input
```
integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")
```

output
```
1/14*sqrt(7)*(pi*sgn(x) + 2*arctan(1/7*sqrt(7)*(x^2 - 1)/x))
```

3.69.9 Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx = \frac{\sqrt{7}\left(\mathrm{atan}\left(\frac{\sqrt{7}\,x^3}{7}+\frac{6\,\sqrt{7}\,x}{7}\right)+\mathrm{atan}\left(\frac{\sqrt{7}\,x}{7}\right)\right)}{7}$$

input
```
int((x^2 + 1)/(5*x^2 + x^4 + 1),x)
```

output
```
(7^(1/2)*(atan((6*7^(1/2)*x)/7 + (7^(1/2)*x^3)/7) + atan((7^(1/2)*x)/7)))/
7
```

3.70 $\int \frac{1+x^2}{1+4x^2+x^4} \, dx$

3.70.1 Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{1+x^2}{1+4x^2+x^4} \, dx = \frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}} + \frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

output
```
1/6*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/6*arctan(x/(1/2*6^(1/2)+
1/2*2^(1/2)))*6^(1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int \frac{1+x^2}{1+4x^2+x^4} \, dx = \frac{\left(-1+\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\left(1+\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

input
```
Integrate[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

output
```
((-1 + Sqrt[3])*ArcTan[x/Sqrt[2 - Sqrt[3]]])/(2*Sqrt[3*(2 - Sqrt[3])]) + (
(1 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3*(2 + Sqrt[3])])
```

3.70.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.84, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4+4x^2+1}\,dx$$

$$\Big\downarrow \; 1477$$

$$\frac{1}{6}\left(3-\sqrt{3}\right)\int \frac{1}{x^2-\sqrt{3}+2}dx+\frac{1}{6}\left(3+\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{3}+2}dx$$

$$\Big\downarrow \; 216$$

$$\frac{\left(3-\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\left(3+\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

input
```
Int[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

output
```
((3 - Sqrt[3])*ArcTan[x/Sqrt[2 - Sqrt[3]]])/(6*Sqrt[2 - Sqrt[3]]) + ((3 +
Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(6*Sqrt[2 + Sqrt[3]])
```

3.70.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

3.70.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{\sqrt{6}\,\arctan\left(\frac{x\sqrt{6}}{6}\right)}{6}+\frac{\sqrt{6}\,\arctan\left(\frac{x^{3}\sqrt{6}}{6}+\frac{5x\sqrt{6}}{6}\right)}{6}$	35
default	$\frac{\left(1+\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\sqrt{6}+3\sqrt{2}}+\frac{\left(\sqrt{3}-1\right)\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}$	70

input
```
int((x^2+1)/(x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*6^(1/2)*arctan(1/6*x*6^(1/2))+1/6*6^(1/2)*arctan(1/6*x^3*6^(1/2)+5/6*x
*6^(1/2))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx = \frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}(x^3+5\,x)\right)+\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}x\right)$$

input
```
integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(6)*arctan(1/6*sqrt(6)*(x^3 + 5*x)) + 1/6*sqrt(6)*arctan(1/6*sqrt(
6)*x)
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx = \frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{6}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{6}x^3}{6}+\frac{5\sqrt{6}x}{6}\right)\right)}{12}$$

input
```
integrate((x**2+1)/(x**4+4*x**2+1),x)
```

output
```
sqrt(6)*(2*atan(sqrt(6)*x/6) + 2*atan(sqrt(6)*x**3/6 + 5*sqrt(6)*x/6))/12
```

3.70.7 Maxima [F]

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx = \int \frac{x^2+1}{x^4+4\,x^2+1}\,dx$$

input
```
integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/(x^4 + 4*x^2 + 1), x)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx = \frac{1}{12}\,\sqrt{6}\left(\pi\operatorname{sgn}(x) + 2\,\arctan\left(\frac{\sqrt{6}(x^2-1)}{6\,x}\right)\right)$$

input
```
integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*(pi*sgn(x) + 2*arctan(1/6*sqrt(6)*(x^2 - 1)/x))
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx = \frac{\sqrt{6}\,\left(\operatorname{atan}\left(\frac{\sqrt{6}\,x^3}{6}+\frac{5\,\sqrt{6}\,x}{6}\right)+\operatorname{atan}\left(\frac{\sqrt{6}\,x}{6}\right)\right)}{6}$$

input
```
int((x^2 + 1)/(4*x^2 + x^4 + 1),x)
```

output
```
(6^(1/2)*(atan((5*6^(1/2)*x)/6 + (6^(1/2)*x^3)/6) + atan((6^(1/2)*x)/6)))/
6
```

3.71 $\int \frac{1+x^2}{1+3x^2+x^4}\,dx$

3.71.1 Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx=\frac{\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{5}}+\frac{\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{\sqrt{5}}$$

output
```
1/5*arctan(x*2^(1/2)/(3+5^(1/2))^(1/2))*5^(1/2)+1/5*arctan(x*(1/2+1/2*5^(1
/2)))*5^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx=\frac{\left(-1+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3-\sqrt{5}}}x\right)}{\sqrt{10\left(3-\sqrt{5}\right)}}+\frac{\left(1+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

input
```
Integrate[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

output
```
((-1 + Sqrt[5])*ArcTan[Sqrt[2/(3 - Sqrt[5])]*x])/Sqrt[10*(3 - Sqrt[5])] +
((1 + Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/Sqrt[10*(3 + Sqrt[5])]
```

3.71.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.82, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4+3x^2+1}dx$$

$$\Big\downarrow \ 1477$$

$$\frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{x^2+\frac{1}{2}\left(3-\sqrt{5}\right)}dx+\frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{x^2+\frac{1}{2}\left(3+\sqrt{5}\right)}dx$$

$$\Big\downarrow \ 216$$

$$\frac{\left(5+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}+\frac{1}{10}\left(5-\sqrt{5}\right)\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)$$

input
```
Int[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

output
```
((5 + Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/(5*Sqrt[2*(3 + Sqrt[5])])
+ ((5 - Sqrt[5])*Sqrt[(3 + Sqrt[5])/2]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/10
```

3.71.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

3.71.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\sqrt{5}\,\arctan\left(\frac{x^{3}\sqrt{5}}{5}+\frac{4x\sqrt{5}}{5}\right)}{5}$	35
default	$\frac{2\left(\sqrt{5}-1\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}+\frac{2\left(\sqrt{5}+1\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}$	66

input
```
int((x^2+1)/(x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/5*arctan(1/5*x*5^(1/2))*5^(1/2)+1/5*5^(1/2)*arctan(1/5*x^3*5^(1/2)+4/5*x
*5^(1/2))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx = \frac{1}{5}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}(x^3+4\,x)\right)+\frac{1}{5}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}x\right)$$

input
```
integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="fricas")
```

output
```
1/5*sqrt(5)*arctan(1/5*sqrt(5)*(x^3 + 4*x)) + 1/5*sqrt(5)*arctan(1/5*sqrt(
5)*x)
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx = \frac{\sqrt{5}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{5}x^3}{5}+\frac{4\sqrt{5}x}{5}\right)\right)}{10}$$

input
```
integrate((x**2+1)/(x**4+3*x**2+1),x)
```

output
```
sqrt(5)*(2*atan(sqrt(5)*x/5) + 2*atan(sqrt(5)*x**3/5 + 4*sqrt(5)*x/5))/10
```

3.71.7 Maxima [F]

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx = \int \frac{x^2+1}{x^4+3\,x^2+1}\,dx$$

input
```
integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/(x^4 + 3*x^2 + 1), x)
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx = \frac{1}{10}\sqrt{5}\left(\pi\text{sgn}(x)+2\,\arctan\left(\frac{\sqrt{5}(x^2-1)}{5\,x}\right)\right)$$

input
```
integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")
```

output
```
1/10*sqrt(5)*(pi*sgn(x) + 2*arctan(1/5*sqrt(5)*(x^2 - 1)/x))
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx = \frac{\sqrt{5}\left(\text{atan}\left(\frac{\sqrt{5}\,x^3}{5}+\frac{4\,\sqrt{5}\,x}{5}\right)+\text{atan}\left(\frac{\sqrt{5}\,x}{5}\right)\right)}{5}$$

input
```
int((x^2 + 1)/(3*x^2 + x^4 + 1),x)
```

output
```
(5^(1/2)*(atan((4*5^(1/2)*x)/5 + (5^(1/2)*x^3)/5) + atan((5^(1/2)*x)/5)))/
5
```

3.72 $\int \frac{1+x^2}{1+2x^2+x^4}\,dx$

3.72.1 Optimal result

Integrand size = 18, antiderivative size = 2

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \arctan(x)$$

output
```
arctan(x)
```

3.72.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \arctan(x)$$

input
```
Integrate[(1 + x^2)/(1 + 2*x^2 + x^4),x]
```

output
```
ArcTan[x]
```

3.72.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1380, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4+2x^2+1} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{1}{x^2+1}dx$$

$$\Big\downarrow \text{216}$$

$$\arctan(x)$$

input
```
Int[(1 + x^2)/(1 + 2*x^2 + x^4),x]
```

output
```
ArcTan[x]
```

3.72.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.72.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\arctan(x)$	3
risch	$\arctan(x)$	3
parallelrisch	$\frac{i\ln(x+i)}{2}-\frac{i\ln(x-i)}{2}$	18

input
```
int((x^2+1)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \arctan(x)$$

input
```
integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
arctan(x)
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \operatorname{atan}(x)$$

input
```
integrate((x**2+1)/(x**4+2*x**2+1),x)
```

output
```
atan(x)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \arctan(x)$$

input
```
integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
arctan(x)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \arctan(x)$$

input
```
integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")
```

output
```
arctan(x)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 13.39 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+2x^2+x^4}\,dx = \operatorname{atan}(x)$$

input
```
int((x^2 + 1)/(2*x^2 + x^4 + 1),x)
```

output
```
atan(x)
```

3.73 $\int \frac{1+x^2}{1+x^2+x^4}\,dx$

3.73.1 Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx=-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)
```

3.73.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.61

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx=\frac{(-i+\sqrt{3})\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1-i\sqrt{3}\right)}}+\frac{(i+\sqrt{3})\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1+i\sqrt{3}\right)}}$$

input
```
Integrate[(1 + x^2)/(1 + x^2 + x^4),x]
```

output
```
((-I + Sqrt[3])*ArcTan[x/Sqrt[(1 - I*Sqrt[3])/2]])/Sqrt[6*(1 - I*Sqrt[3])]
 + ((I + Sqrt[3])*ArcTan[x/Sqrt[(1 + I*Sqrt[3])/2]])/Sqrt[6*(1 + I*Sqrt[3]
)]
```

3.73.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4+x^2+1}\,dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1}{x^2+x+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\int \frac{1}{-(2x+1)^2-3}d(2x+1)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(1 + x^2)/(1 + x^2 + x^4),x]
```

output
```
ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
```

3.73.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.73.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	34
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{x\sqrt{3}}{3}\right)}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{x^3\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{3}$	35

input
```
int((x^2+1)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx=\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x^3+2\,x)\right)+\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)$$

input
```
integrate((x^2+1)/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*x)
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{6}$$

input
```
integrate((x**2+1)/(x**4+x**2+1),x)
```

output
```
sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/6
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)$$

input
```
integrate((x^2+1)/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1))
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\left(\pi\mathrm{sgn}(x)+2\,\arctan\left(\frac{\sqrt{3}(x^2-1)}{3\,x}\right)\right)$$

input
```
integrate((x^2+1)/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*(x^2 - 1)/x))
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\sqrt{3}\,x}{3}\right)+\operatorname{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{3}$$

input
```
int((x^2 + 1)/(x^2 + x^4 + 1),x)
```

output
```
(3^(1/2)*(atan((2*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) + atan((3^(1/2)*x)/3)))/
3
```

3.74 $\int \frac{1+x^2}{1+x^4}\,dx$

3.74.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1+x^2}{1+x^4}\,dx=-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}x\right)}{\sqrt{2}}$$

output
```
1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/2*arctan(1+x*2^(1/2))*2^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{1+x^4}\,dx=\frac{-\arctan\left(1-\sqrt{2}x\right)+\arctan\left(1+\sqrt{2}x\right)}{\sqrt{2}}$$

input
```
Integrate[(1 + x^2)/(1 + x^4),x]
```

output
```
(-ArcTan[1 - Sqrt[2]*x] + ArcTan[1 + Sqrt[2]*x])/Sqrt[2]
```

3.74.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1476, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4+1} \, dx$$
$$\Big\downarrow \ 1476$$
$$\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx$$
$$\Big\downarrow \ 1082$$
$$\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}$$
$$\Big\downarrow \ 217$$
$$\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

input
```
Int[(1 + x^2)/(1 + x^4),x]
```

output
```
-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2]
```

3.74.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.74.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\frac{x^3\sqrt{2}}{2}+\frac{x\sqrt{2}}{2}\right)}{2}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}$
meijerg	$\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2+\sqrt{2}\,(x^4)\ldots}\right.}{4(x^4)^{\frac{3}{4}}}$

input
```
int((x^2+1)/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctan(1/2*x*2^(1/2))+1/2*2^(1/2)*arctan(1/2*x^3*2^(1/2)+1/2*x
*2^(1/2))
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^3+x)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)$$

input
```
integrate((x^2+1)/(x^4+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + x)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)
*x)
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}x^3}{2}+\frac{\sqrt{2}x}{2}\right)\right)}{4}$$

input
```
integrate((x**2+1)/(x**4+1),x)
```

output
```
sqrt(2)*(2*atan(sqrt(2)*x/2) + 2*atan(sqrt(2)*x**3/2 + sqrt(2)*x/2))/4
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)$$

input
```
integrate((x^2+1)/(x^4+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2)))
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)$$

input
```
integrate((x^2+1)/(x^4+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2)))
```

3.74.9 Mupad [B] (verification not implemented)

Time = 13.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\,x^3}{2}+\frac{\sqrt{2}\,x}{2}\right)+\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)\right)}{2}$$

input
```
int((x^2 + 1)/(x^4 + 1),x)
```

output
```
(2^(1/2)*(atan((2^(1/2)*x)/2 + (2^(1/2)*x^3)/2) + atan((2^(1/2)*x)/2)))/2
```

3.75 $\int \frac{1+x^2}{1-x^2+x^4} \, dx$

3.75.1 Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{1+x^2}{1-x^2+x^4} \, dx = -\arctan\left(\sqrt{3}-2x\right)+\arctan\left(\sqrt{3}+2x\right)$$

output
```
arctan(2*x-3^(1/2))+arctan(2*x+3^(1/2))
```

3.75.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{1+x^2}{1-x^2+x^4} \, dx = -\arctan\left(\frac{x}{-1+x^2}\right)$$

input
```
Integrate[(1 + x^2)/(1 - x^2 + x^4),x]
```

output
```
-ArcTan[x/(-1 + x^2)]
```

3.75.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4-x^2+1} \, dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)-\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)$$

$$\Big\downarrow \text{217}$$

$$\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)$$

input
```
Int[(1 + x^2)/(1 - x^2 + x^4),x]
```

output
```
-ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]
```

3.75.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.75.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.35

method	result	size
risch	$\arctan\left(x^3\right)+\arctan\left(x\right)$	8
default	$\arctan\left(2x-\sqrt{3}\right)+\arctan\left(2x+\sqrt{3}\right)$	20
parallelrisch	$\frac{i\ln\left(x^2+ix-1\right)}{2}-\frac{i\ln\left(x^2-ix-1\right)}{2}$	28

input
```
int((x^2+1)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctan(x^3)+arctan(x)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx=\arctan\left(x^3\right)+\arctan\left(x\right)$$

input
```
integrate((x^2+1)/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
arctan(x^3) + arctan(x)
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \operatorname{atan}(x) + \operatorname{atan}(x^3)$$

input
```
integrate((x**2+1)/(x**4-x**2+1),x)
```

output
```
atan(x) + atan(x**3)
```

3.75.7 Maxima [F]

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \int \frac{x^2+1}{x^4-x^2+1}\,dx$$

input
```
integrate((x^2+1)/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/(x^4 - x^2 + 1), x)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \frac{1}{4}\,\pi\operatorname{sgn}(x) + \frac{1}{2}\,\arctan\left(\frac{x^4-3\,x^2+1}{2\,(x^3-x)}\right)$$

input
```
integrate((x^2+1)/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/4*pi*sgn(x) + 1/2*arctan(1/2*(x^4 - 3*x^2 + 1)/(x^3 - x))
```

3.75.9 Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \operatorname{atan}\left(x^3\right)+\operatorname{atan}(x)$$

input
```
int((x^2 + 1)/(x^4 - x^2 + 1),x)
```

output
```
atan(x^3) + atan(x)
```

3.76 $\int \frac{1+x^2}{1-2x^2+x^4}\,dx$

3.76.1 Optimal result

Integrand size = 18, antiderivative size = 11

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx=\frac{x}{1-x^2}$$

output
```
x/(-x^2+1)
```

3.76.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx=-\frac{x}{-1+x^2}$$

input
```
Integrate[(1 + x^2)/(1 - 2*x^2 + x^4),x]
```

output
```
-(x/(-1 + x^2))
```

3.76.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1380, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4-2x^2+1} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^2+1}{(1-x^2)^2}dx$$

$$\Big\downarrow \text{297}$$

$$\frac{x}{1-x^2}$$

input
```
Int[(1 + x^2)/(1 - 2*x^2 + x^4),x]
```

output
```
x/(1 - x^2)
```

3.76.3.1 Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.76.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{x}{x^2-1}$	11
norman	$-\frac{x}{x^2-1}$	11
risch	$-\frac{x}{x^2-1}$	11
parallelrisch	$-\frac{x}{x^2-1}$	11
default	$-\frac{1}{2(x+1)}-\frac{1}{2(x-1)}$	16

input
```
int((x^2+1)/(x^4-2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-x/(x^2-1)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx = -\frac{x}{x^2-1}$$

input
```
integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")
```

output
```
-x/(x^2 - 1)
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx = -\frac{x}{x^2-1}$$

input
```
integrate((x**2+1)/(x**4-2*x**2+1),x)
```

output
```
-x/(x**2 - 1)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx = -\frac{x}{x^2-1}$$

input
```
integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")
```

output
```
-x/(x^2 - 1)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx = -\frac{1}{x-\frac{1}{x}}$$

input
```
integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
```

output
```
-1/(x - 1/x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-2x^2+x^4}\,dx = -\frac{x}{x^2-1}$$

input
```
int((x^2 + 1)/(x^4 - 2*x^2 + 1),x)
```

output
```
-x/(x^2 - 1)
```

3.77 $\int \frac{1+x^2}{1-3x^2+x^4} \, dx$

3.77.1 Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{1+x^2}{1-3x^2+x^4} \, dx = \frac{1}{2}\log\left(1-\sqrt{5}-2x\right)+\frac{1}{2}\log\left(1+\sqrt{5}-2x\right) -\frac{1}{2}\log\left(1-\sqrt{5}+2x\right)-\frac{1}{2}\log\left(1+\sqrt{5}+2x\right)$$

output
```
1/2*ln(1-2*x-5^(1/2))-1/2*ln(1+2*x-5^(1/2))+1/2*ln(1-2*x+5^(1/2))-1/2*ln(1
+2*x+5^(1/2))
```

3.77.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

$$\int \frac{1+x^2}{1-3x^2+x^4} \, dx = -\frac{1}{2}\log\left(1-x-x^2\right)+\frac{1}{2}\log\left(1+x-x^2\right)$$

input
```
Integrate[(1 + x^2)/(1 - 3*x^2 + x^4),x]
```

output
```
-1/2*Log[1 - x - x^2] + Log[1 + x - x^2]/2
```

3.77.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4-3x^2+1}\,dx$$

$$\Big\downarrow \ 1475$$

$$\frac{1}{2}\int \frac{1}{x^2-\sqrt{5}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{5}x+1}dx$$

$$\Big\downarrow \ 1081$$

$$\frac{1}{2}\int \left(-\frac{2}{2x-\sqrt{5}+1}-\frac{2}{-2x+\sqrt{5}+1}\right)dx+\frac{1}{2}\int \left(-\frac{2}{2x+\sqrt{5}+1}-\frac{2}{-2x-\sqrt{5}+1}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\log\left(-2x+\sqrt{5}+1\right)-\log\left(2x-\sqrt{5}+1\right)\right)+\frac{1}{2}\left(\log\left(-2x-\sqrt{5}+1\right)-\log\left(2x+\sqrt{5}+1\right)\right)$$

input
```
Int[(1 + x^2)/(1 - 3*x^2 + x^4),x]
```

output
```
(Log[1 + Sqrt[5] - 2*x] - Log[1 - Sqrt[5] + 2*x])/2 + (Log[1 - Sqrt[5] - 2
*x] - Log[1 + Sqrt[5] + 2*x])/2
```

3.77.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.77.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.34

method	result	size
default	$-\frac{\ln(x^2+x-1)}{2}+\frac{\ln(x^2-x-1)}{2}$	22
norman	$-\frac{\ln(x^2+x-1)}{2}+\frac{\ln(x^2-x-1)}{2}$	22
risch	$-\frac{\ln(x^2+x-1)}{2}+\frac{\ln(x^2-x-1)}{2}$	22
parallelrisch	$-\frac{\ln(x^2+x-1)}{2}+\frac{\ln(x^2-x-1)}{2}$	22

input
```
int((x^2+1)/(x^4-3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x^2+x-1)+1/2*ln(x^2-x-1)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.32

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx = -\frac{1}{2}\log\left(x^2+x-1\right) + \frac{1}{2}\log\left(x^2-x-1\right)$$

input
```
integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="fricas")
```

output
```
-1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.29

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx = \frac{\log\left(x^2-x-1\right)}{2} - \frac{\log\left(x^2+x-1\right)}{2}$$

input
```
integrate((x**2+1)/(x**4-3*x**2+1),x)
```

output
```
log(x**2 - x - 1)/2 - log(x**2 + x - 1)/2
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.32

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx = -\frac{1}{2}\log\left(x^2+x-1\right) + \frac{1}{2}\log\left(x^2-x-1\right)$$

input
```
integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="maxima")
```

output
```
-1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx = -\frac{1}{4}\log\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}-2\right|\right)$$

input
```
integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")
```

output
```
-1/4*log(abs(x + 1/(x - 1/x) - 1/x + 2)) + 1/4*log(abs(x + 1/(x - 1/x) - 1
/x - 2))
```

3.77.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.18

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx = -\operatorname{atanh}\left(\frac{x}{x^2-1}\right)$$

input
```
int((x^2 + 1)/(x^4 - 3*x^2 + 1),x)
```

output
```
-atanh(x/(x^2 - 1))
```

3.78 $\int \frac{1+x^2}{1-4x^2+x^4} \, dx$

3.78.1 Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{1+x^2}{1-4x^2+x^4} \, dx = \frac{\operatorname{arctanh}\left(\sqrt{3}-\sqrt{2}x\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\sqrt{3}+\sqrt{2}x\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(x*2^(1/2)-3^(1/2))*2^(1/2)-1/2*arctanh(x*2^(1/2)+3^(1/2))*2^(
1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1+x^2}{1-4x^2+x^4} \, dx = \frac{\log\left(1+\sqrt{2}x-x^2\right) - \log\left(-1+\sqrt{2}x+x^2\right)}{2\sqrt{2}}$$

input
```
Integrate[(1 + x^2)/(1 - 4*x^2 + x^4),x]
```

output
```
(Log[1 + Sqrt[2]*x - x^2] - Log[-1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])
```

3.78.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 102 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4-4x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-\sqrt{6}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{6}x+1}dx$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{2}\int \left(-\frac{\sqrt{2}}{2x+\sqrt{2}\left(1-\sqrt{3}\right)}-\frac{\sqrt{2}}{-2x+\sqrt{6}+\sqrt{2}}\right)dx+\frac{1}{2}\int \left(\frac{1}{\sqrt{2}x+\sqrt{3}-1}-\frac{\sqrt{2}}{2x+\sqrt{6}+\sqrt{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}x-\sqrt{3}+1\right)}{\sqrt{2}}-\frac{\log\left(2x+\sqrt{6}+\sqrt{2}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-2x+\sqrt{6}+\sqrt{2}\right)}{\sqrt{2}}-\frac{\log\left(\sqrt{2}x-\sqrt{3}+1\right)}{\sqrt{2}}\right)$$

input
```
Int[(1 + x^2)/(1 - 4*x^2 + x^4),x]
```

output
```
(-(Log[Sqrt[2] + Sqrt[6] + 2*x]/Sqrt[2]) + Log[1 - Sqrt[3] - Sqrt[2]*x]/Sq
rt[2])/2 + (Log[Sqrt[2] + Sqrt[6] - 2*x]/Sqrt[2] - Log[1 - Sqrt[3] + Sqrt[
2]*x]/Sqrt[2])/2
```

3.78.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.78.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{\sqrt{2}\,\ln\left(x^2-x\sqrt{2}-1\right)}{4}-\frac{\sqrt{2}\,\ln\left(x^2+x\sqrt{2}-1\right)}{4}$	35
default	$-\frac{\sqrt{3}\,\left(\sqrt{3}+3\right)\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\left(\sqrt{6}-\sqrt{2}\right)}$	70

input
```
int((x^2+1)/(x^4-4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*ln(x^2-x*2^(1/2)-1)-1/4*2^(1/2)*ln(x^2+x*2^(1/2)-1)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^4-2\sqrt{2}(x^3-x)+1}{x^4-4x^2+1}\right)$$

input
```
integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((x^4 - 2*sqrt(2)*(x^3 - x) + 1)/(x^4 - 4*x^2 + 1))
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx = \frac{\sqrt{2}\log\left(x^2-\sqrt{2}x-1\right)}{4} - \frac{\sqrt{2}\log\left(x^2+\sqrt{2}x-1\right)}{4}$$

input
```
integrate((x**2+1)/(x**4-4*x**2+1),x)
```

output
```
sqrt(2)*log(x**2 - sqrt(2)*x - 1)/4 - sqrt(2)*log(x**2 + sqrt(2)*x - 1)/4
```

3.78.7 Maxima [F]

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx = \int \frac{x^2+1}{x^4-4x^2+1}\,dx$$

input
```
integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/(x^4 - 4*x^2 + 1), x)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left|2\,x-2\,\sqrt{2}-\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{2}-\frac{2}{x}\right|}\right)$$

input
```
integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2/x)/abs(2*x + 2*sqrt(2) - 2/x))
```

3.78.9 Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.42

$$\int \frac{1+x^2}{1-4x^2+x^4}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{x^2-1}\right)}{2}$$

input
```
int((x^2 + 1)/(x^4 - 4*x^2 + 1),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*x)/(x^2 - 1)))/2
```

3.79 $\int \frac{1+x^2}{1-5x^2+x^4} \, dx$

3.79.1 Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \frac{1+x^2}{1-5x^2+x^4} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{7}+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
1/3*arctanh(1/3*(-2*x+7^(1/2))*3^(1/2))*3^(1/2)-1/3*arctanh(1/3*(2*x+7^(1/
2))*3^(1/2))*3^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1+x^2}{1-5x^2+x^4} \, dx = \frac{\log\left(1+\sqrt{3}x-x^2\right) - \log\left(-1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

input
```
Integrate[(1 + x^2)/(1 - 5*x^2 + x^4),x]
```

output
```
(Log[1 + Sqrt[3]*x - x^2] - Log[-1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])
```

3.79.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 105 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.28, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4-5x^2+1}\,dx$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-\sqrt{7}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{7}x+1}dx$$

$$\Big\downarrow \text{ 1081}$$

$$\frac{1}{2}\int \left(-\frac{2}{2\sqrt{3}x-\sqrt{21}+3}-\frac{2}{-2\sqrt{3}x+\sqrt{21}+3}\right)dx+\frac{1}{2}\int \left(-\frac{2}{2\sqrt{3}x+\sqrt{21}+3}-\frac{2}{-2\sqrt{3}x-\sqrt{21}+3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{\log\left(-2\sqrt{3}x+\sqrt{21}+3\right)}{\sqrt{3}}-\frac{\log\left(2\sqrt{3}x-\sqrt{21}+3\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{\log\left(-2\sqrt{3}x-\sqrt{21}+3\right)}{\sqrt{3}}-\frac{\log\left(2\sqrt{3}x+\sqrt{21}+3\right)}{\sqrt{3}}\right)$$

input
```
Int[(1 + x^2)/(1 - 5*x^2 + x^4),x]
```

output
```
(Log[3 + Sqrt[21] - 2*Sqrt[3]*x]/Sqrt[3] - Log[3 - Sqrt[21] + 2*Sqrt[3]*x]
/Sqrt[3])/2 + (Log[3 - Sqrt[21] - 2*Sqrt[3]*x]/Sqrt[3] - Log[3 + Sqrt[21]
+ 2*Sqrt[3]*x]/Sqrt[3])/2
```

3.79.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.79.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\sqrt{3}\,\ln\left(x^{2}-x\sqrt{3}-1\right)}{6}-\frac{\sqrt{3}\,\ln\left(x^{2}+x\sqrt{3}-1\right)}{6}$	35
default	$-\frac{2\sqrt{21}\,\left(7+\sqrt{21}\right)\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}-\frac{2\left(-7+\sqrt{21}\right)\sqrt{21}\,\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}$	82

input
```
int((x^2+1)/(x^4-5*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*ln(x^2-x*3^(1/2)-1)-1/6*3^(1/2)*ln(x^2+x*3^(1/2)-1)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2}{1-5x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{x^4+x^2-2\sqrt{3}(x^3-x)+1}{x^4-5\,x^2+1}\right)$$

input
```
integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log((x^4 + x^2 - 2*sqrt(3)*(x^3 - x) + 1)/(x^4 - 5*x^2 + 1))
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2}{1-5x^2+x^4}\,dx = \frac{\sqrt{3}\log\left(x^2-\sqrt{3}x-1\right)}{6} - \frac{\sqrt{3}\log\left(x^2+\sqrt{3}x-1\right)}{6}$$

input
```
integrate((x**2+1)/(x**4-5*x**2+1),x)
```

output
```
sqrt(3)*log(x**2 - sqrt(3)*x - 1)/6 - sqrt(3)*log(x**2 + sqrt(3)*x - 1)/6
```

3.79.7 Maxima [F]

$$\int \frac{1+x^2}{1-5x^2+x^4}\,dx = \int \frac{x^2+1}{x^4-5\,x^2+1}\,dx$$

input
```
integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/(x^4 - 5*x^2 + 1), x)
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2}{1-5x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(\frac{\left|2\,x-2\,\sqrt{3}-\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{3}-\frac{2}{x}\right|}\right)$$

input
```
integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) - 2/x)/abs(2*x + 2*sqrt(3) - 2/x))
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{1+x^2}{1-5x^2+x^4}\,dx = -\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{x^2-1}\right)}{3}$$

input
```
int((x^2 + 1)/(x^4 - 5*x^2 + 1),x)
```

output
```
-(3^(1/2)*atanh((3^(1/2)*x)/(x^2 - 1)))/3
```

3.80 $\int \frac{1-x^2}{1+bx^2+x^4} \, dx$

3.80.1 Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{1-x^2}{1+bx^2+x^4} \, dx = -\frac{\log \left(1-\sqrt{2-b}x+x^2\right)}{2\sqrt{2-b}}+\frac{\log \left(1+\sqrt{2-b}x+x^2\right)}{2\sqrt{2-b}}$$

output
```
-1/2*ln(1+x^2-x*(2-b)^(1/2))/(2-b)^(1/2)+1/2*ln(1+x^2+x*(2-b)^(1/2))/(2-b)
^(1/2)
```

3.80.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 125 vs. 2(62) = 124.

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.02

$$\int \frac{1-x^2}{1+bx^2+x^4} \, dx = \frac{\frac{\left(2+b-\sqrt{-4+b^2}\right) \arctan \left(\frac{\sqrt{2}x}{\sqrt{b-\sqrt{-4+b^2}}}\right)}{\sqrt{b-\sqrt{-4+b^2}}}-\frac{\left(2+b+\sqrt{-4+b^2}\right) \arctan \left(\frac{\sqrt{2}x}{\sqrt{b+\sqrt{-4+b^2}}}\right)}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2}\sqrt{-4+b^2}}$$

input
```
Integrate[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

output
```
(((2 + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/S
qrt[b - Sqrt[-4 + b^2]] - ((2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqr
t[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

3.80.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{bx^2+x^4+1} \, dx$$

$$\Big\downarrow \text{ 1478}$$

$$-\frac{\int -\frac{\sqrt{2-b}-2x}{x^2-\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}-\frac{\int -\frac{2x+\sqrt{2-b}}{x^2+\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sqrt{2-b}-2x}{x^2-\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}+\frac{\int \frac{2x+\sqrt{2-b}}{x^2+\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\log \left(\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}-\frac{\log \left(-\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}$$

input
```
Int[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

output
```
-1/2*Log[1 - Sqrt[2 - b]*x + x^2]/Sqrt[2 - b] + Log[1 + Sqrt[2 - b]*x + x^
2]/(2*Sqrt[2 - b])
```

3.80.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.80.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

method	result	size
risch	$-\frac{\ln\left(-x^{2}\sqrt{2-b}+(2-b)x-\sqrt{2-b}\right)}{2\sqrt{2-b}}+\frac{\ln\left(-x^{2}\sqrt{2-b}+x(b-2)-\sqrt{2-b}\right)}{2\sqrt{2-b}}$	78
default	$\frac{\left(-2-\sqrt{(b-2)(2+b)}-b\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\sqrt{2\sqrt{(b-2)(2+b)}+2b}}+\frac{\left(2-\sqrt{(b-2)(2+b)}+b\right)\arctan\left(\frac{2x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$	128

input
```
int((-x^2+1)/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(2-b)^(1/2)*ln(-x^2*(2-b)^(1/2)+(2-b)*x-(2-b)^(1/2))+1/2/(2-b)^(1/2)*
ln(-x^2*(2-b)^(1/2)+x*(b-2)-(2-b)^(1/2))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = \left[-\frac{\sqrt{-b+2}\log\left(\frac{x^4-(b-4)x^2+2\,(x^3+x)\sqrt{-b+2}+1}{x^4+bx^2+1}\right)}{2\,(b-2)}, \frac{\sqrt{b-2}\arctan\left(\frac{x^3+(b-1)x}{\sqrt{b-2}}\right)-\sqrt{b-2}\arctan\left(\frac{x}{\sqrt{b-2}}\right)}{b-2}\right]$$

input
```
integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b + 2)*log((x^4 - (b - 4)*x^2 + 2*(x^3 + x)*sqrt(-b + 2) + 1)/
(x^4 + b*x^2 + 1))/(b - 2), (sqrt(b - 2)*arctan((x^3 + (b - 1)*x)/sqrt(b -
 2)) - sqrt(b - 2)*arctan(x/sqrt(b - 2)))/(b - 2)]
```

3.80.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = \frac{\sqrt{-\frac{1}{b-2}}\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b-2}}+2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2} - \frac{\sqrt{-\frac{1}{b-2}}\log\left(x^2+x\left(b\sqrt{-\frac{1}{b-2}}-2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2}$$

input
```
integrate((-x**2+1)/(x**4+b*x**2+1),x)
```

output
```
sqrt(-1/(b - 2))*log(x**2 + x*(-b*sqrt(-1/(b - 2)) + 2*sqrt(-1/(b - 2))) +
 1)/2 - sqrt(-1/(b - 2))*log(x**2 + x*(b*sqrt(-1/(b - 2)) - 2*sqrt(-1/(b -
 2))) + 1)/2
```

3.80.7 Maxima [F]

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = \int -\frac{x^2-1}{x^4+bx^2+1}\,dx$$

input
```
integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(x^4 + b*x^2 + 1), x)
```

3.80.8 Giac [F]

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = \int -\frac{x^2-1}{x^4+bx^2+1}\,dx$$

input
```
integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")
```

output
```
sage0*x
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = -\frac{\operatorname{atan}\left(\frac{x}{\sqrt{b-2}}\right)-\operatorname{atan}\left((b-2)\left(x\left(\frac{1}{\sqrt{b-2}}+\frac{\frac{4}{b-2}+1}{\sqrt{b-2}\,(b+2)}\right)+\frac{x^3\left(\frac{2b}{b-2}-1\right)}{\sqrt{b-2}\,(b+2)}\right)\right)}{\sqrt{b-2}}$$

input
```
int(-(x^2 - 1)/(b*x^2 + x^4 + 1),x)
```

output
```
-(atan(x/(b - 2)^(1/2)) - atan((b - 2)*(x*(1/(b - 2)^(1/2) + (4/(b - 2) +
1)/((b - 2)^(1/2)*(b + 2))) + (x^3*((2*b)/(b - 2) - 1))/((b - 2)^(1/2)*(b
+ 2)))))/(b - 2)^(1/2)
```

3.81 $\int \frac{1-x^2}{1+5x^2+x^4}\,dx$

3.81.1 Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx=-\frac{\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{3}}+\frac{\arctan\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)}{\sqrt{3}}$$

output
```
-1/3*arctan(x*2^(1/2)/(5+21^(1/2))^(1/2))*3^(1/2)+1/3*arctan(x*(1/2*7^(1/2
)+1/2*3^(1/2)))*3^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.74

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx=\frac{\left(7-\sqrt{21}\right)\arctan\left(\sqrt{\frac{2}{5-\sqrt{21}}}x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}}+\frac{\left(-7-\sqrt{21}\right)\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

input
```
Integrate[(1 - x^2)/(1 + 5*x^2 + x^4),x]
```

output
```
((7 - Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sqrt[21])]
+ ((-7 - Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[42*(5 + Sqrt[21]
)]
```

3.81.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{x^4+5x^2+1} \, dx$$

$$\Big\downarrow \text{1477}$$

$$-\frac{1}{6}\left(3-\sqrt{21}\right)\int \frac{1}{x^2+\frac{1}{2}\left(5-\sqrt{21}\right)}dx-\frac{1}{6}\left(3+\sqrt{21}\right)\int \frac{1}{x^2+\frac{1}{2}\left(5+\sqrt{21}\right)}dx$$

$$\Big\downarrow \text{216}$$

$$-\frac{\left(3+\sqrt{21}\right)\arctan\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)}{3\sqrt{2\left(5+\sqrt{21}\right)}}-\frac{1}{6}\left(3-\sqrt{21}\right)\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)$$

input
```
Int[(1 - x^2)/(1 + 5*x^2 + x^4),x]
```

output
```
-1/3*((3 + Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[2*(5 + Sqrt[21
])] - ((3 - Sqrt[21])*Sqrt[(5 + Sqrt[21])/2]*ArcTan[Sqrt[(5 + Sqrt[21])/2]
*x])/6
```

3.81.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

3.81.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{\sqrt{3}\, \arctan\left(\frac{x\sqrt{3}}{3}\right)}{3}+\frac{\sqrt{3}\, \arctan\left(\frac{x^{3}\sqrt{3}}{3}+\frac{4x\sqrt{3}}{3}\right)}{3}$	35
default	$-\frac{2\sqrt{21}\, \left(7+\sqrt{21}\right) \arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}-\frac{2\left(-7+\sqrt{21}\right)\sqrt{21}\, \arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}$	82

input
```
int((-x^2+1)/(x^4+5*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*3^(1/2)*arctan(1/3*x*3^(1/2))+1/3*3^(1/2)*arctan(1/3*x^3*3^(1/2)+4/3*
x*3^(1/2))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \frac{1-x^2}{1+5x^2+x^4}\, dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x^3+4\,x)\right)-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)$$

input
```
integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 4*x)) - 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*x)
```

3.81.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1-x^2}{1+5x^2+x^4}\, dx = -\frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{4\sqrt{3}x}{3}\right)\right)}{6}$$

input
```
integrate((-x**2+1)/(x**4+5*x**2+1),x)
```

output
```
-sqrt(3)*(2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 4*sqrt(3)*x/3))/6
```

3.81.7 Maxima [F]

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx = \int -\frac{x^2-1}{x^4+5\,x^2+1}\,dx$$

input
```
integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(x^4 + 5*x^2 + 1), x)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\left(\pi\text{sgn}(x) - 2\,\arctan\left(\frac{\sqrt{3}(x^2+1)}{3\,x}\right)\right)$$

input
```
integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(pi*sgn(x) - 2*arctan(1/3*sqrt(3)*(x^2 + 1)/x))
```

3.81.9 Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx = \frac{\sqrt{3}\left(\text{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{4\,\sqrt{3}\,x}{3}\right)-\text{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{3}$$

input
```
int(-(x^2 - 1)/(5*x^2 + x^4 + 1),x)
```

output
```
(3^(1/2)*(atan((4*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) - atan((3^(1/2)*x)/3)))/
3
```

3.82 $\int \frac{1-x^2}{1+4x^2+x^4} dx$

3.82.1 Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{1-x^2}{1+4x^2+x^4} dx = \frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1/2)-1/2*arctan(x/(1/2*6^(1/2)+
1/2*2^(1/2)))*2^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

$$\int \frac{1-x^2}{1+4x^2+x^4} dx = \frac{-\left(\left(-3+\sqrt{3}\right)\sqrt{2+\sqrt{3}}\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)\right) - \sqrt{2-\sqrt{3}}\left(3+\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

input
```
Integrate[(1 - x^2)/(1 + 4*x^2 + x^4),x]
```

output
```
(-((-3 + Sqrt[3])*Sqrt[2 + Sqrt[3]]*ArcTan[x/Sqrt[2 - Sqrt[3]]]) - Sqrt[2
- Sqrt[3]]*(3 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3])
```

3.82.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{x^4+4x^2+1}dx$$

$$\Big\downarrow \text{1477}$$

$$-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{x^2-\sqrt{3}+2}dx-\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{1}{x^2+\sqrt{3}+2}dx$$

$$\Big\downarrow \text{216}$$

$$-\frac{\left(1-\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{2-\sqrt{3}}}-\frac{\left(1+\sqrt{3}\right)\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}$$

input
```
Int[(1 - x^2)/(1 + 4*x^2 + x^4),x]
```

output
```
-1/2*((1 - Sqrt[3])*ArcTan[x/Sqrt[2 - Sqrt[3]]])/Sqrt[2 - Sqrt[3]] - ((1 +
 Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[2 + Sqrt[3]])
```

3.82.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

3.82.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\frac{x^3\sqrt{2}}{2}+\frac{3x\sqrt{2}}{2}\right)}{2}$	35
default	$-\frac{\sqrt{3}\left(\sqrt{3}+3\right)\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\left(\sqrt{6}-\sqrt{2}\right)}$	70

input
```
int((-x^2+1)/(x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*arctan(1/2*x*2^(1/2))+1/2*2^(1/2)*arctan(1/2*x^3*2^(1/2)+3/2*
x*2^(1/2))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^3+3\,x)\right)-\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)$$

input
```
integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + 3*x)) - 1/2*sqrt(2)*arctan(1/2*sqrt(
2)*x)
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = -\frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{2}x^3}{2}+\frac{3\sqrt{2}x}{2}\right)\right)}{4}$$

input
```
integrate((-x**2+1)/(x**4+4*x**2+1),x)
```

output
```
-sqrt(2)*(2*atan(sqrt(2)*x/2) - 2*atan(sqrt(2)*x**3/2 + 3*sqrt(2)*x/2))/4
```

3.82.7 Maxima [F]

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = \int -\frac{x^2-1}{x^4+4\,x^2+1}\,dx$$

input
```
integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(x^4 + 4*x^2 + 1), x)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = \frac{1}{4}\sqrt{2}\left(\pi\mathrm{sgn}(x) - 2\arctan\left(\frac{\sqrt{2}(x^2+1)}{2\,x}\right)\right)$$

input
```
integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(pi*sgn(x) - 2*arctan(1/2*sqrt(2)*(x^2 + 1)/x))
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = \frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}\,x^3}{2}+\frac{3\sqrt{2}\,x}{2}\right)-\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)\right)}{2}$$

input
```
int(-(x^2 - 1)/(4*x^2 + x^4 + 1),x)
```

output
```
(2^(1/2)*(atan((3*2^(1/2)*x)/2 + (2^(1/2)*x^3)/2) - atan((2^(1/2)*x)/2)))/
2
```

3.83 $\int \frac{1-x^2}{1+3x^2+x^4}\,dx$

3.83.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx=-\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)+\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)$$

output
```
-arctan(x*2^(1/2)/(3+5^(1/2))^(1/2))+arctan(x*(1/2+1/2*5^(1/2)))
```

3.83.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.26

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx=\arctan\left(\frac{x}{1+x^2}\right)$$

input
```
Integrate[(1 - x^2)/(1 + 3*x^2 + x^4),x]
```

output
```
ArcTan[x/(1 + x^2)]
```

3.83.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{x^4+3x^2+1}\,dx$$

$$\Big\downarrow \; 1477$$

$$-\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{1}{x^2+\frac{1}{2}\left(3-\sqrt{5}\right)}dx-\frac{1}{2}\left(1+\sqrt{5}\right)\int \frac{1}{x^2+\frac{1}{2}\left(3+\sqrt{5}\right)}dx$$

$$\Big\downarrow \; 216$$

$$-\frac{\left(1+\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{\sqrt{2\left(3+\sqrt{5}\right)}}-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)$$

input
```
Int[(1 - x^2)/(1 + 3*x^2 + x^4),x]
```

output
```
-(((1 + Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/Sqrt[2*(3 + Sqrt[5])]) -
 ((1 - Sqrt[5])*Sqrt[(3 + Sqrt[5])/2]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/2
```

3.83.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

3.83.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.36

method	result	size
risch	$-\arctan\left(x\right)+\arctan\left(x^{3}+2x\right)$	14
parallelrisch	$\frac{i\ln\left(x^{2}-ix+1\right)}{2}-\frac{i\ln\left(x^{2}+ix+1\right)}{2}$	28
default	$-\frac{2\left(-5+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}-\frac{2\sqrt{5}\left(5+\sqrt{5}\right)\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}$	66

input
```
int((-x^2+1)/(x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-arctan(x)+arctan(x^3+2*x)
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx=\arctan\left(x^3+2\,x\right)-\arctan\left(x\right)$$

input
```
integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="fricas")
```

output
```
arctan(x^3 + 2*x) - arctan(x)
```

3.83.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.26

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx=-\operatorname{atan}\left(x\right)+\operatorname{atan}\left(x^3+2x\right)$$

input
```
integrate((-x**2+1)/(x**4+3*x**2+1),x)
```

output
```
-atan(x) + atan(x**3 + 2*x)
```

3.83.7 Maxima [F]

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx = \int -\frac{x^2-1}{x^4+3\,x^2+1}\,dx$$

input
```
integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(x^4 + 3*x^2 + 1), x)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx = \frac{1}{4}\,\pi\mathrm{sgn}(x) - \frac{1}{2}\,\arctan\left(\frac{x^4+x^2+1}{2\,(x^3+x)}\right)$$

input
```
integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")
```

output
```
1/4*pi*sgn(x) - 1/2*arctan(1/2*(x^4 + x^2 + 1)/(x^3 + x))
```

3.83.9 Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx = \mathrm{atan}\left(x^3+2\,x\right) - \mathrm{atan}(x)$$

input
```
int(-(x^2 - 1)/(3*x^2 + x^4 + 1),x)
```

output
```
atan(2*x + x^3) - atan(x)
```

3.84 $\int \frac{1-x^2}{1+2x^2+x^4} dx$

3.84.1 Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{1-x^2}{1+2x^2+x^4} dx = \frac{x}{1+x^2}$$

output
```
x/(x^2+1)
```

3.84.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{1+2x^2+x^4} dx = \frac{x}{1+x^2}$$

input
```
Integrate[(1 - x^2)/(1 + 2*x^2 + x^4),x]
```

output
```
x/(1 + x^2)
```

3.84.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1380, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{x^4+2x^2+1}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int \frac{1-x^2}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{ 297}$$

$$\frac{x}{x^2+1}$$

input
```
Int[(1 - x^2)/(1 + 2*x^2 + x^4),x]
```

output
```
x/(1 + x^2)
```

3.84.3.1 Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.84.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
gosper	$\frac{x}{x^2+1}$	10
default	$\frac{x}{x^2+1}$	10
norman	$\frac{x}{x^2+1}$	10
risch	$\frac{x}{x^2+1}$	10
parallelrisch	$\frac{x}{x^2+1}$	10

input
```
int((-x^2+1)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
x/(x^2+1)
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{1+2x^2+x^4}\,dx = \frac{x}{x^2+1}$$

input
```
integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
x/(x^2 + 1)
```

3.84.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{1-x^2}{1+2x^2+x^4}\,dx = \frac{x}{x^2+1}$$

input
```
integrate((-x**2+1)/(x**4+2*x**2+1),x)
```

output
```
x/(x**2 + 1)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{1+2x^2+x^4}\,dx = \frac{x}{x^2+1}$$

input
```
integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
x/(x^2 + 1)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1-x^2}{1+2x^2+x^4}\,dx = \frac{1}{x+\frac{1}{x}}$$

input
```
integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")
```

output
```
1/(x + 1/x)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{1+2x^2+x^4}\,dx = \frac{x}{x^2+1}$$

input
```
int(-(x^2 - 1)/(2*x^2 + x^4 + 1),x)
```

output
```
x/(x^2 + 1)
```

3.85 $\int \frac{1-x^2}{1+x^2+x^4} \, dx$

3.85.1 Optimal result

Integrand size = 18, antiderivative size = 25

$$\int \frac{1-x^2}{1+x^2+x^4} \, dx = -\frac{1}{2}\log\left(1-x+x^2\right)+\frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
-1/2*ln(x^2-x+1)+1/2*ln(x^2+x+1)
```

3.85.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{1+x^2+x^4} \, dx = -\frac{1}{2}\log\left(1-x+x^2\right)+\frac{1}{2}\log\left(1+x+x^2\right)$$

input
```
Integrate[(1 - x^2)/(1 + x^2 + x^4),x]
```

output
```
-1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/2
```

3.85.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{x^4+x^2+1} \, dx$$

$$\Big\downarrow \text{1478}$$

$$-\frac{1}{2} \int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2} \int -\frac{2x+1}{x^2+x+1}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \int \frac{1-2x}{x^2-x+1}dx+\frac{1}{2} \int \frac{2x+1}{x^2+x+1}dx$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2} \log \left(x^2+x+1\right)-\frac{1}{2} \log \left(x^2-x+1\right)$$

input
```
Int[(1 - x^2)/(1 + x^2 + x^4),x]
```

output
```
-1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/2
```

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.85.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\ln(x^2-x+1)}{2}+\frac{\ln(x^2+x+1)}{2}$	22
norman	$-\frac{\ln(x^2-x+1)}{2}+\frac{\ln(x^2+x+1)}{2}$	22
risch	$-\frac{\ln(x^2-x+1)}{2}+\frac{\ln(x^2+x+1)}{2}$	22
parallelrisch	$-\frac{\ln(x^2-x+1)}{2}+\frac{\ln(x^2+x+1)}{2}$	22

input
```
int((-x^2+1)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x^2-x+1)+1/2*ln(x^2+x+1)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx = \frac{1}{2}\log\left(x^2+x+1\right) - \frac{1}{2}\log\left(x^2-x+1\right)$$

input
```
integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx = -\frac{\log\left(x^2-x+1\right)}{2} + \frac{\log\left(x^2+x+1\right)}{2}$$

input
```
integrate((-x**2+1)/(x**4+x**2+1),x)
```

output
```
-log(x**2 - x + 1)/2 + log(x**2 + x + 1)/2
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx = \frac{1}{2}\log\left(x^2+x+1\right) - \frac{1}{2}\log\left(x^2-x+1\right)$$

input
```
integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx = \frac{1}{4}\log\left(\left|x+\frac{1}{x+\frac{1}{x}}+\frac{1}{x}+2\right|\right) - \frac{1}{4}\log\left(\left|x+\frac{1}{x+\frac{1}{x}}+\frac{1}{x}-2\right|\right)$$

input
```
integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/4*log(abs(x + 1/(x + 1/x) + 1/x + 2)) - 1/4*log(abs(x + 1/(x + 1/x) + 1/
x - 2))
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.40

$$\int \frac{1-x^2}{1+x^2+x^4}\,dx = \operatorname{atanh}\left(\frac{x}{x^2+1}\right)$$

input
```
int(-(x^2 - 1)/(x^2 + x^4 + 1),x)
```

output
```
atanh(x/(x^2 + 1))
```

3.86 $\int \frac{1-x^2}{1+x^4} dx$

3.86.1 Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{1-x^2}{1+x^4} dx = -\frac{\log \left(1-\sqrt{2}x+x^2\right)}{2\sqrt{2}} + \frac{\log \left(1+\sqrt{2}x+x^2\right)}{2\sqrt{2}}$$

output
```
-1/4*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1-x^2}{1+x^4} dx = \frac{-\log \left(-1+\sqrt{2}x-x^2\right)+\log \left(1+\sqrt{2}x+x^2\right)}{2\sqrt{2}}$$

input
```
Integrate[(1 - x^2)/(1 + x^4),x]
```

output
```
(-Log[-1 + Sqrt[2]*x - x^2] + Log[1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])
```

3.86.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{x^4+1} \, dx$$

$$\Big\downarrow \ 1479$$

$$-\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx$$

$$\Big\downarrow \ 1103$$

$$\frac{\log\left(x^2+\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}$$

input
```
Int[(1 - x^2)/(1 + x^4),x]
```

output
```
-1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2
])
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.86.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\ln\left(1+x^2-x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+x^2+x\sqrt{2}\right)\sqrt{2}}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}-\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}$
meijerg	$-\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4 \cdots\right.}{2+\sqrt{2}\left(x \cdots\right.}\right)}{4\left(x^4\right)^{\frac{3}{4}}}$

input
```
int((-x^2+1)/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1-x^2}{1+x^4}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(\frac{x^4+4\,x^2+2\,\sqrt{2}(x^3+x)+1}{x^4+1}\right)$$

input
```
integrate((-x^2+1)/(x^4+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((x^4 + 4*x^2 + 2*sqrt(2)*(x^3 + x) + 1)/(x^4 + 1))
```

3.86.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1+x^4}\,dx = -\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{4}$$

input
```
integrate((-x**2+1)/(x**4+1),x)
```

output
```
-sqrt(2)*log(x**2 - sqrt(2)*x + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/4
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1-x^2}{1+x^4}\,dx = \frac{1}{4}\,\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{4}\,\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate((-x^2+1)/(x^4+1),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1
)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1-x^2}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) - \frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate((-x^2+1)/(x^4+1),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1
)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{1-x^2}{1+x^4}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{x^2+1}\right)}{2}$$

input
```
int(-(x^2 - 1)/(x^4 + 1),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*x)/(x^2 + 1)))/2
```

3.87 $\int \frac{1-x^2}{1-x^2+x^4} \, dx$

3.87.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{1-x^2}{1-x^2+x^4} \, dx = -\frac{\log \left(1-\sqrt{3}x+x^2\right)}{2\sqrt{3}}+\frac{\log \left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

output
```
-1/6*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1-x^2}{1-x^2+x^4} \, dx = \frac{-\log \left(-1+\sqrt{3}x-x^2\right)+\log \left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

input
```
Integrate[(1 - x^2)/(1 - x^2 + x^4),x]
```

output
```
(-Log[-1 + Sqrt[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])
```

3.87.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{x^4-x^2+1} \, dx$$

$$\Big\downarrow \text{1478}$$

$$-\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}$$

input
```
Int[(1 - x^2)/(1 - x^2 + x^4),x]
```

output
```
-1/2*Log[1 - Sqrt[3]*x + x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3
])
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.87.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	35
risch	$-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	35

input
```
int((-x^2+1)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx=\frac{1}{6}\,\sqrt{3}\log\left(\frac{x^4+5\,x^2+2\,\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)$$

input
```
integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1))
```

3.87.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx = -\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{6}$$

input
```
integrate((-x**2+1)/(x**4-x**2+1),x)
```

output
```
-sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/6
```

3.87.7 Maxima [F]

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx = \int -\frac{x^2-1}{x^4-x^2+1}\,dx$$

input
```
integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx = -\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2x-2\sqrt{3}+\frac{2}{x}\right|}{\left|2x+2\sqrt{3}+\frac{2}{x}\right|}\right)$$

input
```
integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) + 2/x)/abs(2*x + 2*sqrt(3) + 2/x))
```

3.87.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{1-x^2}{1-x^2+x^4}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{x^2+1}\right)}{3}$$

input
```
int(-(x^2 - 1)/(x^4 - x^2 + 1),x)
```

output
```
(3^(1/2)*atanh((3^(1/2)*x)/(x^2 + 1)))/3
```

3.88 $\int \frac{1-x^2}{1-2x^2+x^4}\,dx$

3.88.1 Optimal result

Integrand size = 20, antiderivative size = 2

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = \operatorname{arctanh}(x)$$

output
```
arctanh(x)
```

3.88.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(2) = 4.

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 9.50

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = -\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

input
```
Integrate[(1 - x^2)/(1 - 2*x^2 + x^4),x]
```

output
```
-1/2*Log[1 - x] + Log[1 + x]/2
```

3.88.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1380, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{x^4-2x^2+1} \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int \frac{1}{1-x^2}dx$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}(x)$$

input
```
Int[(1 - x^2)/(1 - 2*x^2 + x^4),x]
```

output
```
ArcTanh[x]
```

3.88.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.88.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\operatorname{arctanh}(x)$	3
norman	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14
risch	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14
parallelrisch	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14

input
```
int((-x^2+1)/(x^4-2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctanh(x)
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(2) = 4.

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

input
```
integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")
```

output
```
1/2*log(x + 1) - 1/2*log(x - 1)
```

3.88.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(2) = 4.

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = -\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

input
```
integrate((-x**2+1)/(x**4-2*x**2+1),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

input
```
integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")
```

output
```
1/2*log(x + 1) - 1/2*log(x - 1)
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(2) = 4$.

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 7.50

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

input
```
integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
```

output
```
1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.88.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{1-2x^2+x^4}\,dx = \operatorname{atanh}(x)$$

input
```
int(-(x^2 - 1)/(x^4 - 2*x^2 + 1),x)
```

output
```
atanh(x)
```

3.89 $\int \frac{1-x^2}{1-3x^2+x^4}\,dx$

3.89.1 Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\operatorname{arctanh}\left(\frac{1+2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

output
```
-1/5*arctanh(1/5*(1-2*x)*5^(1/2))*5^(1/2)+1/5*arctanh(1/5*(1+2*x)*5^(1/2))
*5^(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx=\frac{-\log\left(-1+\sqrt{5}x-x^2\right)+\log\left(1+\sqrt{5}x+x^2\right)}{2\sqrt{5}}$$

input
```
Integrate[(1 - x^2)/(1 - 3*x^2 + x^4),x]
```

output
```
(-Log[-1 + Sqrt[5]*x - x^2] + Log[1 + Sqrt[5]*x + x^2])/(2*Sqrt[5])
```

3.89.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 85 vs. 2(38) = 76.

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{x^4-3x^2+1}\,dx$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{2}\int \frac{1}{x^2-x-1}dx-\frac{1}{2}\int \frac{1}{x^2+x-1}dx$$

$$\Big\downarrow \text{1081}$$

$$-\frac{1}{2}\int \left(\frac{2}{\sqrt{5}\left(-2x-\sqrt{5}+1\right)}-\frac{2}{-2\sqrt{5}x+\sqrt{5}+5}\right)dx-\frac{1}{2}\int \left(\frac{2}{\sqrt{5}\left(2x-\sqrt{5}+1\right)}-\frac{2}{2\sqrt{5}x+\sqrt{5}+5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\log\left(-2x-\sqrt{5}+1\right)}{\sqrt{5}}-\frac{\log\left(-2x+\sqrt{5}+1\right)}{\sqrt{5}}\right)+\frac{1}{2}\left(\frac{\log\left(2x+\sqrt{5}+1\right)}{\sqrt{5}}-\frac{\log\left(2x-\sqrt{5}+1\right)}{\sqrt{5}}\right)$$

input
```
Int[(1 - x^2)/(1 - 3*x^2 + x^4),x]
```

output
```
(Log[1 - Sqrt[5] - 2*x]/Sqrt[5] - Log[1 + Sqrt[5] - 2*x]/Sqrt[5])/2 + (-(L
og[1 - Sqrt[5] + 2*x]/Sqrt[5]) + Log[1 + Sqrt[5] + 2*x]/Sqrt[5])/2
```

3.89.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.89.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{5}$	34
risch	$\frac{\sqrt{5}\,\ln\left(x^2+x\sqrt{5}+1\right)}{10}-\frac{\sqrt{5}\,\ln\left(x^2-x\sqrt{5}+1\right)}{10}$	35

input
```
int((-x^2+1)/(x^4-3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/5*arctanh(1/5*(1+2*x)*5^(1/2))*5^(1/2)+1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5
^(1/2))
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = \frac{1}{10}\,\sqrt{5}\log\left(\frac{x^4+7\,x^2+2\,\sqrt{5}(x^3+x)+1}{x^4-3\,x^2+1}\right)$$

input
```
integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="fricas")
```

output
```
1/10*sqrt(5)*log((x^4 + 7*x^2 + 2*sqrt(5)*(x^3 + x) + 1)/(x^4 - 3*x^2 + 1)
)
```

3.89.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = -\frac{\sqrt{5}\log\left(x^2-\sqrt{5}x+1\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\sqrt{5}x+1\right)}{10}$$

input
```
integrate((-x**2+1)/(x**4-3*x**2+1),x)
```

output
```
-sqrt(5)*log(x**2 - sqrt(5)*x + 1)/10 + sqrt(5)*log(x**2 + sqrt(5)*x + 1)/
10
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = -\frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+1}{2\,x+\sqrt{5}+1}\right)-\frac{1}{10}\,\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}-1}{2\,x+\sqrt{5}-1}\right)$$

input
```
integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="maxima")
```

output
```
-1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/10*sqrt(5)*
log((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1))
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = -\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2\,x-2\,\sqrt{5}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{5}+\frac{2}{x}\right|}\right)$$

input
```
integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")
```

output
```
-1/10*sqrt(5)*log(abs(2*x - 2*sqrt(5) + 2/x)/abs(2*x + 2*sqrt(5) + 2/x))
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.47

$$\int \frac{1-x^2}{1-3x^2+x^4}\,dx = \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,x}{x^2+1}\right)}{5}$$

input
```
int(-(x^2 - 1)/(x^4 - 3*x^2 + 1),x)
```

output
```
(5^(1/2)*atanh((5^(1/2)*x)/(x^2 + 1)))/5
```

3.90 $\int \frac{1-x^2}{1-4x^2+x^4} \, dx$

3.90.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{1-x^2}{1-4x^2+x^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{1-\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}} + \frac{\operatorname{arctanh}\left(\frac{1+\sqrt{2}x}{\sqrt{3}}\right)}{\sqrt{6}}$$

output
```
-1/6*arctanh(1/3*(1-x*2^(1/2))*3^(1/2))*6^(1/2)+1/6*arctanh(1/3*(1+x*2^(1/
2))*3^(1/2))*6^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-4x^2+x^4} \, dx = \frac{-\log\left(-1+\sqrt{6}x-x^2\right)+\log\left(1+\sqrt{6}x+x^2\right)}{2\sqrt{6}}$$

input
```
Integrate[(1 - x^2)/(1 - 4*x^2 + x^4),x]
```

output
```
(-Log[-1 + Sqrt[6]*x - x^2] + Log[1 + Sqrt[6]*x + x^2])/(2*Sqrt[6])
```

3.90.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 102 vs. 2(47) = 94.

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{x^4-4x^2+1}\,dx$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x-1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x-1}dx$$

$$\Big\downarrow \text{1081}$$

$$-\frac{1}{2}\int \left(\frac{\sqrt{\frac{2}{3}}}{2x+\sqrt{2}\left(1-\sqrt{3}\right)}-\frac{\sqrt{\frac{2}{3}}}{2x+\sqrt{6}+\sqrt{2}}\right)dx-\frac{1}{2}\int \left(\frac{1}{\sqrt{3}\left(-\sqrt{2}x-\sqrt{3}+1\right)}-\frac{\sqrt{\frac{2}{3}}}{-2x+\sqrt{6}+\sqrt{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}x-\sqrt{3}+1\right)}{\sqrt{6}}-\frac{\log\left(-2x+\sqrt{6}+\sqrt{2}\right)}{\sqrt{6}}\right)+\frac{1}{2}\left(\frac{\log\left(2x+\sqrt{6}+\sqrt{2}\right)}{\sqrt{6}}-\frac{\log\left(\sqrt{2}x-\sqrt{3}+1\right)}{\sqrt{6}}\right)$$

input
```
Int[(1 - x^2)/(1 - 4*x^2 + x^4),x]
```

output
```
(-(Log[Sqrt[2] + Sqrt[6] - 2*x]/Sqrt[6]) + Log[1 - Sqrt[3] - Sqrt[2]*x]/Sq
rt[6])/2 + (Log[Sqrt[2] + Sqrt[6] + 2*x]/Sqrt[6] - Log[1 - Sqrt[3] + Sqrt[
2]*x]/Sqrt[6])/2
```

3.90.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.90.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{\sqrt{6}\,\ln\left(x^2+x\sqrt{6}+1\right)}{12}-\frac{\sqrt{6}\,\ln\left(x^2-x\sqrt{6}+1\right)}{12}$	35
default	$\frac{\left(1+\sqrt{3}\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\sqrt{6}+3\sqrt{2}}+\frac{\left(\sqrt{3}-1\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}$	70

input
```
int((-x^2+1)/(x^4-4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/12*6^(1/2)*ln(x^2+x*6^(1/2)+1)-1/12*6^(1/2)*ln(x^2-x*6^(1/2)+1)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = \frac{1}{12}\,\sqrt{6}\log\left(\frac{x^4+8\,x^2+2\,\sqrt{6}(x^3+x)+1}{x^4-4\,x^2+1}\right)$$

input
```
integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(6)*log((x^4 + 8*x^2 + 2*sqrt(6)*(x^3 + x) + 1)/(x^4 - 4*x^2 + 1)
)
```

3.90.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = -\frac{\sqrt{6}\log\left(x^2-\sqrt{6}x+1\right)}{12}+\frac{\sqrt{6}\log\left(x^2+\sqrt{6}x+1\right)}{12}$$

input
```
integrate((-x**2+1)/(x**4-4*x**2+1),x)
```

output
```
-sqrt(6)*log(x**2 - sqrt(6)*x + 1)/12 + sqrt(6)*log(x**2 + sqrt(6)*x + 1)/
12
```

3.90.7 Maxima [F]

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = \int -\frac{x^2-1}{x^4-4\,x^2+1}\,dx$$

input
```
integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(x^4 - 4*x^2 + 1), x)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = -\frac{1}{12}\sqrt{6}\log\left(\frac{\left|2\,x-2\,\sqrt{6}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{6}+\frac{2}{x}\right|}\right)$$

input
```
integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2/x)/abs(2*x + 2*sqrt(6) + 2/x))
```

3.90.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{1-x^2}{1-4x^2+x^4}\,dx = \frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{x^2+1}\right)}{6}$$

input
```
int(-(x^2 - 1)/(x^4 - 4*x^2 + 1),x)
```

output
```
(6^(1/2)*atanh((6^(1/2)*x)/(x^2 + 1)))/6
```

3.91 $\int \frac{1-x^2}{1-5x^2+x^4} \, dx$

3.91.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{1-x^2}{1-5x^2+x^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}+2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

output
```
-1/7*arctanh(1/7*(-2*x+3^(1/2))*7^(1/2))*7^(1/2)+1/7*arctanh(1/7*(2*x+3^(1
/2))*7^(1/2))*7^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1-x^2}{1-5x^2+x^4} \, dx = \frac{-\log \left(-1+\sqrt{7}x-x^2\right)+\log \left(1+\sqrt{7}x+x^2\right)}{2\sqrt{7}}$$

input
```
Integrate[(1 - x^2)/(1 - 5*x^2 + x^4),x]
```

output
```
(-Log[-1 + Sqrt[7]*x - x^2] + Log[1 + Sqrt[7]*x + x^2])/(2*Sqrt[7])
```

3.91.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 105 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.28, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{x^4-5x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{2}\int \frac{1}{x^2-\sqrt{3}x-1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x-1}dx$$

$$\Big\downarrow \text{1081}$$

$$-\frac{1}{2}\int \left(-\frac{2}{2\sqrt{7}x-\sqrt{21}+7}-\frac{2}{-2\sqrt{7}x+\sqrt{21}+7}\right)dx-\frac{1}{2}\int \left(-\frac{2}{2\sqrt{7}x+\sqrt{21}+7}-\frac{2}{-2\sqrt{7}x-\sqrt{21}+7}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\log\left(2\sqrt{7}x-\sqrt{21}+7\right)}{\sqrt{7}}-\frac{\log\left(-2\sqrt{7}x+\sqrt{21}+7\right)}{\sqrt{7}}\right)+\frac{1}{2}\left(\frac{\log\left(2\sqrt{7}x+\sqrt{21}+7\right)}{\sqrt{7}}-\frac{\log\left(-2\sqrt{7}x-\sqrt{21}+7\right)}{\sqrt{7}}\right)$$

input
```
Int[(1 - x^2)/(1 - 5*x^2 + x^4),x]
```

output
```
(-(Log[7 + Sqrt[21] - 2*Sqrt[7]*x]/Sqrt[7]) + Log[7 - Sqrt[21] + 2*Sqrt[7]
*x]/Sqrt[7])/2 + (-(Log[7 - Sqrt[21] - 2*Sqrt[7]*x]/Sqrt[7]) + Log[7 + Sqr
t[21] + 2*Sqrt[7]*x]/Sqrt[7])/2
```

3.91.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.91.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\sqrt{7}\,\ln\left(x^{2}+x\sqrt{7}+1\right)}{14}-\frac{\sqrt{7}\,\ln\left(x^{2}-x\sqrt{7}+1\right)}{14}$	35
default	$\frac{2\left(3+\sqrt{21}\right)\sqrt{21}\,\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}+\frac{2\left(-3+\sqrt{21}\right)\sqrt{21}\,\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}$	82

input
```
int((-x^2+1)/(x^4-5*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/14*7^(1/2)*ln(x^2+x*7^(1/2)+1)-1/14*7^(1/2)*ln(x^2-x*7^(1/2)+1)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx = \frac{1}{14}\sqrt{7}\log\left(\frac{x^4+9\,x^2+2\,\sqrt{7}(x^3+x)+1}{x^4-5\,x^2+1}\right)$$

input
```
integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")
```

output
```
1/14*sqrt(7)*log((x^4 + 9*x^2 + 2*sqrt(7)*(x^3 + x) + 1)/(x^4 - 5*x^2 + 1)
)
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx = -\frac{\sqrt{7}\log\left(x^2-\sqrt{7}x+1\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\sqrt{7}x+1\right)}{14}$$

input
```
integrate((-x**2+1)/(x**4-5*x**2+1),x)
```

output
```
-sqrt(7)*log(x**2 - sqrt(7)*x + 1)/14 + sqrt(7)*log(x**2 + sqrt(7)*x + 1)/
14
```

3.91.7 Maxima [F]

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx = \int -\frac{x^2-1}{x^4-5\,x^2+1}\,dx$$

input
```
integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(x^4 - 5*x^2 + 1), x)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx = -\frac{1}{14}\sqrt{7}\log\left(\frac{\left|2\,x-2\,\sqrt{7}+\frac{2}{x}\right|}{\left|2\,x+2\,\sqrt{7}+\frac{2}{x}\right|}\right)$$

input
```
integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")
```

output
```
-1/14*sqrt(7)*log(abs(2*x - 2*sqrt(7) + 2/x)/abs(2*x + 2*sqrt(7) + 2/x))
```

3.91.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{1-x^2}{1-5x^2+x^4}\,dx = \frac{\sqrt{7}\,\mathrm{atanh}\left(\frac{\sqrt{7}\,x}{x^2+1}\right)}{7}$$

input
```
int(-(x^2 - 1)/(x^4 - 5*x^2 + 1),x)
```

output
```
(7^(1/2)*atanh((7^(1/2)*x)/(x^2 + 1)))/7
```

3.92 $\int \frac{-1-3x^2}{1+2x^2+9x^4} \, dx$

3.92.1 Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{-1-3x^2}{1+2x^2+9x^4} \, dx = \frac{\arctan\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\arctan\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

output
```
1/4*arctan(1/2*(1-3*x)*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+3*x)*2^(1/2))*2^
(1/2)
```

3.92.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \frac{-1-3x^2}{1+2x^2+9x^4} \, dx = -\frac{(-i+\sqrt{2})\arctan\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}} - \frac{(i+\sqrt{2})\arctan\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}$$

input
```
Integrate[(-1 - 3*x^2)/(1 + 2*x^2 + 9*x^4),x]
```

output
```
-1/2*((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/Sqrt[2*(1 - (2
*I)*Sqrt[2])] - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*S
qrt[2*(1 + (2*I)*Sqrt[2])])
```

3.92.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-3x^2-1}{9x^4+2x^2+1} \, dx$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{6}\int \frac{1}{x^2-\frac{2x}{3}+\frac{1}{3}}dx-\frac{1}{6}\int \frac{1}{x^2+\frac{2x}{3}+\frac{1}{3}}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\int \frac{1}{-\left(2x-\frac{2}{3}\right)^2-\frac{8}{9}}d\left(2x-\frac{2}{3}\right)+\frac{1}{3}\int \frac{1}{-\left(2x+\frac{2}{3}\right)^2-\frac{8}{9}}d\left(2x+\frac{2}{3}\right)$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{3\left(2x-\frac{2}{3}\right)}{2\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\arctan\left(\frac{3\left(2x+\frac{2}{3}\right)}{2\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Int[(-1 - 3*x^2)/(1 + 2*x^2 + 9*x^4),x]
```

output
```
-1/2*ArcTan[(3*(-2/3 + 2*x))/(2*Sqrt[2])]/Sqrt[2] - ArcTan[(3*(2/3 + 2*x))
/(2*Sqrt[2])]/(2*Sqrt[2])
```

3.92.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.92.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)}{4}$	34
risch	$-\frac{\sqrt{2}\,\arctan\left(\frac{3x\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{9x^3\sqrt{2}}{4}+\frac{5x\sqrt{2}}{4}\right)}{4}$	35

input
```
int((-3*x^2-1)/(9*x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^
(1/2))
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx=-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(9\,x^3+5\,x\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

input
```
integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sq
rt(2)*x)
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx = -\frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\,\mathrm{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

input
```
integrate((-3*x**2-1)/(9*x**4+2*x**2+1),x)
```

output
```
-sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4)
)/8
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x-1)\right)$$

input
```
integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2
)*(3*x - 1))
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x-1)\right)$$

input
```
integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2
)*(3*x - 1))
```

3.92.9 Mupad [B] (verification not implemented)

Time = 13.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx=-\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{9\sqrt{2}\,x^3}{4}+\frac{5\sqrt{2}\,x}{4}\right)+\operatorname{atan}\left(\frac{3\sqrt{2}\,x}{4}\right)\right)}{4}$$

input
```
int(-(3*x^2 + 1)/(2*x^2 + 9*x^4 + 1),x)
```

output
```
-(2^(1/2)*(atan((5*2^(1/2)*x)/4 + (9*2^(1/2)*x^3)/4) + atan((3*2^(1/2)*x)/
4)))/4
```

3.93 $\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx$

3.93.1 Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx=\frac{\arctan\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\arctan\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

output
```
1/4*arctan(1/2*(1-3*x)*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+3*x)*2^(1/2))*2^
(1/2)
```

3.93.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx=-\frac{\left(-i+\sqrt{2}\right)\arctan\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(i+\sqrt{2}\right)\arctan\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}$$

input
```
Integrate[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

output
```
-1/2*((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/Sqrt[2*(1 - (2
*I)*Sqrt[2])] - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*S
qrt[2*(1 + (2*I)*Sqrt[2])])
```

3.93.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+1}{-9x^4-2x^2-1}\,dx$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{6}\int \frac{1}{x^2-\frac{2x}{3}+\frac{1}{3}}dx-\frac{1}{6}\int \frac{1}{x^2+\frac{2x}{3}+\frac{1}{3}}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\int \frac{1}{-\left(2x-\frac{2}{3}\right)^2-\frac{8}{9}}d\left(2x-\frac{2}{3}\right)+\frac{1}{3}\int \frac{1}{-\left(2x+\frac{2}{3}\right)^2-\frac{8}{9}}d\left(2x+\frac{2}{3}\right)$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{3\left(2x-\frac{2}{3}\right)}{2\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\arctan\left(\frac{3\left(2x+\frac{2}{3}\right)}{2\sqrt{2}}\right)}{2\sqrt{2}}$$

input
```
Int[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

output
```
-1/2*ArcTan[(3*(-2/3 + 2*x))/(2*Sqrt[2])]/Sqrt[2] - ArcTan[(3*(2/3 + 2*x))
/(2*Sqrt[2])]/(2*Sqrt[2])
```

3.93.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.93.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)}{4}$	34
risch	$-\frac{\sqrt{2}\,\arctan\left(\frac{3x\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{9x^3\sqrt{2}}{4}+\frac{5x\sqrt{2}}{4}\right)}{4}$	35

input
```
int((3*x^2+1)/(-9*x^4-2*x^2-1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^
(1/2))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx=-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(9\,x^3+5\,x)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

input
```
integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sq
rt(2)*x)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx = -\frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\,\mathrm{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

input
```
integrate((3*x**2+1)/(-9*x**4-2*x**2-1),x)
```

output
```
-sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4)
)/8
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x-1)\right)$$

input
```
integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2
)*(3*x - 1))
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3\,x-1)\right)$$

input
```
integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2
)*(3*x - 1))
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx = -\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{9\sqrt{2}\,x^3}{4}+\frac{5\sqrt{2}\,x}{4}\right)+\operatorname{atan}\left(\frac{3\sqrt{2}\,x}{4}\right)\right)}{4}$$

input
```
int(-(3*x^2 + 1)/(2*x^2 + 9*x^4 + 1),x)
```

output
```
-(2^(1/2)*(atan((5*2^(1/2)*x)/4 + (9*2^(1/2)*x^3)/4) + atan((3*2^(1/2)*x)/
4)))/4
```

3.94 $\int \frac{3+2x^2}{1-2x^2+x^4}\,dx$

3.94.1 Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = \frac{5x}{2\left(1-x^2\right)}+\frac{\operatorname{arctanh}(x)}{2}$$

output
```
5/2*x/(-x^2+1)+1/2*arctanh(x)
```

3.94.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = \frac{1}{4}\left(-\frac{10x}{-1+x^2}-\log(1-x)+\log(1+x)\right)$$

input
```
Integrate[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]
```

output
```
((-10*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

3.94.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1380, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+3}{x^4-2x^2+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{2x^2+3}{(1-x^2)^2}dx$$

$$\Big\downarrow \text{298}$$

$$\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{5x}{2(1-x^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}(x)}{2}+\frac{5x}{2(1-x^2)}$$

input
```
Int[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]
```

output
```
(5*x)/(2*(1 - x^2)) + ArcTanh[x]/2
```

3.94.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

3.94.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

method	result	size
norman	$-\frac{5x}{2(x^2-1)} - \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4}$	24
risch	$-\frac{5x}{2(x^2-1)} - \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4}$	24
default	$-\frac{5}{4(x+1)} + \frac{\ln(x+1)}{4} - \frac{5}{4(x-1)} - \frac{\ln(x-1)}{4}$	28
parallelrisch	$-\frac{\ln(x-1)x^2-\ln(x+1)x^2-\ln(x-1)+\ln(x+1)+10x}{4(x^2-1)}$	41

input
```
int((2*x^2+3)/(x^4-2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-5/2*x/(x^2-1)-1/4*ln(x-1)+1/4*ln(x+1)
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = \frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-10\,x}{4\,(x^2-1)}$$

input
```
integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="fricas")
```

output
```
1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 10*x)/(x^2 - 1)
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = -\frac{5x}{2x^2-2} - \frac{\log(x-1)}{4} + \frac{\log(x+1)}{4}$$

input
```
integrate((2*x**2+3)/(x**4-2*x**2+1),x)
```

output
```
-5*x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = -\frac{5\,x}{2\,(x^2-1)} + \frac{1}{4}\,\log(x+1) - \frac{1}{4}\,\log(x-1)$$

input
```
integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="maxima")
```

output
```
-5/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = -\frac{5\,x}{2\,(x^2-1)} + \frac{1}{4}\,\log(|x+1|) - \frac{1}{4}\,\log(|x-1|)$$

input
```
integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="giac")
```

output
```
-5/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

3.94.9 Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = \frac{\operatorname{atanh}(x)}{2} - \frac{5\,x}{2\,(x^2-1)}$$

input
```
int((2*x^2 + 3)/(x^4 - 2*x^2 + 1),x)
```

output
```
atanh(x)/2 - (5*x)/(2*(x^2 - 1))
```

3.95 $\int \frac{2+3x^2}{5-8x^2+3x^4} \, dx$

3.95.1 Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{2+3x^2}{5-8x^2+3x^4} \, dx = \frac{5\text{arctanh}(x)}{2} - \frac{7}{2}\sqrt{\frac{3}{5}}\text{arctanh}\left(\sqrt{\frac{3}{5}}x\right)$$

output
```
5/2*arctanh(x)-7/10*arctanh(1/5*x*15^(1/2))*15^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{2+3x^2}{5-8x^2+3x^4} \, dx = \frac{1}{20}\left(7\sqrt{15}\log\left(\sqrt{15}-3x\right) - 25\log(1-x) + 25\log(1+x) - 7\sqrt{15}\log\left(\sqrt{15}+3x\right)\right)$$

input
```
Integrate[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

output
```
(7*Sqrt[15]*Log[Sqrt[15] - 3*x] - 25*Log[1 - x] + 25*Log[1 + x] - 7*Sqrt[1
5]*Log[Sqrt[15] + 3*x])/20
```

3.95.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{3x^4-8x^2+5}\,dx$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{21}{2}\int \frac{1}{3x^2-5}dx-\frac{15}{2}\int \frac{1}{3x^2-3}dx$$

$$\Big\downarrow \text{ 220}$$

$$\frac{5\text{arctanh}(x)}{2}-\frac{7}{2}\sqrt{\frac{3}{5}}\text{arctanh}\left(\sqrt{\frac{3}{5}}x\right)$$

input
```
Int[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

output
```
(5*ArcTanh[x])/2 - (7*Sqrt[3/5]*ArcTanh[Sqrt[3/5]*x])/2
```

3.95.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.95.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{5\ln(x+1)}{4}-\frac{7\,\mathrm{arctanh}\left(\frac{x\sqrt{15}}{5}\right)\sqrt{15}}{10}-\frac{5\ln(x-1)}{4}$	26
risch	$-\frac{5\ln(x-1)}{4}+\frac{7\sqrt{15}\,\ln\left(3x-\sqrt{15}\right)}{20}-\frac{7\sqrt{15}\,\ln\left(3x+\sqrt{15}\right)}{20}+\frac{5\ln(x+1)}{4}$	42

input
```
int((3*x^2+2)/(3*x^4-8*x^2+5),x,method=_RETURNVERBOSE)
```

output
```
5/4*ln(x+1)-7/10*arctanh(1/5*x*15^(1/2))*15^(1/2)-5/4*ln(x-1)
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx = \frac{7}{20}\,\sqrt{5}\sqrt{3}\log\left(-\frac{2\,\sqrt{5}\sqrt{3}x-3\,x^2-5}{3\,x^2-5}\right)+\frac{5}{4}\,\log\left(x+1\right)-\frac{5}{4}\,\log\left(x-1\right)$$

input
```
integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="fricas")
```

output
```
7/20*sqrt(5)*sqrt(3)*log(-(2*sqrt(5)*sqrt(3)*x - 3*x^2 - 5)/(3*x^2 - 5)) +
 5/4*log(x + 1) - 5/4*log(x - 1)
```

3.95.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx = -\frac{5\log\left(x-1\right)}{4}+\frac{5\log\left(x+1\right)}{4}+\frac{7\sqrt{15}\log\left(x-\frac{\sqrt{15}}{3}\right)}{20}-\frac{7\sqrt{15}\log\left(x+\frac{\sqrt{15}}{3}\right)}{20}$$

input
```
integrate((3*x**2+2)/(3*x**4-8*x**2+5),x)
```

output
```
-5*log(x - 1)/4 + 5*log(x + 1)/4 + 7*sqrt(15)*log(x - sqrt(15)/3)/20 - 7*s
qrt(15)*log(x + sqrt(15)/3)/20
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx = \frac{7}{20}\sqrt{15}\log\left(\frac{3\,x-\sqrt{15}}{3\,x+\sqrt{15}}\right) + \frac{5}{4}\log\left(x+1\right) - \frac{5}{4}\log\left(x-1\right)$$

input
```
integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="maxima")
```

output
```
7/20*sqrt(15)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 5/4*log(x + 1) - 5/
4*log(x - 1)
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx = \frac{7}{20}\sqrt{15}\log\left(\frac{\left|6\,x-2\,\sqrt{15}\right|}{\left|6\,x+2\,\sqrt{15}\right|}\right) + \frac{5}{4}\log\left(\left|x+1\right|\right) - \frac{5}{4}\log\left(\left|x-1\right|\right)$$

input
```
integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="giac")
```

output
```
7/20*sqrt(15)*log(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15))) + 5/4*log(a
bs(x + 1)) - 5/4*log(abs(x - 1))
```

3.95.9 Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx = \frac{5\,\mathrm{atanh}(x)}{2} - \frac{7\sqrt{15}\,\mathrm{atanh}\left(\frac{\sqrt{15}\,x}{5}\right)}{10}$$

input
```
int((3*x^2 + 2)/(3*x^4 - 8*x^2 + 5),x)
```

output
```
(5*atanh(x))/2 - (7*15^(1/2)*atanh((15^(1/2)*x)/5))/10
```

3.96 $\int \frac{d+ex^2}{5-8x^2+3x^4} \, dx$

3.96.1 Optimal result

Integrand size = 22, antiderivative size = 36

$$\int \frac{d+ex^2}{5-8x^2+3x^4} \, dx = \frac{1}{2}(d+e)\text{arctanh}(x) - \frac{(3d+5e)\text{arctanh}\left(\sqrt{\frac{3}{5}}x\right)}{2\sqrt{15}}$$

output
```
1/2*(d+e)*arctanh(x)-1/30*(3*d+5*e)*arctanh(1/5*x*15^(1/2))*15^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.00

$$\int \frac{d+ex^2}{5-8x^2+3x^4} \, dx = \frac{1}{60}\left(\sqrt{15}(3d+5e)\log\left(\sqrt{15}-3x\right) - 15(d+e)\log(1-x) + 15(d+e)\log(1+x) - \sqrt{15}(3d+5e)\log\left(\sqrt{15}+3x\right)\right)$$

input
```
Integrate[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

output
```
(Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] - 3*x] - 15*(d + e)*Log[1 - x] + 15*(d
+ e)*Log[1 + x] - Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] + 3*x])/60
```

3.96.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{3x^4-8x^2+5}dx$$

$$\Big\downarrow \ 1480$$

$$\frac{1}{2}(3d+5e)\int \frac{1}{3x^2-5}dx-\frac{3}{2}(d+e)\int \frac{1}{3x^2-3}dx$$

$$\Big\downarrow \ 220$$

$$\frac{1}{2}\text{arctanh}(x)(d+e)-\frac{\text{arctanh}\left(\sqrt{\frac{3}{5}}x\right)(3d+5e)}{2\sqrt{15}}$$

input
```
Int[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

output
```
((d + e)*ArcTanh[x])/2 - ((3*d + 5*e)*ArcTanh[Sqrt[3/5]*x])/(2*Sqrt[15])
```

3.96.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.96.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

method	result
default	$\left(\frac{d}{4}+\frac{e}{4}\right)\ln\left(x+1\right)-\frac{\left(\frac{3d}{2}+\frac{5e}{2}\right)\operatorname{arctanh}\left(\frac{x\sqrt{15}}{5}\right)\sqrt{15}}{15}+\left(-\frac{d}{4}-\frac{e}{4}\right)\ln\left(x-1\right)$
risch	$\frac{\sqrt{15}\,\ln\left(-x\sqrt{15}+5\right)d}{20}+\frac{\sqrt{15}\,\ln\left(-x\sqrt{15}+5\right)e}{12}-\frac{\sqrt{15}\,\ln\left(x\sqrt{15}+5\right)d}{20}-\frac{\sqrt{15}\,\ln\left(x\sqrt{15}+5\right)e}{12}+\frac{\ln(x+1)d}{4}+\frac{\ln(x+1)e}{4}-\ln($

input
```
int((e*x^2+d)/(3*x^4-8*x^2+5),x,method=_RETURNVERBOSE)
```

output
```
(1/4*d+1/4*e)*ln(x+1)-1/15*(3/2*d+5/2*e)*arctanh(1/5*x*15^(1/2))*15^(1/2)+
(-1/4*d-1/4*e)*ln(x-1)
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx = \frac{1}{60}\sqrt{15}(3\,d+5\,e)\log\left(\frac{3\,x^2-2\,\sqrt{15}x+5}{3\,x^2-5}\right) + \frac{1}{4}\,(d+e)\log\left(x+1\right)-\frac{1}{4}\,(d+e)\log\left(x-1\right)$$

input
```
integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="fricas")
```

output
```
1/60*sqrt(15)*(3*d + 5*e)*log((3*x^2 - 2*sqrt(15)*x + 5)/(3*x^2 - 5)) + 1/
4*(d + e)*log(x + 1) - 1/4*(d + e)*log(x - 1)
```

3.96.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(31) = 62$.

Time = 0.79 (sec) , antiderivative size = 474, normalized size of antiderivative = 13.17

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx$$
$$= \frac{(d+e)\log\left(x+\frac{-51d^3(d+e)-180d^2e(d+e)-225de^2(d+e)+60d(d+e)^3-100e^3(d+e)+75e(d+e)^3}{9d^4+24d^3e-40de^3-25e^4}\right)}{4}$$
$$-\frac{(d+e)\log\left(x+\frac{51d^3(d+e)+180d^2e(d+e)+225de^2(d+e)-60d(d+e)^3+100e^3(d+e)-75e(d+e)^3}{9d^4+24d^3e-40de^3-25e^4}\right)}{4}$$
$$+\frac{\sqrt{15}\cdot(3d+5e)\log\left(x+\frac{-\frac{17\sqrt{15}d^3\cdot(3d+5e)}{5}-12\sqrt{15}d^2e(3d+5e)-15\sqrt{15}de^2\cdot(3d+5e)+\frac{4\sqrt{15}d(3d+5e)^3}{15}-\frac{20\sqrt{15}e^3\cdot(3d+5e)}{3}+\frac{\sqrt{15}e(3d+5}{3}}{9d^4+24d^3e-40de^3-25e^4}\right.}{60}$$
$$-\frac{\sqrt{15}\cdot(3d+5e)\log\left(x+\frac{\frac{17\sqrt{15}d^3\cdot(3d+5e)}{5}+12\sqrt{15}d^2e(3d+5e)+15\sqrt{15}de^2\cdot(3d+5e)-\frac{4\sqrt{15}d(3d+5e)^3}{15}+\frac{20\sqrt{15}e^3\cdot(3d+5e)}{3}-\frac{\sqrt{15}e(3d+5e)}{3}}{9d^4+24d^3e-40de^3-25e^4}\right.}{60}$$

input
```
integrate((e*x**2+d)/(3*x**4-8*x**2+5),x)
```

output
```
(d + e)*log(x + (-51*d**3*(d + e) - 180*d**2*e*(d + e) - 225*d*e**2*(d + e
) + 60*d*(d + e)**3 - 100*e**3*(d + e) + 75*e*(d + e)**3)/(9*d**4 + 24*d**
3*e - 40*d*e**3 - 25*e**4))/4 - (d + e)*log(x + (51*d**3*(d + e) + 180*d**
2*e*(d + e) + 225*d*e**2*(d + e) - 60*d*(d + e)**3 + 100*e**3*(d + e) - 75
*e*(d + e)**3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/4 + sqrt(15)*(3
*d + 5*e)*log(x + (-17*sqrt(15)*d**3*(3*d + 5*e)/5 - 12*sqrt(15)*d**2*e*(3
*d + 5*e) - 15*sqrt(15)*d*e**2*(3*d + 5*e) + 4*sqrt(15)*d*(3*d + 5*e)**3/1
5 - 20*sqrt(15)*e**3*(3*d + 5*e)/3 + sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4
+ 24*d**3*e - 40*d*e**3 - 25*e**4))/60 - sqrt(15)*(3*d + 5*e)*log(x + (17*
sqrt(15)*d**3*(3*d + 5*e)/5 + 12*sqrt(15)*d**2*e*(3*d + 5*e) + 15*sqrt(15)
*d*e**2*(3*d + 5*e) - 4*sqrt(15)*d*(3*d + 5*e)**3/15 + 20*sqrt(15)*e**3*(3
*d + 5*e)/3 - sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**3*e - 40*d*e**3
 - 25*e**4))/60
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx = \frac{1}{60}\sqrt{15}(3\,d+5\,e)\log\left(\frac{3\,x-\sqrt{15}}{3\,x+\sqrt{15}}\right) + \frac{1}{4}\,(d+e)\log\left(x+1\right) - \frac{1}{4}\,(d+e)\log\left(x-1\right)$$

input
```
integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="maxima")
```

output
```
1/60*sqrt(15)*(3*d + 5*e)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 1/4*(d
+ e)*log(x + 1) - 1/4*(d + e)*log(x - 1)
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(27) = 54.

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx = \frac{1}{60}\sqrt{15}(3\,d+5\,e)\log\left(\frac{|6\,x-2\,\sqrt{15}|}{|6\,x+2\,\sqrt{15}|}\right) + \frac{1}{4}\,(d+e)\log\left(|x+1|\right) - \frac{1}{4}\,(d+e)\log\left(|x-1|\right)$$

input
```
integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="giac")
```

output
```
1/60*sqrt(15)*(3*d + 5*e)*log(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15)))
 + 1/4*(d + e)*log(abs(x + 1)) - 1/4*(d + e)*log(abs(x - 1))
```

3.96.9 Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 290, normalized size of antiderivative = 8.06

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx$$
$$= \frac{\sqrt{15}\,\mathrm{atanh}\left(\frac{54\sqrt{15}\,d^3\,x}{25\left(-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3\right)} - \frac{6\sqrt{15}\,e^3\,x}{-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3} - \frac{18\sqrt{15}\,d\,e^2\,x}{5\left(-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3\right)} + \frac{}{5\left(-\frac{54\,d^3}{5}\right.}\right.}{30}$$
$$- \mathrm{atanh}\left(\frac{18\,d^3\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3} - \frac{30\,e^3\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3}\right.$$
$$\left. - \frac{30\,d\,e^2\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3} + \frac{18\,d^2\,e\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3}\right)\left(\frac{d}{2}+\frac{e}{2}\right)$$

input
```
int((d + e*x^2)/(3*x^4 - 8*x^2 + 5),x)
```

output
```
(15^(1/2)*atanh((54*15^(1/2)*d^3*x)/(25*(18*d*e^2 - 18*d^2*e - (54*d^3)/5
+ 30*e^3)) - (6*15^(1/2)*e^3*x)/(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3
) - (18*15^(1/2)*d*e^2*x)/(5*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3))
+ (18*15^(1/2)*d^2*e*x)/(5*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)))*(
3*d + 5*e))/30 - atanh((18*d^3*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3)
- (30*e^3*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3) - (30*d*e^2*x)/(30*d*
e^2 - 18*d^2*e - 18*d^3 + 30*e^3) + (18*d^2*e*x)/(30*d*e^2 - 18*d^2*e - 18
*d^3 + 30*e^3))*(d/2 + e/2)
```

3.97 $\int \frac{3+x^2}{1+3x^2+x^4} \, dx$

3.97.1 Optimal result

Integrand size = 18, antiderivative size = 74

$$\int \frac{3+x^2}{1+3x^2+x^4} \, dx = -\frac{1}{10}\sqrt{180-80\sqrt{5}}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right) + \frac{\left(3+\sqrt{5}\right)^{3/2}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{10}}$$

output
```
1/20*arctan(x*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)-1/10*arctan(x*
2^(1/2)/(3+5^(1/2))^(1/2))*(10-4*5^(1/2))
```

3.97.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{3+x^2}{1+3x^2+x^4} \, dx = \frac{-\left(3-\sqrt{5}\right)^{3/2}\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)+\left(3+\sqrt{5}\right)^{3/2}\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)}{2\sqrt{10}}$$

input
```
Integrate[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

output
```
(-((3 - Sqrt[5])^(3/2)*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]) + (3 + Sqrt[5])^(3
/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

3.97.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+3}{x^4+3x^2+1}\,dx$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{x^2+\frac{1}{2}\left(3-\sqrt{5}\right)}dx+\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{x^2+\frac{1}{2}\left(3+\sqrt{5}\right)}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{\left(5-3\sqrt{5}\right)\arctan\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)}{5\sqrt{2\left(3+\sqrt{5}\right)}}+\frac{1}{10}\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\left(5+3\sqrt{5}\right)\arctan\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)$$

input
```
Int[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

output
```
((5 - 3*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/(5*Sqrt[2*(3 + Sqrt[5])]
) + (Sqrt[(3 + Sqrt[5])/2]*(5 + 3*Sqrt[5])*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]
)/10
```

3.97.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.97.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+3\_Z^2+1\right)}\frac{\left(\_R^2+3\right)\ln\left(x-\_R\right)}{2\_R^3+3\_R}\right)}{2}$	40
default	$\frac{2\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}+\frac{2\sqrt{5}\,\left(\sqrt{5}-3\right)\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}$	66

input
```
int((x^2+3)/(x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2+3)/(2*_R^3+3*_R)*ln(x-_R),_R=RootOf(_Z^4+3*_Z^2+1))
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(44) = 88.

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99

$$\begin{aligned}\int \frac{3+x^2}{1+3x^2+x^4}\,dx = & -\frac{1}{10}\sqrt{5}\sqrt{4\sqrt{5}-9}\log\left(\sqrt{4\sqrt{5}-9}\left(3\sqrt{5}+7\right)+2x\right) \\ & +\frac{1}{10}\sqrt{5}\sqrt{4\sqrt{5}-9}\log\left(-\sqrt{4\sqrt{5}-9}\left(3\sqrt{5}+7\right)+2x\right) \\ & -\frac{1}{10}\sqrt{5}\sqrt{-4\sqrt{5}-9}\log\left(\left(3\sqrt{5}-7\right)\sqrt{-4\sqrt{5}-9}+2x\right) \\ & +\frac{1}{10}\sqrt{5}\sqrt{-4\sqrt{5}-9}\log\left(-\left(3\sqrt{5}-7\right)\sqrt{-4\sqrt{5}-9}+2x\right)\end{aligned}$$

input
```
integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="fricas")
```

output
```
-1/10*sqrt(5)*sqrt(4*sqrt(5) - 9)*log(sqrt(4*sqrt(5) - 9)*(3*sqrt(5) + 7)
+ 2*x) + 1/10*sqrt(5)*sqrt(4*sqrt(5) - 9)*log(-sqrt(4*sqrt(5) - 9)*(3*sqrt
(5) + 7) + 2*x) - 1/10*sqrt(5)*sqrt(-4*sqrt(5) - 9)*log((3*sqrt(5) - 7)*sq
rt(-4*sqrt(5) - 9) + 2*x) + 1/10*sqrt(5)*sqrt(-4*sqrt(5) - 9)*log(-(3*sqrt
(5) - 7)*sqrt(-4*sqrt(5) - 9) + 2*x)
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx = 2\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2x}{-1+\sqrt{5}}\right)-2\cdot\left(\frac{1}{2}-\frac{\sqrt{5}}{5}\right)\operatorname{atan}\left(\frac{2x}{1+\sqrt{5}}\right)$$

input
```
integrate((x**2+3)/(x**4+3*x**2+1),x)
```

output
```
2*(sqrt(5)/5 + 1/2)*atan(2*x/(-1 + sqrt(5))) - 2*(1/2 - sqrt(5)/5)*atan(2*
x/(1 + sqrt(5)))
```

3.97.7 Maxima [F]

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx = \int \frac{x^2+3}{x^4+3\,x^2+1}\,dx$$

input
```
integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^2 + 3)/(x^4 + 3*x^2 + 1), x)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx = \frac{1}{5}\left(2\sqrt{5}-5\right)\arctan\left(\frac{2x}{\sqrt{5}+1}\right)+\frac{1}{5}\left(2\sqrt{5}+5\right)\arctan\left(\frac{2x}{\sqrt{5}-1}\right)$$

input
```
integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="giac")
```

output
```
1/5*(2*sqrt(5) - 5)*arctan(2*x/(sqrt(5) + 1)) + 1/5*(2*sqrt(5) + 5)*arctan
(2*x/(sqrt(5) - 1))
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx = 2\,\mathrm{atanh}\left(\frac{80\,x\,\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}-56}-\frac{48\sqrt{5}\,x\,\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}-56}\right)\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}} - 2\,\mathrm{atanh}\left(\frac{80\,x\,\sqrt{-\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}+56}+\frac{48\sqrt{5}\,x\,\sqrt{-\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}+56}\right)\sqrt{-\frac{\sqrt{5}}{5}-\frac{9}{20}}$$

input
```
int((x^2 + 3)/(3*x^2 + x^4 + 1),x)
```

output
```
2*atanh((80*x*(5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) - 56) - (48*5^(1/2)*x*
(5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) - 56))*(5^(1/2)/5 - 9/20)^(1/2) - 2*
atanh((80*x*(- 5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) + 56) + (48*5^(1/2)*x*
(- 5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) + 56))*(- 5^(1/2)/5 - 9/20)^(1/2)
```

3.98 $\int \frac{a+bx^2}{1+x^2+x^4}\,dx$

3.98.1 Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx=-\frac{(a+b)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}(a-b)\log\left(1-x+x^2\right)+\frac{1}{4}(a-b)\log\left(1+x+x^2\right)$$

output
```
-1/4*(a-b)*ln(x^2-x+1)+1/4*(a-b)*ln(x^2+x+1)-1/6*(a+b)*arctan(1/3*(1-2*x)*
3^(1/2))*3^(1/2)+1/6*(a+b)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.98.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx=\frac{\left(2ia+\left(-i+\sqrt{3}\right)b\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}}+\frac{\left(-2ia+\left(i+\sqrt{3}\right)b\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[(a + b*x^2)/(1 + x^2 + x^4),x]
```

output
```
(((2*I)*a + (-I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)
*Sqrt[3]] + (((-2*I)*a + (I + Sqrt[3])*b)*ArcTan[((I + Sqrt[3])*x)/2])/Sqr
t[6 - (6*I)*Sqrt[3]]
```

3.98.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^4+x^2+1} \, dx$$

$$\Big\downarrow \text{1483}$$

$$\frac{1}{2}\int \frac{a-(a-b)x}{x^2-x+1}dx+\frac{1}{2}\int \frac{a+(a-b)x}{x^2+x+1}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}(a+b)\int \frac{1}{x^2-x+1}dx-\frac{1}{2}(a-b)\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(a+b)\int \frac{1}{x^2+x+1}dx+\frac{1}{2}(a-b)\int \frac{2x+1}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}(a+b)\int \frac{1}{x^2-x+1}dx+\frac{1}{2}(a-b)\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(a+b)\int \frac{1}{x^2+x+1}dx+\frac{1}{2}(a-b)\int \frac{2x+1}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}(a-b)\int \frac{1-2x}{x^2-x+1}dx-(a+b)\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{1}{2}(a-b)\int \frac{2x+1}{x^2+x+1}dx-(a+b)\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}(a-b)\int \frac{1-2x}{x^2-x+1}dx+\frac{(a+b)\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}(a-b)\int \frac{2x+1}{x^2+x+1}dx+\frac{(a+b)\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{(a+b)\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}(a-b)\log\left(x^2-x+1\right)\right)+$$
$$\frac{1}{2}\left(\frac{(a+b)\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}(a-b)\log\left(x^2+x+1\right)\right)$$

input
```
Int[(a + b*x^2)/(1 + x^2 + x^4),x]
```

output
```
(((a + b)*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - ((a - b)*Log[1 - x + x^2])
/2)/2 + (((a + b)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + ((a - b)*Log[1 + x
+ x^2])/2)/2
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.98.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result
default	$\frac{(-a+b)\ln\left(x^2-x+1\right)}{4}+\frac{\left(\frac{a}{2}+\frac{b}{2}\right)\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{(a-b)\ln\left(x^2+x+1\right)}{4}+\frac{\left(\frac{a}{2}+\frac{b}{2}\right)\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\frac{\sqrt{3}\,a\arctan\left(\frac{2b^2\sqrt{3}\,x}{3\left(a^2-ab+b^2\right)}+\frac{b\sqrt{3}\,a}{3a^2-3ab+3b^2}+\frac{2\sqrt{3}\,a^2x}{3\left(a^2-ab+b^2\right)}-\frac{2\sqrt{3}\,abx}{3\left(a^2-ab+b^2\right)}-\frac{\sqrt{3}\,a^2}{3\left(a^2-ab+b^2\right)}-\frac{\sqrt{3}\,b^2}{3\left(a^2-ab+b^2\right)}\right)}{6}+\frac{\sqrt{3}\,b\arctan\left(\frac{2b^2\sqrt{3}}{3\left(a^2-ab\right.}\right.}{}$

input
```
int((b*x^2+a)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-a+b)*ln(x^2-x+1)+1/3*(1/2*a+1/2*b)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))+1/4*(a-b)*ln(x^2+x+1)+1/3*(1/2*a+1/2*b)*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{a+bx^2}{1+x^2+x^4}\,dx =& \frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\\&+\frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\\&+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*l
og(x^2 - x + 1)
```

3.98.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 740, normalized size of antiderivative = 8.92

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx = \left(-\frac{a}{4}+\frac{b}{4} - \frac{\sqrt{3}i(a+b)}{12}\right)\log\left(x+\frac{2a^3\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)+6a^2b\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)-12ab^2\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}i(a+}{12}\right.}{a^4-a}\right.$$

$$+\left(-\frac{a}{4}+\frac{b}{4} + \frac{\sqrt{3}i(a+b)}{12}\right)\log\left(x+\frac{2a^3\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}i(a+b)}{12}\right)+6a^2b\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}i(a+b)}{12}\right)-12ab^2\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}i(a+}{12}\right.}{a^4-a}\right.$$

$$+\left(\frac{a}{4}-\frac{b}{4} - \frac{\sqrt{3}i(a+b)}{12}\right)\log\left(x+\frac{2a^3\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)+6a^2b\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)-12ab^2\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}i(a+b)}{12}\right)+}{a^4-a^3b+}\right.$$

$$+\left(\frac{a}{4}-\frac{b}{4} + \frac{\sqrt{3}i(a+b)}{12}\right)\log\left(x+\frac{2a^3\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}i(a+b)}{12}\right)+6a^2b\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}i(a+b)}{12}\right)-12ab^2\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}i(a+b)}{12}\right)+}{a^4-a^3b+}\right.$$

input
```
integrate((b*x**2+a)/(x**4+x**2+1),x)
```

output
```
(-a/4 + b/4 - sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(-a/4 + b/4 - sqrt(3)*
I*(a + b)/12) + 6*a**2*b*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12) - 12*a*b**2*(
-a/4 + b/4 - sqrt(3)*I*(a + b)/12) + 24*a*(-a/4 + b/4 - sqrt(3)*I*(a + b)/
12)**3 + 2*b**3*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12) - 48*b*(-a/4 + b/4 - s
qrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4)) + (-a/4 + b/4 +
sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12)
+ 6*a**2*b*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) - 12*a*b**2*(-a/4 + b/4 + s
qrt(3)*I*(a + b)/12) + 24*a*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12)**3 + 2*b**
3*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) - 48*b*(-a/4 + b/4 + sqrt(3)*I*(a +
b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4)) + (a/4 - b/4 - sqrt(3)*I*(a +
b)/12)*log(x + (2*a**3*(a/4 - b/4 - sqrt(3)*I*(a + b)/12) + 6*a**2*b*(a/4
- b/4 - sqrt(3)*I*(a + b)/12) - 12*a*b**2*(a/4 - b/4 - sqrt(3)*I*(a + b)/1
2) + 24*a*(a/4 - b/4 - sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(a/4 - b/4 - sqrt
(3)*I*(a + b)/12) - 48*b*(a/4 - b/4 - sqrt(3)*I*(a + b)/12)**3)/(a**4 - a*
*3*b + a*b**3 - b**4)) + (a/4 - b/4 + sqrt(3)*I*(a + b)/12)*log(x + (2*a**
3*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) + 6*a**2*b*(a/4 - b/4 + sqrt(3)*I*(a
+ b)/12) - 12*a*b**2*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) + 24*a*(a/4 - b/4
+ sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) - 4
8*b*(a/4 - b/4 + sqrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4)
)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}(a-b)\log\left(x^2+x+1\right) - \frac{1}{4}(a-b)\log\left(x^2-x+1\right)$$

input
```
integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*l
og(x^2 - x + 1)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}(a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{4}\,(a-b)\log\left(x^2+x+1\right) - \frac{1}{4}\,(a-b)\log\left(x^2-x+1\right)$$

input
```
integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*ar
ctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*l
og(x^2 - x + 1)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 827, normalized size of antiderivative = 9.96

$$\int \frac{a+bx^2}{1+x^2+x^4}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)/(x^2 + x^4 + 1),x)
```

output
```
- atan(((x*(4*a*b - 4*a^2 + 2*b^2) + (12*a + 24*x*(b/4 - a/4 + (3^(1/2)*a*
1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1
i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i + (x*(4*a*b
 - 4*a^2 + 2*b^2) - (12*a - 24*x*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)
*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/
4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i)/((x*(4*a*b - 4*a^2 + 2*b^2)
 + (12*a + 24*x*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4
- a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i
)/12 + (3^(1/2)*b*1i)/12) - (x*(4*a*b - 4*a^2 + 2*b^2) - (12*a - 24*x*(b/4
 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1
i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i
)/12) - 2*a*b^2 + 2*a^2*b + 2*b^3))*((a*1i)/2 - (b*1i)/2 + (3^(1/2)*a)/6 +
 (3^(1/2)*b)/6) - atan(((x*(4*a*b - 4*a^2 + 2*b^2) + (12*a + 24*x*(a/4 - b
/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/1
2 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12
)*1i + (x*(4*a*b - 4*a^2 + 2*b^2) - (12*a - 24*x*(a/4 - b/4 + (3^(1/2)*a*1
i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i
)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i)/((x*(4*a*b
- 4*a^2 + 2*b^2) + (12*a + 24*x*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*
b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - ...
```

3.98. $\int \frac{a+bx^2}{1+x^2+x^4}\,dx$

3.99 $\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx$

3.99.1 Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx=\frac{x(a+b-(a-2b)x^2)}{6(1+x^2+x^4)}-\frac{(4a+b)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}}-\frac{1}{8}(2a-b)\log\left(1-x+x^2\right)+\frac{1}{8}(2a-b)\log\left(1+x+x^2\right)$$

output
```
1/6*x*(a+b-(a-2*b)*x^2)/(x^4+x^2+1)-1/8*(2*a-b)*ln(x^2-x+1)+1/8*(2*a-b)*ln
(x^2+x+1)-1/36*(4*a+b)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/36*(4*a+b)*ar
ctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.99.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.24

$$\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx=\frac{x(a+b-ax^2+2bx^2)}{6(1+x^2+x^4)}-\frac{\left(\left(-11i+\sqrt{3}\right)a-2\left(-2i+\sqrt{3}\right)b\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{6\sqrt{6+6i\sqrt{3}}}-\frac{\left(\left(11i+\sqrt{3}\right)a-2\left(2i+\sqrt{3}\right)b\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{6\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

output
```
(x*(a + b - a*x^2 + 2*b*x^2))/(6*(1 + x^2 + x^4)) - (((-11*I + Sqrt[3])*a
- 2*(-2*I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/(6*Sqrt[6 + (6*I)*Sq
rt[3]]) - (((11*I + Sqrt[3])*a - 2*(2*I + Sqrt[3])*b)*ArcTan[((I + Sqrt[3]
)*x)/2])/(6*Sqrt[6 - (6*I)*Sqrt[3]])
```

3.99.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1492, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(x^4+x^2+1)^2}dx$$

$$\Big\downarrow \text{ 1492}$$

$$\frac{1}{6}\int \frac{-((a-2b)x^2)+5a-b}{x^4+x^2+1}dx+\frac{x(-(x^2(a-2b))+a+b)}{6(x^4+x^2+1)}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{5a-b-3(2a-b)x}{x^2-x+1}dx+\frac{1}{2}\int \frac{5a-b+3(2a-b)x}{x^2+x+1}dx\right)+\frac{x(-(x^2(a-2b))+a+b)}{6(x^4+x^2+1)}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{2}(4a+b)\int \frac{1}{x^2-x+1}dx-\frac{3}{2}(2a-b)\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(4a+b)\int \frac{1}{x^2+x+1}dx+\frac{3}{2}(2a-b)\int\right.\right.$$
$$\frac{x(-(x^2(a-2b))+a+b)}{6(x^4+x^2+1)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{2}(4a+b)\int \frac{1}{x^2-x+1}dx+\frac{3}{2}(2a-b)\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(4a+b)\int \frac{1}{x^2+x+1}dx+\frac{3}{2}(2a-b)\int\right.\right.$$
$$\frac{x(-(x^2(a-2b))+a+b)}{6(x^4+x^2+1)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{3}{2}(2a-b)\int\frac{1-2x}{x^2-x+1}dx-(4a+b)\int\frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{3}{2}(2a-b)\int\frac{2x+1}{x^2+x+1}dx-(4a\right.\right.$$
$$\frac{x\left(-\left(x^2(a-2b)\right)+a+b\right)}{6\left(x^4+x^2+1\right)}$$

$\Big\downarrow$ 217

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{3}{2}(2a-b)\int\frac{1-2x}{x^2-x+1}dx+\frac{(4a+b)\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{3}{2}(2a-b)\int\frac{2x+1}{x^2+x+1}dx+\frac{(4a+b)\arctan}{\sqrt{3}}\right.\right.$$
$$\frac{x\left(-\left(x^2(a-2b)\right)+a+b\right)}{6\left(x^4+x^2+1\right)}$$

$\Big\downarrow$ 1103

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{(4a+b)\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3}{2}(2a-b)\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{(4a+b)\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}(2a-b)\log\left(x^2\right.\right.\right.$$
$$\frac{x\left(-\left(x^2(a-2b)\right)+a+b\right)}{6\left(x^4+x^2+1\right)}$$

input
```
Int[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

output
```
(x*(a + b - (a - 2*b)*x^2))/(6*(1 + x^2 + x^4)) + ((((4*a + b)*ArcTan[(-1
+ 2*x)/Sqrt[3]])/Sqrt[3] - (3*(2*a - b)*Log[1 - x + x^2])/2)/2 + (((4*a +
b)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*(2*a - b)*Log[1 + x + x^2])/2)/
2)/6
```

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.99.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

method	result
default	$-\frac{\left(\frac{a}{3}-\frac{2b}{3}\right)x-\frac{2a}{3}+\frac{b}{3}}{4(x^2-x+1)}-\frac{(6a-3b)\ln(x^2-x+1)}{24}-\frac{\left(-2a-\frac{b}{2}\right)\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{18}+\frac{\left(-\frac{a}{3}+\frac{2b}{3}\right)x-\frac{2a}{3}+\frac{b}{3}}{4x^2+4x+4}+\frac{(6a-3b)\ln(x^2+}{24}$
risch	$\frac{a\ln(124a^2x^2-100ab\,x^2+28b^2x^2+124a^2x-100abx+28b^2x+124a^2-100ab+28b^2)}{4}-\frac{b\ln(124a^2x^2-100ab\,x^2+28b^2x^2+124a^2x-100}{8}$

input
```
int((b*x^2+a)/(x^4+x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*((1/3*a-2/3*b)*x-2/3*a+1/3*b)/(x^2-x+1)-1/24*(6*a-3*b)*ln(x^2-x+1)-1/
18*(-2*a-1/2*b)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*((-1/3*a+2/3*b)*x-
2/3*a+1/3*b)/(x^2+x+1)+1/24*(6*a-3*b)*ln(x^2+x+1)+1/18*(2*a+1/2*b)*arctan(
1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.55

$$\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx = -\frac{12\,(a-2\,b)x^3 - 2\,\sqrt{3}((4\,a+b)x^4+(4\,a+b)x^2+4\,a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - 2\,\sqrt{3}((4\,a+b)x^4 +$$

input
```
integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/72*(12*(a - 2*b)*x^3 - 2*sqrt(3)*((4*a + b)*x^4 + (4*a + b)*x^2 + 4*a +
 b)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((4*a + b)*x^4 + (4*a + b)*x
^2 + 4*a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(a + b)*x - 9*((2*a - b)*
x^4 + (2*a - b)*x^2 + 2*a - b)*log(x^2 + x + 1) + 9*((2*a - b)*x^4 + (2*a
- b)*x^2 + 2*a - b)*log(x^2 - x + 1))/(x^4 + x^2 + 1)
```

3.99.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 874, normalized size of antiderivative = 7.34

$$\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)/(x**4+x**2+1)**2,x)
```

output
```
(x**3*(-a + 2*b) + x*(a + b))/(6*x**4 + 6*x**2 + 6) + (-a/4 + b/8 - sqrt(3
)*I*(4*a + b)/72)*log(x + (76*a**3*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) +
 948*a**2*b*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(-a/4 + b/8
 - sqrt(3)*I*(4*a + b)/72) + 12096*a*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72)
**3 + 148*b**3*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) - 8640*b*(-a/4 + b/8
- sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a
*b**3 - 7*b**4)) + (-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*
(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72) + 948*a**2*b*(-a/4 + b/8 + sqrt(3)*I
*(4*a + b)/72) - 816*a*b**2*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72) + 12096*
a*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)**3 + 148*b**3*(-a/4 + b/8 + sqrt(3
)*I*(4*a + b)/72) - 8640*b*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)**3)/(248*
a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a*b**3 - 7*b**4)) + (a/4 - b/8 - sqr
t(3)*I*(4*a + b)/72)*log(x + (76*a**3*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72)
 + 948*a**2*b*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(a/4 - b/8
 - sqrt(3)*I*(4*a + b)/72) + 12096*a*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72)*
*3 + 148*b**3*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 8640*b*(a/4 - b/8 - s
qrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a*b*
*3 - 7*b**4)) + (a/4 - b/8 + sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*(a/4
 - b/8 + sqrt(3)*I*(4*a + b)/72) + 948*a**2*b*(a/4 - b/8 + sqrt(3)*I*(4*a
+ b)/72) - 816*a*b**2*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) + 12096*a*(a...
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx = \frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{8}(2a-b)\log\left(x^2+x+1\right) - \frac{1}{8}(2a-b)\log\left(x^2-x+1\right) - \frac{(a-2b)x^3-(a+b)x}{6(x^4+x^2+1)}$$

input
```
integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

output
```
1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(4*a +
 b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^2 + x + 1) - 1/8*(
2*a - b)*log(x^2 - x + 1) - 1/6*((a - 2*b)*x^3 - (a + b)*x)/(x^4 + x^2 + 1
)
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx = \frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{36}\sqrt{3}(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{8}(2a-b)\log\left(x^2+x+1\right) - \frac{1}{8}(2a-b)\log\left(x^2-x+1\right) - \frac{ax^3-2bx^3-ax-bx}{6(x^4+x^2+1)}$$

input
```
integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="giac")
```

output
```
1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(4*a +
 b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^2 + x + 1) - 1/8*(
2*a - b)*log(x^2 - x + 1) - 1/6*(a*x^3 - 2*b*x^3 - a*x - b*x)/(x^4 + x^2 +
 1)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 897, normalized size of antiderivative = 7.54

$$\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)/(x^2 + x^4 + 1)^2,x)
```

output
```
atan((((2*b - 10*a + 24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/
72))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18
- (19*a*b)/9 + b^2/9))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)
*1i + ((10*a - 2*b + 24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/
72))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18
- (19*a*b)/9 + b^2/9))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)
*1i)/((19*a*b^2)/36 - (29*a^2*b)/36 + (31*a^3)/108 - (7*b^3)/54 + ((2*b -
10*a + 24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(b/8 - a/
4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 +
 b^2/9))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - ((10*a - 2*
b + 24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(b/8 - a/4 +
 (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^
2/9))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)))*((a*1i)/2 - (b
*1i)/4 + (3^(1/2)*a)/9 + (3^(1/2)*b)/36) + atan((((2*b - 10*a + 24*x*(a/4
- b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(a/4 - b/8 + (3^(1/2)*a*1i
)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*(a/4 - b
/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)*1i + ((10*a - 2*b + 24*x*(a/4
- b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(a/4 - b/8 + (3^(1/2)*a*1i
)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*(a/4 - b
/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)*1i)/((19*a*b^2)/36 - (29*a^...
```

3.99. $\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx$

3.100 $\int \frac{a+bx^2}{2+x^2+x^4}\,dx$

3.100.1 Optimal result

Integrand size = 18, antiderivative size = 234

$$\begin{aligned}\int \frac{a+bx^2}{2+x^2+x^4}\,dx =& -\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(a+\sqrt{2}b\right)\arctan\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)\\ &+\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(a+\sqrt{2}b\right)\arctan\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)\\ &-\frac{(a-\sqrt{2}b)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}\\ &+\frac{(a-\sqrt{2}b)\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}\end{aligned}$$

output
```
-1/28*arctan((-2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(a+b*2^(1/2)
)*(-14+28*2^(1/2))^(1/2)+1/28*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/
2))^(1/2))*(a+b*2^(1/2))*(-14+28*2^(1/2))^(1/2)-1/4*ln(x^2+2^(1/2)-x*(-1+2
*2^(1/2))^(1/2))*(a-b*2^(1/2))/(-2+4*2^(1/2))^(1/2)+1/4*ln(x^2+2^(1/2)+x*(
-1+2*2^(1/2))^(1/2))*(a-b*2^(1/2))/(-2+4*2^(1/2))^(1/2)
```

3.100.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.47

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx = \frac{\left(-2ia+\left(i+\sqrt{7}\right)b\right)\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{14-14i\sqrt{7}}} + \frac{\left(2ia+\left(-i+\sqrt{7}\right)b\right)\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{14+14i\sqrt{7}}}$$

input
```
Integrate[(a + b*x^2)/(2 + x^2 + x^4),x]
```

output
```
(((-2*I)*a + (I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[14 -
 (14*I)*Sqrt[7]] + (((2*I)*a + (-I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 + I*Sqrt
[7])/2]])/Sqrt[14 + (14*I)*Sqrt[7]]
```

3.100.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^4+x^2+2}\,dx$$

$$\Big\downarrow \text{1483}$$

$$\frac{\int \frac{\sqrt{-1+2\sqrt{2}}a-\left(a-\sqrt{2}b\right)x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}a+\left(a-\sqrt{2}b\right)x}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\sqrt{2\sqrt{2}-1}\left(a+\sqrt{2}b\right)\int \frac{1}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx-\frac{1}{2}\left(a-\sqrt{2}b\right)\int -\frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\frac{1}{2}\sqrt{2\sqrt{2}-1}\left(a+\sqrt{2}b\right)\int \frac{1}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\frac{1}{2}\left(a-\sqrt{2}b\right)\int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\sqrt{2\sqrt{2}-1}\left(a+\sqrt{2}b\right)\int \frac{1}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\frac{1}{2}\left(a-\sqrt{2}b\right)\int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\frac{1}{2}\sqrt{2\sqrt{2}-1}\left(a+\sqrt{2}b\right)\int \frac{1}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\frac{1}{2}\left(a-\sqrt{2}b\right)\int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{1}{2}\left(a-\sqrt{2}b\right)\int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx-\sqrt{2\sqrt{2}-1}\left(a+\sqrt{2}b\right)\int \frac{1}{-\left(2x-\sqrt{-1+2\sqrt{2}}\right)^2-2\sqrt{2}-1}d\left(2x-\sqrt{-1+2\sqrt{2}}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\frac{1}{2}\left(a-\sqrt{2}b\right)\int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx-\sqrt{2\sqrt{2}-1}\left(a+\sqrt{2}b\right)\int \frac{1}{-\left(2x+\sqrt{-1+2\sqrt{2}}\right)^2-2\sqrt{2}-1}d\left(2x+\sqrt{-1+2\sqrt{2}}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}\left(a-\sqrt{2}b\right)\int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}}\left(a+\sqrt{2}b\right)\arctan\left(\frac{2x-\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\frac{1}{2}\left(a-\sqrt{2}b\right)\int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}}\left(a+\sqrt{2}b\right)\arctan\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}}\left(a+\sqrt{2}b\right)\arctan\left(\frac{2x-\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)-\frac{1}{2}\left(a-\sqrt{2}b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}}\left(a+\sqrt{2}b\right)\arctan\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\left(a-\sqrt{2}b\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{2\sqrt{2\left(2\sqrt{2}-1\right)}}$$

3.100. $\int \frac{a+bx^2}{2+x^2+x^4}\,dx$

input
```
Int[(a + b*x^2)/(2 + x^2 + x^4),x]
```

output
```
(Sqrt[(-1 + 2*Sqrt[2])/(1 + 2*Sqrt[2])]*(a + Sqrt[2]*b)*ArcTan[(-Sqrt[-1 +
 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]] - ((a - Sqrt[2]*b)*Log[Sqrt[2] - S
qrt[-1 + 2*Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2*(-1 + 2*Sqrt[2])]) + (Sqrt[(-1
+ 2*Sqrt[2])/(1 + 2*Sqrt[2])]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]]
 + 2*x)/Sqrt[1 + 2*Sqrt[2]]] + ((a - Sqrt[2]*b)*Log[Sqrt[2] + Sqrt[-1 + 2*
Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2*(-1 + 2*Sqrt[2])])
```

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.100.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.16

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2+2\right)} \frac{\left(\_R^2 b+a\right)\ln\left(x-\_R\right)}{2\_R^3+\_R}\right)}{2}$
default	$\frac{\left(\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,a-4\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,b+4\sqrt{-1+2\sqrt{2}}\,a-2\sqrt{-1+2\sqrt{2}}\,b\right)\ln\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)}{56}+\frac{\left(7\sqrt{2}\,a-\frac{\left(\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,a-4\right.}{}\right.}{}$

input
```
int((b*x^2+a)/(x^4+x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2*b+a)/(2*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+_Z^2+2))
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(167) = 334$.

Time = 0.28 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.67

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx$$
$$= -\frac{1}{28}\sqrt{7}\sqrt{a^2-8\,ab+2\,b^2+\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\log\Bigg(-4\left(a^4-a^3b+2\,ab^3-4\,b^4\right)x$$
$$+\sqrt{a^2-8\,ab+2\,b^2+\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\left(\sqrt{7}(a^3-2\,ab^2)+\sqrt{-a^4+4\,a^2b^2-4\,b^4}(a-4\,b)\right)\Bigg)$$
$$+\frac{1}{28}\sqrt{7}\sqrt{a^2-8\,ab+2\,b^2+\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\log\Bigg(-4\left(a^4-a^3b+2\,ab^3-4\,b^4\right)x$$
$$-\sqrt{a^2-8\,ab+2\,b^2+\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\left(\sqrt{7}(a^3-2\,ab^2)+\sqrt{-a^4+4\,a^2b^2-4\,b^4}(a-4\,b)\right)\Bigg)$$
$$-\frac{1}{28}\sqrt{7}\sqrt{a^2-8\,ab+2\,b^2-\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\log\Bigg(-4\left(a^4-a^3b+2\,ab^3-4\,b^4\right)x$$
$$+\sqrt{a^2-8\,ab+2\,b^2-\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\left(\sqrt{7}(a^3-2\,ab^2)-\sqrt{-a^4+4\,a^2b^2-4\,b^4}(a-4\,b)\right)\Bigg)$$
$$+\frac{1}{28}\sqrt{7}\sqrt{a^2-8\,ab+2\,b^2-\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\log\Bigg(-4\left(a^4-a^3b+2\,ab^3-4\,b^4\right)x$$
$$-\sqrt{a^2-8\,ab+2\,b^2-\sqrt{7}\sqrt{-a^4+4\,a^2b^2-4\,b^4}}\left(\sqrt{7}(a^3-2\,ab^2)-\sqrt{-a^4+4\,a^2b^2-4\,b^4}(a-4\,b)\right)\Bigg)$$

input
```
integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="fricas")
```

output
```
-1/28*sqrt(7)*sqrt(a^2 - 8*a*b + 2*b^2 + sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4
*b^4))*log(-4*(a^4 - a^3*b + 2*a*b^3 - 4*b^4)*x + sqrt(a^2 - 8*a*b + 2*b^2
 + sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*b^4))*(sqrt(7)*(a^3 - 2*a*b^2) + sqrt
(-a^4 + 4*a^2*b^2 - 4*b^4)*(a - 4*b))) + 1/28*sqrt(7)*sqrt(a^2 - 8*a*b + 2
*b^2 + sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*b^4))*log(-4*(a^4 - a^3*b + 2*a*b
^3 - 4*b^4)*x - sqrt(a^2 - 8*a*b + 2*b^2 + sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 -
 4*b^4))*(sqrt(7)*(a^3 - 2*a*b^2) + sqrt(-a^4 + 4*a^2*b^2 - 4*b^4)*(a - 4*
b))) - 1/28*sqrt(7)*sqrt(a^2 - 8*a*b + 2*b^2 - sqrt(7)*sqrt(-a^4 + 4*a^2*b
^2 - 4*b^4))*log(-4*(a^4 - a^3*b + 2*a*b^3 - 4*b^4)*x + sqrt(a^2 - 8*a*b +
 2*b^2 - sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*b^4))*(sqrt(7)*(a^3 - 2*a*b^2)
- sqrt(-a^4 + 4*a^2*b^2 - 4*b^4)*(a - 4*b))) + 1/28*sqrt(7)*sqrt(a^2 - 8*a
*b + 2*b^2 - sqrt(7)*sqrt(-a^4 + 4*a^2*b^2 - 4*b^4))*log(-4*(a^4 - a^3*b +
 2*a*b^3 - 4*b^4)*x - sqrt(a^2 - 8*a*b + 2*b^2 - sqrt(7)*sqrt(-a^4 + 4*a^2
*b^2 - 4*b^4))*(sqrt(7)*(a^3 - 2*a*b^2) - sqrt(-a^4 + 4*a^2*b^2 - 4*b^4)*(
a - 4*b)))
```

3.100.6 Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.52

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx$$
$$= \operatorname{RootSum}\left(1568t^4 + t^2\left(-28a^2 + 224ab - 56b^2\right) + a^4 - 2a^3b + 5a^2b^2 - 4ab^3 + 4b^4, \left(t \mapsto t\log\left(x + \frac{112t^3\right.\right.\right.$$

input
```
integrate((b*x**2+a)/(x**4+x**2+2),x)
```

output
```
RootSum(1568*_t**4 + _t**2*(-28*a**2 + 224*a*b - 56*b**2) + a**4 - 2*a**3*
b + 5*a**2*b**2 - 4*a*b**3 + 4*b**4, Lambda(_t, _t*log(x + (112*_t**3*a -
448*_t**3*b + 6*_t*a**3 + 12*_t*a**2*b - 48*_t*a*b**2 + 8*_t*b**3)/(a**4 -
 a**3*b + 2*a*b**3 - 4*b**4))))
```

3.100.7 Maxima [F]

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx = \int \frac{bx^2+a}{x^4+x^2+2}\,dx$$

input
```
integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)/(x^4 + x^2 + 2), x)
```

3.100.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(167) = 334$.

Time = 0.54 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.58

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx =$$

$$-\frac{1}{896}\sqrt{7}\left(\sqrt{7}2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}+4\right)+3\sqrt{7}2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}-4\right)-3\cdot 2^{\frac{3}{4}}b\left(\sqrt{2}+4\right)\sqrt{-2\sqrt{2}+8}\right.$$

$$-\frac{1}{896}\sqrt{7}\left(\sqrt{7}2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}+4\right)+3\sqrt{7}2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}-4\right)-3\cdot 2^{\frac{3}{4}}b\left(\sqrt{2}+4\right)\sqrt{-2\sqrt{2}+8}\right.$$

$$-\frac{1}{1792}\sqrt{7}\left(3\sqrt{7}2^{\frac{3}{4}}b\left(\sqrt{2}+4\right)\sqrt{-2\sqrt{2}+8}+\sqrt{7}2^{\frac{3}{4}}b\left(\sqrt{2}-4\right)\sqrt{-2\sqrt{2}+8}+2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}+4\right)\right.$$

$$\left.+2\cdot 2^{\frac{1}{4}}x\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}+\sqrt{2}\right)$$

$$+\frac{1}{1792}\sqrt{7}\left(3\sqrt{7}2^{\frac{3}{4}}b\left(\sqrt{2}+4\right)\sqrt{-2\sqrt{2}+8}+\sqrt{7}2^{\frac{3}{4}}b\left(\sqrt{2}-4\right)\sqrt{-2\sqrt{2}+8}+2^{\frac{3}{4}}b\sqrt{2\sqrt{2}+8}\left(\sqrt{2}+4\right)\right.$$

$$\left.-2\cdot 2^{\frac{1}{4}}x\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}+\sqrt{2}\right)$$

input
```
integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="giac")
```

output
```
-1/896*sqrt(7)*(sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*sq
rt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 3*2^(3/4)*b*(sqrt(2) +
 4)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) -
8*sqrt(7)*2^(1/4)*a*sqrt(2*sqrt(2) + 8) + 8*2^(1/4)*a*sqrt(-2*sqrt(2) + 8)
)*arctan(2*2^(3/4)*sqrt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(s
qrt(2) + 4)) - 1/896*sqrt(7)*(sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(
2) + 4) + 3*sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 3*2^(3/4
)*b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*s
qrt(2) + 8) - 8*sqrt(7)*2^(1/4)*a*sqrt(2*sqrt(2) + 8) + 8*2^(1/4)*a*sqrt(-
2*sqrt(2) + 8))*arctan(2*2^(3/4)*sqrt(1/2)*(x - 2^(1/4)*sqrt(-1/8*sqrt(2)
+ 1/2))/sqrt(sqrt(2) + 4)) - 1/1792*sqrt(7)*(3*sqrt(7)*2^(3/4)*b*(sqrt(2)
+ 4)*sqrt(-2*sqrt(2) + 8) + sqrt(7)*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2
) + 8) + 2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*2^(3/4)*b*sqrt(2*
sqrt(2) + 8)*(sqrt(2) - 4) - 8*sqrt(7)*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) - 8*
2^(1/4)*a*sqrt(2*sqrt(2) + 8))*log(x^2 + 2*2^(1/4)*x*sqrt(-1/8*sqrt(2) + 1
/2) + sqrt(2)) + 1/1792*sqrt(7)*(3*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-2
*sqrt(2) + 8) + sqrt(7)*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) + 2^(
3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*2^(3/4)*b*sqrt(2*sqrt(2) + 8)
*(sqrt(2) - 4) - 8*sqrt(7)*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) - 8*2^(1/4)*a*sq
rt(2*sqrt(2) + 8))*log(x^2 - 2*2^(1/4)*x*sqrt(-1/8*sqrt(2) + 1/2) + sqr...
```

3.100. $\int \frac{a+bx^2}{2+x^2+x^4}\, dx$

3.100.9 Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.29

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx = -\operatorname{atan}\left(\frac{a^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}\,7\mathrm{i}}{\frac{\sqrt{7}\,a^3\,1\mathrm{i}}{2}-a\,b^2-2\,a^2\,b+\frac{a^3}{2}+4\,b^3-\sqrt{7}\,a\,b^2\,1\mathrm{i}} - \frac{b^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}\,14\mathrm{i}}{\frac{\sqrt{7}\,a^3\,1\mathrm{i}}{2}-a\,b^2-2\,a^2\,b+\frac{a^3}{2}+4\,b^3-\sqrt{7}\,a\,b^2\,1\mathrm{i}} + \frac{\sqrt{7}\,a^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}}{\frac{\sqrt{7}\,a^3\,1\mathrm{i}}{2}-a\,b^2-2\,a^2\,b+\frac{a^3}{2}+4\,b^3-\sqrt{7}\,a\,b^2\,1\mathrm{i}} - \frac{2\,\sqrt{7}\,b^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}}{\frac{\sqrt{7}\,a^3\,1\mathrm{i}}{2}-a\,b^2-2\,a^2\,b+\frac{a^3}{2}+4\,b^3-\sqrt{7}\,a\,b^2\,1\mathrm{i}}\right)\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}+\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}-\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}\,2\mathrm{i} - 2\operatorname{atanh}\left(\frac{7\,a^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}-\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}+\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}}{\frac{\sqrt{7}\,a^3\,1\mathrm{i}}{2}+a\,b^2+2\,a^2\,b-\frac{a^3}{2}-4\,b^3-\sqrt{7}\,a\,b^2\,1\mathrm{i}} - \frac{14\,b^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}-\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}+\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}}{\frac{\sqrt{7}\,a^3\,1\mathrm{i}}{2}+a\,b^2+2\,a^2\,b-\frac{a^3}{2}-4\,b^3-\sqrt{7}\,a\,b^2\,1\mathrm{i}} + \frac{\sqrt{7}\,a^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}-\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}+\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}\,1\mathrm{i}}{\frac{\sqrt{7}\,a^3\,1\mathrm{i}}{2}+a\,b^2+2\,a^2\,b-\frac{a^3}{2}-4\,b^3-\sqrt{7}\,a\,b^2\,1\mathrm{i}} - \frac{\sqrt{7}\,b^2\,x\,\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}-\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}+\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}\,2\mathrm{i}}{\frac{\sqrt{7}\,a^3\,1\mathrm{i}}{2}+a\,b^2+2\,a^2\,b-\frac{a^3}{2}-4\,b^3-\sqrt{7}\,a\,b^2\,1\mathrm{i}}\right)\sqrt{\frac{a^2}{112}-\frac{a\,b}{14}+\frac{b^2}{56}-\frac{\sqrt{7}\,a^2\,1\mathrm{i}}{112}+\frac{\sqrt{7}\,b^2\,1\mathrm{i}}{56}}$$

input
```
int((a + b*x^2)/(x^2 + x^4 + 2),x)
```

output

```
- atan((a^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^2
/112 + b^2/56)^(1/2)*7i)/((7^(1/2)*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4
*b^3 - 7^(1/2)*a*b^2*1i) - (b^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1
/2)*b^2*1i)/56 + a^2/112 + b^2/56)^(1/2)*14i)/((7^(1/2)*a^3*1i)/2 - a*b^2
- 2*a^2*b + a^3/2 + 4*b^3 - 7^(1/2)*a*b^2*1i) + (7^(1/2)*a^2*x*((7^(1/2)*a
^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^2/112 + b^2/56)^(1/2))/((7
^(1/2)*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4*b^3 - 7^(1/2)*a*b^2*1i) - (
2*7^(1/2)*b^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a
^2/112 + b^2/56)^(1/2))/((7^(1/2)*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4*
b^3 - 7^(1/2)*a*b^2*1i))*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1
i)/56 + a^2/112 + b^2/56)^(1/2)*2i - 2*atanh((7*a^2*x*((7^(1/2)*b^2*1i)/56
 - (7^(1/2)*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2))/((7^(1/2)*a^
3*1i)/2 + a*b^2 + 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i) - (14*b^2*x*
((7^(1/2)*b^2*1i)/56 - (7^(1/2)*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)
^(1/2))/((7^(1/2)*a^3*1i)/2 + a*b^2 + 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*
b^2*1i) + (7^(1/2)*a^2*x*((7^(1/2)*b^2*1i)/56 - (7^(1/2)*a^2*1i)/112 - (a*
b)/14 + a^2/112 + b^2/56)^(1/2)*1i)/((7^(1/2)*a^3*1i)/2 + a*b^2 + 2*a^2*b
- a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i) - (7^(1/2)*b^2*x*((7^(1/2)*b^2*1i)/56
- (7^(1/2)*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2)*2i)/((7^(1/2)*
a^3*1i)/2 + a*b^2 + 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i))*((7^(1...
```

3.100. $\int \frac{a+bx^2}{2+x^2+x^4}\,dx$

3.101 $\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx$

3.101.1 Optimal result

Integrand size = 18, antiderivative size = 316

$$\begin{aligned}\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx &= \frac{x(3a+2b-(a-4b)x^2)}{28\,(2+x^2+x^4)} - \frac{1}{56}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(\left(11-\sqrt{2}\right)a\right.\\ &\qquad \left.-\left(2-4\sqrt{2}\right)b\right)\arctan\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)\\ &\quad + \frac{1}{56}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(\left(11-\sqrt{2}\right)a\right.\\ &\qquad \left.-\left(2-4\sqrt{2}\right)b\right)\arctan\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)\\ &\quad - \frac{(11a+\sqrt{2}(a-4b)-2b)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{112\sqrt{2\left(-1+2\sqrt{2}\right)}}\\ &\quad + \frac{\left(\left(11+\sqrt{2}\right)a-2\left(b+2\sqrt{2}b\right)\right)\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{112\sqrt{2\left(-1+2\sqrt{2}\right)}}\end{aligned}$$

output
```
1/28*x*(3*a+2*b-(a-4*b)*x^2)/(x^4+x^2+2)-1/784*arctan((-2*x+(-1+2*2^(1/2))
^(1/2))/(1+2*2^(1/2))^(1/2))*(-b*(2-4*2^(1/2))+a*(11-2^(1/2)))*(-14+28*2^(
1/2))^(1/2)+1/784*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(
-b*(2-4*2^(1/2))+a*(11-2^(1/2)))*(-14+28*2^(1/2))^(1/2)-1/112*ln(x^2+2^(1/
2)-x*(-1+2*2^(1/2))^(1/2))*(11*a-2*b+(a-4*b)*2^(1/2))/(-2+4*2^(1/2))^(1/2)
+1/112*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(a*(11+2^(1/2))-2*b-4*b*2^(1
/2))/(-2+4*2^(1/2))^(1/2)
```

3.101.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.53

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx=\frac{3ax+2bx-ax^3+4bx^3}{28(2+x^2+x^4)}-\frac{\left(\left(23i+\sqrt{7}\right)a-4\left(2i+\sqrt{7}\right)b\right)\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{28\sqrt{14-14i\sqrt{7}}}-\frac{\left(\left(-23i+\sqrt{7}\right)a-4\left(-2i+\sqrt{7}\right)b\right)\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{28\sqrt{14+14i\sqrt{7}}}$$

input
```
Integrate[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```

output
```
(3*a*x + 2*b*x - a*x^3 + 4*b*x^3)/(28*(2 + x^2 + x^4)) - (((23*I + Sqrt[7]
)*a - 4*(2*I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/(28*Sqrt[14
- (14*I)*Sqrt[7]]) - (((-23*I + Sqrt[7])*a - 4*(-2*I + Sqrt[7])*b)*ArcTan[
x/Sqrt[(1 + I*Sqrt[7])/2]])/(28*Sqrt[14 + (14*I)*Sqrt[7]])
```

3.101.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1492, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{\left(x^4+x^2+2\right)^2}dx$$

$$\Big\downarrow \; 1492$$

$$\frac{1}{28}\int \frac{-\left((a-4b)x^2\right)+11a-2b}{x^4+x^2+2}dx+\frac{x\left(-\left(x^2(a-4b)\right)+3a+2b\right)}{28\left(x^4+x^2+2\right)}$$

$$\Big\downarrow \; 1483$$

$$\frac{1}{28}\left(\frac{\int \frac{\sqrt{-1+2\sqrt{2}}(11a-2b)-\left(11a+\sqrt{2}(a-4b)-2b\right)x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\int \frac{\sqrt{-1+2\sqrt{2}}(11a-2b)+\left(\left(11+\sqrt{2}\right)a-2\left(2\sqrt{2}b+b\right)\right)x}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx}{2\sqrt{2\left(2\sqrt{2}-1\right)}}\right)+\frac{x\left(-\left(x^2(a-4b)\right)+3a+2b\right)}{28\left(x^4+x^2+2\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{28}\left(\frac{\frac{1}{2}\sqrt{2\sqrt{2}-1}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\int \frac{1}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx-\frac{1}{2}\left(\sqrt{2}(a-4b)+11a-2b\right)\int -\frac{\sqrt{-1+2\sqrt{2}\cdots}}{x^2-\sqrt{-1+2\cdots}}\cdots}{2\sqrt{2\left(2\sqrt{2}-1\right)}}\cdots\right.$$
$$\frac{x\left(-\left(x^2(a-4b)\right)+3a+2b\right)}{28\left(x^4+x^2+2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{28}\left(\frac{\frac{1}{2}\sqrt{2\sqrt{2}-1}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\int \frac{1}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\frac{1}{2}\left(\sqrt{2}(a-4b)+11a-2b\right)\int \frac{\sqrt{-1+2\sqrt{2}}-\cdots}{x^2-\sqrt{-1+2\sqrt{2}}x\cdots}\cdots}{2\sqrt{2\left(2\sqrt{2}-1\right)}}\cdots\right.$$
$$\frac{x\left(-\left(x^2(a-4b)\right)+3a+2b\right)}{28\left(x^4+x^2+2\right)}$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{28}\left(\frac{\frac{1}{2}\left(\sqrt{2}(a-4b)+11a-2b\right)\int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx-\sqrt{2\sqrt{2}-1}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\int \frac{1}{-\left(2x-\sqrt{-1+2\sqrt{\right.}}}}{2\sqrt{2\left(2\sqrt{2}-1\right)}}\right.$$

$$\frac{x\left(-\left(x^2(a-4b)\right)+3a+2b\right)}{28\left(x^4+x^2+2\right)}$$

$\Big\downarrow$ 217

$$\frac{1}{28}\left(\frac{\frac{1}{2}\left(\sqrt{2}(a-4b)+11a-2b\right)\int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}}dx+\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\arctan\left(\frac{2x-\sqrt{2\sqrt{2}-}}{\sqrt{1+2\sqrt{2}}}\right.}{2\sqrt{2\left(2\sqrt{2}-1\right)}}\right.$$

$$\frac{x\left(-\left(x^2(a-4b)\right)+3a+2b\right)}{28\left(x^4+x^2+2\right)}$$

$\Big\downarrow$ 1103

$$\frac{1}{28}\left(\frac{\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\arctan\left(\frac{2x-\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)-\frac{1}{2}\left(\sqrt{2}(a-4b)+11a-2b\right)\log\left(x^2-\sqrt{2\sqrt{2}-}\right.}{2\sqrt{2\left(2\sqrt{2}-1\right)}}\right.$$

$$\frac{x\left(-\left(x^2(a-4b)\right)+3a+2b\right)}{28\left(x^4+x^2+2\right)}$$

input
```
Int[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```

output
```
(x*(3*a + 2*b - (a - 4*b)*x^2))/(28*(2 + x^2 + x^4)) + ((Sqrt[(-1 + 2*Sqrt
[2])/(1 + 2*Sqrt[2])]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(-Sqrt
[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]] - ((11*a + Sqrt[2]*(a - 4*b)
- 2*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2*(-1 + 2*S
qrt[2])]) + (Sqrt[(-1 + 2*Sqrt[2])/(1 + 2*Sqrt[2])]*((11 - Sqrt[2])*a - (2
 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]]
+ (((11 + Sqrt[2])*a - 2*(b + 2*Sqrt[2]*b))*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt
[2]]*x + x^2])/2)/(2*Sqrt[2*(-1 + 2*Sqrt[2])]))/28
```

3.101. $\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx$

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.101.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\left(\frac{b}{7}-\frac{a}{28}\right)x^{3}+\left(\frac{b}{14}+\frac{3a}{28}\right)x}{x^{4}+x^{2}+2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+\_Z^{2}+2\right)}\frac{\left(\left(-a+4b\right)\_R^{2}-2b+11a\right)\ln\left(x-\_R\right)}{2\_R^{3}+\_R}\right)}{56}$
default	$\frac{\frac{\left(-14a-28\sqrt{2}\,a+112b\sqrt{2}+56b\right)x}{1+2\sqrt{2}}+\frac{\sqrt{-1+2\sqrt{2}}\left(-70a-42\sqrt{2}\,a+56b\sqrt{2}+28b\right)}{1+2\sqrt{2}}}{784x\sqrt{-1+2\sqrt{2}}+784x^{2}+784\sqrt{2}}+\frac{\left(107\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,a-50\sqrt{-1+2\sqrt{2}}\,\sqrt{2}\,b+106\sqrt{-1+2\sqrt{2}}\,a-88\sqrt{-\right.}{2}$

input
```
int((b*x^2+a)/(x^4+x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
((1/7*b-1/28*a)*x^3+(1/14*b+3/28*a)*x)/(x^4+x^2+2)+1/56*sum(((-a+4*b)*_R^2
-2*b+11*a)/(2*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+_Z^2+2))
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. 2(235) = 470.

Time = 0.29 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.02

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx =$$
$$-\frac{28\,(a-4\,b)x^3+\sqrt{7}(x^4+x^2+2)\sqrt{211\,a^2-428\,ab+100\,b^2+7\,\sqrt{7}\sqrt{-289\,a^4+136\,a^3b+120\,a^2b^2-3}}{}$$

input
```
integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="fricas")
```

output
```
-1/784*(28*(a - 4*b)*x^3 + sqrt(7)*(x^4 + x^2 + 2)*sqrt(211*a^2 - 428*a*b
+ 100*b^2 + 7*sqrt(7)*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 -
 16*b^4))*log(-8*(1139*a^4 - 1169*a^3*b + 318*a^2*b^2 + 124*a*b^3 - 88*b^4
)*x + sqrt(211*a^2 - 428*a*b + 100*b^2 + 7*sqrt(7)*sqrt(-289*a^4 + 136*a^3
*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4))*(sqrt(7)*(187*a^3 - 78*a^2*b - 36*a
*b^2 + 8*b^3) + 3*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*
b^4)*(5*a - 6*b))) - sqrt(7)*(x^4 + x^2 + 2)*sqrt(211*a^2 - 428*a*b + 100*
b^2 + 7*sqrt(7)*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^
4))*log(-8*(1139*a^4 - 1169*a^3*b + 318*a^2*b^2 + 124*a*b^3 - 88*b^4)*x -
sqrt(211*a^2 - 428*a*b + 100*b^2 + 7*sqrt(7)*sqrt(-289*a^4 + 136*a^3*b + 1
20*a^2*b^2 - 32*a*b^3 - 16*b^4))*(sqrt(7)*(187*a^3 - 78*a^2*b - 36*a*b^2 +
 8*b^3) + 3*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4)*(
5*a - 6*b))) + sqrt(7)*(x^4 + x^2 + 2)*sqrt(211*a^2 - 428*a*b + 100*b^2 -
7*sqrt(7)*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4))*lo
g(-8*(1139*a^4 - 1169*a^3*b + 318*a^2*b^2 + 124*a*b^3 - 88*b^4)*x + sqrt(2
11*a^2 - 428*a*b + 100*b^2 - 7*sqrt(7)*sqrt(-289*a^4 + 136*a^3*b + 120*a^2
*b^2 - 32*a*b^3 - 16*b^4))*(sqrt(7)*(187*a^3 - 78*a^2*b - 36*a*b^2 + 8*b^3
) - 3*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4)*(5*a -
6*b))) - sqrt(7)*(x^4 + x^2 + 2)*sqrt(211*a^2 - 428*a*b + 100*b^2 - 7*sqrt
(7)*sqrt(-289*a^4 + 136*a^3*b + 120*a^2*b^2 - 32*a*b^3 - 16*b^4))*log(-...
```

3.101.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.52

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx = \frac{x^3(-a+4b)+x(3a+2b)}{28x^4+28x^2+56}$$
$$+\mathrm{RootSum}\left(240945152t^4+t^2\left(-1157968a^2+2348864ab-548800b^2\right)+4489a^4-7102a^3b+5757a^2b^2-\right.$$

input
```
integrate((b*x**2+a)/(x**4+x**2+2)**2,x)
```

output
```
(x**3*(-a + 4*b) + x*(3*a + 2*b))/(28*x**4 + 28*x**2 + 56) + RootSum(24094
5152*_t**4 + _t**2*(-1157968*a**2 + 2348864*a*b - 548800*b**2) + 4489*a**4
 - 7102*a**3*b + 5757*a**2*b**2 - 2332*a*b**3 + 484*b**4, Lambda(_t, _t*lo
g(x + (2634240*_t**3*a - 3161088*_t**3*b + 11996*_t*a**3 + 12792*_t*a**2*b
 - 21936*_t*a*b**2 + 4384*_t*b**3)/(1139*a**4 - 1169*a**3*b + 318*a**2*b**
2 + 124*a*b**3 - 88*b**4)))
```

3.101.7 Maxima [F]

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx=\int \frac{bx^2+a}{(x^4+x^2+2)^2}\,dx$$

input
```
integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="maxima")
```

output
```
-1/28*((a - 4*b)*x^3 - (3*a + 2*b)*x)/(x^4 + x^2 + 2) + 1/28*integrate(-((
a - 4*b)*x^2 - 11*a + 2*b)/(x^4 + x^2 + 2), x)
```

3.101.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. 2(235) = 470.

Time = 0.57 (sec) , antiderivative size = 1112, normalized size of antiderivative = 3.52

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="giac")
```

output
```
1/25088*sqrt(7)*(sqrt(7)*2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) - 4*s
qrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*sqrt(7)*2^(3/4)*a*s
qrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 12*sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8
)*(sqrt(2) - 4) - 3*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) + 12*2^(3
/4)*b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*a*(sqrt(2) - 4)*sqrt(-2
*sqrt(2) + 8) + 4*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) + 88*sqrt(7
)*2^(1/4)*a*sqrt(2*sqrt(2) + 8) - 16*sqrt(7)*2^(1/4)*b*sqrt(2*sqrt(2) + 8)
 - 88*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) + 16*2^(1/4)*b*sqrt(-2*sqrt(2) + 8))*
arctan(2*2^(3/4)*sqrt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqr
t(2) + 4)) + 1/25088*sqrt(7)*(sqrt(7)*2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(
2) + 4) - 4*sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*sqrt(7
)*2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 12*sqrt(7)*2^(3/4)*b*sqrt(
2*sqrt(2) + 8)*(sqrt(2) - 4) - 3*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-2*sqrt(2) +
 8) + 12*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*a*(sqrt(2)
 - 4)*sqrt(-2*sqrt(2) + 8) + 4*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8
) + 88*sqrt(7)*2^(1/4)*a*sqrt(2*sqrt(2) + 8) - 16*sqrt(7)*2^(1/4)*b*sqrt(2
*sqrt(2) + 8) - 88*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) + 16*2^(1/4)*b*sqrt(-2*s
qrt(2) + 8))*arctan(2*2^(3/4)*sqrt(1/2)*(x - 2^(1/4)*sqrt(-1/8*sqrt(2) + 1
/2))/sqrt(sqrt(2) + 4)) + 1/50176*sqrt(7)*(3*sqrt(7)*2^(3/4)*a*(sqrt(2) +
4)*sqrt(-2*sqrt(2) + 8) - 12*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-2*sq...
```

3.101.9 Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 1491, normalized size of antiderivative = 4.72

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)/(x^2 + x^4 + 2)^2,x)
```

output
```
atan((b^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/
3136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2)*1i)
/(4*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3*a*b^2)/1568 - (18
3*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (
7^(1/2)*a^2*b*39i)/3136)) - (a^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21
952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*
a*b*1i)/3136)^(1/2)*17i)/(16*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/7
84 + (3*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7
^(1/2)*a*b^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)) + (a*b*x*((7^(1/2)*a^2*
17i)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (
25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/(4*((7^(1/2)*a^3*187i)/62
72 + (7^(1/2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/
6272 + (9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136))
- (17*7^(1/2)*a^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 - (7^(1/2)*
b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(
1/2))/(112*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3*a*b^2)/156
8 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1
568 - (7^(1/2)*a^2*b*39i)/3136)) + (7^(1/2)*b^2*x*((7^(1/2)*a^2*17i)/12544
 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21
952 - (7^(1/2)*a*b*1i)/3136)^(1/2))/(28*((7^(1/2)*a^3*187i)/6272 + (7^(...
```

3.101. $\int \frac{a+bx^2}{(2+x^2+x^4)^2} \, dx$

3.102 $\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx$

3.102.1 Optimal result

Integrand size = 29, antiderivative size = 160

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}} - \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right) + \frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)$$

output
```
-1/8*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(4+2*2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2+2
^(1/2))^(1/2))*(4+2*2^(1/2))^(1/2)-1/2*arctan((-2*x+(2+2^(1/2))^(1/2))/(2-
2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)+1/2*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^
(1/2))^(1/2))/(2+2^(1/2))^(1/2)
```

3.102.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = \frac{\sqrt{-1-i}\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{-1-i}}\right)+\sqrt{-1+i}\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{-1+i}}\right)}{2^{3/4}}$$

input
```
Integrate[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

output
```
(Sqrt[-1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[-1 - I]] + Sqrt[-1 + I]*ArcTan[(2^(1
/4)*x)/Sqrt[-1 + I]])/2^(3/4)
```

3.102.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2}-x^2}{x^4-\sqrt{2}x^2+1}dx$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\int \frac{\left(1+\sqrt{2}\right)x+\sqrt{2\left(2+\sqrt{2}\right)}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2-\sqrt{2+\sqrt{2}}x+1}dx-\frac{1}{2}\left(1+\sqrt{2}\right)\int -\frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2+\sqrt{2+\sqrt{2}}x+1}dx+\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2-\sqrt{2+\sqrt{2}}x+1}dx+\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{x^2+\sqrt{2+\sqrt{2}}x+1}dx+\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx}{2\sqrt{2+\sqrt{2}}}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\frac{1}{2}(1+\sqrt{2})\int\frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx-\sqrt{2-\sqrt{2}}\int\frac{1}{-\left(2x-\sqrt{2+\sqrt{2}}\right)^2+\sqrt{2}-2}d\left(2x-\sqrt{2+\sqrt{2}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}(1+\sqrt{2})\int\frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx-\sqrt{2-\sqrt{2}}\int\frac{1}{-\left(2x+\sqrt{2+\sqrt{2}}\right)^2+\sqrt{2}-2}d\left(2x+\sqrt{2+\sqrt{2}}\right)}{2\sqrt{2+\sqrt{2}}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}(1+\sqrt{2})\int\frac{\sqrt{2+\sqrt{2}}-2x}{x^2-\sqrt{2+\sqrt{2}}x+1}dx+\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\frac{1}{2}(1+\sqrt{2})\int\frac{2x+\sqrt{2+\sqrt{2}}}{x^2+\sqrt{2+\sqrt{2}}x+1}dx+\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}(1+\sqrt{2})\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{2}(1+\sqrt{2})\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)}{2\sqrt{2+\sqrt{2}}}$$

input
```
Int[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

output
```
(ArcTan[(-Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]] - ((1 + Sqrt[2])*Log
[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[2]]) + (ArcTan[(Sqrt[
2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]] + ((1 + Sqrt[2])*Log[1 + Sqrt[2 + S
qrt[2]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[2]])
```

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.102.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2\,\mathrm{RootOf}\left(\_Z^2-2,\mathrm{index}=1\right)+1\right)}\frac{\left(\_R^2-\sqrt{2}\right)\ln\left(x-\_R\right)}{-2\_R^3+\_R\sqrt{2}}\right)}{2}$
default	$\frac{\sqrt{2}\left(\frac{\sqrt{2+\sqrt{2}}\,\ln\left(1+x^2+x\sqrt{2+\sqrt{2}}\right)}{2}+\frac{2\left(1-\frac{\sqrt{2}}{2}\right)\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}\right)}{4}+\frac{\sqrt{2}\left(-\frac{\sqrt{2+\sqrt{2}}\,\ln\left(1+x^2-x\sqrt{2+\sqrt{2}}\right)}{2}+\frac{2\left(1-\frac{\sqrt{2}}{2}\right)\arctan\left(\frac{2x-\sqrt{\cdots}}{\sqrt{2-\cdots}}\right)}{\sqrt{2-\sqrt{2}}}\cdots\right.}{4}$

input
```
int((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2-2^(1/2))/(-2*_R^3+_R*2^(1/2))*ln(x-_R),_R=RootOf(_Z^4-_Z^2*R
ootOf(_Z^2-2,index=1)+1))
```

3.102.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = \frac{1}{4}\sqrt{(i+1)\sqrt{2}}\log\left(2x+\sqrt{2}\sqrt{(i+1)\sqrt{2}}\right) - \frac{1}{4}\sqrt{(i+1)\sqrt{2}}\log\left(2x-\sqrt{2}\sqrt{(i+1)\sqrt{2}}\right) + \frac{1}{4}\sqrt{-(i-1)\sqrt{2}}\log\left(2x+\sqrt{2}\sqrt{-(i-1)\sqrt{2}}\right) - \frac{1}{4}\sqrt{-(i-1)\sqrt{2}}\log\left(2x-\sqrt{2}\sqrt{-(i-1)\sqrt{2}}\right)$$

input
```
integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="fricas")
```

output
```
1/4*sqrt((I + 1)*sqrt(2))*log(2*x + sqrt(2)*sqrt((I + 1)*sqrt(2))) - 1/4*s
qrt((I + 1)*sqrt(2))*log(2*x - sqrt(2)*sqrt((I + 1)*sqrt(2))) + 1/4*sqrt(-
(I - 1)*sqrt(2))*log(2*x + sqrt(2)*sqrt(-(I - 1)*sqrt(2))) - 1/4*sqrt(-(I
- 1)*sqrt(2))*log(2*x - sqrt(2)*sqrt(-(I - 1)*sqrt(2)))
```

3.102.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((-x**2+2**(1/2))/(1+x**4-x**2*2**(1/2)),x)
```

output
```
Exception raised: PolynomialError >> 1/(128*_t**4 - 16*sqrt(2)*_t**2 + 1)
contains an element of the set of generators.
```

3.102.7 Maxima [F]

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = \int -\frac{x^2-\sqrt{2}}{x^4-\sqrt{2}x^2+1}\,dx$$

input
```
integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="maxima")
```

output
```
-integrate((x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1), x)
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.76

$$\begin{aligned}\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx =& \frac{1}{4}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) \\ &+ \frac{1}{4}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right) \\ &+ \frac{1}{8}\sqrt{2\sqrt{2}+4}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right) \\ &- \frac{1}{8}\sqrt{2\sqrt{2}+4}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)\end{aligned}$$

input
```
integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="giac")
```

output
```
1/4*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) +
2)) + 1/4*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt
(2) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1
/8*sqrt(2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx = -\mathrm{atan}\left(x\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} - \frac{\sqrt{2}\,\sqrt{8}\,x\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}}{2}\right)\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} - \mathrm{atan}\left(x\sqrt{\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} + \frac{\sqrt{2}\,\sqrt{8}\,x\sqrt{\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}}{2}\right)\sqrt{\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i}$$

input
```
int((2^(1/2) - x^2)/(x^4 - 2^(1/2)*x^2 + 1),x)
```

output
```
- atan(x*(2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*2i - (2^(1/2)*8^(1/2)*x*(2^(
1/2)/16 - (8^(1/2)*1i)/32)^(1/2))/2)*(2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*
2i - atan(x*(2^(1/2)/16 + (8^(1/2)*1i)/32)^(1/2)*2i + (2^(1/2)*8^(1/2)*x*(
2^(1/2)/16 + (8^(1/2)*1i)/32)^(1/2))/2)*(2^(1/2)/16 + (8^(1/2)*1i)/32)^(1/
2)*2i
```

3.103 $\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx$

3.103.1 Optimal result

Integrand size = 26, antiderivative size = 172

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}} - \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right) + \frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)$$

output
```
-1/8*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2-2
^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-1/2*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+
2^(1/2))^(1/2))/(2-2^(1/2))^(1/2)+1/2*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2))/(2-2^(1/2))^(1/2)
```

3.103.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \frac{\sqrt{1-i}\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1-i}}\right)+\sqrt{1+i}\arctan\left(\frac{\sqrt[4]{2}x}{\sqrt{1+i}}\right)}{2^{3/4}}$$

input
```
Integrate[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

output
```
(Sqrt[1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[1 - I]] + Sqrt[1 + I]*ArcTan[(2^(1/4)
*x)/Sqrt[1 + I]])/2^(3/4)
```

3.103.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+\sqrt{2}}{x^4+\sqrt{2}x^2+1}dx$$

$$\Big\downarrow \text{1483}$$

$$\frac{\int \frac{\left(1-\sqrt{2}\right)x+\sqrt{2\left(2-\sqrt{2}\right)}}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(1-\sqrt{2}\right)x}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2-\sqrt{2-\sqrt{2}}x+1}dx+\frac{1}{2}\left(1-\sqrt{2}\right)\int -\frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+$$
$$\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2+\sqrt{2-\sqrt{2}}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2-\sqrt{2-\sqrt{2}}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+$$
$$\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{x^2+\sqrt{2-\sqrt{2}}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{1}{2}(1-\sqrt{2})\int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx-\sqrt{2+\sqrt{2}}\int \frac{1}{-\left(2x-\sqrt{2-\sqrt{2}}\right)^2-\sqrt{2}-2}d\left(2x-\sqrt{2-\sqrt{2}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{-\frac{1}{2}(1-\sqrt{2})\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx-\sqrt{2+\sqrt{2}}\int \frac{1}{-\left(2x+\sqrt{2-\sqrt{2}}\right)^2-\sqrt{2}-2}d\left(2x+\sqrt{2-\sqrt{2}}\right)}{2\sqrt{2-\sqrt{2}}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}(1-\sqrt{2})\int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2-\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}(1-\sqrt{2})\int \frac{2x+\sqrt{2-\sqrt{2}}}{x^2+\sqrt{2-\sqrt{2}}x+1}dx}{2\sqrt{2-\sqrt{2}}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}(1-\sqrt{2})\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}(1-\sqrt{2})\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)}{2\sqrt{2-\sqrt{2}}}$$

input
```
Int[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

output
```
(ArcTan[(-Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]] + ((1 - Sqrt[2])*Log
[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[2]]) + (ArcTan[(Sqrt[
2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]] - ((1 - Sqrt[2])*Log[1 + Sqrt[2 - S
qrt[2]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[2]])
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.103.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(1+\_Z^4+\_Z^2\operatorname{RootOf}\left(\_Z^2-2,\mathit{index}=1\right)\right)}\frac{\left(\_R^2+\sqrt{2}\right)\ln\left(x-\_R\right)}{2\_R^3+\_R\sqrt{2}}\right)}{2}$
default	$\frac{\sqrt{2}\left(-\frac{\sqrt{2-\sqrt{2}}\,\ln\left(1+x^2-x\sqrt{2-\sqrt{2}}\right)}{2}+\frac{2\left(\frac{\sqrt{2}}{2}+1\right)\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{\sqrt{2+\sqrt{2}}}\right)}{4}+\frac{\sqrt{2}\left(\frac{\sqrt{2-\sqrt{2}}\,\ln\left(1+x^2+x\sqrt{2-\sqrt{2}}\right)}{2}+\frac{2\left(\frac{\sqrt{2}}{2}+1\right)\arctan\left(\frac{2x+\sqrt{}}{\sqrt{2}}\right.}{\sqrt{2+\sqrt{2}}}\right.}{4}$

input
```
int((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2+2^(1/2))/(2*_R^3+_R*2^(1/2))*ln(x-_R),_R=RootOf(1+_Z^4+_Z^2*
RootOf(_Z^2-2,index=1)))
```

3.103.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \frac{1}{4}\sqrt{(i-1)\sqrt{2}}\log\left(2x+\sqrt{2}\sqrt{(i-1)\sqrt{2}}\right) - \frac{1}{4}\sqrt{(i-1)\sqrt{2}}\log\left(2x-\sqrt{2}\sqrt{(i-1)\sqrt{2}}\right) + \frac{1}{4}\sqrt{-(i+1)\sqrt{2}}\log\left(2x+\sqrt{2}\sqrt{-(i+1)\sqrt{2}}\right) - \frac{1}{4}\sqrt{-(i+1)\sqrt{2}}\log\left(2x-\sqrt{2}\sqrt{-(i+1)\sqrt{2}}\right)$$

input
```
integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="fricas")
```

output
```
1/4*sqrt((I - 1)*sqrt(2))*log(2*x + sqrt(2)*sqrt((I - 1)*sqrt(2))) - 1/4*s
qrt((I - 1)*sqrt(2))*log(2*x - sqrt(2)*sqrt((I - 1)*sqrt(2))) + 1/4*sqrt(-
(I + 1)*sqrt(2))*log(2*x + sqrt(2)*sqrt(-(I + 1)*sqrt(2))) - 1/4*sqrt(-(I
+ 1)*sqrt(2))*log(2*x - sqrt(2)*sqrt(-(I + 1)*sqrt(2)))
```

3.103.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((x**2+2**(1/2))/(1+x**4+x**2*2**(1/2)),x)
```

output
```
Exception raised: PolynomialError >> 1/(128*_t**4 + 16*sqrt(2)*_t**2 + 1)
contains an element of the set of generators.
```

3.103.7 Maxima [F]

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \int \frac{x^2+\sqrt{2}}{x^4+\sqrt{2}x^2+1}\,dx$$

input
```
integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="maxima")
```

output
```
integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1), x)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \frac{1}{4}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) + \frac{1}{4}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right) + \frac{1}{8}\sqrt{-2\sqrt{2}+4}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right) - \frac{1}{8}\sqrt{-2\sqrt{2}+4}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

input
```
integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="giac")
```

output
```
1/4*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2
)) + 1/4*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2
) + 2)) + 1/8*sqrt(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1
/8*sqrt(-2*sqrt(2) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx = \operatorname{atan}\left(x\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} + \frac{\sqrt{2}\,\sqrt{8}\,x\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}}{2}\right)\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} + \operatorname{atan}\left(x\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i} - \frac{\sqrt{2}\,\sqrt{8}\,x\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}}{2}\right)\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i}$$

input
```
int((2^(1/2) + x^2)/(2^(1/2)*x^2 + x^4 + 1),x)
```

output
```
atan(x*(- 2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*2i + (2^(1/2)*8^(1/2)*x*(- 2
^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2))/2)*(- 2^(1/2)/16 - (8^(1/2)*1i)/32)^(1
/2)*2i + atan(x*((8^(1/2)*1i)/32 - 2^(1/2)/16)^(1/2)*2i - (2^(1/2)*8^(1/2)
*x*((8^(1/2)*1i)/32 - 2^(1/2)/16)^(1/2))/2)*((8^(1/2)*1i)/32 - 2^(1/2)/16)
^(1/2)*2i
```

3.104 $\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx$

3.104.1 Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \frac{\left(1-\sqrt{2}\right)\arctan\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{\left(1-\sqrt{2}\right)\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{\left(1+\sqrt{2}\right)\log\left(1-\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} + \frac{\left(1+\sqrt{2}\right)\log\left(1+\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}}$$

output
```
-1/4*ln(1+x^2-x*(2-b)^(1/2))*(1+2^(1/2))/(2-b)^(1/2)+1/4*ln(1+x^2+x*(2-b)^
(1/2))*(1+2^(1/2))/(2-b)^(1/2)+1/2*arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))*
(1-2^(1/2))/(2+b)^(1/2)-1/2*arctan((2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1-2^(1/
2))/(2+b)^(1/2)
```

3.104.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \$\text{Aborted}$$

input
```
Integrate[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
```

output
```
$Aborted
```

3.104.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2}-x^2}{bx^2+x^4+1}dx$$

$$\Big\downarrow \text{1483}$$

$$\frac{\int \frac{\sqrt{2}\sqrt{2-b}-\left(1+\sqrt{2}\right)x}{x^2-\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}+\frac{\int \frac{\left(1+\sqrt{2}\right)x+\sqrt{2}\sqrt{2-b}}{x^2+\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{-\frac{1}{2}\left(1-\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{x^2-\sqrt{2-b}x+1}dx-\frac{1}{2}\left(1+\sqrt{2}\right)\int -\frac{\sqrt{2-b}-2x}{x^2-\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2-b}}{x^2+\sqrt{2-b}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{x^2+\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2-b}-2x}{x^2-\sqrt{2-b}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{x^2-\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2-b}}{x^2+\sqrt{2-b}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{x^2+\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2-b}-2x}{x^2-\sqrt{2-b}x+1}dx+\left(1-\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{-\left(2x-\sqrt{2-b}\right)^2-b-2}d\left(2x-\sqrt{2-b}\right)}{2\sqrt{2-b}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2-b}}{x^2+\sqrt{2-b}x+1}dx+\left(1-\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{-\left(2x+\sqrt{2-b}\right)^2-b-2}d\left(2x+\sqrt{2-b}\right)}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2-b}-2x}{x^2-\sqrt{2-b}x+1}dx-\frac{\left(1-\sqrt{2}\right)\sqrt{2-b}\arctan\left(\frac{2x-\sqrt{2-b}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}}{2\sqrt{2-b}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{2x+\sqrt{2-b}}{x^2+\sqrt{2-b}x+1}dx-\frac{\left(1-\sqrt{2}\right)\sqrt{2-b}\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\frac{\left(1-\sqrt{2}\right)\sqrt{2-b}\arctan\left(\frac{2x-\sqrt{2-b}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(-\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\sqrt{2-b}x+x^2+1\right)-\frac{\left(1-\sqrt{2}\right)\sqrt{2-b}\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}}{2\sqrt{2-b}}$$

input
```
Int[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
```

output
```
(-(((1 - Sqrt[2])*Sqrt[2 - b]*ArcTan[(-Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/Sq
rt[2 + b]) - ((1 + Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/2)/(2*Sqrt[2 - b
]) + (-(((1 - Sqrt[2])*Sqrt[2 - b]*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]]
)/Sqrt[2 + b]) + ((1 + Sqrt[2])*Log[1 + Sqrt[2 - b]*x + x^2])/2)/(2*Sqrt[2
 - b])
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.104.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4+\_Z^2 b+1\right)} \frac{\left(-\_R^2+\sqrt{2}\right)\ln\left(x-\_R\right)}{2\_R^3+\_R b}}{2}$	44
default	$\frac{\left(-\sqrt{(b-2)(2+b)}-b-2\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\sqrt{2\sqrt{(b-2)(2+b)}+2b}}+\frac{\left(-\sqrt{(b-2)(2+b)}+b+2\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$	136

input
```
int((-x^2+2^(1/2))/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((-_R^2+2^(1/2))/(2*_R^3+_R*b)*ln(x-_R),_R=RootOf(_Z^4+_Z^2*b+1))
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. 2(123) = 246.

Time = 0.26 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.23

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b+4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(2\,(2\,b^2-9)x + \sqrt{\frac{1}{2}}\left(2\,b^3-3\,\sqrt{2}(b^2-4)-8\,b-\frac{2\,b^4-14\,b^2-\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b+4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b+4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(2\,(2\,b^2-9)x - \sqrt{\frac{1}{2}}\left(2\,b^3-3\,\sqrt{2}(b^2-4)-8\,b-\frac{2\,b^4-14\,b^2-\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b+4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b+4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\left(2\,(2\,b^2-9)x + \sqrt{\frac{1}{2}}\left(2\,b^3-3\,\sqrt{2}(b^2-4)-8\,b+\frac{2\,b^4-14\,b^2-\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b+4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b+4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\left(2\,(2\,b^2-9)x - \sqrt{\frac{1}{2}}\left(2\,b^3-3\,\sqrt{2}(b^2-4)-8\,b+\frac{2\,b^4-14\,b^2-\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b+4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\right)$$

input
```
integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")
```

3.104. $\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx$

output
```
-1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2
*b^2 - 9)*x + sqrt(1/2)*(2*b^3 - 3*sqrt(2)*(b^2 - 4) - 8*b - (2*b^4 - 14*b
^2 - sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b + 4*sqrt(2) + sqr
t(b^2 - 4))/(b^2 - 4))) + 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2
- 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*x - sqrt(1/2)*(2*b^3 - 3*sqrt(2)*(b^2 -
 4) - 8*b - (2*b^4 - 14*b^2 - sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqr
t(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) - 1/2*sqrt(1/2)*sqrt(-(3*
b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*x + sqrt(1/2)*
(2*b^3 - 3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*b^2 - sqrt(2)*(b^3 - 4*b)
 + 24)/sqrt(b^2 - 4))*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4)))
+ 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(
2*b^2 - 9)*x - sqrt(1/2)*(2*b^3 - 3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*
b^2 - sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b + 4*sqrt(2) - sq
rt(b^2 - 4))/(b^2 - 4)))
```

3.104.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(128) = 256$.

Time = 1.34 (sec) , antiderivative size = 1469, normalized size of antiderivative = 9.18

$$\int \frac{\sqrt{2} - x^2}{1 + bx^2 + x^4} \, dx = \text{Too large to display}$$

input
```
integrate((-x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

output
```
-RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 + 16*sqrt(2)*b*
*2 - 48*b - 64*sqrt(2)) + 2*b**2 + 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**
3*(64*b**12/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 64
70*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 +
729) + 672*sqrt(2)*b**11/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt
(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 -
 3402*b**2 + 729) + 5760*b**10/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 214
4*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*
b**3 - 3402*b**2 + 729) + 12064*sqrt(2)*b**9/(8*b**10 + 88*sqrt(2)*b**9 +
828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 -
 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + 17744*b**8/(8*b**10 + 88*sqrt(2)*b
**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*
b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 27480*sqrt(2)*b**7/(8*b**10
+ 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2
)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 154608*b**6/(8
*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310
*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 141376*
sqrt(2)*b**5/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6
470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 +
 729) - 69072*b**4/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)...
```

3.104.7 Maxima [F]

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \int -\frac{x^2-\sqrt{2}}{x^4+bx^2+1}\,dx$$

input
```
integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")
```

output
```
-integrate((x^2 - sqrt(2))/(x^4 + b*x^2 + 1), x)
```

3.104.8 Giac [F]

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \int -\frac{x^2-\sqrt{2}}{x^4+bx^2+1}\,dx$$

input
```
integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")
```

output
```
sage0*x
```

3.104.9 Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 1227, normalized size of antiderivative = 7.67

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx =$$
$$-\mathrm{atan}\left(\frac{x\sqrt{\frac{12\,b+16\,\sqrt{2}-4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}}\,32\mathrm{i}-b\,x\left(\frac{12\,b+16\,\sqrt{2}-4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}\right)^{3/2}256$$

input
```
int((2^(1/2) - x^2)/(b*x^2 + x^4 + 1),x)
```

output
```
atan((x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 +
b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*(-(4*2^(1/2)*b^2
- 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 -
 64*b^2 + 128))^(3/2)*256i + b^2*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b +
3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*
8i - b^4*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4
 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*(-(4*2^(1/2)*
b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b
^4 - 64*b^2 + 128))^(3/2)*128i - b^5*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*
b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3
/2)*16i + 2^(1/2)*b*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b
^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - 2^(1/2)
*b^3*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b
^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(2^(1/2)*b^3 - 4*2^(1/2)
*b + 2^(1/2)*(48*b^2 - 12*b^4 + b^6 - 64)^(1/2) + 2*b^2 - 8))*(-(4*2^(1/2)
*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*
b^4 - 64*b^2 + 128))^(1/2)*2i - atan((x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^
2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1
/2)*32i - b*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b
^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3/2)*256i + b^2*x*((12*b...
```

3.104. $\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx$

3.105 $\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx$

3.105.1 Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = -\frac{\left(1+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}}+\frac{\left(1+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} + \frac{\left(1-\sqrt{2}\right)\log\left(1-\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}} - \frac{\left(1-\sqrt{2}\right)\log\left(1+\sqrt{2-b}x+x^2\right)}{4\sqrt{2-b}}$$

output
```
1/4*ln(1+x^2-x*(2-b)^(1/2))*(1-2^(1/2))/(2-b)^(1/2)-1/4*ln(1+x^2+x*(2-b)^(
1/2))*(1-2^(1/2))/(2-b)^(1/2)-1/2*arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))*(
1+2^(1/2))/(2+b)^(1/2)+1/2*arctan((2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1+2^(1/2
))/(2+b)^(1/2)
```

3.105.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = \$\text{Aborted}$$

input
```
Integrate[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
```

output
```
$Aborted
```

3.105.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+\sqrt{2}}{bx^2+x^4+1}\,dx$$

$$\Big\downarrow \text{1483}$$

$$\frac{\int \frac{\left(1-\sqrt{2}\right)x+\sqrt{2}\sqrt{2-b}}{x^2-\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}+\frac{\int \frac{\sqrt{2}\sqrt{2-b}-\left(1-\sqrt{2}\right)x}{x^2+\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{x^2-\sqrt{2-b}x+1}dx+\frac{1}{2}\left(1-\sqrt{2}\right)\int -\frac{\sqrt{2-b}-2x}{x^2-\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{x^2+\sqrt{2-b}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{2x+\sqrt{2-b}}{x^2+\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{x^2-\sqrt{2-b}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-b}-2x}{x^2-\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}+\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{x^2+\sqrt{2-b}x+1}dx-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{2x+\sqrt{2-b}}{x^2+\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-b}-2x}{x^2-\sqrt{2-b}x+1}dx-\left(1+\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{-\left(2x-\sqrt{2-b}\right)^2-b-2}d\left(2x-\sqrt{2-b}\right)}{2\sqrt{2-b}}+\frac{-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{2x+\sqrt{2-b}}{x^2+\sqrt{2-b}x+1}dx-\left(1+\sqrt{2}\right)\sqrt{2-b}\int \frac{1}{-\left(2x+\sqrt{2-b}\right)^2-b-2}d\left(2x+\sqrt{2-b}\right)}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\left(1+\sqrt{2}\right)\sqrt{2-b}\arctan\left(\frac{2x-\sqrt{2-b}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-b}-2x}{x^2-\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}+\frac{\frac{\left(1+\sqrt{2}\right)\sqrt{2-b}\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{2x+\sqrt{2-b}}{x^2+\sqrt{2-b}x+1}dx}{2\sqrt{2-b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{(1+\sqrt{2})\sqrt{2-b}\arctan\left(\frac{2x-\sqrt{2-b}}{\sqrt{b+2}}\right)}{\sqrt{b+2}}+\frac{1}{2}(1-\sqrt{2})\log\left(-\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}+\frac{\frac{(1+\sqrt{2})\sqrt{2-b}\arctan\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{1}{2}(1-\sqrt{2})\log\left(\sqrt{2-b}x+x^2+1\right)}{2\sqrt{2-b}}$$

input
```
Int[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
```

output
```
(((1 + Sqrt[2])*Sqrt[2 - b]*ArcTan[(-Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/Sqrt
[2 + b] + ((1 - Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/2)/(2*Sqrt[2 - b])
+ (((1 + Sqrt[2])*Sqrt[2 - b]*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/Sqr
t[2 + b] - ((1 - Sqrt[2])*Log[1 + Sqrt[2 - b]*x + x^2])/2)/(2*Sqrt[2 - b])
```

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.105.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.26

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2 b+1\right)}\frac{\left(\_R^2+\sqrt{2}\right)\ln\left(x-\_R\right)}{2\_R^3+\_R b}\right)}{2}$	42
default	$\frac{\left(\sqrt{(b-2)(2+b)}+b-2\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\,\sqrt{2\sqrt{(b-2)(2+b)}+2b}}+\frac{\left(\sqrt{(b-2)(2+b)}-b+2\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\,\sqrt{-2\sqrt{(b-2)(2+b)}+2b}}$	132

input
```
int((x^2+2^(1/2))/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2+2^(1/2))/(2*_R^3+_R*b)*ln(x-_R),_R=RootOf(_Z^4+_Z^2*b+1))
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(123) = 246.

Time = 0.28 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.21

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b-4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(2\left(2\,b^2-9\right)x + \sqrt{\frac{1}{2}}\left(2\,b^3+3\,\sqrt{2}(b^2-4)-8\,b-\frac{2\,b^4-14\,b^2+\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b-4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b-4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(2\left(2\,b^2-9\right)x - \sqrt{\frac{1}{2}}\left(2\,b^3+3\,\sqrt{2}(b^2-4)-8\,b-\frac{2\,b^4-14\,b^2+\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b-4\,\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b-4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\left(2\left(2\,b^2-9\right)x + \sqrt{\frac{1}{2}}\left(2\,b^3+3\,\sqrt{2}(b^2-4)-8\,b+\frac{2\,b^4-14\,b^2+\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b-4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3\,b-4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\left(2\left(2\,b^2-9\right)x - \sqrt{\frac{1}{2}}\left(2\,b^3+3\,\sqrt{2}(b^2-4)-8\,b+\frac{2\,b^4-14\,b^2+\sqrt{2}(b^3-4\,b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3\,b-4\,\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\right)$$

input
```
integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")
```

3.105. $\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx$

output
```
1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*
b^2 - 9)*x + sqrt(1/2)*(2*b^3 + 3*sqrt(2)*(b^2 - 4) - 8*b - (2*b^4 - 14*b^
2 + sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b - 4*sqrt(2) + sqrt
(b^2 - 4))/(b^2 - 4))) - 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 -
 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*x - sqrt(1/2)*(2*b^3 + 3*sqrt(2)*(b^2 -
4) - 8*b - (2*b^4 - 14*b^2 + sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt
(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) + 1/2*sqrt(1/2)*sqrt(-(3*b
 - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*x + sqrt(1/2)*(
2*b^3 + 3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*b^2 + sqrt(2)*(b^3 - 4*b)
+ 24)/sqrt(b^2 - 4))*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))) -
 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2
*b^2 - 9)*x - sqrt(1/2)*(2*b^3 + 3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*b
^2 + sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b - 4*sqrt(2) - sqr
t(b^2 - 4))/(b^2 - 4)))
```

3.105.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(128) = 256$.

Time = 1.39 (sec) , antiderivative size = 1467, normalized size of antiderivative = 9.17

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = \text{Too large to display}$$

input
```
integrate((x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

output
```
RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 - 16*sqrt(2)*b**
2 - 48*b + 64*sqrt(2)) + 2*b**2 - 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**3
*(64*b**12/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 647
0*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 7
29) - 672*sqrt(2)*b**11/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(
2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 -
3402*b**2 + 729) + 5760*b**10/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144
*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b
**3 - 3402*b**2 + 729) - 12064*sqrt(2)*b**9/(8*b**10 - 88*sqrt(2)*b**9 + 8
28*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 +
2322*sqrt(2)*b**3 - 3402*b**2 + 729) + 17744*b**8/(8*b**10 - 88*sqrt(2)*b*
*9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b
**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + 27480*sqrt(2)*b**7/(8*b**10 -
 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)
*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 154608*b**6/(8*
b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*
sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + 141376*s
qrt(2)*b**5/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 64
70*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 +
729) - 69072*b**4/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*...
```

3.105.7 Maxima [F]

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = \int \frac{x^2+\sqrt{2}}{x^4+bx^2+1}\,dx$$

input
```
integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^2 + sqrt(2))/(x^4 + b*x^2 + 1), x)
```

3.105.8 Giac [F]

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = \int \frac{x^2+\sqrt{2}}{x^4+bx^2+1}\,dx$$

input
```
integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")
```

output
```
sage0*x
```

3.105.9 Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 1227, normalized size of antiderivative = 7.67

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx =$$
$$-\mathrm{atan}\left(\frac{x\sqrt{\frac{12\,b-16\,\sqrt{2}+4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}}\,32\mathrm{i}-b\,x\left(\frac{12\,b-16\,\sqrt{2}+4\,\sqrt{2}\,b^2-3\,b^3+\sqrt{b^6-12\,b^4+48\,b^2-64}}{8\,b^4-64\,b^2+128}\right)^{3/2}256$$

input
```
int((2^(1/2) + x^2)/(b*x^2 + x^4 + 1),x)
```

output
```
atan((x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 +
b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*(-(16*2^(1/2) - 1
2*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 -
 64*b^2 + 128))^(3/2)*256i + b^2*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 +
3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*
8i - b^4*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4
 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*(-(16*2^(1/2)
 - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b
^4 - 64*b^2 + 128))^(3/2)*128i - b^5*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^
2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3
/2)*16i - 2^(1/2)*b*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b
^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i + 2^(1/2)
*b^3*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b
^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(2^(1/2)*b^3 - 4*2^(1/2)
*b + 2^(1/2)*(48*b^2 - 12*b^4 + b^6 - 64)^(1/2) - 2*b^2 + 8))*(-(16*2^(1/2
) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*
b^4 - 64*b^2 + 128))^(1/2)*2i - atan((x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^
2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1
/2)*32i - b*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b
^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3/2)*256i + b^2*x*((12*b...
```

3.105. $\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4} \, dx$

3.106 $\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx$

3.106.1 Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = -\frac{\arctan\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\arctan\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} -\frac{\sqrt{3}\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{a}}$$

output
```
1/2*arctan(-3^(1/2)+2*x/a^(1/2))/a^(1/2)+1/2*arctan(3^(1/2)+2*x/a^(1/2))/a
^(1/2)-1/4*ln(a+x^2-x*3^(1/2)*a^(1/2))*3^(1/2)/a^(1/2)+1/4*ln(a+x^2+x*3^(1
/2)*a^(1/2))*3^(1/2)/a^(1/2)
```

3.106.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = \frac{\sqrt[4]{-1}\left(-\sqrt{i+\sqrt{3}}(3i+\sqrt{3})\arctan\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\sqrt{a}}\right)+\sqrt{-i+\sqrt{3}}(-3i+\sqrt{3})\,\mathrm{arctanh}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt{a}}\right)\right)}{2\sqrt{6}\sqrt{a}}$$

input
```
Integrate[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

output
```
((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[
-I + Sqrt[3]]*Sqrt[a])]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1
 + I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])]))/(2*Sqrt[6]*Sqrt[a])
```

3.106.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1483, 27, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx$$

$$\Big\downarrow \ 1483$$

$$\frac{\int \frac{a\left(2\sqrt{3}\sqrt{a}-3x\right)}{x^2-\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{3}a^{3/2}}+\frac{\int \frac{\sqrt{3}a\left(\sqrt{3}x+2\sqrt{a}\right)}{x^2+\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{3}a^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2\sqrt{3}\sqrt{a}-3x}{x^2-\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{3}\sqrt{a}}+\frac{\int \frac{\sqrt{3}x+2\sqrt{a}}{x^2+\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{1}{2}\sqrt{3}\sqrt{a}\int \frac{1}{x^2-\sqrt{3}\sqrt{a}x+a}dx-\frac{3}{2}\int -\frac{\sqrt{3}\sqrt{a}-2x}{x^2-\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{3}\sqrt{a}}+\frac{\frac{1}{2}\sqrt{a}\int \frac{1}{x^2+\sqrt{3}\sqrt{a}x+a}dx+\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}\sqrt{a}}{x^2+\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{2}\sqrt{3}\sqrt{a}\int \frac{1}{x^2-\sqrt{3}\sqrt{a}x+a}dx+\frac{3}{2}\int \frac{\sqrt{3}\sqrt{a}-2x}{x^2-\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{3}\sqrt{a}}+\frac{\frac{1}{2}\sqrt{a}\int \frac{1}{x^2+\sqrt{3}\sqrt{a}x+a}dx+\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}\sqrt{a}}{x^2+\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{3}{2}\int \frac{\sqrt{3}\sqrt{a}-2x}{x^2-\sqrt{3}\sqrt{a}x+a}dx+\int \frac{1}{-\left(1-\frac{2x}{\sqrt{3}\sqrt{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2x}{\sqrt{3}\sqrt{a}}\right)}{2\sqrt{3}\sqrt{a}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}\sqrt{a}}{x^2+\sqrt{3}\sqrt{a}x+a}dx-\frac{\int \frac{1}{-\left(\frac{2x}{\sqrt{3}\sqrt{a}}+1\right)^2-\frac{1}{3}}d\left(\frac{2x}{\sqrt{3}\sqrt{a}}+1\right)}{\sqrt{3}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{3}{2}\int \frac{\sqrt{3}\sqrt{a}-2x}{x^2-\sqrt{3}\sqrt{a}x+a}dx-\sqrt{3}\arctan\left(\sqrt{3}\left(1-\frac{2x}{\sqrt{3}\sqrt{a}}\right)\right)}{2\sqrt{3}\sqrt{a}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}\sqrt{a}}{x^2+\sqrt{3}\sqrt{a}x+a}dx+\arctan\left(\sqrt{3}\left(\frac{2x}{\sqrt{3}\sqrt{a}}+1\right)\right)}{2\sqrt{a}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\sqrt{3}\arctan\left(\sqrt{3}\left(1-\frac{2x}{\sqrt{3}\sqrt{a}}\right)\right)-\frac{3}{2}\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{2\sqrt{3}\sqrt{a}}+\frac{\arctan\left(\sqrt{3}\left(\frac{2x}{\sqrt{3}\sqrt{a}}+1\right)\right)+\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{2\sqrt{a}}$$

input
```
Int[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

output
```
(-(Sqrt[3]*ArcTan[Sqrt[3]*(1 - (2*x)/(Sqrt[3]*Sqrt[a]))]) - (3*Log[a - Sqr
t[3]*Sqrt[a]*x + x^2])/2)/(2*Sqrt[3]*Sqrt[a]) + (ArcTan[Sqrt[3]*(1 + (2*x)
/(Sqrt[3]*Sqrt[a]))] + (Sqrt[3]*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/2)/(2*Sq
rt[a])
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.106.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.42

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-a\_Z^{2}+a^{2}\right)}\frac{\left(-\_R^{2}+2a\right)\ln\left(x-\_R\right)}{2\_R^{3}-\_Ra}\right)}{2}$	48
default	$\frac{\frac{\sqrt{3}\,\ln\left(a+x^{2}+x\sqrt{3}\,\sqrt{a}\right)}{2}+\arctan\left(\frac{2x+\sqrt{3}\,\sqrt{a}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{-\frac{\sqrt{3}\,\ln\left(x\sqrt{3}\,\sqrt{a}-x^{2}-a\right)}{2}-\arctan\left(\frac{\sqrt{3}\,\sqrt{a}-2x}{\sqrt{a}}\right)}{2\sqrt{a}}$	90

input
```
int((-x^2+2*a)/(x^4-a*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((-_R^2+2*a)/(2*_R^3-_R*a)*ln(x-_R),_R=RootOf(_Z^4-_Z^2*a+a^2))
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(80) = 160$.

Time = 0.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.92

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}+1}{a}}\log\left(\sqrt{\frac{1}{2}}a\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}+1}{a}}+x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}+1}{a}}\log\left(-\sqrt{\frac{1}{2}}a\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}+1}{a}}+x\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}-1}{a}}\log\left(\sqrt{\frac{1}{2}}a\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}-1}{a}}+x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}-1}{a}}\log\left(-\sqrt{\frac{1}{2}}a\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a^2}}-1}{a}}+x\right)$$

input
```
integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1/a^2) + 1)/a)*log(sqrt(1/2)*a*sqrt((s
qrt(3)*a*sqrt(-1/a^2) + 1)/a) + x) - 1/2*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1
/a^2) + 1)/a)*log(-sqrt(1/2)*a*sqrt((sqrt(3)*a*sqrt(-1/a^2) + 1)/a) + x) +
 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a^2) - 1)/a)*log(sqrt(1/2)*a*sqrt(
-(sqrt(3)*a*sqrt(-1/a^2) - 1)/a) + x) - 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqr
t(-1/a^2) - 1)/a)*log(-sqrt(1/2)*a*sqrt(-(sqrt(3)*a*sqrt(-1/a^2) - 1)/a) +
 x)
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.24

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = -\operatorname{RootSum}\left(16t^4a^2-4t^2a+1,\left(t\mapsto t\log\left(-2ta+x\right)\right)\right)$$

input
```
integrate((-x**2+2*a)/(x**4-a*x**2+a**2),x)
```

output
```
-RootSum(16*_t**4*a**2 - 4*_t**2*a + 1, Lambda(_t, _t*log(-2*_t*a + x)))
```

3.106.7 Maxima [F]

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = \int -\frac{x^2-2\,a}{x^4-ax^2+a^2}\,dx$$

input
```
integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 2*a)/(x^4 - a*x^2 + a^2), x)
```

3.106.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.56 (sec) , antiderivative size = 4217, normalized size of antiderivative = 36.99

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = \text{Too large to display}$$

input
```
integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*(3*sqrt(3)*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/ab
s(a))))^2*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^3*sin(1/2*real_part(ar
ccos(1/2*a/abs(a)))) - sqrt(3)*a^2*abs(a)^(3/2)*cosh(1/2*imag_part(arccos(
1/2*a/abs(a))))^3*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3 - 9*sqrt(3)*a
^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag_p
art(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh
(1/2*imag_part(arccos(1/2*a/abs(a)))) + 3*sqrt(3)*a^2*abs(a)^(3/2)*cosh(1/
2*imag_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a)
)))^3*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) + 9*sqrt(3)*a^2*abs(a)^(3/
2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag_part(arccos(1/
2*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*imag_part(
arccos(1/2*a/abs(a))))^2 - 3*sqrt(3)*a^2*abs(a)^(3/2)*cosh(1/2*imag_part(a
rccos(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3*sinh(1/2*
imag_part(arccos(1/2*a/abs(a))))^2 - 3*sqrt(3)*a^2*abs(a)^(3/2)*cos(1/2*re
al_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))*
sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 + sqrt(3)*a^2*abs(a)^(3/2)*sin
(1/2*real_part(arccos(1/2*a/abs(a))))^3*sinh(1/2*imag_part(arccos(1/2*a/ab
s(a))))^3 + a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))^3*co
sh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 - 3*a^3*sqrt(abs(a))*cos(1/2*rea
l_part(arccos(1/2*a/abs(a))))*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))...
```

3.106.9 Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = -\frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{\frac{1}{8\,a}+\frac{\sqrt{3}\,1\mathrm{i}}{8\,a}}\,1\mathrm{i}+\sqrt{3}\,x\,\sqrt{\frac{1}{8\,a}+\frac{\sqrt{3}\,1\mathrm{i}}{8\,a}}\right)\,\sqrt{\frac{1+\sqrt{3}\,1\mathrm{i}}{a}}\,1\mathrm{i}}{4} - \frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{\frac{1}{8\,a}-\frac{\sqrt{3}\,1\mathrm{i}}{8\,a}}\,1\mathrm{i}-\sqrt{3}\,x\,\sqrt{\frac{1}{8\,a}-\frac{\sqrt{3}\,1\mathrm{i}}{8\,a}}\right)\,\sqrt{-\frac{-1+\sqrt{3}\,1\mathrm{i}}{a}}\,1\mathrm{i}}{4}$$

input
```
int((2*a - x^2)/(a^2 - a*x^2 + x^4),x)
```

output
```
- (8^(1/2)*atan(x*((3^(1/2)*1i)/(8*a) + 1/(8*a))^(1/2)*1i + 3^(1/2)*x*((3^
(1/2)*1i)/(8*a) + 1/(8*a))^(1/2))*((3^(1/2)*1i + 1)/a)^(1/2)*1i)/4 - (8^(1
/2)*atan(x*(1/(8*a) - (3^(1/2)*1i)/(8*a))^(1/2)*1i - 3^(1/2)*x*(1/(8*a) -
(3^(1/2)*1i)/(8*a))^(1/2))*(-(3^(1/2)*1i - 1)/a)^(1/2)*1i)/4
```

3.107 $\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx$

3.107.1 Optimal result

Integrand size = 31, antiderivative size = 122

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx=-\frac{\arctan\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\arctan\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\sqrt{3}\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt[4]{a}}$$

output
```
1/2*arctan(2*x/a^(1/4)-3^(1/2))/a^(1/4)+1/2*arctan(2*x/a^(1/4)+3^(1/2))/a^
(1/4)-1/4*ln(x^2-a^(1/4)*x*3^(1/2)+a^(1/2))*3^(1/2)/a^(1/4)+1/4*ln(x^2+a^(
1/4)*x*3^(1/2)+a^(1/2))*3^(1/2)/a^(1/4)
```

3.107.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx=\frac{\sqrt[4]{-1}\left(-\sqrt{i+\sqrt{3}}(3i+\sqrt{3})\arctan\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\sqrt[4]{a}}\right)+\sqrt{-i+\sqrt{3}}(-3i+\sqrt{3})\,\text{arctanh}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt[4]{a}}\right)\right)}{2\sqrt{6}\sqrt[4]{a}}$$

input
```
Integrate[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

output
```
((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[
-I + Sqrt[3]]*a^(1/4))]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1
 + I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))]))/(2*Sqrt[6]*a^(1/4))
```

3.107.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1483, 27, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2\sqrt{a}-x^2}{-\sqrt{a}x^2+a+x^4}dx$$

$$\Big\downarrow \text{1483}$$

$$\frac{\int \frac{\sqrt{a}\left(2\sqrt{3}\sqrt[4]{a}-3x\right)}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt{3}a^{3/4}}+\frac{\int \frac{\sqrt{3}\sqrt{a}\left(\sqrt{3}x+2\sqrt[4]{a}\right)}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt{3}a^{3/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2\sqrt{3}\sqrt[4]{a}-3x}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt{3}\sqrt[4]{a}}+\frac{\int \frac{\sqrt{3}x+2\sqrt[4]{a}}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt[4]{a}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}\int \frac{1}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx-\frac{3}{2}\int -\frac{\sqrt{3}\sqrt[4]{a}-2x}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt{3}\sqrt[4]{a}}+\frac{\frac{1}{2}\sqrt[4]{a}\int \frac{1}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx+\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}\sqrt[4]{a}}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt[4]{a}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}\int \frac{1}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx+\frac{3}{2}\int \frac{\sqrt{3}\sqrt[4]{a}-2x}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt{3}\sqrt[4]{a}}+\frac{\frac{1}{2}\sqrt[4]{a}\int \frac{1}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx+\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}\sqrt[4]{a}}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt[4]{a}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{3}{2}\int \frac{\sqrt{3}\sqrt[4]{a}-2x}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx+\int \frac{1}{-\left(1-\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)}{2\sqrt{3}\sqrt[4]{a}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}\sqrt[4]{a}}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx-\frac{\int \frac{1}{-\left(\frac{2x}{\sqrt{3}\sqrt[4]{a}}+1\right)^2-\frac{1}{3}}d\left(\frac{2x}{\sqrt{3}\sqrt[4]{a}}+1\right)}{\sqrt{3}}}{2\sqrt[4]{a}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{3}{2}\int \frac{\sqrt{3}\sqrt[4]{a}-2x}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx-\sqrt{3}\arctan\left(\sqrt{3}\left(1-\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)\right)}{2\sqrt{3}\sqrt[4]{a}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2x+\sqrt{3}\sqrt[4]{a}}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx+\arctan\left(\sqrt{3}\left(\frac{2x}{\sqrt{3}\sqrt[4]{a}}+1\right)\right)}{2\sqrt[4]{a}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\sqrt{3}\arctan\left(\sqrt{3}\left(1-\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)\right)-\frac{3}{2}\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{2\sqrt{3}\sqrt[4]{a}}+\frac{\arctan\left(\sqrt{3}\left(\frac{2x}{\sqrt{3}\sqrt[4]{a}}+1\right)\right)+\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{2\sqrt[4]{a}}$$

input
```
Int[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

output
```
(-(Sqrt[3]*ArcTan[Sqrt[3]*(1 - (2*x)/(Sqrt[3]*a^(1/4)))]) - (3*Log[Sqrt[a]
 - Sqrt[3]*a^(1/4)*x + x^2])/2)/(2*Sqrt[3]*a^(1/4)) + (ArcTan[Sqrt[3]*(1 +
 (2*x)/(Sqrt[3]*a^(1/4)))] + (Sqrt[3]*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^
2])/2)/(2*a^(1/4))
```

3.107. $\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx$

3.107.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.107.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\frac{\sqrt{3}\,\ln\left(x^{2}+a^{\frac{1}{4}} x \sqrt{3}+\sqrt{a}\right)}{2}+\arctan\left(\frac{2x+a^{\frac{1}{4}}\sqrt{3}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{1}{4}}}+\frac{-\frac{\sqrt{3}\,\ln\left(x^{2}-a^{\frac{1}{4}} x \sqrt{3}+\sqrt{a}\right)}{2}+\arctan\left(\frac{2x-a^{\frac{1}{4}}\sqrt{3}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{1}{4}}}$	90

input
```
int((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^(1/4)*(1/2*3^(1/2)*ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))+arctan((2*x+a^(
1/4)*3^(1/2))/a^(1/4)))+1/2/a^(1/4)*(-1/2*3^(1/2)*ln(x^2-a^(1/4)*x*3^(1/2)
+a^(1/2))+arctan((2*x-a^(1/4)*3^(1/2))/a^(1/4)))
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(84) = 168.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.06

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}+x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(-\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}+x\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}\log\left(\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}+x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}\log\left(-\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{-\frac{\sqrt{3}a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}+x\right)$$

input
```
integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a)*log(sqrt(1/2)*sqrt(
a)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a) + x) - 1/2*sqrt(1/2)*sqrt((sqr
t(3)*a*sqrt(-1/a) + sqrt(a))/a)*log(-sqrt(1/2)*sqrt(a)*sqrt((sqrt(3)*a*sqr
t(-1/a) + sqrt(a))/a) + x) + 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - s
qrt(a))/a)*log(sqrt(1/2)*sqrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a)
 + x) - 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a)*log(-sqrt(
1/2)*sqrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + x)
```

3.107. $\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx$

3.107.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((-x**2+2*a**(1/2))/(a+x**4-x**2*a**(1/2)),x)
```

output
```
Exception raised: PolynomialError >> 1/(64*_t**4*a - 16*_t**2*sqrt(a) + 1)
 contains an element of the set of generators.
```

3.107.7 Maxima [F]

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = \int -\frac{x^2-2\sqrt{a}}{x^4-\sqrt{a}x^2+a}\,dx$$

input
```
integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 2*sqrt(a))/(x^4 - sqrt(a)*x^2 + a), x)
```

3.107.8 Giac [F(-2)]

Exception generated.

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.107.9 Mupad [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.30

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx = 2\,\mathrm{atanh}\left(x\sqrt{\frac{1}{8\sqrt{a}}-\frac{\sqrt{-27\,a^3}}{24\,a^2}} - \frac{9\,a^{3/2}\,x\sqrt{\frac{1}{8\sqrt{a}}-\frac{\sqrt{-27\,a^3}}{24\,a^2}}}{\sqrt{-27\,a^3}}\right)\sqrt{\frac{1}{8\sqrt{a}}-\frac{\sqrt{-27\,a^3}}{24\,a^2}} + 2\,\mathrm{atanh}\left(x\sqrt{\frac{\sqrt{-27\,a^3}}{24\,a^2}+\frac{1}{8\sqrt{a}}} + \frac{9\,a^{3/2}\,x\sqrt{\frac{\sqrt{-27\,a^3}}{24\,a^2}+\frac{1}{8\sqrt{a}}}}{\sqrt{-27\,a^3}}\right)\sqrt{\frac{\sqrt{-27\,a^3}}{24\,a^2}+\frac{1}{8\sqrt{a}}}$$

input
```
int((2*a^(1/2) - x^2)/(a + x^4 - a^(1/2)*x^2),x)
```

output
```
2*atanh(x*(1/(8*a^(1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2) - (9*a^(3/2)*x*
(1/(8*a^(1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2))/(-27*a^3)^(1/2))*(1/(8*a
^(1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2) + 2*atanh(x*((-27*a^3)^(1/2)/(24
*a^2) + 1/(8*a^(1/2)))^(1/2) + (9*a^(3/2)*x*((-27*a^3)^(1/2)/(24*a^2) + 1/
(8*a^(1/2)))^(1/2))/(-27*a^3)^(1/2))*((-27*a^3)^(1/2)/(24*a^2) + 1/(8*a^(1
/2)))^(1/2)
```

3.108 $\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx$

3.108.1 Optimal result

Integrand size = 32, antiderivative size = 124

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}-\frac{\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{4\sqrt[3]{b}}$$

output
```
-1/4*ln(b^(2/3)-b^(1/3)*x+x^2)/b^(1/3)+1/4*ln(b^(2/3)+b^(1/3)*x+x^2)/b^(1/
3)-1/2*arctan(1/3*(b^(1/3)-2*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)+1/2*arcta
n(1/3*(b^(1/3)+2*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

3.108.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx=\frac{\sqrt[4]{-1}\left(\sqrt{-i+\sqrt{3}}(-3i+\sqrt{3})\arctan\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt[3]{b}}\right)-\sqrt{i+\sqrt{3}}(3i+\sqrt{3})\arctan\right.}{2\sqrt{6}\sqrt[3]{b}}$$

input
```
Integrate[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

output
```
((-1)^(1/4)*(Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[
I + Sqrt[3]]*b^(1/3))] - Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTanh[((1 + I
)*x)/(Sqrt[-I + Sqrt[3]]*b^(1/3))]))/(2*Sqrt[6]*b^(1/3))
```

3.108.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1483, 27, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2b^{2/3}+x^2}{b^{2/3}x^2+b^{4/3}+x^4}dx$$

$$\Big\downarrow \text{1483}$$

$$\frac{\int \frac{b^{2/3}\left(2\sqrt[3]{b}-x\right)}{x^2-\sqrt[3]{b}x+b^{2/3}}dx}{2b}+\frac{\int \frac{b^{2/3}\left(x+2\sqrt[3]{b}\right)}{x^2+\sqrt[3]{b}x+b^{2/3}}dx}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2\sqrt[3]{b}-x}{x^2-\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{b}}+\frac{\int \frac{x+2\sqrt[3]{b}}{x^2+\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{b}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{x^2-\sqrt[3]{b}x+b^{2/3}}dx-\frac{1}{2}\int -\frac{\sqrt[3]{b}-2x}{x^2-\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{b}}+\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{x^2+\sqrt[3]{b}x+b^{2/3}}dx+\frac{1}{2}\int \frac{2x+\sqrt[3]{b}}{x^2+\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{x^2-\sqrt[3]{b}x+b^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{b}-2x}{x^2-\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{b}}+\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{x^2+\sqrt[3]{b}x+b^{2/3}}dx+\frac{1}{2}\int \frac{2x+\sqrt[3]{b}}{x^2+\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2x}{x^2-\sqrt[3]{b}x+b^{2/3}}dx+3\int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2x}{\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt[3]{b}}{x^2+\sqrt[3]{b}x+b^{2/3}}dx-3\int \frac{1}{-\left(\frac{2x}{\sqrt[3]{b}}+1\right)^2-3}d\left(\frac{2x}{\sqrt[3]{b}}+1\right)}{2\sqrt[3]{b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2x}{x^2-\sqrt[3]{b}x+b^{2/3}}dx-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2\sqrt[3]{b}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt[3]{b}}{x^2+\sqrt[3]{b}x+b^{2/3}}dx+\sqrt{3}\arctan\left(\frac{\frac{2x}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(b^{2/3}-\sqrt[3]{b}x+x^2\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\arctan\left(\frac{\frac{2x}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(b^{2/3}+\sqrt[3]{b}x+x^2\right)}{2\sqrt[3]{b}}$$

input
```
Int[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

output
```
(-(Sqrt[3]*ArcTan[(1 - (2*x)/b^(1/3))/Sqrt[3]]) - Log[b^(2/3) - b^(1/3)*x
+ x^2]/2)/(2*b^(1/3)) + (Sqrt[3]*ArcTan[(1 + (2*x)/b^(1/3))/Sqrt[3]] + Log
[b^(2/3) + b^(1/3)*x + x^2]/2)/(2*b^(1/3))
```

3.108.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.108.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{-\frac{\ln\left(b^{\frac{2}{3}}-b^{\frac{1}{3}}x+x^{2}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-b^{\frac{1}{3}}+2x\right)\sqrt{3}}{3b^{\frac{1}{3}}}\right)}{2b^{\frac{1}{3}}}+\frac{\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}x+x^{2}\right)}{2}+\arctan\left(\frac{\left(b^{\frac{1}{3}}+2x\right)\sqrt{3}}{3b^{\frac{1}{3}}}\right)\sqrt{3}}{2b^{\frac{1}{3}}}$	87

input
```
int((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^(1/3)*(-1/2*ln(b^(2/3)-b^(1/3)*x+x^2)+3^(1/2)*arctan(1/3*(-b^(1/3)+2
*x)*3^(1/2)/b^(1/3)))+1/2/b^(1/3)*(1/2*ln(b^(2/3)+b^(1/3)*x+x^2)+arctan(1/
3*(b^(1/3)+2*x)/b^(1/3)*3^(1/2))*3^(1/2))
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.13

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = \left[\frac{\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\,x^3+\sqrt{3}\left(2\,b^{\frac{2}{3}}x^2+bx-b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3\,b^{\frac{2}{3}}x-b}{x^3+b}\right)+\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\,x^3+\sqrt{3}\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*x^3 + sqrt(3)*(2*b^(2/3)*x^2 + b*x
 - b^(4/3))*sqrt(-1/b^(2/3)) - 3*b^(2/3)*x - b)/(x^3 + b)) + sqrt(3)*b*sqr
t(-1/b^(2/3))*log((2*x^3 + sqrt(3)*(2*b^(2/3)*x^2 - b*x - b^(4/3))*sqrt(-1
/b^(2/3)) - 3*b^(2/3)*x + b)/(x^3 - b)) + b^(2/3)*log(x^2 + b^(1/3)*x + b^
(2/3)) - b^(2/3)*log(x^2 - b^(1/3)*x + b^(2/3)))/b, 1/4*(2*sqrt(3)*b^(2/3)
*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/b^(1/3)) - 2*sqrt(3)*b^(2/3)*arctan(-1
/3*sqrt(3)*(2*x - b^(1/3))/b^(1/3)) + b^(2/3)*log(x^2 + b^(1/3)*x + b^(2/3
)) - b^(2/3)*log(x^2 - b^(1/3)*x + b^(2/3)))/b]
```

3.108.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = \frac{\left(-\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}-\frac{\sqrt{3}i}{4}\right)+x\right)+\left(-\frac{1}{4}+\frac{\sqrt{3}i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}+\frac{\sqrt{3}i}{4}\right)+\cdots\right.}{\sqrt[3]{\cdots}}$$

input
```
integrate((2*b**(2/3)+x**2)/(b**(4/3)+b**(2/3)*x**2+x**4),x)
```

output
```
((-1/4 - sqrt(3)*I/4)*log(2*b**(1/3)*(-1/4 - sqrt(3)*I/4) + x) + (-1/4 + s
qrt(3)*I/4)*log(2*b**(1/3)*(-1/4 + sqrt(3)*I/4) + x) + (1/4 - sqrt(3)*I/4)
*log(2*b**(1/3)*(1/4 - sqrt(3)*I/4) + x) + (1/4 + sqrt(3)*I/4)*log(2*b**(1
/3)*(1/4 + sqrt(3)*I/4) + x))/b**(1/3)
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.71

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx=\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{2\,b^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{2\,b^{\frac{1}{3}}}+\frac{\log\left(x^2+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}}-\frac{\log\left(x^2-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}}$$

input
```
integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="maxima")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/b^(1/3))/b^(1/3) + 1/2*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x - b^(1/3))/b^(1/3))/b^(1/3) + 1/4*log(x^2 + b^
(1/3)*x + b^(2/3))/b^(1/3) - 1/4*log(x^2 - b^(1/3)*x + b^(2/3))/b^(1/3)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.74

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx=\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x+b^{\frac{1}{3}}\right)}{3\,|b|^{\frac{1}{3}}}\right)}{2\,|b|^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-b^{\frac{1}{3}}\right)}{3\,|b|^{\frac{1}{3}}}\right)}{2\,|b|^{\frac{1}{3}}}+\frac{\log\left(x^2+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}}-\frac{\log\left(x^2-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}}$$

input
```
integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3)
+ 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3
) + 1/4*log(x^2 + b^(1/3)*x + b^(2/3))/b^(1/3) - 1/4*log(x^2 - b^(1/3)*x +
 b^(2/3))/b^(1/3)
```

3.108.9 Mupad [B] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = \frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{-\frac{1}{8\,b^{2/3}}-\frac{\sqrt{3}\,1\mathrm{i}}{8\,b^{2/3}}}\,1\mathrm{i}+\sqrt{3}\,x\,\sqrt{-\frac{1}{8\,b^{2/3}}-\frac{\sqrt{3}\,1\mathrm{i}}{8\,b^{2/3}}}\right)\,\sqrt{-\frac{1+\sqrt{3}\,1\mathrm{i}}{b^{2/3}}}\,1\mathrm{i}}{4} + \frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{-\frac{1}{8\,b^{2/3}}+\frac{\sqrt{3}\,1\mathrm{i}}{8\,b^{2/3}}}\,1\mathrm{i}-\sqrt{3}\,x\,\sqrt{-\frac{1}{8\,b^{2/3}}+\frac{\sqrt{3}\,1\mathrm{i}}{8\,b^{2/3}}}\right)\,\sqrt{\frac{-1+\sqrt{3}\,1\mathrm{i}}{b^{2/3}}}\,1\mathrm{i}}{4}$$

input
```
int((2*b^(2/3) + x^2)/(b^(4/3) + x^4 + b^(2/3)*x^2),x)
```

output
```
(8^(1/2)*atan(x*(- (3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)))^(1/2)*1i + 3^
(1/2)*x*(- (3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)))^(1/2))*(-(3^(1/2)*1i
+ 1)/b^(2/3))^(1/2)*1i)/4 + (8^(1/2)*atan(x*((3^(1/2)*1i)/(8*b^(2/3)) - 1/
(8*b^(2/3)))^(1/2)*1i - 3^(1/2)*x*((3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)
))^(1/2))*((3^(1/2)*1i - 1)/b^(2/3))^(1/2)*1i)/4
```

3.109 $\int \frac{A+Bx^2}{a^2-ax^2+x^4} \, dx$

3.109.1 Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4} \, dx = -\frac{(A+aB)\arctan\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(A+aB)\arctan\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}} -\frac{(A-aB)\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}} +\frac{(A-aB)\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}}$$

output
```
1/2*(B*a+A)*arctan(-3^(1/2)+2*x/a^(1/2))/a^(3/2)+1/2*(B*a+A)*arctan(3^(1/2
)+2*x/a^(1/2))/a^(3/2)-1/12*(-B*a+A)*ln(a+x^2-x*3^(1/2)*a^(1/2))/a^(3/2)*3
^(1/2)+1/12*(-B*a+A)*ln(a+x^2+x*3^(1/2)*a^(1/2))/a^(3/2)*3^(1/2)
```

3.109.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4} \, dx = \frac{\sqrt[4]{-1}\left(\frac{\left(-2iA+\left(-i+\sqrt{3}\right)aB\right)\arctan\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\sqrt{a}}\right)}{\sqrt{-i+\sqrt{3}}}-\frac{\left(2iA+\left(i+\sqrt{3}\right)aB\right)\text{arctanh}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt{a}}\right)}{\sqrt{i+\sqrt{3}}}\right)}{\sqrt{6}a^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

output
```
((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*a*B)*ArcTan[((1 + I)*x)/(Sqrt[-I
+ Sqrt[3]]*Sqrt[a])])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (I + Sqrt[3])*a*B)*
ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])])/Sqrt[I + Sqrt[3]]))/(Sqr
t[6]*a^(3/2))
```

3.109.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1483, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}dx$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{\int \frac{\sqrt{3}\sqrt{a}A-(A-aB)x}{x^2-\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{3}a^{3/2}}+\frac{\int \frac{\sqrt{3}\sqrt{a}A+(A-aB)x}{x^2+\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{3}a^{3/2}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{1}{2}\sqrt{3}\sqrt{a}(aB+A)\int \frac{1}{x^2-\sqrt{3}\sqrt{a}x+a}dx-\frac{1}{2}(A-aB)\int -\frac{\sqrt{3}\sqrt{a}-2x}{x^2-\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{3}a^{3/2}}+\frac{\frac{1}{2}\sqrt{3}\sqrt{a}(aB+A)\int \frac{1}{x^2+\sqrt{3}\sqrt{a}x+a}dx+\frac{1}{2}(A-aB)\int \frac{2x+\sqrt{3}\sqrt{a}}{x^2+\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{3}a^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{1}{2}\sqrt{3}\sqrt{a}(aB+A)\int \frac{1}{x^2-\sqrt{3}\sqrt{a}x+a}dx+\frac{1}{2}(A-aB)\int \frac{\sqrt{3}\sqrt{a}-2x}{x^2-\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{3}a^{3/2}}+\frac{\frac{1}{2}\sqrt{3}\sqrt{a}(aB+A)\int \frac{1}{x^2+\sqrt{3}\sqrt{a}x+a}dx+\frac{1}{2}(A-aB)\int \frac{2x+\sqrt{3}\sqrt{a}}{x^2+\sqrt{3}\sqrt{a}x+a}dx}{2\sqrt{3}a^{3/2}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\frac{1}{2}(A-aB)\int \frac{\sqrt{3}\sqrt{a}-2x}{x^2-\sqrt{3}\sqrt{a}x+a}dx+(aB+A)\int \frac{1}{-\left(1-\frac{2x}{\sqrt{3}\sqrt{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2x}{\sqrt{3}\sqrt{a}}\right)}{2\sqrt{3}a^{3/2}}+\frac{\frac{1}{2}(A-aB)\int \frac{2x+\sqrt{3}\sqrt{a}}{x^2+\sqrt{3}\sqrt{a}x+a}dx-(aB+A)\int \frac{1}{-\left(\frac{2x}{\sqrt{3}\sqrt{a}}+1\right)^2-\frac{1}{3}}d\left(\frac{2x}{\sqrt{3}\sqrt{a}}+1\right)}{2\sqrt{3}a^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}(A-aB)\int \frac{\sqrt{3}\sqrt{a}-2x}{x^2-\sqrt{3}\sqrt{a}x+a}dx-\sqrt{3}(aB+A)\arctan\left(\sqrt{3}\left(1-\frac{2x}{\sqrt{3}\sqrt{a}}\right)\right)}{2\sqrt{3}a^{3/2}}+\frac{\frac{1}{2}(A-aB)\int \frac{2x+\sqrt{3}\sqrt{a}}{x^2+\sqrt{3}\sqrt{a}x+a}dx+\sqrt{3}(aB+A)\arctan\left(\sqrt{3}\left(\frac{2x}{\sqrt{3}\sqrt{a}}+1\right)\right)}{2\sqrt{3}a^{3/2}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\sqrt{3}(aB+A)\arctan\left(\sqrt{3}\left(1-\frac{2x}{\sqrt{3}\sqrt{a}}\right)\right)-\frac{1}{2}(A-aB)\log\left(-\sqrt{3}\sqrt{a}x+a+x^2\right)}{2\sqrt{3}a^{3/2}}+\frac{\sqrt{3}(aB+A)\arctan\left(\sqrt{3}\left(\frac{2x}{\sqrt{3}\sqrt{a}}+1\right)\right)+\frac{1}{2}(A-aB)\log\left(\sqrt{3}\sqrt{a}x+a+x^2\right)}{2\sqrt{3}a^{3/2}}$$

input
```
Int[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

output
```
(-(Sqrt[3]*(A + a*B)*ArcTan[Sqrt[3]*(1 - (2*x)/(Sqrt[3]*Sqrt[a]))]) - ((A
- a*B)*Log[a - Sqrt[3]*Sqrt[a]*x + x^2])/2)/(2*Sqrt[3]*a^(3/2)) + (Sqrt[3]
*(A + a*B)*ArcTan[Sqrt[3]*(1 + (2*x)/(Sqrt[3]*Sqrt[a]))] + ((A - a*B)*Log[
a + Sqrt[3]*Sqrt[a]*x + x^2])/2)/(2*Sqrt[3]*a^(3/2))
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.109.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-a\_Z^{2}+a^{2}\right)}\frac{\left(B\_R^{2}+A\right)\ln\left(x-\_R\right)}{2\_R^{3}-\_Ra}\right)}{2}$
default	$\frac{\frac{\left(-B\sqrt{3}\,a^{2}+A\sqrt{3}\,a\right)\ln\left(a+x^{2}+x\sqrt{3}\,\sqrt{a}\right)}{2}+\frac{2\left(3Aa^{\frac{3}{2}}-\frac{\left(-B\sqrt{3}\,a^{2}+A\sqrt{3}\,a\right)\sqrt{3}\,\sqrt{a}}{2}\right)\arctan\left(\frac{2x+\sqrt{3}\,\sqrt{a}}{\sqrt{a}}\right)}{\sqrt{a}}}{6a^{\frac{5}{2}}}+\frac{-\frac{\left(-B\sqrt{3}\,a^{2}+A\sqrt{3}\,a\right)\ln\left(x\sqrt{3}\,\sqrt{a}-a\right.}{2}}{}$

input
```
int((B*x^2+A)/(x^4-a*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((B*_R^2+A)/(2*_R^3-_R*a)*ln(x-_R),_R=RootOf(_Z^4-_Z^2*a+a^2))
```

3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(104) = 208$.

Time = 0.29 (sec) , antiderivative size = 903, normalized size of antiderivative = 6.64

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx$$
$$=\frac{1}{2}\sqrt{\frac{1}{6}}\sqrt{-\frac{B^2a^2+3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}+4ABa+A^2}{a^3}}\log\left(2\left(B^4a^4+AB^3a^3-A^3Ba-A^4\right)x\right.$$
$$+3\sqrt{\frac{1}{6}}\left(AB^2a^4-A^3a^2-\sqrt{\frac{1}{3}}\left(2Ba^6+Aa^5\right)\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}\right)\sqrt{-\frac{B^2a^2+3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4}{}}}{}}$$
$$-\frac{1}{2}\sqrt{\frac{1}{6}}\sqrt{-\frac{B^2a^2+3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}+4ABa+A^2}{a^3}}\log\left(2\left(B^4a^4+AB^3a^3-A^3Ba-A^4\right)x\right.$$
$$-3\sqrt{\frac{1}{6}}\left(AB^2a^4-A^3a^2-\sqrt{\frac{1}{3}}\left(2Ba^6+Aa^5\right)\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}\right)\sqrt{-\frac{B^2a^2+3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4}{}}}{}}$$
$$+\frac{1}{2}\sqrt{\frac{1}{6}}\sqrt{-\frac{B^2a^2-3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}+4ABa+A^2}{a^3}}\log\left(2\left(B^4a^4+AB^3a^3-A^3Ba-A^4\right)x\right.$$
$$+3\sqrt{\frac{1}{6}}\left(AB^2a^4-A^3a^2+\sqrt{\frac{1}{3}}\left(2Ba^6+Aa^5\right)\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}\right)\sqrt{-\frac{B^2a^2-3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4}{}}}{}}$$
$$-\frac{1}{2}\sqrt{\frac{1}{6}}\sqrt{-\frac{B^2a^2-3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}+4ABa+A^2}{a^3}}\log\left(2\left(B^4a^4+AB^3a^3-A^3Ba-A^4\right)x\right.$$
$$-3\sqrt{\frac{1}{6}}\left(AB^2a^4-A^3a^2+\sqrt{\frac{1}{3}}\left(2Ba^6+Aa^5\right)\sqrt{-\frac{B^4a^4-2A^2B^2a^2+A^4}{a^6}}\right)\sqrt{-\frac{B^2a^2-3\sqrt{\frac{1}{3}}a^3\sqrt{-\frac{B^4a^4}{}}}{}}$$

input
```
integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/6)*sqrt(-(B^2*a^2 + 3*sqrt(1/3)*a^3*sqrt(-(B^4*a^4 - 2*A^2*B^2*
a^2 + A^4)/a^6) + 4*A*B*a + A^2)/a^3)*log(2*(B^4*a^4 + A*B^3*a^3 - A^3*B*a
 - A^4)*x + 3*sqrt(1/6)*(A*B^2*a^4 - A^3*a^2 - sqrt(1/3)*(2*B*a^6 + A*a^5)
*sqrt(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt(-(B^2*a^2 + 3*sqrt(1/3)*
a^3*sqrt(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) + 4*A*B*a + A^2)/a^3)) - 1/
2*sqrt(1/6)*sqrt(-(B^2*a^2 + 3*sqrt(1/3)*a^3*sqrt(-(B^4*a^4 - 2*A^2*B^2*a^
2 + A^4)/a^6) + 4*A*B*a + A^2)/a^3)*log(2*(B^4*a^4 + A*B^3*a^3 - A^3*B*a -
 A^4)*x - 3*sqrt(1/6)*(A*B^2*a^4 - A^3*a^2 - sqrt(1/3)*(2*B*a^6 + A*a^5)*s
qrt(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt(-(B^2*a^2 + 3*sqrt(1/3)*a^
3*sqrt(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) + 4*A*B*a + A^2)/a^3)) + 1/2*
sqrt(1/6)*sqrt(-(B^2*a^2 - 3*sqrt(1/3)*a^3*sqrt(-(B^4*a^4 - 2*A^2*B^2*a^2
+ A^4)/a^6) + 4*A*B*a + A^2)/a^3)*log(2*(B^4*a^4 + A*B^3*a^3 - A^3*B*a - A
^4)*x + 3*sqrt(1/6)*(A*B^2*a^4 - A^3*a^2 + sqrt(1/3)*(2*B*a^6 + A*a^5)*sqr
t(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt(-(B^2*a^2 - 3*sqrt(1/3)*a^3*
sqrt(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) + 4*A*B*a + A^2)/a^3)) - 1/2*sq
rt(1/6)*sqrt(-(B^2*a^2 - 3*sqrt(1/3)*a^3*sqrt(-(B^4*a^4 - 2*A^2*B^2*a^2 +
A^4)/a^6) + 4*A*B*a + A^2)/a^3)*log(2*(B^4*a^4 + A*B^3*a^3 - A^3*B*a - A^4
)*x - 3*sqrt(1/6)*(A*B^2*a^4 - A^3*a^2 + sqrt(1/3)*(2*B*a^6 + A*a^5)*sqrt(
-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt(-(B^2*a^2 - 3*sqrt(1/3)*a^3*sq
rt(-(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) + 4*A*B*a + A^2)/a^3))
```

3.109.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx$$
$$= \text{RootSum}\left(144t^4a^6+t^2\cdot\left(12A^2a^3+48ABa^4+12B^2a^5\right)+A^4+2A^3Ba+3A^2B^2a^2+2AB^3a^3+B^4a^4,\ \left(\right.\right.$$

input
```
integrate((B*x**2+A)/(x**4-a*x**2+a**2),x)
```

output
```
RootSum(144*_t**4*a**6 + _t**2*(12*A**2*a**3 + 48*A*B*a**4 + 12*B**2*a**5)
 + A**4 + 2*A**3*B*a + 3*A**2*B**2*a**2 + 2*A*B**3*a**3 + B**4*a**4, Lambd
a(_t, _t*log(x + (24*_t**3*A*a**5 + 48*_t**3*B*a**6 - 2*_t*A**3*a**2 + 6*_
t*A**2*B*a**3 + 12*_t*A*B**2*a**4 + 2*_t*B**3*a**5)/(-A**4 - A**3*B*a + A*
B**3*a**3 + B**4*a**4))))
```

3.109.7 Maxima [F]

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx=\int \frac{Bx^2+A}{x^4-ax^2+a^2}\,dx$$

input
```
integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2), x)
```

3.109.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.84 (sec) , antiderivative size = 4293, normalized size of antiderivative = 31.57

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(3*sqrt(3)*B*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/a
bs(a))))^2*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^3*sin(1/2*real_part(a
rccos(1/2*a/abs(a)))) - sqrt(3)*B*a^2*abs(a)^(3/2)*cosh(1/2*imag_part(arcc
os(1/2*a/abs(a))))^3*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3 - 9*sqrt(3
)*B*a^2*abs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*i
mag_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a/abs(a))))
*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) + 3*sqrt(3)*B*a^2*abs(a)^(3/2)*
cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2*a
/abs(a))))^3*sinh(1/2*imag_part(arccos(1/2*a/abs(a)))) + 9*sqrt(3)*B*a^2*a
bs(a)^(3/2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))^2*cosh(1/2*imag_part(
arccos(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))*sinh(1/2*i
mag_part(arccos(1/2*a/abs(a))))^2 - 3*sqrt(3)*B*a^2*abs(a)^(3/2)*cosh(1/2*
imag_part(arccos(1/2*a/abs(a))))*sin(1/2*real_part(arccos(1/2*a/abs(a))))^
3*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^2 - 3*sqrt(3)*B*a^2*abs(a)^(3/
2)*cos(1/2*real_part(arccos(1/2*a/abs(a))))^2*sin(1/2*real_part(arccos(1/2
*a/abs(a))))*sinh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 + sqrt(3)*B*a^2*a
bs(a)^(3/2)*sin(1/2*real_part(arccos(1/2*a/abs(a))))^3*sinh(1/2*imag_part(
arccos(1/2*a/abs(a))))^3 + B*a^3*sqrt(abs(a))*cos(1/2*real_part(arccos(1/2
*a/abs(a))))^3*cosh(1/2*imag_part(arccos(1/2*a/abs(a))))^3 - 3*B*a^3*sqrt(
abs(a))*cos(1/2*real_part(arccos(1/2*a/abs(a))))*cosh(1/2*imag_part(arc...
```

3.109.9 Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 1007, normalized size of antiderivative = 7.40

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx = \operatorname{atan}\left(\frac{A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}\,6\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}} + \frac{2\,\sqrt{3}\,A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}} - \frac{B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}\,6\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}} - \frac{2\,\sqrt{3}\,B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}}\right)\sqrt{-\frac{A^2+B^2\,a^2+4\,A\,B\,a+\sqrt{3}\,A^2\,1\mathrm{i}-\sqrt{}}{24\,a^3}}$$

$$+\operatorname{atan}\left(\frac{A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}\,6\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}} - \frac{2\,\sqrt{3}\,A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}} - \frac{B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}\,6\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}} + \frac{2\,\sqrt{3}\,B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1\mathrm{i}}\right)\sqrt{-\frac{A^2+B^2\,a^2+4\,A\,B\,a-\sqrt{3}\,A^2\,1\mathrm{i}+\sqrt{}}{24\,a^3}}$$

input
```
int((A + B*x^2)/(a^2 - a*x^2 + x^4),x)
```

output
```
atan((A^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (3^(1/2)*A^2*1i)/(24*a
^3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2
 + (3^(1/2)*A^3*1i)/a - A*B^2*a - 3^(1/2)*A*B^2*a*1i) + (2*3^(1/2)*A^2*x*(
(3^(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24
*a^3) - (A*B)/(6*a^2))^(1/2))/(2*A^2*B + A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*
1i)/a - A*B^2*a - 3^(1/2)*A*B^2*a*1i) - (B^2*a^2*x*((3^(1/2)*B^2*1i)/(24*a
) - B^2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24*a^3) - (A*B)/(6*a^2))
^(1/2)*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*1i)/a - A*B^2*a - 3
^(1/2)*A*B^2*a*1i) - (2*3^(1/2)*B^2*a^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^2/(
24*a) - (3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2))/(
2*A^2*B + A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*1i)/a - A*B^2*a - 3^(1/2)*A*B^2
*a*1i))*(-(3^(1/2)*A^2*1i + A^2 + B^2*a^2 - 3^(1/2)*B^2*a^2*1i + 4*A*B*a)/
(24*a^3))^(1/2)*2i + atan((A^2*x*((3^(1/2)*A^2*1i)/(24*a^3) - B^2/(24*a) -
 A^2/(24*a^3) - (3^(1/2)*B^2*1i)/(24*a) - (A*B)/(6*a^2))^(1/2)*6i)/(2*A^2*
B + A^3/a - 2*B^3*a^2 - (3^(1/2)*A^3*1i)/a - A*B^2*a + 3^(1/2)*A*B^2*a*1i)
 - (2*3^(1/2)*A^2*x*((3^(1/2)*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3)
 - (3^(1/2)*B^2*1i)/(24*a) - (A*B)/(6*a^2))^(1/2))/(2*A^2*B + A^3/a - 2*B^
3*a^2 - (3^(1/2)*A^3*1i)/a - A*B^2*a + 3^(1/2)*A*B^2*a*1i) - (B^2*a^2*x*((
3^(1/2)*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3) - (3^(1/2)*B^2*1i)/(2
4*a) - (A*B)/(6*a^2))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 - (3^(1/2)...
```

3.109. $\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx$

3.110 $\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx$

3.110.1 Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx = -\frac{\left(A+\sqrt{a}B\right)\arctan\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} + \frac{\left(A+\sqrt{a}B\right)\arctan\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}} - \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt{3}a^{3/4}} + \frac{\left(A-\sqrt{a}B\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}x+x^2\right)}{4\sqrt{3}a^{3/4}}$$

output
```
-1/12*ln(x^2-a^(1/4)*x*3^(1/2)+a^(1/2))*(A-B*a^(1/2))/a^(3/4)*3^(1/2)+1/12
*ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))*(A-B*a^(1/2))/a^(3/4)*3^(1/2)+1/2*arcta
n(2*x/a^(1/4)-3^(1/2))*(A+B*a^(1/2))/a^(3/4)+1/2*arctan(2*x/a^(1/4)+3^(1/2
))*(A+B*a^(1/2))/a^(3/4)
```

3.110.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx = \frac{\sqrt[4]{-1}\left(\frac{\left(-2iA+\left(-i+\sqrt{3}\right)\sqrt{a}B\right)\arctan\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\sqrt[4]{a}}\right)}{\sqrt{-i+\sqrt{3}}}-\frac{\left(2iA+\left(i+\sqrt{3}\right)\sqrt{a}B\right)\text{arctanh}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt[4]{a}}\right)}{\sqrt{i+\sqrt{3}}}\right)}{\sqrt{6}a^{3/4}}$$

input
```
Integrate[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

output
```
((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*Sqrt[a]*B)*ArcTan[((1 + I)*x)/(Sq
rt[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (I + Sqrt[3])
*Sqrt[a]*B)*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))])/Sqrt[I + Sqr
t[3]]))/(Sqrt[6]*a^(3/4))
```

3.110.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1483, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{-\sqrt{a}x^2+a+x^4}\,dx$$

$$\Big\downarrow \text{1483}$$

$$\frac{\int \frac{\sqrt{3}\sqrt[4]{a}A-\left(A-\sqrt{a}B\right)x}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt{3}a^{3/4}}+\frac{\int \frac{\sqrt{3}\sqrt[4]{a}A+\left(A-\sqrt{a}B\right)x}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt{3}a^{3/4}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}(\sqrt{a}B+A)\int\frac{1}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx-\frac{1}{2}(A-\sqrt{a}B)\int-\frac{\sqrt{3}\sqrt[4]{a}-2x}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt{3}a^{3/4}}+\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}(\sqrt{a}B+A)\int\frac{1}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx+\frac{1}{2}(A-\sqrt{a}B)\int\frac{2x+\sqrt{3}\sqrt[4]{a}}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt{3}a^{3/4}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}(\sqrt{a}B+A)\int\frac{1}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx+\frac{1}{2}(A-\sqrt{a}B)\int\frac{\sqrt{3}\sqrt[4]{a}-2x}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt{3}a^{3/4}}+\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}(\sqrt{a}B+A)\int\frac{1}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx+\frac{1}{2}(A-\sqrt{a}B)\int\frac{2x+\sqrt{3}\sqrt[4]{a}}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx}{2\sqrt{3}a^{3/4}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{1}{2}(A-\sqrt{a}B)\int\frac{\sqrt{3}\sqrt[4]{a}-2x}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx+(\sqrt{a}B+A)\int\frac{1}{-\left(1-\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)}{2\sqrt{3}a^{3/4}}+\frac{\frac{1}{2}(A-\sqrt{a}B)\int\frac{2x+\sqrt{3}\sqrt[4]{a}}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx-(\sqrt{a}B+A)\int\frac{1}{-\left(\frac{2x}{\sqrt{3}\sqrt[4]{a}}+1\right)^2-\frac{1}{3}}d\left(\frac{2x}{\sqrt{3}\sqrt[4]{a}}+1\right)}{2\sqrt{3}a^{3/4}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}(A-\sqrt{a}B)\int\frac{\sqrt{3}\sqrt[4]{a}-2x}{x^2-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx-\sqrt{3}(\sqrt{a}B+A)\arctan\left(\sqrt{3}\left(1-\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)\right)}{2\sqrt{3}a^{3/4}}+\frac{\frac{1}{2}(A-\sqrt{a}B)\int\frac{2x+\sqrt{3}\sqrt[4]{a}}{x^2+\sqrt{3}\sqrt[4]{a}x+\sqrt{a}}dx+\sqrt{3}(\sqrt{a}B+A)\arctan\left(\sqrt{3}\left(\frac{2x}{\sqrt{3}\sqrt[4]{a}}+1\right)\right)}{2\sqrt{3}a^{3/4}}$$

$$\Big\downarrow \; 1103$$

$$\frac{-\sqrt{3}(\sqrt{a}B+A)\arctan\left(\sqrt{3}\left(1-\frac{2x}{\sqrt{3}\sqrt[4]{a}}\right)\right)-\frac{1}{2}(A-\sqrt{a}B)\log\left(-\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{2\sqrt{3}a^{3/4}}+\frac{\sqrt{3}(\sqrt{a}B+A)\arctan\left(\sqrt{3}\left(\frac{2x}{\sqrt{3}\sqrt[4]{a}}+1\right)\right)+\frac{1}{2}(A-\sqrt{a}B)\log\left(\sqrt{3}\sqrt[4]{a}x+\sqrt{a}+x^2\right)}{2\sqrt{3}a^{3/4}}$$

input
```
Int[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

3.110. $\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx$

output
```
(-(Sqrt[3]*(A + Sqrt[a]*B)*ArcTan[Sqrt[3]*(1 - (2*x)/(Sqrt[3]*a^(1/4)))])
- ((A - Sqrt[a]*B)*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/2)/(2*Sqrt[3]*a
^(3/4)) + (Sqrt[3]*(A + Sqrt[a]*B)*ArcTan[Sqrt[3]*(1 + (2*x)/(Sqrt[3]*a^(1
/4)))] + ((A - Sqrt[a]*B)*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/2)/(2*Sq
rt[3]*a^(3/4))
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.110.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18

method	result
default	$\frac{\frac{\left(A\sqrt{a}\,\sqrt{3}-B\sqrt{3}\,a\right)\ln\left(x^{2}+a^{\frac{1}{4}}x\sqrt{3}+\sqrt{a}\right)}{2}+\frac{2\left(3Aa^{\frac{3}{4}}-\frac{\left(A\sqrt{a}\,\sqrt{3}-B\sqrt{3}\,a\right)a^{\frac{1}{4}}\sqrt{3}}{2}\right)\arctan\left(\frac{2x+a^{\frac{1}{4}}\sqrt{3}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}}{6a^{\frac{5}{4}}}+\frac{\frac{\left(-A\sqrt{a}\,\sqrt{3}+B\sqrt{3}\,a\right)\ln\left(x^{2}-a^{\frac{1}{4}}x\sqrt{3}\right.}{2}}{}$

input
```
int((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^(5/4)*(1/2*(A*a^(1/2)*3^(1/2)-B*3^(1/2)*a)*ln(x^2+a^(1/4)*x*3^(1/2)+
a^(1/2))+2*(3*A*a^(3/4)-1/2*(A*a^(1/2)*3^(1/2)-B*3^(1/2)*a)*a^(1/4)*3^(1/2
))/a^(1/4)*arctan((2*x+a^(1/4)*3^(1/2))/a^(1/4)))+1/6/a^(5/4)*(1/2*(-A*a^(
1/2)*3^(1/2)+B*3^(1/2)*a)*ln(x^2-a^(1/4)*x*3^(1/2)+a^(1/2))+2*(3*A*a^(3/4)
+1/2*(-A*a^(1/2)*3^(1/2)+B*3^(1/2)*a)*a^(1/4)*3^(1/2))/a^(1/4)*arctan((2*x
-a^(1/4)*3^(1/2))/a^(1/4)))
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. 2(116) = 232.

Time = 0.38 (sec) , antiderivative size = 1141, normalized size of antiderivative = 7.13

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*
a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^3 - A^6)*x + 3*sq
rt(1/6)*(A*B^4*a^3 - A^5*a - sqrt(1/3)*(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*
a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - sqrt(1/3)*(A*B^2*
a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4
*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a
 + A^2)*sqrt(a))/a^2)) - 1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sq
rt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(2
*(B^6*a^3 - A^6)*x - 3*sqrt(1/6)*(A*B^4*a^3 - A^5*a - sqrt(1/3)*(2*B^3*a^4
 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^
4*B*a - sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4
)/a^3))*sqrt(a))*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B
^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) + 1/2*sqrt(1/6)*sqrt(-(4*A
*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a +
 A^2)*sqrt(a))/a^2)*log(2*(B^6*a^3 - A^6)*x + 3*sqrt(1/6)*(A*B^4*a^3 - A^5
*a + sqrt(1/3)*(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)
/a^3) - (A^2*B^3*a^2 - A^4*B*a + sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^
4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^2
*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) -
 1/2*sqrt(1/6)*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*...
```

3.110.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((B*x**2+A)/(a+x**4-x**2*a**(1/2)),x)
```

output
```
Exception raised: PolynomialError >> 1/(64*_t**4*a - 16*_t**2*B**2*sqrt(a)
 + B**4) contains an element of the set of generators.
```

3.110.7 Maxima [F]

$$\int \frac{A + Bx^2}{a - \sqrt{a}x^2 + x^4} \, dx = \int \frac{Bx^2 + A}{x^4 - \sqrt{a}x^2 + a} \, dx$$

input
```
integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(x^4 - sqrt(a)*x^2 + a), x)
```

3.110.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx^2}{a - \sqrt{a}x^2 + x^4} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.110.9 Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 1155, normalized size of antiderivative = 7.22

$$\int \frac{A+Bx^2}{a-\sqrt{a}x^2+x^4}\,dx$$
$$= -2\,\mathrm{atanh}\left(\frac{6\,A^2\,x\,\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-\frac{A^2\,\sqrt{-27\,a^3}}{72\,a^3}-\frac{A^2}{24\,a^{3/2}}-\frac{A\,B}{6\,a}}}{2\,A^2\,B-2\,B^3\,a+\frac{A^3}{\sqrt{a}}-A\,B^2\,\sqrt{a}+\frac{A^3\,\sqrt{-27\,a^3}}{3\,a^2}-\frac{A\,B^2\,\sqrt{-27\,a^3}}{3\,a}} - \frac{6\,B^2\,a\,x\,\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-\frac{A^2\,\sqrt{-27\,a^3}}{72\,a^3}-\frac{A^2}{24\,a^{3/2}}-\frac{A\,B}{6\,a}}}{2\,A^2\,B-2\,B^3\,a+\frac{A^3}{\sqrt{a}}-A\,B^2\,\sqrt{a}+\frac{A^3\,\sqrt{-27\,a^3}}{3\,a^2}-\frac{A\,B^2\,\sqrt{-27\,a^3}}{3\,a}} - \frac{2\,A^2\,x\,\sqrt{-27\,a^3}\,\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-\frac{A^2\,\sqrt{-27\,a^3}}{72\,a^3}-\frac{A^2}{24\,a^{3/2}}-\frac{A\,B}{6\,a}}}{3\,a^{3/2}\,\left(2\,A^2\,B-2\,B^3\,a+\frac{A^3}{\sqrt{a}}-A\,B^2\,\sqrt{a}+\frac{A^3\,\sqrt{-27\,a^3}}{3\,a^2}-\frac{A\,B^2\,\sqrt{-27\,a^3}}{3\,a}\right)} + \frac{2\,B^2\,x\,\sqrt{-27\,a^3}\,\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-\frac{A^2\,\sqrt{-27\,a^3}}{72\,a^3}-\frac{A^2}{24\,a^{3/2}}-\frac{A\,B}{6\,a}}}{3\,\sqrt{a}\,\left(2\,A^2\,B-2\,B^3\,a+\frac{A^3}{\sqrt{a}}-A\,B^2\,\sqrt{a}+\frac{A^3\,\sqrt{-27\,a^3}}{3\,a^2}-\frac{A\,B^2\,\sqrt{-27\,a^3}}{3\,a}\right)}\right)\sqrt{\frac{B^2\,\sqrt{-27\,a^3}}{72\,a^2}-\frac{B^2}{24\,\sqrt{a}}-\frac{A^2}{}}$$

input
```
int((A + B*x^2)/(a + x^4 - a^(1/2)*x^2),x)
```

output
```
- 2*atanh((6*A^2*x*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A
^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(2*A
^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^
2) - (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (6*B^2*a*x*((B^2*(-27*a^3)^(1/2))/(7
2*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/
2)) - (A*B)/(6*a))^(1/2))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2)
 + (A^3*(-27*a^3)^(1/2))/(3*a^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (2*A^2
*x*(-27*a^3)^(1/2)*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A
^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(3*a
^(3/2)*(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(
1/2))/(3*a^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a))) + (2*B^2*x*(-27*a^3)^(1/2)
*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2)
)/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(3*a^(1/2)*(2*A^2*B -
2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^2) - (A
*B^2*(-27*a^3)^(1/2))/(3*a))))*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a
^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))
^(1/2) - 2*atanh((6*A^2*x*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2
)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2
))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2)
)/(3*a^2) + (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (6*B^2*a*x*((A^2*(-27*a^3)...
```

3.111 $\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4} \, dx$

3.111.1 Optimal result

Integrand size = 29, antiderivative size = 414

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4} \, dx = -\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}} + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}} - \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}} + \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x+\sqrt{c}x^2\right)}{4\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

output
```
-1/2*arctan((-2*x*c^(1/2)+(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))/(2*a^(1/2
)*c^(1/2)-(a*c)^(1/2))^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(1/2)/c^(1/2)/(2*a^(
1/2)*c^(1/2)-(a*c)^(1/2))^(1/2)+1/2*arctan((2*x*c^(1/2)+(2*a^(1/2)*c^(1/2)
+(a*c)^(1/2))^(1/2))/(2*a^(1/2)*c^(1/2)-(a*c)^(1/2))^(1/2))*(B*a^(1/2)+A*c
^(1/2))/a^(1/2)/c^(1/2)/(2*a^(1/2)*c^(1/2)-(a*c)^(1/2))^(1/2)-1/4*ln(a^(1/
2)+x^2*c^(1/2)-x*(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))*(A-B*a^(1/2)/c^(1/
2))/a^(1/2)/(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2)+1/4*ln(a^(1/2)+x^2*c^(1/
2)+x*(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))*(A-B*a^(1/2)/c^(1/2))/a^(1/2)/
(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2)
```

3.111.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx$$
$$= \frac{\text{RootSum}\left[a^2+ac\#1^4+c^2\#1^8\&,\frac{aA\log(x-\#1)+aB\log(x-\#1)\#1^2+A\sqrt{ac}\log(x-\#1)\#1^2+Ac\log(x-\#1)\#1^4+B\sqrt{ac}\log\ldots}{a\#1^3+2c\#1^7}\right.}{4c}$$

input
```
Integrate[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```

output
```
RootSum[a^2 + a*c*#1^4 + c^2*#1^8 & , (a*A*Log[x - #1] + a*B*Log[x - #1]*#
1^2 + A*Sqrt[a*c]*Log[x - #1]*#1^2 + A*c*Log[x - #1]*#1^4 + B*Sqrt[a*c]*Lo
g[x - #1]*#1^4 + B*c*Log[x - #1]*#1^6)/(a*#1^3 + 2*c*#1^7) & ]/(4*c)
```

3.111.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1483, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{-x^2\sqrt{ac}+a+cx^4}\,dx$$
$$\Big\downarrow \text{1483}$$
$$\frac{\int \frac{A\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}A+\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\sqrt{c}x}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}A+(\sqrt{a}B-A\sqrt{c})x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\int \frac{A\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-(\sqrt{a}B-A\sqrt{c})x}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$
$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\left(\sqrt{a}B+A\sqrt{c}\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{1}{2}\left(\sqrt{a}B-A\sqrt{c}\right)\int -\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\left(\sqrt{a}B+A\sqrt{c}\right)\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{1}{2}\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\left(\sqrt{a}B+A\sqrt{c}\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{1}{2}\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\left(\sqrt{a}B+A\sqrt{c}\right)\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{1}{2}\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\left(\sqrt{a}B+A\sqrt{c}\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\left(\sqrt{a}B+A\sqrt{c}\right)\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

$$\Big\downarrow \; 1083$$

$$\frac{-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\left(\sqrt{a}B+A\sqrt{c}\right)\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}\right)^2-\frac{2\sqrt{a}\sqrt{c}-\sqrt{ac}}{c}}d\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}\right)}{\sqrt{c}}}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}\left(\sqrt{a}B+A\sqrt{c}\right)\int \frac{1}{-\left(2x+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}\right)^2-\frac{2\sqrt{a}\sqrt{c}-\sqrt{ac}}{c}}d\left(2x+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}\right)}{\sqrt{c}}}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

$$\Big\downarrow \; 217$$

3.111. $\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx$

$$\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}(\sqrt{a}B+A\sqrt{c})\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}-\frac{(\sqrt{a}B-A\sqrt{c})\int\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}(\sqrt{a}B+A\sqrt{c})\arctan\left(\frac{\sqrt{c}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}+2x\right)}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}-\frac{(\sqrt{a}B-A\sqrt{c})\int\frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}(\sqrt{a}B+A\sqrt{c})\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}+\frac{1}{2}(\sqrt{a}B-A\sqrt{c})\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}+\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}(\sqrt{a}B+A\sqrt{c})\arctan\left(\frac{\sqrt{c}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}+2x\right)}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}-\sqrt{ac}}}-\frac{1}{2}(\sqrt{a}B-A\sqrt{c})\log\left(x\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+\sqrt{ac}}}$$

input
```
Int[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```

output
```
(((Sqrt[a]*B + A*Sqrt[c])*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*ArcTan[(Sqrt
[c]*(-(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]/Sqrt[c]) + 2*x))/Sqrt[2*Sqrt[a]
*Sqrt[c] - Sqrt[a*c]]])/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]] + ((Sqrt[a]*B
- A*Sqrt[c])*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]
*x^2])/2)/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]) + (((Sqr
t[a]*B + A*Sqrt[c])*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*ArcTan[(Sqrt[c]*(S
qrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]/Sqrt[c] + 2*x))/Sqrt[2*Sqrt[a]*Sqrt[c]
- Sqrt[a*c]]])/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]] - ((Sqrt[a]*B - A*Sqrt[
c])*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2])/2)
/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]])
```

3.111. $\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx$

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.111.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.84

method	result
default	$\frac{-\frac{\left(-B\sqrt{3}\,(ac)^{\frac{3}{4}} a^{\frac{3}{2}}+A\sqrt{3}\,(ac)^{\frac{3}{4}}\sqrt{c}\,a\right)\ln\left(x\sqrt{3}\,(ac)^{\frac{1}{4}}-x^{2}\sqrt{c}-\sqrt{a}\right)}{2\sqrt{c}}+\frac{2\left(-3Aca^{2}+\frac{\left(-B\sqrt{3}\,(ac)^{\frac{3}{4}} a^{\frac{3}{2}}+A\sqrt{3}\,(ac)^{\frac{3}{4}}\sqrt{c}\,a\right)\sqrt{3}\,(ac)^{\frac{1}{4}}}{2\sqrt{c}}\right)\arctan\left(\frac{\sqrt{3}\,(\cdots}{\sqrt{4\sqrt{\cdots}}}\right)}{\sqrt{4\sqrt{a}\,\sqrt{c}-3\sqrt{ac}}}\cdots}{6a^{\frac{5}{2}}c}$

input
```
int((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^(5/2)/c*(-1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/4)*
c^(1/2)*a)/c^(1/2)*ln(x*3^(1/2)*(a*c)^(1/4)-x^2*c^(1/2)-a^(1/2))+2*(-3*A*c
*a^2+1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/4)*c^(1/2)*a)*
3^(1/2)*(a*c)^(1/4)/c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arcta
n((3^(1/2)*(a*c)^(1/4)-2*x*c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2
)))+1/6/a^(5/2)/c*(1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/
4)*c^(1/2)*a)/c^(1/2)*ln(x^2*c^(1/2)+x*3^(1/2)*(a*c)^(1/4)+a^(1/2))+2*(3*A
*c*a^2-1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/4)*c^(1/2)*a
)*3^(1/2)*(a*c)^(1/4)/c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arc
tan((2*x*c^(1/2)+3^(1/2)*(a*c)^(1/4))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1
/2)))
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1457 vs. 2(289) = 578.

Time = 0.60 (sec) , antiderivative size = 1457, normalized size of antiderivative = 3.52

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c +
 A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*l
og(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(
1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*
c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2
- A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(
a*c))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) + 1/2*sqr
t(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2
)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B
^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3)*(2*
B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^
3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^
2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sq
rt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^
3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) - 1/2*sqrt(1/6)*s
qrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^
3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 -
A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 + sqrt(1/3)*(2*B^3*a^4*c
^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)...
```

3.111.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((B*x**2+A)/(a+c*x**4-x**2*(a*c)**(1/2)),x)
```

output
```
Exception raised: PolynomialError >> 1/(64*_t**4*a*c**3 - 16*_t**2*B**2*c*
sqrt(a*c) + B**4) contains an element of the set of generators.
```

3.111.7 Maxima [F]

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx=\int \frac{Bx^2+A}{cx^4-\sqrt{ac}x^2+a}\,dx$$

input
```
integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(c*x^4 - sqrt(a*c)*x^2 + a), x)
```

3.111.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.111.9 Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 3285, normalized size of antiderivative = 7.93

$$\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx=\text{Too large to display}$$

input
```
int((A + B*x^2)/(a + c*x^4 - x^2*(a*c)^(1/2)),x)
```

output

```
- atan(((((12*A*a)/c^2 - (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(
B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2)
- A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2
*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A
^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*
a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))
^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(B^2*a*(
-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c
*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c
)^(1/2))/(72*a^3*c^3))^(1/2)*1i - (((12*A*a)/c^2 + (2*x*(4*c*(a*c)^(3/2) -
 16*a*c^2*(a*c)^(1/2))*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^
(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c
^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(B^2
*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A
^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*
(a*c)^(1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*
c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) -
 B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c
)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)*1i)/((((12*A*a)/c^2
 - (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(B^2*a*(-27*a^3*c^3)...
```

3.111. $\int \frac{A+Bx^2}{a-\sqrt{ac}x^2+cx^4}\,dx$

3.112 $\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx$

3.112.1 Optimal result

Integrand size = 32, antiderivative size = 234

$$\begin{aligned}\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = &-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}\\ &+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}\\ &-\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}\\ &+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{3}a^{3/4}\sqrt[4]{c}}\end{aligned}$$

output
```
-1/12*ln(-a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c^(1/2))*(A-B*a^(1/2)/c^(1
/2))/a^(3/4)/c^(1/4)*3^(1/2)+1/12*ln(a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2
*c^(1/2))*(A-B*a^(1/2)/c^(1/2))/a^(3/4)/c^(1/4)*3^(1/2)+1/2*arctan(2*c^(1/
4)*x/a^(1/4)-3^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(3/4)/c^(3/4)+1/2*arctan(2*c
^(1/4)*x/a^(1/4)+3^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(3/4)/c^(3/4)
```

3.112.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = \frac{\sqrt[4]{-1}\left(\frac{\left(\left(-i+\sqrt{3}\right)\sqrt{a}B-2iA\sqrt{c}\right)\arctan\left(\frac{(1+i)\sqrt[4]{c}x}{\sqrt{-i+\sqrt{3}}\sqrt[4]{a}}\right)}{\sqrt{-i+\sqrt{3}}}-\frac{\left(\left(i+\sqrt{3}\right)\sqrt{a}B+2iA\sqrt{c}\right)\operatorname{arctanh}\left(\frac{(1+i)\sqrt[4]{c}x}{\sqrt{i+\sqrt{3}}\sqrt[4]{a}}\right)}{\sqrt{i+\sqrt{3}}}\right)}{\sqrt{6}a^{3/4}c^{3/4}}$$

input
```
Integrate[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```

output
```
((-1)^(1/4)*((((-I + Sqrt[3])*Sqrt[a]*B - (2*I)*A*Sqrt[c])*ArcTan[((1 + I)
*c^(1/4)*x)/(Sqrt[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((I + Sqr
t[3])*Sqrt[a]*B + (2*I)*A*Sqrt[c])*ArcTanh[((1 + I)*c^(1/4)*x)/(Sqrt[I + S
qrt[3]]*a^(1/4))])/Sqrt[I + Sqrt[3]]))/(Sqrt[6]*a^(3/4)*c^(3/4))
```

3.112.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1483, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{-\sqrt{a}\sqrt{c}x^2+a+cx^4}\,dx$$

$$\Big\downarrow \ 1483$$

$$\frac{\int \frac{\sqrt{3}\sqrt[4]{a}A-\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{3}a^{3/4}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{3}\sqrt[4]{a}A+\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{3}a^{3/4}\sqrt[4]{c}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{3}\sqrt[4]{a}A-\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\sqrt[4]{c}x}{x^2-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{3}a^{3/4}\sqrt{c}}+\frac{\int \frac{\sqrt{3}\sqrt[4]{a}A+\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\sqrt[4]{c}x}{x^2+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{3}a^{3/4}\sqrt{c}}$$

$\downarrow$ 1142

$$\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{x^2-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx-\frac{1}{2}\sqrt[4]{c}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int -\frac{\sqrt{3}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{3}a^{3/4}\sqrt{c}}+\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{x^2+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\sqrt[4]{c}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{2\sqrt[4]{c}x+\sqrt{3}\sqrt[4]{a}}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{3}a^{3/4}\sqrt{c}}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{x^2-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\sqrt[4]{c}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{\sqrt{3}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{3}a^{3/4}\sqrt{c}}+\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{x^2+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\sqrt[4]{c}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{2\sqrt[4]{c}x+\sqrt{3}\sqrt[4]{a}}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{3}a^{3/4}\sqrt{c}}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{x^2-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{\sqrt{3}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{3}a^{3/4}\sqrt{c}}+\frac{\frac{1}{2}\sqrt{3}\sqrt[4]{a}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{x^2+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{2\sqrt[4]{c}x+\sqrt{3}\sqrt[4]{a}}{x^2+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{3}a^{3/4}\sqrt{c}}$$

$\downarrow$ 1082

$$\frac{\frac{1}{2}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{\sqrt{3}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\sqrt[4]{c}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{-\left(1-\frac{2\sqrt[4]{c}x}{\sqrt{3}\sqrt[4]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[4]{c}x}{\sqrt{3}\sqrt[4]{a}}\right)}{2\sqrt{3}a^{3/4}\sqrt{c}}+\frac{\frac{1}{2}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{2\sqrt[4]{c}x+\sqrt{3}\sqrt[4]{a}}{x^2+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx-\sqrt[4]{c}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{-\left(\frac{2\sqrt[4]{c}x}{\sqrt{3}\sqrt[4]{a}}+1\right)^2-\frac{1}{3}}d\left(\frac{2\sqrt[4]{c}x}{\sqrt{3}\sqrt[4]{a}}+1\right)}{2\sqrt{3}a^{3/4}\sqrt{c}}$$

$\downarrow$ 217

3.112. $\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx$

$$\frac{\frac{1}{2}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{\sqrt{3}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx-\sqrt{3}\sqrt[4]{c}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[4]{c}x}{\sqrt{3}\sqrt[4]{a}}\right)\right)}{2\sqrt{3}a^{3/4}\sqrt{c}}+\frac{\frac{1}{2}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{2\sqrt[4]{c}x+\sqrt{3}\sqrt[4]{a}}{x^2+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\sqrt{3}\sqrt[4]{c}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\arctan\left(\sqrt{3}\left(\frac{2\sqrt[4]{c}x}{\sqrt{3}\sqrt[4]{a}}+1\right)\right)}{2\sqrt{3}a^{3/4}\sqrt{c}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\sqrt{3}\sqrt[4]{c}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[4]{c}x}{\sqrt{3}\sqrt[4]{a}}\right)\right)-\frac{1}{2}\sqrt[4]{c}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{3}a^{3/4}\sqrt{c}}+\frac{\sqrt{3}\sqrt[4]{c}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\arctan\left(\sqrt{3}\left(\frac{2\sqrt[4]{c}x}{\sqrt{3}\sqrt[4]{a}}+1\right)\right)+\frac{1}{2}\sqrt[4]{c}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{3}a^{3/4}\sqrt{c}}$$

input
```
Int[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```

output
```
(-(Sqrt[3]*(A + (Sqrt[a]*B)/Sqrt[c])*c^(1/4)*ArcTan[Sqrt[3]*(1 - (2*c^(1/4
)*x)/(Sqrt[3]*a^(1/4)))]) - ((A - (Sqrt[a]*B)/Sqrt[c])*c^(1/4)*Log[Sqrt[a]
 - Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/2)/(2*Sqrt[3]*a^(3/4)*Sqrt[c]
) + (Sqrt[3]*(A + (Sqrt[a]*B)/Sqrt[c])*c^(1/4)*ArcTan[Sqrt[3]*(1 + (2*c^(1
/4)*x)/(Sqrt[3]*a^(1/4)))] + ((A - (Sqrt[a]*B)/Sqrt[c])*c^(1/4)*Log[Sqrt[a
] + Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/2)/(2*Sqrt[3]*a^(3/4)*Sqrt[c
])
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

3.112.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4-\_Z^2\operatorname{RootOf}\left(\_Z^2-a,\mathrm{index}=1\right)\operatorname{RootOf}\left(\_Z^2-c,\mathrm{index}=1\right)+a\right)}\frac{\left(-B\_R^2-A\right)\ln\left(x-\_R\right)}{-2c\_R^3+\_R\sqrt{a}\,\sqrt{c}}\right)}{2}$
default	$\frac{\frac{\left(A\sqrt{a}\,\sqrt{3}\,c-B\sqrt{c}\,\sqrt{3}\,a\right)\ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{3}+\sqrt{a}+x^2\sqrt{c}\right)}{2\sqrt{c}}+\frac{2\left(3Ac^{\frac{3}{4}}a^{\frac{3}{4}}-\frac{\left(A\sqrt{a}\,\sqrt{3}\,c-B\sqrt{c}\,\sqrt{3}\,a\right)a^{\frac{1}{4}}\sqrt{3}}{2c^{\frac{1}{4}}}\right)\arctan\left(\frac{a^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{3}+2x\sqrt{c}}{\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{c}}}}{6c^{\frac{3}{4}}a^{\frac{5}{4}}}+\frac{\left(-A\sqrt{a}\,\sqrt{3}\right.}{}$

input
```
int((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((-B*_R^2-A)/(-2*c*_R^3+_R*a^(1/2)*c^(1/2))*ln(x-_R),_R=RootOf(c*_Z
^4-_Z^2*RootOf(_Z^2-a,index=1)*RootOf(_Z^2-c,index=1)+a))
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(160) = 320$.

Time = 0.98 (sec) , antiderivative size = 1469, normalized size of antiderivative = 6.28

$$\int \frac{A + Bx^2}{a - \sqrt{a}\sqrt{c}x^2 + cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c +
 A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c
^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
 (A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sq
rt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) - sqrt
(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^3*c^3)))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*
c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^
2*c^2))) + 1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2
*B^2*a*c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(
c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A
^5*a*c^3 - (A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a
^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt
(c) - sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2
*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*
A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sq
rt(c))/(a^2*c^2))) - 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^
2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt
(a)*sqrt(c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*
a^3*c - A^5*a*c^3 - (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3...
```

3.112.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((B*x**2+A)/(a+c*x**4-x**2*a**(1/2)*c**(1/2)),x)
```

output
```
Exception raised: PolynomialError >> 1/(64*_t**4*a*c**5 - 16*_t**2*B**2*sq
rt(a)*c**(7/2) + B**4*c**2) contains an element of the set of generators.
```

3.112.7 Maxima [F]

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = \int \frac{Bx^2+A}{cx^4-\sqrt{a}\sqrt{c}x^2+a}\,dx$$

input
```
integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(c*x^4 - sqrt(a)*sqrt(c)*x^2 + a), x)
```

3.112.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.112.9 Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 1575, normalized size of antiderivative = 6.73

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}x^2+cx^4}\,dx=\text{Too large to display}$$

input
```
int((A + B*x^2)/(a + c*x^4 - a^(1/2)*c^(1/2)*x^2),x)
```

output
```
- 2*atanh((6*A^2*x*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/
2)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2
/(24*a^(3/2)*c^(1/2)))^(1/2))/((2*A^2*B)/c - (2*B^3*a)/c^2 + A^3/(a^(1/2)*
c^(1/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^2) - (A*B^2*a^(1/2))/c^(3/2)
 - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^3)) - (6*B^2*a*x*((B^2*(-27*a^3*c^3)
^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-2
7*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(24*a^(3/2)*c^(1/2)))^(1/2))/(2*A^2*B
 - (2*B^3*a)/c + (A^3*c^(1/2))/a^(1/2) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*
c) - (A*B^2*a^(1/2))/c^(1/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^2)) - (2
*A^2*x*(-27*a^3*c^3)^(1/2)*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(
24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^
2) - A^2/(24*a^(3/2)*c^(1/2)))^(1/2))/(3*a^(3/2)*c^(7/2)*((2*A^2*B)/c^3 -
(2*B^3*a)/c^4 + A^3/(a^(1/2)*c^(5/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c
^4) - (A*B^2*a^(1/2))/c^(7/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))) +
(2*B^2*x*(-27*a^3*c^3)^(1/2)*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2
/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*
c^2) - A^2/(24*a^(3/2)*c^(1/2)))^(1/2))/(3*a^(1/2)*c^(9/2)*((2*A^2*B)/c^3
- (2*B^3*a)/c^4 + A^3/(a^(1/2)*c^(5/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2
*c^4) - (A*B^2*a^(1/2))/c^(7/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))))
*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2)*c^(3/2)) - (...
```

3.113 $\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx$

3.113.1 Optimal result

Integrand size = 22, antiderivative size = 96

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = -\sqrt{\frac{1}{2}\left(-1+\sqrt{13}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right) + \sqrt{7+2\sqrt{13}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right),\frac{1}{6}\left(-7-\sqrt{13}\right)\right)$$

output
```
-1/2*EllipticE(x*2^(1/2)/(1+13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))*
(-2+2*13^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(1+13^(1/2))^(1/2),1/6*I*3^(1/2)
+1/6*I*39^(1/2))*(7+2*13^(1/2))^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = -\frac{i\left(\left(1+\sqrt{13}\right)E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)-\left(-5+\sqrt{13}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right.\right.\right.}{\sqrt{2\left(1+\sqrt{13}\right)}}$$

input
```
Integrate[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
```

output
```
((-I)*((1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]*x], (-7
+ Sqrt[13])/6] - (-5 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13]
)]*x], (-7 + Sqrt[13])/6]))/Sqrt[2*(1 + Sqrt[13])]
```

3.113.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-x^2}{\sqrt{-x^4+x^2+3}}dx$$

$$\Big\downarrow \text{1494}$$

$$2\int \frac{3-x^2}{\sqrt{-2x^2+\sqrt{13}+1}\sqrt{2x^2+\sqrt{13}-1}}dx$$

$$\Big\downarrow \text{399}$$

$$2\left(\frac{1}{2}\left(5+\sqrt{13}\right)\int \frac{1}{\sqrt{-2x^2+\sqrt{13}+1}\sqrt{2x^2+\sqrt{13}-1}}dx-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{13}-1}}{\sqrt{-2x^2+\sqrt{13}+1}}dx\right)$$

$$\Big\downarrow \text{321}$$

$$2\left(\frac{\left(5+\sqrt{13}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right),\frac{1}{6}\left(-7-\sqrt{13}\right)\right)}{2\sqrt{2\left(\sqrt{13}-1\right)}}-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{13}-1}}{\sqrt{-2x^2+\sqrt{13}+1}}dx\right)$$

$$\Big\downarrow \text{327}$$

$$2\left(\frac{\left(5+\sqrt{13}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right),\frac{1}{6}\left(-7-\sqrt{13}\right)\right)}{2\sqrt{2\left(\sqrt{13}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{13}-1\right)}E\left(\arcsin\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-\right.\right.\right.$$

input
```
Int[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
```

output
```
2*(-1/2*(Sqrt[(-1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(1 + Sqrt[13])]*x
], (-7 - Sqrt[13])/6]) + ((5 + Sqrt[13])*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt
[13])]*x], (-7 - Sqrt[13])/6])/(2*Sqrt[2*(-1 + Sqrt[13])]))
```

3.113.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.113.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(74) = 148$.

Time = 2.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.08

method	result
default	$\frac{18\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-6+6\sqrt{13}}}{6},\frac{i\sqrt{3}}{6}+\frac{i\sqrt{39}}{6}\right)}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x^4+x^2+3}}+\frac{36\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(F\left(\frac{x\sqrt{-}\cdots}{}\right.\right.}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x\cdots}}$
elliptic	$\frac{18\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-6+6\sqrt{13}}}{6},\frac{i\sqrt{3}}{6}+\frac{i\sqrt{39}}{6}\right)}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x^4+x^2+3}}+\frac{36\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(F\left(\frac{x\sqrt{-}\cdots}{}\right.\right.}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x\cdots}}$

input
```
int((-x^2+3)/(-x^4+x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
18/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*13
^(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)*EllipticF(1/6*x*(-6+6*13^(1/2))^(1/2
),1/6*I*3^(1/2)+1/6*I*39^(1/2))+36/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6*13^(
1/2))*x^2)^(1/2)*(1-(-1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)/(1+1
3^(1/2))*(EllipticF(1/6*x*(-6+6*13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/
2))-EllipticE(1/6*x*(-6+6*13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2)))
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.09

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = \frac{-2i\sqrt{2}x\sqrt{\sqrt{13}+1}F(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}+1}}{2x}\right) \mid \frac{1}{6}\sqrt{13}-\frac{7}{6}) + \left(i\sqrt{13}\sqrt{2}x+i\sqrt{2}x\right)\sqrt{\sqrt{13}+1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\cdots}}{2\cdots}\right.\cdots}{4x}$$

input
```
integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(-2*I*sqrt(2)*x*sqrt(sqrt(13) + 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(
sqrt(13) + 1)/x), 1/6*sqrt(13) - 7/6) + (I*sqrt(13)*sqrt(2)*x + I*sqrt(2)*
x)*sqrt(sqrt(13) + 1)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) + 1)/x),
 1/6*sqrt(13) - 7/6) + 4*sqrt(-x^4 + x^2 + 3))/x
```

3.113.6 Sympy [F]

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = -\int \frac{x^2}{\sqrt{-x^4+x^2+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4+x^2+3}}\right)dx$$

input
```
integrate((-x**2+3)/(-x**4+x**2+3)**(1/2),x)
```

output
```
-Integral(x**2/sqrt(-x**4 + x**2 + 3), x) - Integral(-3/sqrt(-x**4 + x**2
+ 3), x)
```

3.113.7 Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

3.113.8 Giac [F]

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = -\int \frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

input
```
int(-(x^2 - 3)/(x^2 - x^4 + 3)^(1/2),x)
```

output
```
-int((x^2 - 3)/(x^2 - x^4 + 3)^(1/2), x)
```

3.114 $\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}} \, dx$

3.114.1 Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}} \, dx = -E\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right) + 4\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-3\right)$$

output
```
-EllipticE(1/3*x*3^(1/2),I*3^(1/2))+4*EllipticF(1/3*x*3^(1/2),I*3^(1/2))
```

3.114.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}} \, dx = -i\sqrt{3}E\left(i\text{arcsinh}(x)\middle|-\frac{1}{3}\right)$$

input
```
Integrate[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
```

output
```
(-I)*Sqrt[3]*EllipticE[I*ArcSinh[x], -1/3]
```

3.114.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1387, 326, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-x^2}{\sqrt{-x^4+2x^2+3}}dx$$

$$\Big\downarrow \text{ 1387}$$

$$\int \frac{\sqrt{3-x^2}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 326}$$

$$4\int \frac{1}{\sqrt{3-x^2}\sqrt{x^2+1}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{3-x^2}}dx$$

$$\Big\downarrow \text{ 321}$$

$$4\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-3\right)-\int \frac{\sqrt{x^2+1}}{\sqrt{3-x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$4\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{3}}\right),-3\right)-E\left(\arcsin\left(\frac{x}{\sqrt{3}}\right)\Big|-3\right)$$

input
```
Int[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
```

output
```
-EllipticE[ArcSin[x/Sqrt[3]], -3] + 4*EllipticF[ArcSin[x/Sqrt[3]], -3]
```

3.114.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

3.114.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(31) = 62$.

Time = 2.02 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.52

method	result	size
default	$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)}{\sqrt{-x^4+2x^2+3}}+\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)-E\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)\right)}{3\sqrt{-x^4+2x^2+3}}$	113
elliptic	$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)}{\sqrt{-x^4+2x^2+3}}+\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)-E\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)\right)}{3\sqrt{-x^4+2x^2+3}}$	113

input
```
int((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3^(1/2)*(-3*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)*EllipticF(1/3*
x*3^(1/2),I*3^(1/2))+1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^
2+3)^(1/2)*(EllipticF(1/3*x*3^(1/2),I*3^(1/2))-EllipticE(1/3*x*3^(1/2),I*3
^(1/2)))
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx = \frac{3i\sqrt{3}xE\left(\arcsin\left(\frac{\sqrt{3}}{x}\right)|-\frac{1}{3}\right)-2i\sqrt{3}xF\left(\arcsin\left(\frac{\sqrt{3}}{x}\right)|-\frac{1}{3}\right)+\sqrt{-x^4+2\,x^2+3}}{x}$$

input
```
integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
(3*I*sqrt(3)*x*elliptic_e(arcsin(sqrt(3)/x), -1/3) - 2*I*sqrt(3)*x*ellipti
c_f(arcsin(sqrt(3)/x), -1/3) + sqrt(-x^4 + 2*x^2 + 3))/x
```

3.114.6 Sympy [F]

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx = -\int \frac{x^2}{\sqrt{-x^4+2x^2+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4+2x^2+3}}\right)dx$$

input
```
integrate((-x**2+3)/(-x**4+2*x**2+3)**(1/2),x)
```

output
```
-Integral(x**2/sqrt(-x**4 + 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 2*x
**2 + 3), x)
```

3.114.7 Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4+2\,x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

3.114.8 Giac [F]

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx=\int -\frac{x^2-3}{\sqrt{-x^4+2\,x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx=\int -\frac{x^2-3}{\sqrt{-x^4+2\,x^2+3}}\,dx$$

input
```
int(-(x^2 - 3)/(2*x^2 - x^4 + 3)^(1/2),x)
```

output
```
int(-(x^2 - 3)/(2*x^2 - x^4 + 3)^(1/2), x)
```

3.115 $\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx$

3.115.1 Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx=-\sqrt{\frac{1}{2}\left(-3+\sqrt{21}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)+\sqrt{9+2\sqrt{21}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right),\frac{1}{2}\left(-5-\sqrt{21}\right)\right)$$

output
```
-1/2*EllipticE(x*2^(1/2)/(3+21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))*(
-6+2*21^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(3+21^(1/2))^(1/2),1/2*I*3^(1/2)+
1/2*I*7^(1/2))*(9+2*21^(1/2))^(1/2)
```

3.115.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx=-\frac{i\left(\left(3+\sqrt{21}\right)E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)-\left(-3+\sqrt{21}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right.\right.\right.}{\sqrt{2\left(3+\sqrt{21}\right)}}$$

input
```
Integrate[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
```

output
```
((-I)*((3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21])]*x], (-5
+ Sqrt[21])/2] - (-3 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21]
)]*x], (-5 + Sqrt[21])/2]))/Sqrt[2*(3 + Sqrt[21])]
```

3.115.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-x^2}{\sqrt{-x^4+3x^2+3}}dx$$

$$\Big\downarrow \text{1494}$$

$$2\int \frac{3-x^2}{\sqrt{-2x^2+\sqrt{21}+3}\sqrt{2x^2+\sqrt{21}-3}}dx$$

$$\Big\downarrow \text{399}$$

$$2\left(\frac{1}{2}\left(3+\sqrt{21}\right)\int \frac{1}{\sqrt{-2x^2+\sqrt{21}+3}\sqrt{2x^2+\sqrt{21}-3}}dx-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{21}-3}}{\sqrt{-2x^2+\sqrt{21}+3}}dx\right)$$

$$\Big\downarrow \text{321}$$

$$2\left(\frac{\left(3+\sqrt{21}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right),\frac{1}{2}\left(-5-\sqrt{21}\right)\right)}{2\sqrt{2\left(\sqrt{21}-3\right)}}-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{21}-3}}{\sqrt{-2x^2+\sqrt{21}+3}}dx\right)$$

$$\Big\downarrow \text{327}$$

$$2\left(\frac{\left(3+\sqrt{21}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right),\frac{1}{2}\left(-5-\sqrt{21}\right)\right)}{2\sqrt{2\left(\sqrt{21}-3\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{21}-3\right)}E\left(\arcsin\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-\right.\right.\right.$$

input
```
Int[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
```

output
```
2*(-1/2*(Sqrt[(-3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(3 + Sqrt[21])]*x
], (-5 - Sqrt[21])/2]) + ((3 + Sqrt[21])*EllipticF[ArcSin[Sqrt[2/(3 + Sqrt
[21])]*x], (-5 - Sqrt[21])/2])/(2*Sqrt[2*(-3 + Sqrt[21])]))
```

3.115.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.115.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(74) = 148$.

Time = 2.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.12

method	result
default	$\frac{18\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-18+6\sqrt{21}}}{6},\frac{i\sqrt{3}}{2}+\frac{i\sqrt{7}}{2}\right)}{\sqrt{-18+6\sqrt{21}}\,\sqrt{-x^4+3x^2+3}}+\frac{36\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\left(F\left(\frac{x\sqrt{-}}{}\right.\right.}{\sqrt{-18+6\sqrt{21}}\,\sqrt{-x}}$
elliptic	$\frac{18\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,F\left(\frac{x\sqrt{-18+6\sqrt{21}}}{6},\frac{i\sqrt{3}}{2}+\frac{i\sqrt{7}}{2}\right)}{\sqrt{-18+6\sqrt{21}}\,\sqrt{-x^4+3x^2+3}}+\frac{36\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\left(F\left(\frac{x\sqrt{-}}{}\right.\right.}{\sqrt{-18+6\sqrt{21}}\,\sqrt{-x}}$

input
```
int((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
18/(-18+6*21^(1/2))^(1/2)*(1-(-1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*2
1^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^(1/2)*EllipticF(1/6*x*(-18+6*21^(1/2))^
(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))+36/(-18+6*21^(1/2))^(1/2)*(1-(-1/2+1/6*
21^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^(1/2
)/(3+21^(1/2))*(EllipticF(1/6*x*(-18+6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I
*7^(1/2))-EllipticE(1/6*x*(-18+6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/
2)))
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx$$
$$= \frac{-6i\sqrt{2}x\sqrt{\sqrt{21}+3}F\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{21}+3}}{2\,x}\right)\mid\frac{1}{2}\sqrt{21}-\frac{5}{2}\right)+\left(i\sqrt{21}\sqrt{2}x+3i\sqrt{2}x\right)\sqrt{\sqrt{21}+3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{}}{}\right.\right.}{4\,x}$$

input
```
integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(-6*I*sqrt(2)*x*sqrt(sqrt(21) + 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(
sqrt(21) + 3)/x), 1/2*sqrt(21) - 5/2) + (I*sqrt(21)*sqrt(2)*x + 3*I*sqrt(2
)*x)*sqrt(sqrt(21) + 3)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(21) + 3)/x
), 1/2*sqrt(21) - 5/2) + 4*sqrt(-x^4 + 3*x^2 + 3))/x
```

3.115.6 Sympy [F]

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx = -\int \frac{x^2}{\sqrt{-x^4+3x^2+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4+3x^2+3}}\right)dx$$

input
```
integrate((-x**2+3)/(-x**4+3*x**2+3)**(1/2),x)
```

output
```
-Integral(x**2/sqrt(-x**4 + 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 3*x
**2 + 3), x)
```

3.115.7 Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4+3\,x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

3.115.8 Giac [F]

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4+3\,x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4+3\,x^2+3}}\,dx$$

input
```
int(-(x^2 - 3)/(3*x^2 - x^4 + 3)^(1/2),x)
```

output
```
int(-(x^2 - 3)/(3*x^2 - x^4 + 3)^(1/2), x)
```

3.116 $\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx$

3.116.1 Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx=-\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)+\sqrt{5+2\sqrt{13}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right),\frac{1}{6}\left(-7+\sqrt{13}\right)\right)$$

output
```
-1/2*EllipticE(x*2^(1/2)/(-1+13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))
*(2+2*13^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(-1+13^(1/2))^(1/2),1/6*I*39^(1/
2)-1/6*I*3^(1/2))*(5+2*13^(1/2))^(1/2)
```

3.116.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx=-\frac{i\left(\left(-1+\sqrt{13}\right)E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right)|-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)-\left(-7+\sqrt{13}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{1+\sqrt{13}}}x\right),-\right.\right.}{\sqrt{2\left(-1+\sqrt{13}\right)}}$$

input
```
Integrate[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```

output
```
((-I)*((-1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]*x], -7/6
 - Sqrt[13]/6] - (-7 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])
]*x], -7/6 - Sqrt[13]/6]))/Sqrt[2*(-1 + Sqrt[13])]
```

3.116.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-x^2}{\sqrt{-x^4-x^2+3}}dx$$

$$\Big\downarrow \ 1494$$

$$2\int \frac{3-x^2}{\sqrt{-2x^2+\sqrt{13}-1}\sqrt{2x^2+\sqrt{13}+1}}dx$$

$$\Big\downarrow \ 399$$

$$2\left(\frac{1}{2}\left(7+\sqrt{13}\right)\int \frac{1}{\sqrt{-2x^2+\sqrt{13}-1}\sqrt{2x^2+\sqrt{13}+1}}dx-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{13}+1}}{\sqrt{-2x^2+\sqrt{13}-1}}dx\right)$$

$$\Big\downarrow \ 321$$

$$2\left(\frac{\left(7+\sqrt{13}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right),\frac{1}{6}\left(-7+\sqrt{13}\right)\right)}{2\sqrt{2\left(1+\sqrt{13}\right)}}-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{13}+1}}{\sqrt{-2x^2+\sqrt{13}-1}}dx\right)$$

$$\Big\downarrow \ 327$$

$$2\left(\frac{\left(7+\sqrt{13}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right),\frac{1}{6}\left(-7+\sqrt{13}\right)\right)}{2\sqrt{2\left(1+\sqrt{13}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{-1+\sqrt{13}}}x\right)|\frac{1}{6}\right.\right.$$

input
```
Int[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```

output
```
2*(-1/2*(Sqrt[(1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(-1 + Sqrt[13])]*x
], (-7 + Sqrt[13])/6]) + ((7 + Sqrt[13])*EllipticF[ArcSin[Sqrt[2/(-1 + Sqr
t[13])]*x], (-7 + Sqrt[13])/6])/(2*Sqrt[2*(1 + Sqrt[13])]))
```

3.116.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.116.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(74) = 148$.

Time = 2.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.22

method	result
default	$\frac{18\sqrt{1-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{6+6\sqrt{13}}}{6},\frac{i\sqrt{39}}{6}-\frac{i\sqrt{3}}{6}\right)}{\sqrt{6+6\sqrt{13}}\,\sqrt{-x^4-x^2+3}}+\frac{36\sqrt{1-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{6+6\sqrt{13}}}{6},\right.\right.}{\sqrt{6+6\sqrt{13}}\,\sqrt{-x^4-x^2+3}\,(}$
elliptic	$\frac{18\sqrt{1-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,F\left(\frac{x\sqrt{6+6\sqrt{13}}}{6},\frac{i\sqrt{39}}{6}-\frac{i\sqrt{3}}{6}\right)}{\sqrt{6+6\sqrt{13}}\,\sqrt{-x^4-x^2+3}}+\frac{36\sqrt{1-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{6+6\sqrt{13}}}{6},\right.\right.}{\sqrt{6+6\sqrt{13}}\,\sqrt{-x^4-x^2+3}\,(}$

input
```
int((-x^2+3)/(-x^4-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
18/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*13^(1
/2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)*EllipticF(1/6*x*(6+6*13^(1/2))^(1/2),1/
6*I*39^(1/2)-1/6*I*3^(1/2))+36/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1/2))*
x^2)^(1/2)*(1-(1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)/(-1+13^(1/2
))*(EllipticF(1/6*x*(6+6*13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))-Ell
ipticE(1/6*x*(6+6*13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2)))
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx$$
$$=\frac{2i\sqrt{2}x\sqrt{\sqrt{13}-1}F\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{13}-1}}{2x}\right)\mid -\frac{1}{6}\sqrt{13}-\frac{7}{6}\right)+\left(i\sqrt{13}\sqrt{2}x-i\sqrt{2}x\right)\sqrt{\sqrt{13}-1}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{}}{2}\right.\right.}{4x}$$

input
```
integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*I*sqrt(2)*x*sqrt(sqrt(13) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(s
qrt(13) - 1)/x), -1/6*sqrt(13) - 7/6) + (I*sqrt(13)*sqrt(2)*x - I*sqrt(2)*
x)*sqrt(sqrt(13) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(13) - 1)/x),
 -1/6*sqrt(13) - 7/6) + 4*sqrt(-x^4 - x^2 + 3))/x
```

3.116.6 Sympy [F]

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = -\int \frac{x^2}{\sqrt{-x^4-x^2+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4-x^2+3}}\right)dx$$

input
```
integrate((-x**2+3)/(-x**4-x**2+3)**(1/2),x)
```

output
```
-Integral(x**2/sqrt(-x**4 - x**2 + 3), x) - Integral(-3/sqrt(-x**4 - x**2
+ 3), x)
```

3.116.7 Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

3.116.8 Giac [F]

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

input
```
int(-(x^2 - 3)/(3 - x^4 - x^2)^(1/2),x)
```

output
```
int(-(x^2 - 3)/(3 - x^4 - x^2)^(1/2), x)
```

3.117 $\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx$

3.117.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = -\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)+2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)$$

output
```
-EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+2*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

3.117.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = -i\left(E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right)\left|-3\right.\right)+2\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)\right)$$

input
```
Integrate[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
```

output
```
(-I)*(EllipticE[I*ArcSinh[x/Sqrt[3]], -3] + 2*EllipticF[I*ArcSinh[x/Sqrt[3
]], -3])
```

3.117.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-x^2}{\sqrt{-x^4-2x^2+3}}dx$$

$$\Big\downarrow \text{ 1494}$$

$$2\int \frac{3-x^2}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{3-x^2}{\sqrt{1-x^2}\sqrt{x^2+3}}dx$$

$$\Big\downarrow \text{ 399}$$

$$6\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx-\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 321}$$

$$2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)$$

input
```
Int[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
```

output
```
-(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) + 2*Sqrt[3]*EllipticF[ArcSin[x], -1/
3]
```

3.117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.117.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(27) = 54$.

Time = 1.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.52

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(F\left(x,\frac{i\sqrt{3}}{3}\right)-E\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{\sqrt{-x^4-2x^2+3}}$	95
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(F\left(x,\frac{i\sqrt{3}}{3}\right)-E\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{\sqrt{-x^4-2x^2+3}}$	95

input
```
int((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3^(1
/2))+(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*
I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = \frac{i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|-3\right)+2i\,xF\left(\arcsin\left(\frac{1}{x}\right)\,|-3\right)+\sqrt{-x^4-2\,x^2+3}}{x}$$

input
```
integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
(I*x*elliptic_e(arcsin(1/x), -3) + 2*I*x*elliptic_f(arcsin(1/x), -3) + sqr
t(-x^4 - 2*x^2 + 3))/x
```

3.117.6 Sympy [F]

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = -\int \frac{x^2}{\sqrt{-x^4-2x^2+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4-2x^2+3}}\right)dx$$

input
```
integrate((-x**2+3)/(-x**4-2*x**2+3)**(1/2),x)
```

output
```
-Integral(x**2/sqrt(-x**4 - 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 2*x
**2 + 3), x)
```

3.117.7 Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

3.117.8 Giac [F]

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

input
```
int(-(x^2 - 3)/(3 - x^4 - 2*x^2)^(1/2),x)
```

output
```
int(-(x^2 - 3)/(3 - x^4 - 2*x^2)^(1/2), x)
```

3.118 $\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx$

3.118.1 Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = -\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right) + \sqrt{3+2\sqrt{21}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right),\frac{1}{2}\left(-5+\sqrt{21}\right)\right)$$

output
```
-1/2*EllipticE(x*2^(1/2)/(-3+21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))*
(6+2*21^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(-3+21^(1/2))^(1/2),1/2*I*7^(1/2)
-1/2*I*3^(1/2))*(3+2*21^(1/2))^(1/2)
```

3.118.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = -\frac{i\left(\left(-3+\sqrt{21}\right)E\left(i\text{arcsinh}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right)|-\frac{5}{2}-\frac{\sqrt{21}}{2}\right)-\left(-9+\sqrt{21}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{2}{3+\sqrt{21}}}x\right),-\ldots\right.\right.}{\sqrt{2\left(-3+\sqrt{21}\right)}}$$

input
```
Integrate[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

output
```
((-I)*((-3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(3 + Sqrt[21])]*x], -5/2
 - Sqrt[21]/2] - (-9 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(3 + Sqrt[21])
]*x], -5/2 - Sqrt[21]/2]))/Sqrt[2*(-3 + Sqrt[21])]
```

3.118.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1494, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-x^2}{\sqrt{-x^4-3x^2+3}}dx$$

$$\Big\downarrow \text{1494}$$

$$2\int \frac{3-x^2}{\sqrt{-2x^2+\sqrt{21}-3}\sqrt{2x^2+\sqrt{21}+3}}dx$$

$$\Big\downarrow \text{399}$$

$$2\left(\frac{1}{2}\left(9+\sqrt{21}\right)\int \frac{1}{\sqrt{-2x^2+\sqrt{21}-3}\sqrt{2x^2+\sqrt{21}+3}}dx-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{21}+3}}{\sqrt{-2x^2+\sqrt{21}-3}}dx\right)$$

$$\Big\downarrow \text{321}$$

$$2\left(\frac{\left(9+\sqrt{21}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right),\frac{1}{2}\left(-5+\sqrt{21}\right)\right)}{2\sqrt{2\left(3+\sqrt{21}\right)}}-\frac{1}{2}\int \frac{\sqrt{2x^2+\sqrt{21}+3}}{\sqrt{-2x^2+\sqrt{21}-3}}dx\right)$$

$$\Big\downarrow \text{327}$$

$$2\left(\frac{\left(9+\sqrt{21}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right),\frac{1}{2}\left(-5+\sqrt{21}\right)\right)}{2\sqrt{2\left(3+\sqrt{21}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}E\left(\arcsin\left(\sqrt{\frac{2}{-3+\sqrt{21}}}x\right)|\frac{1}{2}\right.\right.$$

input
```
Int[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

output
```
2*(-1/2*(Sqrt[(3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(-3 + Sqrt[21])]*x
], (-5 + Sqrt[21])/2]) + ((9 + Sqrt[21])*EllipticF[ArcSin[Sqrt[2/(-3 + Sqr
t[21])]*x], (-5 + Sqrt[21])/2])/(2*Sqrt[2*(3 + Sqrt[21])]))
```

3.118.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] && !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.118.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(74) = 148$.

Time = 2.05 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.22

method	result
default	$\frac{18\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,F\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\frac{i\sqrt{7}}{2}-\frac{i\sqrt{3}}{2}\right)}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}}+\frac{36\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\right.\right.}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}}$
elliptic	$\frac{18\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,F\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\frac{i\sqrt{7}}{2}-\frac{i\sqrt{3}}{2}\right)}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}}+\frac{36\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\left(F\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\right.\right.}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}}$

input
```
int((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
18/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*21^(
1/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*(18+6*21^(1/2))^(1/2
),1/2*I*7^(1/2)-1/2*I*3^(1/2))+36/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*21^(1/
2))*x^2)^(1/2)*(1-(1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)/(-3+2
1^(1/2))*(EllipticF(1/6*x*(18+6*21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2
))-EllipticE(1/6*x*(18+6*21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2)))
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = \frac{6i\sqrt{2}x\sqrt{\sqrt{21}-3}F\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{21}-3}}{2x}\right)\mid -\frac{1}{2}\sqrt{21}-\frac{5}{2}\right)+\left(i\sqrt{21}\sqrt{2}x-3i\sqrt{2}x\right)\sqrt{\sqrt{21}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{}}{}\right.\right.}{4x}$$

input
```
integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(6*I*sqrt(2)*x*sqrt(sqrt(21) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(s
qrt(21) - 3)/x), -1/2*sqrt(21) - 5/2) + (I*sqrt(21)*sqrt(2)*x - 3*I*sqrt(2
)*x)*sqrt(sqrt(21) - 3)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(21) - 3)/x
), -1/2*sqrt(21) - 5/2) + 4*sqrt(-x^4 - 3*x^2 + 3))/x
```

3.118.6 Sympy [F]

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = -\int \frac{x^2}{\sqrt{-x^4-3x^2+3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4-3x^2+3}}\right)dx$$

input
```
integrate((-x**2+3)/(-x**4-3*x**2+3)**(1/2),x)
```

output
```
-Integral(x**2/sqrt(-x**4 - 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 3*x
**2 + 3), x)
```

3.118.7 Maxima [F]

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

3.118.8 Giac [F]

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = \int -\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

input
```
integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx=\int -\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

input
```
int(-(x^2 - 3)/(3 - x^4 - 3*x^2)^(1/2),x)
```

output
```
int(-(x^2 - 3)/(3 - x^4 - 3*x^2)^(1/2), x)
```

3.119 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

3.119.1 Optimal result

Integrand size = 39, antiderivative size = 296

$$\begin{aligned}\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2} \\ &- \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}} \\ &+ \frac{\left(b+2\sqrt{a}\sqrt{c}-\sqrt{b^2-4ac}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\end{aligned}$$

output
```
2*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/(a^(1/2)+x^2*c^(1/2))-2*a^(1/4)*c^(1/4)*
(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4))
)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/
2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/2)-(
-4*a*c+b^2)^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)
/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

3.119.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.63

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{2i\sqrt{2}a\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((-2*I)*Sqrt[2]*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
(b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^2 + c*x^4]
```

3.119.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1511}$$

$$\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-2\sqrt{a}\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-2\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-2\sqrt{c}\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \; 1509$$

$$\frac{\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-2\sqrt{c}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)$$

input
```
Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
-2*Sqrt[c]*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1
/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4
])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])) + ((b + 2*Sqrt[a]*Sqrt[c] - Sqrt[b^2
 - 4*a*c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt
[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.119.4 Maple [A] (verified)

Time = 7.02 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.74

method	result
default	$\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{ca\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{\right.}}{}}}{}$
elliptic	$-\frac{\left(-2c\,x^{2}+\sqrt{-4ac+b^{2}}-b\right)\sqrt{-\left(c\,x^{4}+b\,x^{2}+a\right)\left(4ac-b^{2}\right)}\left(\frac{\left(4ac-b^{2}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}\right)x^{2}}{a\left(4ac-b^{2}\right)}}\,\sqrt{4+\frac{2\left(\left(-4ac+b^{2}\right)^{\frac{3}{2}}+4abc-\right.}{a\left(4ac-b^{2}\right)}}}{4\sqrt{\frac{\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}}{a\left(4ac-b^{2}\right)}}\,\sqrt{}}\right.}{}$

input
```
int((b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/4*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))-1/4*(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*
EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(
-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.09

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{2\sqrt{\frac{1}{2}}\left(acx\sqrt{\frac{b^2-4\,ac}{c^2}}-abx\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac})-\sqrt{\frac{1}{2}}\left(\sqrt{b^2-\cdots}\right.}{\cdots}$$

input
```
integrate((b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(c*x^4+b*x^2+a)^(1/2),x, algorith
m="fricas")
```

output
```
1/2*(2*sqrt(1/2)*(a*c*x*sqrt((b^2 - 4*a*c)/c^2) - a*b*x)*sqrt(c)*sqrt((c*s
qrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b
^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*
c)/(a*c)) - sqrt(1/2)*(sqrt(b^2 - 4*a*c)*b*x - (2*a*b + b^2)*x + ((2*a - b
)*c*x + sqrt(b^2 - 4*a*c)*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sq
rt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^
2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c
)/(a*c)) + 4*sqrt(c*x^4 + b*x^2 + a)*a*c)/(a*c*x)
```

3.119.6 Sympy [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{b+2cx^2-\sqrt{-4ac+b^2}}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((b+2*c*x**2-(-4*a*c+b**2)**(1/2))/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/sqrt(a + b*x**2 + c*x**4), x
)
```

3.119.7 Maxima [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(c*x^4+b*x^2+a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)
```

3.119.8 Giac [F]

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(c*x^4+b*x^2+a)^(1/2),x, algorith
m="giac")
```

output
```
integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{b+2\,c\,x^2-\sqrt{b^2-4\,a\,c}}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.120 $\int (d+ex^2)^4 (a+cx^4) \, dx$

3.120.1 Optimal result

Integrand size = 17, antiderivative size = 106

$$\int (d+ex^2)^4 (a+cx^4) \, dx = ad^4x + \frac{4}{3}ad^3ex^3 + \frac{1}{5}d^2(cd^2+6ae^2)x^5 + \frac{4}{7}de(cd^2+ae^2)x^7 + \frac{1}{9}e^2(6cd^2+ae^2)x^9 + \frac{4}{11}cde^3x^{11} + \frac{1}{13}ce^4x^{13}$$

output
```
a*d^4*x+4/3*a*d^3*e*x^3+1/5*d^2*(6*a*e^2+c*d^2)*x^5+4/7*d*e*(a*e^2+c*d^2)*
x^7+1/9*e^2*(a*e^2+6*c*d^2)*x^9+4/11*c*d*e^3*x^11+1/13*c*e^4*x^13
```

3.120.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^4 (a+cx^4) \, dx = ad^4x + \frac{4}{3}ad^3ex^3 + \frac{1}{5}d^2(cd^2+6ae^2)x^5 + \frac{4}{7}de(cd^2+ae^2)x^7 + \frac{1}{9}e^2(6cd^2+ae^2)x^9 + \frac{4}{11}cde^3x^{11} + \frac{1}{13}ce^4x^{13}$$

input
```
Integrate[(d + e*x^2)^4*(a + c*x^4),x]
```

output
```
a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^
2 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 +
(c*e^4*x^13)/13
```

3.120.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)(d+ex^2)^4\,dx$$

$$\Big\downarrow \ 1468$$

$$\int \left(e^2x^8(ae^2+6cd^2)+4dex^6(ae^2+cd^2)+d^2x^4(6ae^2+cd^2)+ad^4+4ad^3ex^2+4cde^3x^{10}+ce^4x^{12}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{9}e^2x^9(ae^2+6cd^2)+\frac{4}{7}dex^7(ae^2+cd^2)+\frac{1}{5}d^2x^5(6ae^2+cd^2)+ad^4x+\frac{4}{3}ad^3ex^3+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

input
```
Int[(d + e*x^2)^4*(a + c*x^4),x]
```

output
```
a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^
2 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 +
(c*e^4*x^13)/13
```

3.120.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.120.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

method	result
norman	$\frac{c\,e^4x^{13}}{13}+\frac{4cd\,e^3x^{11}}{11}+\left(\frac{1}{9}e^4a+\frac{2}{3}e^2d^2c\right)x^9+\left(\frac{4}{7}d\,e^3a+\frac{4}{7}d^3ec\right)x^7+\left(\frac{6}{5}e^2d^2a+\frac{1}{5}d^4c\right)x^5+\frac{4a\,d^3e\,x}{3}$
default	$\frac{c\,e^4x^{13}}{13}+\frac{4cd\,e^3x^{11}}{11}+\frac{(e^4a+6e^2d^2c)x^9}{9}+\frac{(4d\,e^3a+4d^3ec)x^7}{7}+\frac{(6e^2d^2a+d^4c)x^5}{5}+\frac{4a\,d^3e\,x^3}{3}+a\,d^4x$
gosper	$\frac{1}{13}c\,e^4x^{13}+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{2}{3}x^9e^2d^2c+\frac{4}{7}x^7d\,e^3a+\frac{4}{7}x^7d^3ec+\frac{6}{5}x^5e^2d^2a+\frac{1}{5}x^5d^4c+\frac{4}{3}$
risch	$\frac{1}{13}c\,e^4x^{13}+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{2}{3}x^9e^2d^2c+\frac{4}{7}x^7d\,e^3a+\frac{4}{7}x^7d^3ec+\frac{6}{5}x^5e^2d^2a+\frac{1}{5}x^5d^4c+\frac{4}{3}$
parallelrisch	$\frac{1}{13}c\,e^4x^{13}+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{2}{3}x^9e^2d^2c+\frac{4}{7}x^7d\,e^3a+\frac{4}{7}x^7d^3ec+\frac{6}{5}x^5e^2d^2a+\frac{1}{5}x^5d^4c+\frac{4}{3}$

input
```
int((e*x^2+d)^4*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/13*c*e^4*x^13+4/11*c*d*e^3*x^11+(1/9*e^4*a+2/3*e^2*d^2*c)*x^9+(4/7*d*e^3
*a+4/7*d^3*e*c)*x^7+(6/5*e^2*d^2*a+1/5*d^4*c)*x^5+4/3*a*d^3*e*x^3+a*d^4*x
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \left(d+ex^2\right)^4\left(a+cx^4\right)\,dx=\frac{1}{13}ce^4x^{13}+\frac{4}{11}cde^3x^{11}+\frac{1}{9}\left(6\,cd^2e^2+ae^4\right)x^9+\frac{4}{3}ad^3ex^3+\frac{4}{7}\left(cd^3e+ade^3\right)x^7+ad^4x+\frac{1}{5}\left(cd^4+6\,ad^2e^2\right)x^5$$

input
```
integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="fricas")
```

output
```
1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/9*(6*c*d^2*e^2 + a*e^4)*x^9 + 4/3*
a*d^3*e*x^3 + 4/7*(c*d^3*e + a*d*e^3)*x^7 + a*d^4*x + 1/5*(c*d^4 + 6*a*d^2
*e^2)*x^5
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

$$\int \left(d+ex^2\right)^4 \left(a+cx^4\right)\,dx = ad^4x+\frac{4ad^3ex^3}{3}+\frac{4cde^3x^{11}}{11}+\frac{ce^4x^{13}}{13}+x^9\left(\frac{ae^4}{9}+\frac{2cd^2e^2}{3}\right)+x^7\cdot\left(\frac{4ade^3}{7}+\frac{4cd^3e}{7}\right)+x^5\cdot\left(\frac{6ad^2e^2}{5}+\frac{cd^4}{5}\right)$$

input
```
integrate((e*x**2+d)**4*(c*x**4+a),x)
```

output
```
a*d**4*x + 4*a*d**3*e*x**3/3 + 4*c*d*e**3*x**11/11 + c*e**4*x**13/13 + x**
9*(a*e**4/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 4*c*d**3*e/7) + x**5
*(6*a*d**2*e**2/5 + c*d**4/5)
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \left(d+ex^2\right)^4 \left(a+cx^4\right)\,dx = \frac{1}{13}\,ce^4x^{13}+\frac{4}{11}\,cde^3x^{11}+\frac{1}{9}\left(6\,cd^2e^2+ae^4\right)x^9+\frac{4}{3}\,ad^3ex^3+\frac{4}{7}\left(cd^3e+ade^3\right)x^7+ad^4x+\frac{1}{5}\left(cd^4+6\,ad^2e^2\right)x^5$$

input
```
integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="maxima")
```

output
```
1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/9*(6*c*d^2*e^2 + a*e^4)*x^9 + 4/3*
a*d^3*e*x^3 + 4/7*(c*d^3*e + a*d*e^3)*x^7 + a*d^4*x + 1/5*(c*d^4 + 6*a*d^2
*e^2)*x^5
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^4 \left(a+cx^4\right)\,dx = \frac{1}{13}\,ce^4x^{13}+\frac{4}{11}\,cde^3x^{11}+\frac{2}{3}\,cd^2e^2x^9+\frac{1}{9}\,ae^4x^9+\frac{4}{7}\,cd^3ex^7+\frac{4}{7}\,ade^3x^7+\frac{1}{5}\,cd^4x^5+\frac{6}{5}\,ad^2e^2x^5+\frac{4}{3}\,ad^3ex^3+ad^4x$$

input
```
integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="giac")
```

output
```
1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 2/3*c*d^2*e^2*x^9 + 1/9*a*e^4*x^9 +
4/7*c*d^3*e*x^7 + 4/7*a*d*e^3*x^7 + 1/5*c*d^4*x^5 + 6/5*a*d^2*e^2*x^5 + 4/
3*a*d^3*e*x^3 + a*d^4*x
```

3.120.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)^4\left(a+cx^4\right)\,dx = x^5\left(\frac{c\,d^4}{5}+\frac{6\,a\,d^2\,e^2}{5}\right)+x^9\left(\frac{2\,c\,d^2\,e^2}{3}+\frac{a\,e^4}{9}\right)+x^7\left(\frac{4\,c\,d^3\,e}{7}+\frac{4\,a\,d\,e^3}{7}\right)+\frac{c\,e^4\,x^{13}}{13}+a\,d^4\,x+\frac{4\,a\,d^3\,e\,x^3}{3}+\frac{4\,c\,d\,e^3\,x^{11}}{11}$$

input
```
int((a + c*x^4)*(d + e*x^2)^4,x)
```

output
```
x^5*((c*d^4)/5 + (6*a*d^2*e^2)/5) + x^9*((a*e^4)/9 + (2*c*d^2*e^2)/3) + x^
7*((4*a*d*e^3)/7 + (4*c*d^3*e)/7) + (c*e^4*x^13)/13 + a*d^4*x + (4*a*d^3*e
*x^3)/3 + (4*c*d*e^3*x^11)/11
```

3.121 $\int (d+ex^2)^3 (a+cx^4) \, dx$

3.121.1 Optimal result

Integrand size = 17, antiderivative size = 79

$$\int (d+ex^2)^3 (a+cx^4) \, dx = ad^3x + ad^2ex^3 + \frac{1}{5}d(cd^2+3ae^2)\,x^5 + \frac{1}{7}e(3cd^2+ae^2)\,x^7 + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11}$$

output
```
a*d^3*x+a*d^2*e*x^3+1/5*d*(3*a*e^2+c*d^2)*x^5+1/7*e*(a*e^2+3*c*d^2)*x^7+1/
3*c*d*e^2*x^9+1/11*c*e^3*x^11
```

3.121.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^3 (a+cx^4) \, dx = ad^3x + ad^2ex^3 + \frac{1}{5}d(cd^2+3ae^2)\,x^5 + \frac{1}{7}e(3cd^2+ae^2)\,x^7 + \frac{1}{3}cde^2x^9 + \frac{1}{11}ce^3x^{11}$$

input
```
Integrate[(d + e*x^2)^3*(a + c*x^4),x]
```

output
```
a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)
*x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

3.121.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)\left(d+ex^2\right)^3\,dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(ex^6\left(ae^2+3cd^2\right)+dx^4\left(3ae^2+cd^2\right)+ad^3+3ad^2ex^2+3cde^2x^8+ce^3x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}ex^7\left(ae^2+3cd^2\right)+\frac{1}{5}dx^5\left(3ae^2+cd^2\right)+ad^3x+ad^2ex^3+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

input
```
Int[(d + e*x^2)^3*(a + c*x^4),x]
```

output
```
a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)
*x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

3.121.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.121.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{c\,e^{3} x^{11}}{11}+\frac{c d\,e^{2} x^{9}}{3}+\frac{\left(a\,e^{3}+3 c\,d^{2} e \right) x^{7}}{7}+\frac{\left(3 d\,e^{2} a +d^{3} c \right) x^{5}}{5}+a\,d^{2} e\,x^{3}+a\,d^{3} x$	72
norman	$\frac{c\,e^{3} x^{11}}{11}+\frac{c d\,e^{2} x^{9}}{3}+\left(\frac{1}{7} a\,e^{3}+\frac{3}{7} c\,d^{2} e \right) x^{7}+\left(\frac{3}{5} d\,e^{2} a +\frac{1}{5} d^{3} c \right) x^{5}+a\,d^{2} e\,x^{3}+a\,d^{3} x$	72
gosper	$\frac{1}{11} c\,e^{3} x^{11}+\frac{1}{3} c d\,e^{2} x^{9}+\frac{1}{7} x^{7} a\,e^{3}+\frac{3}{7} x^{7} c\,d^{2} e +\frac{3}{5} x^{5} d\,e^{2} a +\frac{1}{5} x^{5} d^{3} c +a\,d^{2} e\,x^{3}+a\,d^{3} x$	74
risch	$\frac{1}{11} c\,e^{3} x^{11}+\frac{1}{3} c d\,e^{2} x^{9}+\frac{1}{7} x^{7} a\,e^{3}+\frac{3}{7} x^{7} c\,d^{2} e +\frac{3}{5} x^{5} d\,e^{2} a +\frac{1}{5} x^{5} d^{3} c +a\,d^{2} e\,x^{3}+a\,d^{3} x$	74
parallelrisch	$\frac{1}{11} c\,e^{3} x^{11}+\frac{1}{3} c d\,e^{2} x^{9}+\frac{1}{7} x^{7} a\,e^{3}+\frac{3}{7} x^{7} c\,d^{2} e +\frac{3}{5} x^{5} d\,e^{2} a +\frac{1}{5} x^{5} d^{3} c +a\,d^{2} e\,x^{3}+a\,d^{3} x$	74

input
```
int((e*x^2+d)^3*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/11*c*e^3*x^11+1/3*c*d*e^2*x^9+1/7*(a*e^3+3*c*d^2*e)*x^7+1/5*(3*a*d*e^2+c
*d^3)*x^5+a*d^2*e*x^3+a*d^3*x
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \left(d+ex^2\right)^3\left(a+cx^4\right)\,dx=\frac{1}{11}\,ce^3x^{11}+\frac{1}{3}\,cde^2x^9+\frac{1}{7}\,(3\,cd^2e+ae^3)x^7+ad^2ex^3+\frac{1}{5}\,(cd^3+3\,ade^2)x^5+ad^3x$$

input
```
integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="fricas")
```

output
```
1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 1/7*(3*c*d^2*e + a*e^3)*x^7 + a*d^2*e*
x^3 + 1/5*(c*d^3 + 3*a*d*e^2)*x^5 + a*d^3*x
```

3.121.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int (d+ex^2)^3 (a+cx^4)\, dx = ad^3x + ad^2ex^3 + \frac{cde^2x^9}{3} + \frac{ce^3x^{11}}{11} + x^7\left(\frac{ae^3}{7} + \frac{3cd^2e}{7}\right) + x^5\cdot\left(\frac{3ade^2}{5} + \frac{cd^3}{5}\right)$$

input
```
integrate((e*x**2+d)**3*(c*x**4+a),x)
```

output
```
a*d**3*x + a*d**2*e*x**3 + c*d*e**2*x**9/3 + c*e**3*x**11/11 + x**7*(a*e**
3/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + c*d**3/5)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int (d+ex^2)^3 (a+cx^4)\, dx = \frac{1}{11}ce^3x^{11} + \frac{1}{3}cde^2x^9 + \frac{1}{7}(3cd^2e + ae^3)x^7 + ad^2ex^3 + \frac{1}{5}(cd^3 + 3ade^2)x^5 + ad^3x$$

input
```
integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="maxima")
```

output
```
1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 1/7*(3*c*d^2*e + a*e^3)*x^7 + a*d^2*e*
x^3 + 1/5*(c*d^3 + 3*a*d*e^2)*x^5 + a*d^3*x
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int (d+ex^2)^3 (a+cx^4)\, dx = \frac{1}{11}ce^3x^{11} + \frac{1}{3}cde^2x^9 + \frac{3}{7}cd^2ex^7 + \frac{1}{7}ae^3x^7 + \frac{1}{5}cd^3x^5 + \frac{3}{5}ade^2x^5 + ad^2ex^3 + ad^3x$$

input
```
integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="giac")
```

output
```
1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 3/7*c*d^2*e*x^7 + 1/7*a*e^3*x^7 + 1/5*
c*d^3*x^5 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + a*d^3*x
```

3.121.9 Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int (d+ex^2)^3 (a+cx^4)\,dx = x^5\left(\frac{c\,d^3}{5}+\frac{3\,a\,d\,e^2}{5}\right)+x^7\left(\frac{3\,c\,d^2\,e}{7}+\frac{a\,e^3}{7}\right)+\frac{c\,e^3\,x^{11}}{11}+a\,d^3\,x+a\,d^2\,e\,x^3+\frac{c\,d\,e^2\,x^9}{3}$$

input
```
int((a + c*x^4)*(d + e*x^2)^3,x)
```

output
```
x^5*((c*d^3)/5 + (3*a*d*e^2)/5) + x^7*((a*e^3)/7 + (3*c*d^2*e)/7) + (c*e^3
*x^11)/11 + a*d^3*x + a*d^2*e*x^3 + (c*d*e^2*x^9)/3
```

3.122 $\int (d+ex^2)^2 (a+cx^4) \, dx$

3.122.1 Optimal result

Integrand size = 17, antiderivative size = 56

$$\int (d+ex^2)^2 (a+cx^4) \, dx = ad^2x + \frac{2}{3}adex^3 + \frac{1}{5}(cd^2+ae^2)x^5 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

output
```
a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9
```

3.122.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^2 (a+cx^4) \, dx = ad^2x + \frac{2}{3}adex^3 + \frac{1}{5}(cd^2+ae^2)x^5 + \frac{2}{7}cdex^7 + \frac{1}{9}ce^2x^9$$

input
```
Integrate[(d + e*x^2)^2*(a + c*x^4),x]
```

output
```
a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c
*e^2*x^9)/9
```

3.122.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)\left(d+ex^2\right)^2 dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(x^4\left(ae^2+cd^2\right)+ad^2+2adex^2+2cdex^6+ce^2x^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

input
```
Int[(d + e*x^2)^2*(a + c*x^4),x]
```

output
```
a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c
*e^2*x^9)/9
```

3.122.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.122.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result	size
default	$a\,d^2 x+\frac{2 a d e\,x^3}{3}+\frac{(a\,e^2+c\,d^2) x^5}{5}+\frac{2 c d e\,x^7}{7}+\frac{c\,e^2 x^9}{9}$	49
norman	$\frac{c\,e^2 x^9}{9}+\frac{2 c d e\,x^7}{7}+\left(\frac{a\,e^2}{5}+\frac{c\,d^2}{5}\right) x^5+\frac{2 a d e\,x^3}{3}+a\,d^2 x$	50
gosper	$\frac{1}{9} c\,e^2 x^9+\frac{2}{7} c d e\,x^7+\frac{1}{5} x^5 a\,e^2+\frac{1}{5} x^5 c\,d^2+\frac{2}{3} a d e\,x^3+a\,d^2 x$	51
risch	$\frac{1}{9} c\,e^2 x^9+\frac{2}{7} c d e\,x^7+\frac{1}{5} x^5 a\,e^2+\frac{1}{5} x^5 c\,d^2+\frac{2}{3} a d e\,x^3+a\,d^2 x$	51
parallelrisch	$\frac{1}{9} c\,e^2 x^9+\frac{2}{7} c d e\,x^7+\frac{1}{5} x^5 a\,e^2+\frac{1}{5} x^5 c\,d^2+\frac{2}{3} a d e\,x^3+a\,d^2 x$	51

input
```
int((e*x^2+d)^2*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (d+ex^2)^2 (a+cx^4)\,dx = \frac{1}{9}\,ce^2x^9+\frac{2}{7}\,cdex^7+\frac{2}{3}\,adex^3+\frac{1}{5}\,(cd^2+ae^2)x^5+ad^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="fricas")
```

output
```
1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 2/3*a*d*e*x^3 + 1/5*(c*d^2 + a*e^2)*x^5 +
a*d^2*x
```

3.122.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^2 (a+cx^4)\,dx = ad^2x+\frac{2adex^3}{3}+\frac{2cdex^7}{7}+\frac{ce^2x^9}{9}+x^5\left(\frac{ae^2}{5}+\frac{cd^2}{5}\right)$$

input
```
integrate((e*x**2+d)**2*(c*x**4+a),x)
```

output
```
a*d**2*x + 2*a*d*e*x**3/3 + 2*c*d*e*x**7/7 + c*e**2*x**9/9 + x**5*(a*e**2/
5 + c*d**2/5)
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)\,dx=\frac{1}{9}\,ce^2x^9+\frac{2}{7}\,cdex^7+\frac{2}{3}\,adex^3+\frac{1}{5}\,\left(cd^2+ae^2\right)x^5+ad^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="maxima")
```

output
```
1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 2/3*a*d*e*x^3 + 1/5*(c*d^2 + a*e^2)*x^5 +
a*d^2*x
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)\,dx=\frac{1}{9}\,ce^2x^9+\frac{2}{7}\,cdex^7+\frac{1}{5}\,cd^2x^5+\frac{1}{5}\,ae^2x^5+\frac{2}{3}\,adex^3+ad^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="giac")
```

output
```
1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 1/5*c*d^2*x^5 + 1/5*a*e^2*x^5 + 2/3*a*d*e*
x^3 + a*d^2*x
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)\,dx=x^5\left(\frac{c\,d^2}{5}+\frac{a\,e^2}{5}\right)+\frac{c\,e^2\,x^9}{9}+a\,d^2\,x+\frac{2\,a\,d\,e\,x^3}{3}+\frac{2\,c\,d\,e\,x^7}{7}$$

input
```
int((a + c*x^4)*(d + e*x^2)^2,x)
```

output
```
x^5*((a*e^2)/5 + (c*d^2)/5) + (c*e^2*x^9)/9 + a*d^2*x + (2*a*d*e*x^3)/3 +
(2*c*d*e*x^7)/7
```

3.123 $\int (d+ex^2)(a+cx^4)\,dx$

3.123.1 Optimal result

Integrand size = 15, antiderivative size = 32

$$\int (d+ex^2)(a+cx^4)\,dx = adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

output
```
a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

3.123.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (d+ex^2)(a+cx^4)\,dx = adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

input
```
Integrate[(d + e*x^2)*(a + c*x^4),x]
```

output
```
a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

3.123.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)\left(d+ex^2\right)\,dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(ad+aex^2+cdx^4+cex^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

input
```
Int[(d + e*x^2)*(a + c*x^4),x]
```

output
```
a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

3.123.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.123.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
gosper	$adx+\frac{1}{3}ae\,x^{3}+\frac{1}{5}cd\,x^{5}+\frac{1}{7}ce\,x^{7}$	27
default	$adx+\frac{1}{3}ae\,x^{3}+\frac{1}{5}cd\,x^{5}+\frac{1}{7}ce\,x^{7}$	27
norman	$adx+\frac{1}{3}ae\,x^{3}+\frac{1}{5}cd\,x^{5}+\frac{1}{7}ce\,x^{7}$	27
risch	$adx+\frac{1}{3}ae\,x^{3}+\frac{1}{5}cd\,x^{5}+\frac{1}{7}ce\,x^{7}$	27
parallelrisch	$adx+\frac{1}{3}ae\,x^{3}+\frac{1}{5}cd\,x^{5}+\frac{1}{7}ce\,x^{7}$	27

input
```
int((e*x^2+d)*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=\frac{1}{7}\,cex^7+\frac{1}{5}\,cdx^5+\frac{1}{3}\,aex^3+adx$$

input
```
integrate((e*x^2+d)*(c*x^4+a),x, algorithm="fricas")
```

output
```
1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

3.123.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \left(d+ex^2\right)\left(a+cx^4\right)\,dx=adx+\frac{aex^3}{3}+\frac{cdx^5}{5}+\frac{cex^7}{7}$$

input
```
integrate((e*x**2+d)*(c*x**4+a),x)
```

output
```
a*d*x + a*e*x**3/3 + c*d*x**5/5 + c*e*x**7/7
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int (d+ex^2)(a+cx^4)\,dx = \frac{1}{7}cex^7 + \frac{1}{5}cdx^5 + \frac{1}{3}aex^3 + adx$$

input
```
integrate((e*x^2+d)*(c*x^4+a),x, algorithm="maxima")
```

output
```
1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int (d+ex^2)(a+cx^4)\,dx = \frac{1}{7}cex^7 + \frac{1}{5}cdx^5 + \frac{1}{3}aex^3 + adx$$

input
```
integrate((e*x^2+d)*(c*x^4+a),x, algorithm="giac")
```

output
```
1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/3*a*e*x^3 + a*d*x
```

3.123.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int (d+ex^2)(a+cx^4)\,dx = \frac{c\,e\,x^7}{7} + \frac{c\,d\,x^5}{5} + \frac{a\,e\,x^3}{3} + a\,d\,x$$

input
```
int((a + c*x^4)*(d + e*x^2),x)
```

output
```
a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

3.124 $\int \frac{a+cx^4}{d+ex^2}\,dx$

3.124.1 Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{a+cx^4}{d+ex^2}\,dx=-\frac{cdx}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

output
```
-c*d*x/e^2+1/3*c*x^3/e+(a*e^2+c*d^2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/d^(
1/2)
```

3.124.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{a+cx^4}{d+ex^2}\,dx=-\frac{cdx}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2),x]
```

output
```
-((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(Sqrt[d]*e^(5/2))
```

3.124.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{d+ex^2} \, dx$$

$$\Big\downarrow \text{ 1468}$$

$$\int \left(\frac{ae^2+cd^2}{e^2(d+ex^2)}-\frac{cd}{e^2}+\frac{cx^2}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(ae^2+cd^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

input
```
Int[(a + c*x^4)/(d + e*x^2),x]
```

output
```
-((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(Sqrt[d]*e^(5/2))
```

3.124.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.124.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{c\left(-\frac{1}{3}e\,x^{3}+dx\right)}{e^{2}}+\frac{\left(a\,e^{2}+c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{e^{2}\sqrt{ed}}$	47
risch	$\frac{c\,x^{3}}{3e}-\frac{cdx}{e^{2}}-\frac{\ln\left(ex+\sqrt{-ed}\right)a}{2\sqrt{-ed}}-\frac{\ln\left(ex+\sqrt{-ed}\right)c\,d^{2}}{2e^{2}\sqrt{-ed}}+\frac{\ln\left(-ex+\sqrt{-ed}\right)a}{2\sqrt{-ed}}+\frac{\ln\left(-ex+\sqrt{-ed}\right)c\,d^{2}}{2e^{2}\sqrt{-ed}}$	113

input
```
int((c*x^4+a)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-c/e^2*(-1/3*e*x^3+d*x)+(a*e^2+c*d^2)/e^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/
2))
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.38

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \left[\frac{2\,cde^2x^3-6\,cd^2ex-3\,(cd^2+ae^2)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)}{6\,de^3}, \frac{cde^2x^3-3\,cd^2ex+3\,(cd^2+ae^2)\sqrt{de}\,\text{arcta}\right.$$

input
```
integrate((c*x^4+a)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/6*(2*c*d*e^2*x^3 - 6*c*d^2*e*x - 3*(c*d^2 + a*e^2)*sqrt(-d*e)*log((e*x^
2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d*e^3), 1/3*(c*d*e^2*x^3 - 3*c*d^2*
e*x + 3*(c*d^2 + a*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d*e^3)]
```

3.124.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(49) = 98$.

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.89

$$\int \frac{a+cx^4}{d+ex^2}\,dx = -\frac{cdx}{e^2} + \frac{cx^3}{3e} - \frac{\sqrt{-\frac{1}{de^5}}\,(ae^2+cd^2)\log\left(-de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}}\,(ae^2+cd^2)\log\left(de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}$$

input
```
integrate((c*x**4+a)/(e*x**2+d),x)
```

output
```
-c*d*x/e**2 + c*x**3/(3*e) - sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(-d*e*
*2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(d*e*
*2*sqrt(-1/(d*e**5)) + x)/2
```

3.124.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \frac{(cd^2+ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^2} + \frac{ce^2x^3-3\,cdex}{3\,e^3}$$

input
```
integrate((c*x^4+a)/(e*x^2+d),x, algorithm="giac")
```

output
```
(c*d^2 + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) + 1/3*(c*e^2*x^3 - 3
*c*d*e*x)/e^3
```

3.124.9 Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{a+cx^4}{d+ex^2}\,dx = \frac{c\,x^3}{3\,e} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(c\,d^2+a\,e^2)}{\sqrt{d}\,e^{5/2}} - \frac{c\,d\,x}{e^2}$$

input
```
int((a + c*x^4)/(d + e*x^2),x)
```

output
```
(c*x^3)/(3*e) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c*d^2))/(d^(1/2)*e^(5/
2)) - (c*d*x)/e^2
```

3.125 $\int \frac{a+cx^4}{(d+ex^2)^2} \, dx$

3.125.1 Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{a+cx^4}{(d+ex^2)^2} \, dx = \frac{cx}{e^2}+\frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d(d+ex^2)}-\frac{(3cd^2-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

output
```
c*x/e^2+1/2*(a+c*d^2/e^2)*x/d/(e*x^2+d)-1/2*(-a*e^2+3*c*d^2)*arctan(x*e^(1
/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^4}{(d+ex^2)^2} \, dx = \frac{cx}{e^2}+\frac{(cd^2+ae^2)x}{2de^2(d+ex^2)}-\frac{(3cd^2-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2)^2,x]
```

output
```
(c*x)/e^2 + ((c*d^2 + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 - a*e^2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

3.125.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1472, 25, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \textbf{1472}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d(d+ex^2)}-\frac{\int -\frac{2cdx^2+\left(a-\frac{cd^2}{e^2}\right)e}{e(ex^2+d)}dx}{2d}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{2cdx^2+\left(a-\frac{cd^2}{e^2}\right)e}{e(ex^2+d)}dx}{2d}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d(d+ex^2)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{2cdx^2+\left(a-\frac{cd^2}{e^2}\right)e}{ex^2+d}dx}{2de}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d(d+ex^2)}$$

$$\Big\downarrow \textbf{299}$$

$$\frac{\frac{2cdx}{e}-\frac{(3cd^2-ae^2)\int \frac{1}{ex^2+d}dx}{e}}{2de}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d(d+ex^2)}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{\frac{2cdx}{e}-\frac{(3cd^2-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}}{2de}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d(d+ex^2)}$$

input
```
Int[(a + c*x^4)/(d + e*x^2)^2,x]
```

output
```
((a + (c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) + ((2*c*d*x)/e - ((3*c*d^2 - a*e^2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(3/2)))/(2*d*e)
```

3.125.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.125.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{cx}{e^2}+\frac{\frac{\left(a\,e^2+c\,d^2\right)x}{2d\left(e\,x^2+d\right)}+\frac{\left(a\,e^2-3c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^2}$	70
risch	$\frac{cx}{e^2}+\frac{\left(a\,e^2+c\,d^2\right)x}{2d\,e^2\left(e\,x^2+d\right)}-\frac{\ln\left(ex+\sqrt{-ed}\right)a}{4\sqrt{-ed}\,d}+\frac{3d\ln\left(ex+\sqrt{-ed}\right)c}{4e^2\sqrt{-ed}}+\frac{\ln\left(-ex+\sqrt{-ed}\right)a}{4\sqrt{-ed}\,d}-\frac{3d\ln\left(-ex+\sqrt{-ed}\right)c}{4e^2\sqrt{-ed}}$	133

input
```
int((c*x^4+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*x/e^2+1/e^2*(1/2*(a*e^2+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2-3*c*d^2)/d/(e*d)
^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.00

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx$$
$$= \left[\frac{4\,cd^2e^2x^3 + (3\,cd^3 - ade^2 + (3\,cd^2e - ae^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right) + 2\,(3\,cd^3e + ade^3)x}{4\,(d^2e^4x^2 + d^3e^3)}, \frac{2\,cd^2e^2x^3}{}\right.$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - a*d*e^2 + (3*c*d^2*e - a*e^3)*x^2)*sqrt
(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^3*e + a*d*
e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2*e^2*x^3 - (3*c*d^3 - a*d*e^2
 + (3*c*d^2*e - a*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e +
 a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3)]
```

3.125.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(68) = 136.

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.86

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} + \frac{x(ae^2+cd^2)}{2d^2e^2+2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2-3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2-3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

input
```
integrate((c*x**4+a)/(e*x**2+d)**2,x)
```

output
```
c*x/e**2 + x*(a*e**2 + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt(-1/(d*
*3*e**5))*(a*e**2 - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4 +
 sqrt(-1/(d**3*e**5))*(a*e**2 - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5
)) + x)/4
```

3.125.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + cx^4}{(d + ex^2)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{a + cx^4}{(d + ex^2)^2} \, dx = \frac{cx}{e^2} - \frac{(3\,cd^2 - ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^2} + \frac{cd^2x + ae^2x}{2\,(ex^2 + d)de^2}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
c*x/e^2 - 1/2*(3*c*d^2 - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^2) +
1/2*(c*d^2*x + a*e^2*x)/((e*x^2 + d)*d*e^2)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{a + cx^4}{(d + ex^2)^2} \, dx = \frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(a\,e^2 - 3\,c\,d^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2 + a\,e^2\right)}{2\,d\,\left(e^3\,x^2 + d\,e^2\right)}$$

input
```
int((a + c*x^4)/(d + e*x^2)^2,x)
```

output
```
(c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2))/(2*d^(3/2)*e^(5/
2)) + (x*(a*e^2 + c*d^2))/(2*d*(d*e^2 + e^3*x^2))
```

3.126 $\int \frac{a+cx^4}{\left(d+ex^2\right)^3} dx$

3.126.1 Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3} dx = \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2}+\frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8\left(d+ex^2\right)}+\frac{3(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

output
```
1/4*(a+c*d^2/e^2)*x/d/(e*x^2+d)^2+1/8*(3*a/d^2-5*c/e^2)*x/(e*x^2+d)+3/8*(a
*e^2+c*d^2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(5/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3} dx = \frac{ae^2x(5d+3ex^2)-cd^2x(3d+5ex^2)}{8d^2e^2\left(d+ex^2\right)^2}+\frac{3(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2)^3,x]
```

output
```
(a*e^2*x*(5*d + 3*e*x^2) - c*d^2*x*(3*d + 5*e*x^2))/(8*d^2*e^2*(d + e*x^2)
^2) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

3.126.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1472, 25, 25, 27, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}-\frac{\int -\frac{4cdx^2+\left(3a-\frac{cd^2}{e^2}\right)e}{e(ex^2+d)^2}dx}{4d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{\frac{cd^2}{e}-4cx^2d-3ae}{e(ex^2+d)^2}dx}{4d}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}-\frac{\int \frac{\frac{cd^2}{e}-4cx^2d-3ae}{e(ex^2+d)^2}dx}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}-\frac{\int \frac{\frac{cd^2}{e}-4cx^2d-3ae}{(ex^2+d)^2}dx}{4de}$$

$$\Big\downarrow \text{298}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}-\frac{\frac{x\left(\frac{5cd}{e}-\frac{3ae}{d}\right)}{2(d+ex^2)}-\frac{3(ae^2+cd^2)\int \frac{1}{ex^2+d}dx}{2de}}{4de}$$

$$\Big\downarrow \text{218}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d(d+ex^2)^2}-\frac{\frac{x\left(\frac{5cd}{e}-\frac{3ae}{d}\right)}{2(d+ex^2)}-\frac{3(ae^2+cd^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{3/2}}}{4de}$$

input
```
Int[(a + c*x^4)/(d + e*x^2)^3,x]
```

output
```
((a + (c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) - ((((5*c*d)/e - (3*a*e)/d)*x)/(
2*(d + e*x^2)) - (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2
)*e^(3/2)))/(4*d*e)
```

3.126.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.126.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\frac{(3a e^2-5c d^2)x^3}{8d^2 e}+\frac{(5a e^2-3c d^2)x}{8d e^2}}{(e x^2+d)^2}+\frac{3(a e^2+c d^2)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8e^2 d^2\sqrt{ed}}$	92
risch	$\frac{\frac{(3a e^2-5c d^2)x^3}{8d^2 e}+\frac{(5a e^2-3c d^2)x}{8d e^2}}{(e x^2+d)^2}-\frac{3\ln(ex+\sqrt{-ed})a}{16\sqrt{-ed}\,d^2}-\frac{3\ln(ex+\sqrt{-ed})c}{16\sqrt{-ed}\,e^2}+\frac{3\ln(-ex+\sqrt{-ed})a}{16\sqrt{-ed}\,d^2}+\frac{3\ln(-ex+\sqrt{-ed})c}{16\sqrt{-ed}\,e^2}$	153

input
```
int((c*x^4+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*a*e^2-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-3*c*d^2)/d/e^2*x)/(e*x^2+d)^
2+3/8*(a*e^2+c*d^2)/e^2/d^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.29

$$\int \frac{a+cx^4}{(d+ex^2)^3}\,dx = \left[-\frac{2\,(5\,cd^3e^2-3\,ade^4)x^3+3\,(cd^4+ad^2e^2+(cd^2e^2+ae^4)x^4+2\,(cd^3e+ade^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x-}{ex^2+d}\right.}{16\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}\right.$$
$$-\frac{(5\,cd^3e^2-3\,ade^4)x^3-3\,(cd^4+ad^2e^2+(cd^2e^2+ae^4)x^4+2\,(cd^3e+ade^3)x^2)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)+(3\,c}{8\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(5*c*d^3*e^2 - 3*a*d*e^4)*x^3 + 3*(c*d^4 + a*d^2*e^2 + (c*d^2*e^
2 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt
(-d*e)*x - d)/(e*x^2 + d)) + 2*(3*c*d^4*e - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 +
 2*d^4*e^4*x^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - 3*a*d*e^4)*x^3 - 3*(c*d^4
+ a*d^2*e^2 + (c*d^2*e^2 + a*e^4)*x^4 + 2*(c*d^3*e + a*d*e^3)*x^2)*sqrt(d*
e)*arctan(sqrt(d*e)*x/d) + (3*c*d^4*e - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d
^4*e^4*x^2 + d^5*e^3)]
```

3.126.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(90) = 180$.

Time = 0.35 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.35

$$\int \frac{a+cx^4}{(d+ex^2)^3}\,dx = -\frac{3\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)\log\left(-\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)}{3ae^2+3cd^2}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)\log\left(\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}(ae^2+cd^2)}{3ae^2+3cd^2}+x\right)}{16} + \frac{x^3\cdot(3ae^3-5cd^2e)+x(5ade^2-3cd^3)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

input
```
integrate((c*x**4+a)/(e*x**2+d)**3,x)
```

output
```
-3*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)*log(-3*d**3*e**2*sqrt(-1/(d**5*e
**5))*(a*e**2 + c*d**2)/(3*a*e**2 + 3*c*d**2) + x)/16 + 3*sqrt(-1/(d**5*e*
*5))*(a*e**2 + c*d**2)*log(3*d**3*e**2*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d*
*2)/(3*a*e**2 + 3*c*d**2) + x)/16 + (x**3*(3*a*e**3 - 5*c*d**2*e) + x*(5*a
*d*e**2 - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**4)
```

3.126.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{3\left(cd^2+ae^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}d^2e^2} - \frac{5\,cd^2ex^3-3\,ae^3x^3+3\,cd^3x-5\,ade^2x}{8\left(ex^2+d\right)^2d^2e^2}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
3/8*(c*d^2 + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^2) - 1/8*(5*c*d
^2*e*x^3 - 3*a*e^3*x^3 + 3*c*d^3*x - 5*a*d*e^2*x)/((e*x^2 + d)^2*d^2*e^2)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 13.73 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{\frac{x^3\left(3\,a\,e^2-5\,c\,d^2\right)}{8\,d^2\,e}+\frac{x\left(5\,a\,e^2-3\,c\,d^2\right)}{8\,d\,e^2}}{d^2+2\,d\,e\,x^2+e^2\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(c\,d^2+a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}}$$

input
```
int((a + c*x^4)/(d + e*x^2)^3,x)
```

output
```
((x^3*(3*a*e^2 - 5*c*d^2))/(8*d^2*e) + (x*(5*a*e^2 - 3*c*d^2))/(8*d*e^2))/
(d^2 + e^2*x^4 + 2*d*e*x^2) + (3*atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c*d^2)
)/(8*d^(5/2)*e^(5/2))
```

3.127 $\int \frac{a+cx^4}{(d+ex^2)^4} dx$

3.127.1 Optimal result

Integrand size = 17, antiderivative size = 123

$$\int \frac{a+cx^4}{(d+ex^2)^4} dx = \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d(d+ex^2)^3}+\frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24(d+ex^2)^2}+\frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d(d+ex^2)}+\frac{(cd^2+5ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

output
```
1/6*(a+c*d^2/e^2)*x/d/(e*x^2+d)^3+1/24*(5*a/d^2-7*c/e^2)*x/(e*x^2+d)^2+1/1
6*(5*a/d^2+c/e^2)*x/d/(e*x^2+d)+1/16*(5*a*e^2+c*d^2)*arctan(x*e^(1/2)/d^(1
/2))/d^(7/2)/e^(5/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^4}{(d+ex^2)^4} dx = \frac{x(cd^2(-3d^2-8dex^2+3e^2x^4)+ae^2(33d^2+40dex^2+15e^2x^4))}{48d^3e^2(d+ex^2)^3} + \frac{(cd^2+5ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

input
```
Integrate[(a + c*x^4)/(d + e*x^2)^4,x]
```

output
```
(x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + a*e^2*(33*d^2 + 40*d*e*x^2 +
15*e^2*x^4)))/(48*d^3*e^2*(d + e*x^2)^3) + ((c*d^2 + 5*a*e^2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

3.127.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {1472, 25, 25, 27, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{(d+ex^2)^4}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d(d+ex^2)^3}-\frac{\int -\frac{6cdx^2+\left(5a-\frac{cd^2}{e^2}\right)e}{e(ex^2+d)^3}dx}{6d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{\frac{cd^2}{e}-6cx^2d-5ae}{e(ex^2+d)^3}dx}{6d}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d(d+ex^2)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d(d+ex^2)^3}-\frac{\int \frac{\frac{cd^2}{e}-6cx^2d-5ae}{e(ex^2+d)^3}dx}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d(d+ex^2)^3}-\frac{\int \frac{\frac{cd^2}{e}-6cx^2d-5ae}{(ex^2+d)^3}dx}{6de}$$

$$\Big\downarrow \text{298}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d(d+ex^2)^3}-\frac{\frac{x\left(\frac{7cd}{e}-\frac{5ae}{d}\right)}{4(d+ex^2)^2}-\frac{3}{4}\left(\frac{5ae}{d}+\frac{cd}{e}\right)\int \frac{1}{(ex^2+d)^2}dx}{6de}$$

$$\Big\downarrow \text{215}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d(d+ex^2)^3}-\frac{\frac{x\left(\frac{7cd}{e}-\frac{5ae}{d}\right)}{4(d+ex^2)^2}-\frac{3}{4}\left(\frac{5ae}{d}+\frac{cd}{e}\right)\left(\frac{\int \frac{1}{ex^2+d}dx}{2d}+\frac{x}{2d(d+ex^2)}\right)}{6de}$$

$$\Big\downarrow \text{218}$$

$$\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}-\frac{\frac{x\left(\frac{7cd}{e}-\frac{5ae}{d}\right)}{4(d+ex^2)^2}-\frac{3}{4}\left(\frac{5ae}{d}+\frac{cd}{e}\right)\left(\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}+\frac{x}{2d(d+ex^2)}\right)}{6de}$$

input
```
Int[(a + c*x^4)/(d + e*x^2)^4,x]
```

output
```
((a + (c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) - ((((7*c*d)/e - (5*a*e)/d)*x)/(
4*(d + e*x^2)^2) - (3*((c*d)/e + (5*a*e)/d)*(x/(2*d*(d + e*x^2)) + ArcTan[
(Sqrt[e]*x)/Sqrt[d]]/(2*d^(3/2)*Sqrt[e])))/4)/(6*d*e)
```

3.127.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.127.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

method	result
default	$\frac{\frac{\left(5a\,e^2+c\,d^2\right)x^5}{16d^3}+\frac{\left(5a\,e^2-c\,d^2\right)x^3}{6d^2e}+\frac{\left(11a\,e^2-c\,d^2\right)x}{16d\,e^2}}{\left(e\,x^2+d\right)^3}+\frac{\left(5a\,e^2+c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{16d^3e^2\sqrt{ed}}$
risch	$\frac{\frac{\left(5a\,e^2+c\,d^2\right)x^5}{16d^3}+\frac{\left(5a\,e^2-c\,d^2\right)x^3}{6d^2e}+\frac{\left(11a\,e^2-c\,d^2\right)x}{16d\,e^2}}{\left(e\,x^2+d\right)^3}-\frac{5\ln\left(ex+\sqrt{-ed}\right)a}{32\sqrt{-ed}\,d^3}-\frac{\ln\left(ex+\sqrt{-ed}\right)c}{32\sqrt{-ed}\,e^2d}+\frac{5\ln\left(-ex+\sqrt{-ed}\right)a}{32\sqrt{-ed}\,d^3}+\frac{\ln\left(-ex+\sqrt{-\right.}}{32\sqrt{-ed}\,e}$

input
```
int((c*x^4+a)/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*a*e^2+c*d^2)/d^3*x^5+1/6*(5*a*e^2-c*d^2)/d^2/e*x^3+1/16*(11*a*e^2
-c*d^2)/d/e^2*x)/(e*x^2+d)^3+1/16*(5*a*e^2+c*d^2)/d^3/e^2/(e*d)^(1/2)*arct
an(e*x/(e*d)^(1/2))
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.45

$$\int \frac{a+cx^4}{\left(d+ex^2\right)^4}\,dx$$
$$=\left[\frac{6\,(cd^3e^3+5\,ade^5)x^5-16\,(cd^4e^2-5\,ad^2e^4)x^3-3\,((cd^2e^3+5\,ae^5)x^6+cd^5+5\,ad^3e^2+3\,(cd^3e^2+5\,ade^4}{96\,(d^4e^6x^6+3\,d^5e^5x^4+3\,d^6e^4x^2+}\right.$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="fricas")
```

output
```
[1/96*(6*(c*d^3*e^3 + 5*a*d*e^5)*x^5 - 16*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 -
3*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5 + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*
e^4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-
d*e)*x - d)/(e*x^2 + d)) - 6*(c*d^5*e - 11*a*d^3*e^3)*x)/(d^4*e^6*x^6 + 3*
d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3), 1/48*(3*(c*d^3*e^3 + 5*a*d*e^5)*x^
5 - 8*(c*d^4*e^2 - 5*a*d^2*e^4)*x^3 + 3*((c*d^2*e^3 + 5*a*e^5)*x^6 + c*d^5
 + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + 5*a*d^2*e^3)
*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*d^5*e - 11*a*d^3*e^3)*x)/(d^4
*e^6*x^6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]
```

3.127.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.66

$$\int \frac{a+cx^4}{(d+ex^2)^4}\,dx = -\frac{\sqrt{-\frac{1}{d^7e^5}}\cdot(5ae^2+cd^2)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7e^5}}\cdot(5ae^2+cd^2)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32} + \frac{x^5\cdot(15ae^4+3cd^2e^2)+x^3\cdot(40ade^3-8cd^3e)+x(33ad^2e^2-3cd^4)}{48d^6e^2+144d^5e^3x^2+144d^4e^4x^4+48d^3e^5x^6}$$

input
```
integrate((c*x**4+a)/(e*x**2+d)**4,x)
```

output
```
-sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(-d**4*e**2*sqrt(-1/(d**7*e**
5)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(d**4*e**2*sqrt(
-1/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*c*d**2*e**2) + x**3*(40*a*d
*e**3 - 8*c*d**3*e) + x*(33*a*d**2*e**2 - 3*c*d**4))/(48*d**6*e**2 + 144*d
**5*e**3*x**2 + 144*d**4*e**4*x**4 + 48*d**3*e**5*x**6)
```

3.127.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^4}{(d+ex^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^4}{(d+ex^2)^4}\,dx = \frac{(cd^2+5\,ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}d^3e^2} + \frac{3\,cd^2e^2x^5+15\,ae^4x^5-8\,cd^3ex^3+40\,ade^3x^3-3\,cd^4x+33\,ad^2e^2x}{48\,(ex^2+d)^3d^3e^2}$$

input
```
integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="giac")
```

output
```
1/16*(c*d^2 + 5*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e^2) + 1/48*(3
*c*d^2*e^2*x^5 + 15*a*e^4*x^5 - 8*c*d^3*e*x^3 + 40*a*d*e^3*x^3 - 3*c*d^4*x
 + 33*a*d^2*e^2*x)/((e*x^2 + d)^3*d^3*e^2)
```

3.127.9 Mupad [B] (verification not implemented)

Time = 13.97 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^4}{(d+ex^2)^4}\,dx = \frac{\frac{x^5\,(c\,d^2+5\,a\,e^2)}{16\,d^3}+\frac{x^3\,(5\,a\,e^2-c\,d^2)}{6\,d^2\,e}+\frac{x\,(11\,a\,e^2-c\,d^2)}{16\,d\,e^2}}{d^3+3\,d^2\,e\,x^2+3\,d\,e^2\,x^4+e^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(c\,d^2+5\,a\,e^2)}{16\,d^{7/2}\,e^{5/2}}$$

input
```
int((a + c*x^4)/(d + e*x^2)^4,x)
```

output
```
((x^5*(5*a*e^2 + c*d^2))/(16*d^3) + (x^3*(5*a*e^2 - c*d^2))/(6*d^2*e) + (x
*(11*a*e^2 - c*d^2))/(16*d*e^2))/(d^3 + e^3*x^6 + 3*d^2*e*x^2 + 3*d*e^2*x^
4) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2))/(16*d^(7/2)*e^(5/2))
```

3.128 $\int (d+ex^2)^3 (a+cx^4)^2 \, dx$

3.128.1 Optimal result

Integrand size = 19, antiderivative size = 133

$$\int (d+ex^2)^3 (a+cx^4)^2 \, dx = a^2d^3x + a^2d^2ex^3 + \frac{1}{5}ad(2cd^2+3ae^2)x^5 + \frac{1}{7}ae(6cd^2+ae^2)x^7 + \frac{1}{9}cd(cd^2+6ae^2)x^9 + \frac{1}{11}ce(3cd^2+2ae^2)x^{11} + \frac{3}{13}c^2de^2x^{13} + \frac{1}{15}c^2e^3x^{15}$$

output
```
a^2*d^3*x+a^2*d^2*e*x^3+1/5*a*d*(3*a*e^2+2*c*d^2)*x^5+1/7*a*e*(a*e^2+6*c*d
^2)*x^7+1/9*c*d*(6*a*e^2+c*d^2)*x^9+1/11*c*e*(2*a*e^2+3*c*d^2)*x^11+3/13*c
^2*d*e^2*x^13+1/15*c^2*e^3*x^15
```

3.128.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^3 (a+cx^4)^2 \, dx = a^2d^3x + a^2d^2ex^3 + \frac{1}{5}ad(2cd^2+3ae^2)x^5 + \frac{1}{7}ae(6cd^2+ae^2)x^7 + \frac{1}{9}cd(cd^2+6ae^2)x^9 + \frac{1}{11}ce(3cd^2+2ae^2)x^{11} + \frac{3}{13}c^2de^2x^{13} + \frac{1}{15}c^2e^3x^{15}$$

input
```
Integrate[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

output
```
a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^
2 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^
2)*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

3.128.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2 \left(d+ex^2\right)^3 \, dx$$

$$\Big\downarrow \text{ 1468}$$

$$\int \left(a^2d^3+3a^2d^2ex^2+cex^{10}\left(2ae^2+3cd^2\right)+cdx^8\left(6ae^2+cd^2\right)+aex^6\left(ae^2+6cd^2\right)+adx^4\left(3ae^2+2cd^2\right)+3c^2de$$

$$\Big\downarrow \text{ 2009}$$

$$a^2d^3x+a^2d^2ex^3+\frac{1}{11}cex^{11}\left(2ae^2+3cd^2\right)+\frac{1}{9}cdx^9\left(6ae^2+cd^2\right)+\frac{1}{7}aex^7\left(ae^2+6cd^2\right)+\frac{1}{5}adx^5\left(3ae^2+2cd^2\right)+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}$$

input
```
Int[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

output
```
a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^
2 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^
2)*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

3.128.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.128.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

method	result
norman	$a^2d^3x + a^2d^2e\,x^3 + \left(\frac{3}{5}d\,e^2a^2 + \frac{2}{5}d^3ac\right)x^5 + \left(\frac{1}{7}e^3a^2 + \frac{6}{7}d^2eac\right)x^7 + \left(\frac{2}{3}acd\,e^2 + \frac{1}{9}c^2d^3\right)x^9 + ($
default	$\frac{c^2e^3x^{15}}{15} + \frac{3c^2d\,e^2x^{13}}{13} + \frac{(2e^3ac+3d^2e\,c^2)x^{11}}{11} + \frac{(6acd\,e^2+c^2d^3)x^9}{9} + \frac{(e^3a^2+6d^2eac)x^7}{7} + \frac{(3d\,e^2a^2+2d^3ac)x^5}{5} + a^2$
gosper	$a^2d^3x + a^2d^2e\,x^3 + \frac{3}{5}x^5d\,e^2a^2 + \frac{2}{5}x^5d^3ac + \frac{1}{7}x^7e^3a^2 + \frac{6}{7}x^7d^2eac + \frac{2}{3}x^9acd\,e^2 + \frac{1}{9}x^9c^2d^3 + \frac{2}{11}$
risch	$a^2d^3x + a^2d^2e\,x^3 + \frac{3}{5}x^5d\,e^2a^2 + \frac{2}{5}x^5d^3ac + \frac{1}{7}x^7e^3a^2 + \frac{6}{7}x^7d^2eac + \frac{2}{3}x^9acd\,e^2 + \frac{1}{9}x^9c^2d^3 + \frac{2}{11}$
parallelrisch	$a^2d^3x + a^2d^2e\,x^3 + \frac{3}{5}x^5d\,e^2a^2 + \frac{2}{5}x^5d^3ac + \frac{1}{7}x^7e^3a^2 + \frac{6}{7}x^7d^2eac + \frac{2}{3}x^9acd\,e^2 + \frac{1}{9}x^9c^2d^3 + \frac{2}{11}$

input
```
int((e*x^2+d)^3*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^3*x+a^2*d^2*e*x^3+(3/5*d*e^2*a^2+2/5*d^3*a*c)*x^5+(1/7*e^3*a^2+6/7*d
^2*e*a*c)*x^7+(2/3*a*c*d*e^2+1/9*c^2*d^3)*x^9+(2/11*e^3*a*c+3/11*d^2*e*c^2
)*x^11+3/13*c^2*d*e^2*x^13+1/15*c^2*e^3*x^15
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int \left(d+ex^2\right)^3 \left(a+cx^4\right)^2 \,dx = \frac{1}{15}\,c^2e^3x^{15} + \frac{3}{13}\,c^2de^2x^{13} + \frac{1}{11}\left(3\,c^2d^2e + 2\,ace^3\right)x^{11} + \frac{1}{9}\left(c^2d^3 + 6\,acde^2\right)x^9 + a^2d^2ex^3 + \frac{1}{7}\left(6\,acd^2e + a^2e^3\right)x^7 + a^2d^3x + \frac{1}{5}\left(2\,acd^3 + 3\,a^2de^2\right)x^5$$

input
```
integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 1/11*(3*c^2*d^2*e + 2*a*c*e^3)*x
^11 + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + a^2*d^2*e*x^3 + 1/7*(6*a*c*d^2*e +
 a^2*e^3)*x^7 + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*a^2*d*e^2)*x^5
```

3.128.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

$$\int (d+ex^2)^3 (a+cx^4)^2 \, dx = a^2d^3x + a^2d^2ex^3 + \frac{3c^2de^2x^{13}}{13} + \frac{c^2e^3x^{15}}{15} + x^{11} \cdot \left(\frac{2ace^3}{11} + \frac{3c^2d^2e}{11}\right) + x^9 \cdot \left(\frac{2acde^2}{3} + \frac{c^2d^3}{9}\right) + x^7\left(\frac{a^2e^3}{7} + \frac{6acd^2e}{7}\right) + x^5 \cdot \left(\frac{3a^2de^2}{5} + \frac{2acd^3}{5}\right)$$

input
```
integrate((e*x**2+d)**3*(c*x**4+a)**2,x)
```

output
```
a**2*d**3*x + a**2*d**2*e*x**3 + 3*c**2*d*e**2*x**13/13 + c**2*e**3*x**15/
15 + x**11*(2*a*c*e**3/11 + 3*c**2*d**2*e/11) + x**9*(2*a*c*d*e**2/3 + c**
2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*c*d**2*e/7) + x**5*(3*a**2*d*e**2/5 +
2*a*c*d**3/5)
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int (d+ex^2)^3 (a+cx^4)^2 \, dx = \frac{1}{15}c^2e^3x^{15} + \frac{3}{13}c^2de^2x^{13} + \frac{1}{11}(3c^2d^2e + 2ace^3)x^{11} + \frac{1}{9}(c^2d^3 + 6acde^2)x^9 + a^2d^2ex^3 + \frac{1}{7}(6acd^2e + a^2e^3)x^7 + a^2d^3x + \frac{1}{5}(2acd^3 + 3a^2de^2)x^5$$

input
```
integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 1/11*(3*c^2*d^2*e + 2*a*c*e^3)*x
^11 + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + a^2*d^2*e*x^3 + 1/7*(6*a*c*d^2*e +
 a^2*e^3)*x^7 + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*a^2*d*e^2)*x^5
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int (d+ex^2)^3 (a+cx^4)^2 \, dx = \frac{1}{15} c^2e^3x^{15} + \frac{3}{13} c^2de^2x^{13} + \frac{3}{11} c^2d^2ex^{11} + \frac{2}{11} ace^3x^{11} + \frac{1}{9} c^2d^3x^9 + \frac{2}{3} acde^2x^9 + \frac{6}{7} acd^2ex^7 + \frac{1}{7} a^2e^3x^7 + \frac{2}{5} acd^3x^5 + \frac{3}{5} a^2de^2x^5 + a^2d^2ex^3 + a^2d^3x$$

input
```
integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 3/11*c^2*d^2*e*x^11 + 2/11*a*c*e
^3*x^11 + 1/9*c^2*d^3*x^9 + 2/3*a*c*d*e^2*x^9 + 6/7*a*c*d^2*e*x^7 + 1/7*a^
2*e^3*x^7 + 2/5*a*c*d^3*x^5 + 3/5*a^2*d*e^2*x^5 + a^2*d^2*e*x^3 + a^2*d^3*
x
```

3.128.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int (d+ex^2)^3 (a+cx^4)^2 \, dx = x^5 \left(\frac{3\,a^2\,d\,e^2}{5} + \frac{2\,c\,a\,d^3}{5}\right) + x^7 \left(\frac{a^2\,e^3}{7} + \frac{6\,c\,a\,d^2\,e}{7}\right) + x^9 \left(\frac{c^2\,d^3}{9} + \frac{2\,a\,c\,d\,e^2}{3}\right) + x^{11} \left(\frac{3\,c^2\,d^2\,e}{11} + \frac{2\,a\,c\,e^3}{11}\right) + a^2\,d^3\,x + \frac{c^2\,e^3\,x^{15}}{15} + a^2\,d^2\,e\,x^3 + \frac{3\,c^2\,d\,e^2\,x^{13}}{13}$$

input
```
int((a + c*x^4)^2*(d + e*x^2)^3,x)
```

output
```
x^5*((3*a^2*d*e^2)/5 + (2*a*c*d^3)/5) + x^7*((a^2*e^3)/7 + (6*a*c*d^2*e)/7
) + x^9*((c^2*d^3)/9 + (2*a*c*d*e^2)/3) + x^11*((3*c^2*d^2*e)/11 + (2*a*c*
e^3)/11) + a^2*d^3*x + (c^2*e^3*x^15)/15 + a^2*d^2*e*x^3 + (3*c^2*d*e^2*x^
13)/13
```

3.129 $\int (d+ex^2)^2 (a+cx^4)^2 \, dx$

3.129.1 Optimal result

Integrand size = 19, antiderivative size = 97

$$\int (d+ex^2)^2 (a+cx^4)^2 \, dx = a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{5}a(2cd^2+ae^2)\,x^5 + \frac{4}{7}acdex^7 + \frac{1}{9}c(cd^2+2ae^2)\,x^9 + \frac{2}{11}c^2dex^{11} + \frac{1}{13}c^2e^2x^{13}$$

output
```
a^2*d^2*x+2/3*a^2*d*e*x^3+1/5*a*(a*e^2+2*c*d^2)*x^5+4/7*a*c*d*e*x^7+1/9*c*
(2*a*e^2+c*d^2)*x^9+2/11*c^2*d*e*x^11+1/13*c^2*e^2*x^13
```

3.129.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^2 (a+cx^4)^2 \, dx = a^2d^2x + \frac{2}{3}a^2dex^3 + \frac{1}{5}a(2cd^2+ae^2)\,x^5 + \frac{4}{7}acdex^7 + \frac{1}{9}c(cd^2+2ae^2)\,x^9 + \frac{2}{11}c^2dex^{11} + \frac{1}{13}c^2e^2x^{13}$$

input
```
Integrate[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

output
```
a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x
^7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)
/13
```

3.129.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2 \left(d+ex^2\right)^2 \, dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(a^2d^2+2a^2dex^2+cx^8\left(2ae^2+cd^2\right)+ax^4\left(ae^2+2cd^2\right)+4acdex^6+2c^2dex^{10}+c^2e^2x^{12}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

input
```
Int[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

output
```
a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x
^7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)
/13
```

3.129.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.129.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
default	$\frac{c^2e^2x^{13}}{13}+\frac{2c^2de\,x^{11}}{11}+\frac{(2e^2ac+c^2d^2)x^9}{9}+\frac{4acde\,x^7}{7}+\frac{(e^2a^2+2d^2ac)x^5}{5}+\frac{2a^2de\,x^3}{3}+a^2d^2x$
norman	$\frac{c^2e^2x^{13}}{13}+\frac{2c^2de\,x^{11}}{11}+\left(\frac{2}{9}e^2ac+\frac{1}{9}c^2d^2\right)x^9+\frac{4acde\,x^7}{7}+\left(\frac{1}{5}e^2a^2+\frac{2}{5}d^2ac\right)x^5+\frac{2a^2de\,x^3}{3}+a^2d^2x$
gosper	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9c^2d^2+\frac{4}{7}acde\,x^7+\frac{1}{5}x^5e^2a^2+\frac{2}{5}x^5d^2ac+\frac{2}{3}a^2de\,x^3+$
risch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9c^2d^2+\frac{4}{7}acde\,x^7+\frac{1}{5}x^5e^2a^2+\frac{2}{5}x^5d^2ac+\frac{2}{3}a^2de\,x^3+$
parallelrisch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9c^2d^2+\frac{4}{7}acde\,x^7+\frac{1}{5}x^5e^2a^2+\frac{2}{5}x^5d^2ac+\frac{2}{3}a^2de\,x^3+$

input
```
int((e*x^2+d)^2*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*c^2*e^2*x^13+2/11*c^2*d*e*x^11+1/9*(2*a*c*e^2+c^2*d^2)*x^9+4/7*a*c*d*
e*x^7+1/5*(a^2*e^2+2*a*c*d^2)*x^5+2/3*a^2*d*e*x^3+a^2*d^2*x
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int\left(d+ex^2\right)^2\left(a+cx^4\right)^2\,dx=\frac{1}{13}\,c^2e^2x^{13}+\frac{2}{11}\,c^2dex^{11}+\frac{4}{7}\,acdex^7+\frac{1}{9}\,\left(c^2d^2+2\,ace^2\right)x^9+\frac{2}{3}\,a^2dex^3+\frac{1}{5}\,\left(2\,acd^2+a^2e^2\right)x^5+a^2d^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 4/7*a*c*d*e*x^7 + 1/9*(c^2*d^2 + 2
*a*c*e^2)*x^9 + 2/3*a^2*d*e*x^3 + 1/5*(2*a*c*d^2 + a^2*e^2)*x^5 + a^2*d^2*
x
```

3.129.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = a^2d^2x + \frac{2a^2dex^3}{3} + \frac{4acdex^7}{7} + \frac{2c^2dex^{11}}{11} + \frac{c^2e^2x^{13}}{13} + x^9 \cdot \left(\frac{2ace^2}{9} + \frac{c^2d^2}{9}\right) + x^5\left(\frac{a^2e^2}{5} + \frac{2acd^2}{5}\right)$$

input
```
integrate((e*x**2+d)**2*(c*x**4+a)**2,x)
```

output
```
a**2*d**2*x + 2*a**2*d*e*x**3/3 + 4*a*c*d*e*x**7/7 + 2*c**2*d*e*x**11/11 +
 c**2*e**2*x**13/13 + x**9*(2*a*c*e**2/9 + c**2*d**2/9) + x**5*(a**2*e**2/
5 + 2*a*c*d**2/5)
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = \frac{1}{13}\,c^2e^2x^{13} + \frac{2}{11}\,c^2dex^{11} + \frac{4}{7}\,acdex^7 + \frac{1}{9}\,\left(c^2d^2 + 2\,ace^2\right)x^9 + \frac{2}{3}\,a^2dex^3 + \frac{1}{5}\,\left(2\,acd^2 + a^2e^2\right)x^5 + a^2d^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 4/7*a*c*d*e*x^7 + 1/9*(c^2*d^2 + 2
*a*c*e^2)*x^9 + 2/3*a^2*d*e*x^3 + 1/5*(2*a*c*d^2 + a^2*e^2)*x^5 + a^2*d^2*
x
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = \frac{1}{13}\,c^2e^2x^{13} + \frac{2}{11}\,c^2dex^{11} + \frac{1}{9}\,c^2d^2x^9 + \frac{2}{9}\,ace^2x^9 + \frac{4}{7}\,acdex^7 + \frac{2}{5}\,acd^2x^5 + \frac{1}{5}\,a^2e^2x^5 + \frac{2}{3}\,a^2dex^3 + a^2d^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 1/9*c^2*d^2*x^9 + 2/9*a*c*e^2*x^9
+ 4/7*a*c*d*e*x^7 + 2/5*a*c*d^2*x^5 + 1/5*a^2*e^2*x^5 + 2/3*a^2*d*e*x^3 +
a^2*d^2*x
```

3.129.9 Mupad [B] (verification not implemented)

Time = 14.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(d+ex^2\right)^2 \left(a+cx^4\right)^2 \, dx = x^5 \left(\frac{a^2 e^2}{5}+\frac{2 c a d^2}{5}\right)+x^9 \left(\frac{c^2 d^2}{9}+\frac{2 a c e^2}{9}\right)+a^2 d^2 x + \frac{c^2 e^2 x^{13}}{13}+\frac{2 a^2 d e x^3}{3}+\frac{2 c^2 d e x^{11}}{11}+\frac{4 a c d e x^7}{7}$$

input
```
int((a + c*x^4)^2*(d + e*x^2)^2,x)
```

output
```
x^5*((a^2*e^2)/5 + (2*a*c*d^2)/5) + x^9*((c^2*d^2)/9 + (2*a*c*e^2)/9) + a^
2*d^2*x + (c^2*e^2*x^13)/13 + (2*a^2*d*e*x^3)/3 + (2*c^2*d*e*x^11)/11 + (4
*a*c*d*e*x^7)/7
```

3.130 $\int (d+ex^2)(a+cx^4)^2\,dx$

3.130.1 Optimal result

Integrand size = 17, antiderivative size = 60

$$\int (d+ex^2)(a+cx^4)^2\,dx=a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

output
```
a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1/11*c^2*e
*x^11
```

3.130.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (d+ex^2)(a+cx^4)^2\,dx=a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

input
```
Integrate[(d + e*x^2)*(a + c*x^4)^2,x]
```

output
```
a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/
9 + (c^2*e*x^11)/11
```

3.130.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2 \left(d+ex^2\right) \, dx$$

$$\Big\downarrow \; 1468$$

$$\int \left(a^2d+a^2ex^2+2acdx^4+2acex^6+c^2dx^8+c^2ex^{10}\right) dx$$

$$\Big\downarrow \; 2009$$

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

input
```
Int[(d + e*x^2)*(a + c*x^4)^2,x]
```

output
```
a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/
9 + (c^2*e*x^11)/11
```

3.130.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.130.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
gosper	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
default	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
norman	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
risch	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51
parallelrisch	$a^2dx+\frac{1}{3}a^2e\,x^3+\frac{2}{5}acd\,x^5+\frac{2}{7}ace\,x^7+\frac{1}{9}c^2d\,x^9+\frac{1}{11}c^2e\,x^{11}$	51

input
```
int((e*x^2+d)*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1/11*c^2*e
*x^11
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (d+ex^2)\left(a+cx^4\right)^2\,dx=\frac{1}{11}\,c^2ex^{11}+\frac{1}{9}\,c^2dx^9+\frac{2}{7}\,acex^7+\frac{2}{5}\,acdx^5+\frac{1}{3}\,a^2ex^3+a^2dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*
e*x^3 + a^2*d*x
```

3.130.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (d+ex^2)\left(a+cx^4\right)^2\,dx=a^2dx+\frac{a^2ex^3}{3}+\frac{2acdx^5}{5}+\frac{2acex^7}{7}+\frac{c^2dx^9}{9}+\frac{c^2ex^{11}}{11}$$

input
```
integrate((e*x**2+d)*(c*x**4+a)**2,x)
```

output
```
a**2*d*x + a**2*e*x**3/3 + 2*a*c*d*x**5/5 + 2*a*c*e*x**7/7 + c**2*d*x**9/9
 + c**2*e*x**11/11
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (d+ex^2)(a+cx^4)^2\,dx = \frac{1}{11}c^2ex^{11} + \frac{1}{9}c^2dx^9 + \frac{2}{7}acex^7 + \frac{2}{5}acdx^5 + \frac{1}{3}a^2ex^3 + a^2dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*
e*x^3 + a^2*d*x
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (d+ex^2)(a+cx^4)^2\,dx = \frac{1}{11}c^2ex^{11} + \frac{1}{9}c^2dx^9 + \frac{2}{7}acex^7 + \frac{2}{5}acdx^5 + \frac{1}{3}a^2ex^3 + a^2dx$$

input
```
integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/7*a*c*e*x^7 + 2/5*a*c*d*x^5 + 1/3*a^2*
e*x^3 + a^2*d*x
```

3.130.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (d+ex^2)(a+cx^4)^2\,dx = \frac{e\,a^2\,x^3}{3} + d\,a^2\,x + \frac{2\,e\,a\,c\,x^7}{7} + \frac{2\,d\,a\,c\,x^5}{5} + \frac{e\,c^2\,x^{11}}{11} + \frac{d\,c^2\,x^9}{9}$$

input
```
int((a + c*x^4)^2*(d + e*x^2),x)
```

output
```
(a^2*e*x^3)/3 + (c^2*d*x^9)/9 + (c^2*e*x^11)/11 + a^2*d*x + (2*a*c*d*x^5)/
5 + (2*a*c*e*x^7)/7
```

3.131 $\int (a + cx^4)^2 \, dx$

3.131.1 Optimal result

Integrand size = 9, antiderivative size = 25

$$\int (a + cx^4)^2 \, dx = a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

output
```
a^2*x+2/5*a*c*x^5+1/9*c^2*x^9
```

3.131.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a + cx^4)^2 \, dx = a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

input
```
Integrate[(a + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

3.131.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2 dx$$

$$\Big\downarrow \ 747$$

$$\int \left(a^2+2acx^4+c^2x^8\right) dx$$

$$\Big\downarrow \ 2009$$

$$a^2x+\frac{2}{5}acx^5+\frac{c^2x^9}{9}$$

input
```
Int[(a + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

3.131.3.1 Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.131.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$a^2x + \frac{2}{5}x^5ac + \frac{1}{9}c^2x^9$	22
default	$a^2x + \frac{2}{5}x^5ac + \frac{1}{9}c^2x^9$	22
norman	$a^2x + \frac{2}{5}x^5ac + \frac{1}{9}c^2x^9$	22
risch	$a^2x + \frac{2}{5}x^5ac + \frac{1}{9}c^2x^9$	22
parallelrisch	$a^2x + \frac{2}{5}x^5ac + \frac{1}{9}c^2x^9$	22

input
```
int((c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/5*x^5*a*c+1/9*c^2*x^9
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a+cx^4)^2\,dx = \frac{1}{9}\,c^2x^9 + \frac{2}{5}\,acx^5 + a^2x$$

input
```
integrate((c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

3.131.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (a+cx^4)^2\,dx = a^2x + \frac{2acx^5}{5} + \frac{c^2x^9}{9}$$

input
```
integrate((c*x**4+a)**2,x)
```

output
```
a**2*x + 2*a*c*x**5/5 + c**2*x**9/9
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a+cx^4)^2 \, dx = \frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$$

input
```
integrate((c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a+cx^4)^2 \, dx = \frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$$

input
```
integrate((c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

3.131.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a+cx^4)^2 \, dx = a^2 x + \frac{2 a c x^5}{5} + \frac{c^2 x^9}{9}$$

input
```
int((a + c*x^4)^2,x)
```

output
```
a^2*x + (c^2*x^9)/9 + (2*a*c*x^5)/5
```

3.132 $\int \frac{(a+cx^4)^2}{d+ex^2} dx$

3.132.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{(a+cx^4)^2}{d+ex^2} dx = -\frac{cd(cd^2+2ae^2)x}{e^4} + \frac{c(cd^2+2ae^2)x^3}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + \frac{(cd^2+ae^2)^2 \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

output
```
-c*d*(2*a*e^2+c*d^2)*x/e^4+1/3*c*(2*a*e^2+c*d^2)*x^3/e^3-1/5*c^2*d*x^5/e^2
+1/7*c^2*x^7/e+(a*e^2+c*d^2)^2*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{(a+cx^4)^2}{d+ex^2} dx = \frac{cx(70ae^2(-3d+ex^2)+c(-105d^3+35d^2ex^2-21de^2x^4+15e^3x^6))}{105e^4} + \frac{(cd^2+ae^2)^2 \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2),x]
```

output
```
(c*x*(70*a*e^2*(-3*d + e*x^2) + c*(-105*d^3 + 35*d^2*e*x^2 - 21*d*e^2*x^4
+ 15*e^3*x^6)))/(105*e^4) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*e^(9/2))
```

3.132.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{d+ex^2} \, dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(\frac{a^2e^4+2acd^2e^2+c^2d^4}{e^4(d+ex^2)}-\frac{cd(2ae^2+cd^2)}{e^4}+\frac{cx^2(2ae^2+cd^2)}{e^3}-\frac{c^2dx^4}{e^2}+\frac{c^2x^6}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ae^2+cd^2)^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}-\frac{cdx(2ae^2+cd^2)}{e^4}+\frac{cx^3(2ae^2+cd^2)}{3e^3}-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}$$

input
```
Int[(a + c*x^4)^2/(d + e*x^2),x]
```

output
```
-((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^3)/(3*e^3) - (c^
2*d*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)
/Sqrt[d]])/(Sqrt[d]*e^(9/2))
```

3.132.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.132.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96

method	result
default	$-\frac{c\left(-\frac{c\,x^{7} e^{3}}{7}+\frac{c d\,x^{5} e^{2}}{5}-\frac{\left(2 a\,e^{2}+c\,d^{2}\right) x^{3} e}{3}+d\left(2 a\,e^{2}+c\,d^{2}\right) x\right)}{e^{4}}+\frac{\left(a^{2} e^{4}+2 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \arctan\left(\frac{e x}{\sqrt{e d}}\right)}{e^{4} \sqrt{e d}}$
risch	$\frac{c^{2} x^{7}}{7 e}-\frac{c^{2} d\,x^{5}}{5 e^{2}}+\frac{2 c a\,x^{3}}{3 e}+\frac{c^{2} d^{2} x^{3}}{3 e^{3}}-\frac{2 c a d x}{e^{2}}-\frac{c^{2} d^{3} x}{e^{4}}-\frac{\ln\left(e x+\sqrt{-e d}\right) a^{2}}{2 \sqrt{-e d}}-\frac{\ln\left(e x+\sqrt{-e d}\right) a c\,d^{2}}{e^{2} \sqrt{-e d}}-\frac{\ln\left(e x+\sqrt{-e d}\right) c^{2} d^{4}}{2 e^{4} \sqrt{-e d}}+$

input
```
int((c*x^4+a)^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-c/e^4*(-1/7*c*x^7*e^3+1/5*c*d*x^5*e^2-1/3*(2*a*e^2+c*d^2)*x^3*e+d*(2*a*e^
2+c*d^2)*x)+(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^4/(e*d)^(1/2)*arctan(e*x/(e*
d)^(1/2))
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.48

$$\int \frac{\left(a+cx^4\right)^2}{d+ex^2}\,dx = \left[\frac{30\,c^2de^4x^7-42\,c^2d^2e^3x^5+70\,(c^2d^3e^2+2\,acde^4)x^3-105\,(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x}{ex^2+d}\right.}{210\,de^5}\right.$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/210*(30*c^2*d*e^4*x^7 - 42*c^2*d^2*e^3*x^5 + 70*(c^2*d^3*e^2 + 2*a*c*d*
e^4)*x^3 - 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-d*e)*log((e*x^2 -
 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 210*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*
e^5), 1/105*(15*c^2*d*e^4*x^7 - 21*c^2*d^2*e^3*x^5 + 35*(c^2*d^3*e^2 + 2*a
*c*d*e^4)*x^3 + 105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e)*arctan(s
qrt(d*e)*x/d) - 105*(c^2*d^4*e + 2*a*c*d^2*e^3)*x)/(d*e^5)]
```

3.132.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(100) = 200$.

Time = 0.23 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.19

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = -\frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + x^3\cdot\left(\frac{2ac}{3e} + \frac{c^2d^2}{3e^3}\right) + x\left(-\frac{2acd}{e^2} - \frac{c^2d^3}{e^4}\right) - \frac{\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2\log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2}{a^2e^4+2acd^2e^2+c^2d^4}+x\right)}{2} + \frac{\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2\log\left(\frac{de^4\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2}{a^2e^4+2acd^2e^2+c^2d^4}+x\right)}{2}$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d),x)
```

output
```
-c**2*d*x**5/(5*e**2) + c**2*x**7/(7*e) + x**3*(2*a*c/(3*e) + c**2*d**2/(3
*e**3)) + x*(-2*a*c*d/e**2 - c**2*d**3/e**4) - sqrt(-1/(d*e**9))*(a*e**2 +
 c*d**2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4
+ 2*a*c*d**2*e**2 + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 + c*d**2
)**2*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2*a*c*
d**2*e**2 + c**2*d**4) + x)/2
```

3.132.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = \frac{(c^2d^4+2\,acd^2e^2+a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^4} + \frac{15\,c^2e^6x^7-21\,c^2de^5x^5+35\,c^2d^2e^4x^3+70\,ace^6x^3-105\,c^2d^3e^3x-210\,acde^5x}{105\,e^7}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="giac")
```

output
```
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4)
+ 1/105*(15*c^2*e^6*x^7 - 21*c^2*d*e^5*x^5 + 35*c^2*d^2*e^4*x^3 + 70*a*c*e
^6*x^3 - 105*c^2*d^3*e^3*x - 210*a*c*d*e^5*x)/e^7
```

3.132.9 Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.31

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = x^3\left(\frac{c^2\,d^2}{3\,e^3}+\frac{2\,a\,c}{3\,e}\right)+\frac{c^2\,x^7}{7\,e}-\frac{c^2\,d\,x^5}{5\,e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,\left(c\,d^2+a\,e^2\right)^2}{\sqrt{d}\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}\right)\,\left(c\,d^2+a\,e^2\right)^2}{\sqrt{d}\,e^{9/2}}-\frac{d\,x\,\left(\frac{c^2\,d^2}{e^3}+\frac{2\,a\,c}{e}\right)}{e}$$

input
```
int((a + c*x^4)^2/(d + e*x^2),x)
```

output
```
x^3*((c^2*d^2)/(3*e^3) + (2*a*c)/(3*e)) + (c^2*x^7)/(7*e) - (c^2*d*x^5)/(5
*e^2) + (atan((e^(1/2)*x*(a*e^2 + c*d^2)^2)/(d^(1/2)*(a^2*e^4 + c^2*d^4 +
2*a*c*d^2*e^2)))*(a*e^2 + c*d^2)^2)/(d^(1/2)*e^(9/2)) - (d*x*((c^2*d^2)/e^
3 + (2*a*c)/e))/e
```

3.133 $\int \frac{(a+cx^4)^2}{(d+ex^2)^2} dx$

3.133.1 Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2} dx = \frac{c(3cd^2+2ae^2)x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{(7cd^2-ae^2)(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

output
```
c*(2*a*e^2+3*c*d^2)*x/e^4-2/3*c^2*d*x^3/e^3+1/5*c^2*x^5/e^2+1/2*(a*e^2+c*d
^2)^2*x/d/e^4/(e*x^2+d)-1/2*(-a*e^2+7*c*d^2)*(a*e^2+c*d^2)*arctan(x*e^(1/2
)/d^(1/2))/d^(3/2)/e^(9/2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2} dx = \frac{c(3cd^2+2ae^2)x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{(7c^2d^4+6acd^2e^2-a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

output
```
(c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2)
+ ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c^2*d^4 + 6*a*c*d^2*e^
2 - a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

3.133.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1472, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x(ae^2+cd^2)^2}{2de^4(d+ex^2)}-\frac{\int -\frac{\frac{2c^2dx^6}{e}-\frac{2c^2d^2x^4}{e^2}+\frac{2cd(cd^2+2ae^2)x^2}{e^3}+a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{ex^2+d}dx}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{2c^2dx^6}{e}-\frac{2c^2d^2x^4}{e^2}+\frac{2cd(cd^2+2ae^2)x^2}{e^3}+a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{ex^2+d}dx}{2d}+\frac{x(ae^2+cd^2)^2}{2de^4(d+ex^2)}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\int \left(\frac{2c^2dx^4}{e^2}-\frac{4c^2d^2x^2}{e^3}+\frac{2cd(3cd^2+2ae^2)}{e^4}+\frac{-7c^2d^4-6ace^2d^2+a^2e^4}{e^4(ex^2+d)}\right)dx}{2d}+\frac{x(ae^2+cd^2)^2}{2de^4(d+ex^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(7cd^2-ae^2)(ae^2+cd^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}+\frac{2cdx(2ae^2+3cd^2)}{e^4}-\frac{4c^2d^2x^3}{3e^3}+\frac{2c^2dx^5}{5e^2}}{2d}+\frac{x(ae^2+cd^2)^2}{2de^4(d+ex^2)}$$

input
```
Int[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

output
```
((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) + ((2*c*d*(3*c*d^2 + 2*a*e^2)*
x)/e^4 - (4*c^2*d^2*x^3)/(3*e^3) + (2*c^2*d*x^5)/(5*e^2) - ((7*c*d^2 - a*e
^2)*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2)))/(2*d)
```

3.133. $\int \frac{(a+cx^4)^2}{(d+ex^2)^2}dx$

3.133.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.133.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

method	result
default	$\frac{c\left(\frac{1}{5}c\,x^{5}e^{2}-\frac{2}{3}dc\,x^{3}e+2a\,e^{2}x+3c\,d^{2}x\right)}{e^{4}}+\frac{\frac{\left(a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}\right)x}{2d\left(e\,x^{2}+d\right)}+\frac{\left(a^{2}e^{4}-6ac\,d^{2}e^{2}-7c^{2}d^{4}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^{4}}$
risch	$\frac{c^{2}x^{5}}{5e^{2}}-\frac{2c^{2}d\,x^{3}}{3e^{3}}+\frac{2cax}{e^{2}}+\frac{3c^{2}d^{2}x}{e^{4}}+\frac{\left(a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}\right)x}{2d\,e^{4}\left(e\,x^{2}+d\right)}-\frac{\ln\left(ex+\sqrt{-ed}\right)a^{2}}{4\sqrt{-ed}\,d}+\frac{3d\ln\left(ex+\sqrt{-ed}\right)ac}{2e^{2}\sqrt{-ed}}+\frac{7d^{3}\ln(ex+\sqrt{-}$

input
```
int((c*x^4+a)^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c/e^4*(1/5*c*x^5*e^2-2/3*d*c*x^3*e+2*a*e^2*x+3*c*d^2*x)+1/e^4*(1/2*(a^2*e^
4+2*a*c*d^2*e^2+c^2*d^4)/d*x/(e*x^2+d)+1/2*(a^2*e^4-6*a*c*d^2*e^2-7*c^2*d^
4)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.01

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx$$
$$= \left[\frac{12\,c^2d^2e^4x^7 - 28\,c^2d^3e^3x^5 + 20\,(7\,c^2d^4e^2 + 6\,acd^2e^4)x^3 + 15\,(7\,c^2d^5 + 6\,acd^3e^2 - a^2de^4 + (7\,c^2d^4e + 6\,ac}{60\,(d^2e^6x^2 + d^3e^5)}\right.$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*c^2*d^2*e^4*x^7 - 28*c^2*d^3*e^3*x^5 + 20*(7*c^2*d^4*e^2 + 6*a*c
*d^2*e^4)*x^3 + 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (7*c^2*d^4*e +
 6*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)
/(e*x^2 + d)) + 30*(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x)/(d^2*e^6*x
^2 + d^3*e^5), 1/30*(6*c^2*d^2*e^4*x^7 - 14*c^2*d^3*e^3*x^5 + 10*(7*c^2*d^
4*e^2 + 6*a*c*d^2*e^4)*x^3 - 15*(7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^4 + (
7*c^2*d^4*e + 6*a*c*d^2*e^3 - a^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d
) + 15*(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*x)/(d^2*e^6*x^2 + d^3*e^5
)]
```

3.133.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(122) = 244.

Time = 0.41 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.40

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx$$
$$= -\frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + x\left(\frac{2ac}{e^2} + \frac{3c^2d^2}{e^4}\right) + \frac{x(a^2e^4 + 2acd^2e^2 + c^2d^4)}{2d^2e^4 + 2de^5x^2}$$
$$- \frac{\sqrt{-\frac{1}{d^3e^9}}(ae^2 - 7cd^2)(ae^2 + cd^2)\log\left(-\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}(ae^2-7cd^2)(ae^2+cd^2)}{a^2e^4-6acd^2e^2-7c^2d^4} + x\right)}{4}$$
$$+ \frac{\sqrt{-\frac{1}{d^3e^9}}(ae^2 - 7cd^2)(ae^2 + cd^2)\log\left(\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}(ae^2-7cd^2)(ae^2+cd^2)}{a^2e^4-6acd^2e^2-7c^2d^4} + x\right)}{4}$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**2,x)
```

output
```
-2*c**2*d*x**3/(3*e**3) + c**2*x**5/(5*e**2) + x*(2*a*c/e**2 + 3*c**2*d**2
/e**4) + x*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)/(2*d**2*e**4 + 2*d*e*
*5*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(
-d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**
2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4 + sqrt(-1/(d**3*e**9))*(a*e
**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e*
*2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**
4) + x)/4
```

3.133.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = -\frac{(7\,c^2d^4+6\,acd^2e^2-a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^4}+\frac{c^2d^4x+2\,acd^2e^2x+a^2e^4x}{2\,(ex^2+d)de^4}+\frac{3\,c^2e^8x^5-10\,c^2de^7x^3+45\,c^2d^2e^6x+30\,ace^8x}{15\,e^{10}}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e
)*d*e^4) + 1/2*(c^2*d^4*x + 2*a*c*d^2*e^2*x + a^2*e^4*x)/((e*x^2 + d)*d*e^
4) + 1/15*(3*c^2*e^8*x^5 - 10*c^2*d*e^7*x^3 + 45*c^2*d^2*e^6*x + 30*a*c*e^
8*x)/e^10
```

3.133.9 Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.40

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx = x\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,a\,c}{e^2}\right)+\frac{c^2\,x^5}{5\,e^2}-\frac{2\,c^2\,d\,x^3}{3\,e^3}+\frac{x\,(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4)}{2\,d\,(e^5\,x^2+d\,e^4)} - \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x\,(c\,d^2+a\,e^2)\,(a\,e^2-7\,c\,d^2)}{\sqrt{d}\,(-a^2\,e^4+6\,a\,c\,d^2\,e^2+7\,c^2\,d^4)}\right)\,(c\,d^2+a\,e^2)\,(a\,e^2-7\,c\,d^2)}{2\,d^{3/2}\,e^{9/2}}$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^2,x)
```

output
```
x*((3*c^2*d^2)/e^4 + (2*a*c)/e^2) + (c^2*x^5)/(5*e^2) - (2*c^2*d*x^3)/(3*e
^3) + (x*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(2*d*(d*e^4 + e^5*x^2)) - (a
tan((e^(1/2)*x*(a*e^2 + c*d^2)*(a*e^2 - 7*c*d^2))/(d^(1/2)*(7*c^2*d^4 - a^
2*e^4 + 6*a*c*d^2*e^2)))*(a*e^2 + c*d^2)*(a*e^2 - 7*c*d^2))/(2*d^(3/2)*e^(
9/2))
```

3.134 $\int \frac{(a+cx^4)^2}{(d+ex^2)^3} dx$

3.134.1 Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^3} dx = -\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2+ae^2)^2 x}{4de^4(d+ex^2)^2} + \frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2(d+ex^2)} + \frac{(35c^2d^4+6acd^2e^2+3a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}$$

output
```
-3*c^2*d*x/e^4+1/3*c^2*x^3/e^3+1/4*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^2+1/8
*(3*a^2-13*c^2*d^4/e^4-10*a*c*d^2/e^2)*x/d^2/(e*x^2+d)+1/8*(3*a^2*e^4+6*a*
c*d^2*e^2+35*c^2*d^4)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(9/2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^3} dx = \frac{x(3a^2e^4(5d+3ex^2)-6acd^2e^2(3d+5ex^2)-c^2d^2(105d^3+175d^2ex^2+56de^2x^4-8e^3x^6))}{24d^2e^4(d+ex^2)^2} + \frac{(35c^2d^4+6acd^2e^2+3a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

output
```
(x*(3*a^2*e^4*(5*d + 3*e*x^2) - 6*a*c*d^2*e^2*(3*d + 5*e*x^2) - c^2*d^2*(1
05*d^3 + 175*d^2*e*x^2 + 56*d*e^2*x^4 - 8*e^3*x^6)))/(24*d^2*e^4*(d + e*x^
2)^2) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(8*d^(5/2)*e^(9/2))
```

3.134.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1472, 25, 2345, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}-\frac{\int -\frac{\frac{4c^2dx^6}{e}-\frac{4c^2d^2x^4}{e^2}+\frac{4cd\left(cd^2+2ae^2\right)x^2}{e^3}+3a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^2}dx}{4d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{4c^2dx^6}{e}-\frac{4c^2d^2x^4}{e^2}+\frac{4cd\left(cd^2+2ae^2\right)x^2}{e^3}+3a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{\left(ex^2+d\right)^2}dx}{4d}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{2d(d+ex^2)}-\frac{\int -\frac{\frac{11c^2d^4}{e^4}-\frac{16c^2x^2d^3}{e^3}+\frac{8c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+3a^2}{ex^2+d}dx}{2d}}{4d}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\frac{11c^2d^4}{e^4}-\frac{16c^2x^2d^3}{e^3}+\frac{8c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+3a^2}{ex^2+d}dx}{2d}+\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{2d(d+ex^2)}}{4d}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{1467}$$

$$\frac{\frac{\int \left(-\frac{24c^2d^3}{e^4}+\frac{8c^2x^2d^2}{e^3}+\frac{35c^2d^4+6ace^2d^2+3a^2e^4}{e^4\left(ex^2+d\right)}\right)dx}{2d}+\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{2d(d+ex^2)}}{4d}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\frac{\left(3a^2e^4+6acd^2e^2+35c^2d^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}-\frac{24c^2d^3x}{e^4}+\frac{8c^2d^2x^3}{3e^3}}{2d}+\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{2d(d+ex^2)}}{4d}+\frac{x\left(ae^2+cd^2\right)^2}{4de^4\left(d+ex^2\right)^2}$$

input
```
Int[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

output
```
((c*d^2 + a*e^2)^2*x)/(4*d*e^4*(d + e*x^2)^2) + (((3*a^2 - (13*c^2*d^4)/e^
4 - (10*a*c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) + ((-24*c^2*d^3*x)/e^4 + (8*c^2
*d^2*x^3)/(3*e^3) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2)))/(2*d))/(4*d)
```

3.134.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.134. $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx$

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.134.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.98

method	result
default	$-\frac{c^{2}\left(-\frac{1}{3}e\,x^{3}+3dx\right)}{e^{4}}+\frac{\frac{\frac{e\left(3a^{2}e^{4}-10ac\,d^{2}e^{2}-13c^{2}d^{4}\right)x^{3}}{8d^{2}}+\frac{\left(5a^{2}e^{4}-6ac\,d^{2}e^{2}-11c^{2}d^{4}\right)x}{8d}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(3a^{2}e^{4}+6ac\,d^{2}e^{2}+35c^{2}d^{4}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8d^{2}\sqrt{ed}}}{e^{4}}$
risch	$\frac{c^{2}x^{3}}{3e^{3}}-\frac{3c^{2}dx}{e^{4}}+\frac{\frac{e\left(3a^{2}e^{4}-10ac\,d^{2}e^{2}-13c^{2}d^{4}\right)x^{3}}{8d^{2}}+\frac{\left(5a^{2}e^{4}-6ac\,d^{2}e^{2}-11c^{2}d^{4}\right)x}{8d}}{e^{4}\left(e\,x^{2}+d\right)^{2}}-\frac{3\ln\left(ex+\sqrt{-ed}\right)a^{2}}{16\sqrt{-ed}\,d^{2}}-\frac{3\ln\left(ex+\sqrt{-ed}\right)ac}{8e^{2}\sqrt{-ed}}-35$

input
```
int((c*x^4+a)^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-c^2/e^4*(-1/3*e*x^3+3*d*x)+1/e^4*((1/8*e*(3*a^2*e^4-10*a*c*d^2*e^2-13*c^2
*d^4)/d^2*x^3+1/8*(5*a^2*e^4-6*a*c*d^2*e^2-11*c^2*d^4)/d*x)/(e*x^2+d)^2+1/
8*(3*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)/d^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1
/2)))
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.33

$$\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^3}\,dx = \left[\frac{16\,c^2d^3e^4x^7-112\,c^2d^4e^3x^5-2\,(175\,c^2d^5e^2+30\,acd^3e^4-9\,a^2de^6)x^3-3\,(35\,c^2d^6+6\,acd^4e^2+3\,a^2d^2e^4+}{}\right.$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*c^2*d^3*e^4*x^7 - 112*c^2*d^4*e^3*x^5 - 2*(175*c^2*d^5*e^2 + 30*
a*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 - 3*(35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2
*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*e^6)*x^4 + 2*(35*c^2*d^5*e
+ 6*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x
 - d)/(e*x^2 + d)) - 6*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x)/(
d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 - 56*c^2*d
^4*e^3*x^5 - (175*c^2*d^5*e^2 + 30*a*c*d^3*e^4 - 9*a^2*d*e^6)*x^3 + 3*(35*
c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e^4 + (35*c^2*d^4*e^2 + 6*a*c*d^2*e^4
+ 3*a^2*e^6)*x^4 + 2*(35*c^2*d^5*e + 6*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*sqr
t(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2
*e^5)*x)/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]
```

3.134.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.66

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx = -\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} - \frac{\sqrt{-\frac{1}{d^5e^9}}\cdot(3a^2e^4+6acd^2e^2+35c^2d^4)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^9}}\cdot(3a^2e^4+6acd^2e^2+35c^2d^4)\log\left(d^3e^4\sqrt{-\frac{1}{d^5e^9}}+x\right)}{16} + \frac{x^3\cdot(3a^2e^5-10acd^2e^3-13c^2d^4e)+x(5a^2de^4-6acd^3e^2-11c^2d^5)}{8d^4e^4+16d^3e^5x^2+8d^2e^6x^4}$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**3,x)
```

output
```
-3*c**2*d*x/e**4 + c**2*x**3/(3*e**3) - sqrt(-1/(d**5*e**9))*(3*a**2*e**4
+ 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)
/16 + sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*
log(d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + (x**3*(3*a**2*e**5 - 10*a*c*d
**2*e**3 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 6*a*c*d**3*e**2 - 11*c**2*
d**5))/(8*d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)
```

3.134.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx = \frac{(35\,c^2d^4+6\,acd^2e^2+3\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^2e^4} - \frac{13\,c^2d^4ex^3+10\,acd^2e^3x^3-3\,a^2e^5x^3+11\,c^2d^5x+6\,acd^3e^2x-5\,a^2de^4x}{8\,(ex^2+d)^2d^2e^4} + \frac{c^2e^6x^3-9\,c^2de^5x}{3\,e^9}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
1/8*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d
*e)*d^2*e^4) - 1/8*(13*c^2*d^4*e*x^3 + 10*a*c*d^2*e^3*x^3 - 3*a^2*e^5*x^3
+ 11*c^2*d^5*x + 6*a*c*d^3*e^2*x - 5*a^2*d*e^4*x)/((e*x^2 + d)^2*d^2*e^4)
+ 1/3*(c^2*e^6*x^3 - 9*c^2*d*e^5*x)/e^9
```

3.134.9 Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.06

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx = \frac{c^2\,x^3}{3\,e^3} - \frac{\frac{x^3\,(-3\,a^2\,e^5+10\,a\,c\,d^2\,e^3+13\,c^2\,d^4\,e)}{8\,d^2} + \frac{x\,(-5\,a^2\,e^4+6\,a\,c\,d^2\,e^2+11\,c^2\,d^4)}{8\,d}}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(3\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4)}{8\,d^{5/2}\,e^{9/2}} - \frac{3\,c^2\,d\,x}{e^4}$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^3,x)
```

output
```
(c^2*x^3)/(3*e^3) - ((x^3*(13*c^2*d^4*e - 3*a^2*e^5 + 10*a*c*d^2*e^3))/(8*
d^2) + (x*(11*c^2*d^4 - 5*a^2*e^4 + 6*a*c*d^2*e^2))/(8*d))/(d^2*e^4 + e^6*
x^4 + 2*d*e^5*x^2) + (atan((e^(1/2)*x)/d^(1/2))*(3*a^2*e^4 + 35*c^2*d^4 +
6*a*c*d^2*e^2))/(8*d^(5/2)*e^(9/2)) - (3*c^2*d*x)/e^4
```

3.135 $\int \frac{(a+cx^4)^2}{(d+ex^2)^4} \, dx$

3.135.1 Optimal result

Integrand size = 19, antiderivative size = 184

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4} \, dx = \frac{c^2x}{e^4} + \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2 - \frac{19c^2d^4}{e^4} - \frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\left(5a^2 + \frac{29c^2d^4}{e^4} + \frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)} - \frac{(35c^2d^4 - 2acd^2e^2 - 5a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}$$

output
```
c^2*x/e^4+1/6*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^3+1/24*(5*a^2-19*c^2*d^4/e
^4-14*a*c*d^2/e^2)*x/d^2/(e*x^2+d)^2+1/16*(5*a^2+29*c^2*d^4/e^4+2*a*c*d^2/
e^2)*x/d^3/(e*x^2+d)-1/16*(-5*a^2*e^4-2*a*c*d^2*e^2+35*c^2*d^4)*arctan(x*e
^(1/2)/d^(1/2))/d^(7/2)/e^(9/2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.95

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4} \, dx = \frac{x(-2acd^2e^2(3d^2+8dex^2-3e^2x^4) + a^2e^4(33d^2+40dex^2+15e^2x^4) + c^2d^3(105d^3+280d^2ex^2+231de^2x^4}{48d^3e^4(d+ex^2)^3} - \frac{(35c^2d^4-2acd^2e^2-5a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

output
```
(x*(-2*a*c*d^2*e^2*(3*d^2 + 8*d*e*x^2 - 3*e^2*x^4) + a^2*e^4*(33*d^2 + 40*
d*e*x^2 + 15*e^2*x^4) + c^2*d^3*(105*d^3 + 280*d^2*e*x^2 + 231*d*e^2*x^4 +
 48*e^3*x^6)))/(48*d^3*e^4*(d + e*x^2)^3) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 -
 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

3.135.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1472, 25, 2345, 27, 1471, 25, 25, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}dx$$

$$\Big\downarrow \text{1472}$$

$$\frac{x(ae^2+cd^2)^2}{6de^4(d+ex^2)^3}-\frac{\int -\frac{\frac{6c^2dx^6}{e}-\frac{6c^2d^2x^4}{e^2}+\frac{6cd(cd^2+2ae^2)x^2}{e^3}+5a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^3}dx}{6d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{6c^2dx^6}{e}-\frac{6c^2d^2x^4}{e^2}+\frac{6cd(cd^2+2ae^2)x^2}{e^3}+5a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^3}dx}{6d}+\frac{x(ae^2+cd^2)^2}{6de^4(d+ex^2)^3}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}-\frac{\int -\frac{3\left(\frac{8c^2d^2x^4}{e^2}-\frac{16c^2d^3x^2}{e^3}+\frac{5c^2d^4+2ace^2d^2+5a^2e^4}{e^4}\right)}{(ex^2+d)^2}dx}{4d}}{6d}+\frac{x(ae^2+cd^2)^2}{6de^4(d+ex^2)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{\frac{5c^2d^4}{e^4}-\frac{16c^2x^2d^3}{e^3}+\frac{8c^2x^4d^2}{e^2}+\frac{2acd^2}{e^2}+5a^2}{(ex^2+d)^2}dx}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x(ae^2+cd^2)^2}{6de^4(d+ex^2)^3}$$

$$\Big\downarrow \; 1471$$

$$\frac{3\left(\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d(d+ex^2)}-\frac{\int -\frac{16c^2x^2d^3+\left(-\frac{19c^2d^4}{e^4}+\frac{2acd^2}{e^2}+5a^2\right)e^3}{e^3(ex^2+d)}dx}{2d}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(\frac{\int -\frac{\frac{19c^2d^4}{e}-16c^2x^2d^3-2aced^2-5a^2e^3}{e^3(ex^2+d)}dx}{2d}+\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d(d+ex^2)}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d(d+ex^2)}-\frac{\int \frac{\frac{19c^2d^4}{e}-16c^2x^2d^3-2aced^2-5a^2e^3}{e^3(ex^2+d)}dx}{2d}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d(d+ex^2)}-\frac{\int \frac{\frac{19c^2d^4}{e}-16c^2x^2d^3-2aced^2-5a^2e^3}{ex^2+d}dx}{2de^3}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{3\left(\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d(d+ex^2)}-\frac{\frac{\left(-5a^2e^4-2acd^2e^2+35c^2d^4\right)\int\frac{1}{ex^2+d}dx}{e}-\frac{16c^2d^3x}{e}}{2de^3}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

$$\Big\downarrow \; 218$$

3.135. $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^4}\,dx$

$$\frac{\frac{3\left(\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{2d(d+ex^2)}-\frac{\frac{\left(-5a^2e^4-2acd^2e^2+35c^2d^4\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{16c^2d^3x}{e}}{2de^3}\right)}{4d}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}}{6d}+\frac{x\left(ae^2+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}$$

input
```
Int[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

output
```
((c*d^2 + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) + (((5*a^2 - (19*c^2*d^4)/e^
4 - (14*a*c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) + (3*(((5*a^2 + (29*c^2*d^4)/
e^4 + (2*a*c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) - ((-16*c^2*d^3*x)/e + ((35*c^
2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e
^(3/2)))/(2*d*e^3)))/(4*d))/(6*d)
```

3.135.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.135.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97

method	result
default	$\frac{c^2x}{e^4}+\frac{\frac{\frac{e^2\left(5a^2e^4+2ac\,d^2e^2+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4-2ac\,d^2e^2+17c^2d^4\right)x^3}{6d^2}+\frac{\left(11a^2e^4-2ac\,d^2e^2+19c^2d^4\right)x}{16d}}{\left(e\,x^2+d\right)^3}+\frac{\left(5a^2e^4+2ac\,d^2e^2-35c^2d^4\right)\arctan}{16d^3\sqrt{ed}}}{e^4}$
risch	$\frac{c^2x}{e^4}+\frac{\frac{e^2\left(5a^2e^4+2ac\,d^2e^2+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4-2ac\,d^2e^2+17c^2d^4\right)x^3}{6d^2}+\frac{\left(11a^2e^4-2ac\,d^2e^2+19c^2d^4\right)x}{16d}}{e^4(e\,x^2+d)^3}-\frac{5\ln\left(ex+\sqrt{-ed}\right)a^2}{32\sqrt{-ed}\,d^3}-\frac{\ln(ex}{16e}$

input
```
int((c*x^4+a)^2/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

output
```
c^2*x/e^4+1/e^4*((1/16*e^2*(5*a^2*e^4+2*a*c*d^2*e^2+29*c^2*d^4)/d^3*x^5+1/
6*e*(5*a^2*e^4-2*a*c*d^2*e^2+17*c^2*d^4)/d^2*x^3+1/16*(11*a^2*e^4-2*a*c*d^
2*e^2+19*c^2*d^4)/d*x)/(e*x^2+d)^3+1/16*(5*a^2*e^4+2*a*c*d^2*e^2-35*c^2*d^
4)/d^3/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.60

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx$$
$$= \left[\frac{96\,c^2d^4e^4x^7+6\,(77\,c^2d^5e^3+2\,acd^3e^5+5\,a^2de^7)x^5+16\,(35\,c^2d^6e^2-2\,acd^4e^4+5\,a^2d^2e^6)x^3+3\,(35\,c^2d^7}{}\right.$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="fricas")
```

output
```
[1/96*(96*c^2*d^4*e^4*x^7 + 6*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^
7)*x^5 + 16*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 + 3*(35*c
^2*d^7 - 2*a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 -
 5*a^2*e^7)*x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3
*(35*c^2*d^6*e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(-d*e)*log((e*x^2
 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11
*a^2*d^3*e^5)*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5),
1/48*(48*c^2*d^4*e^4*x^7 + 3*(77*c^2*d^5*e^3 + 2*a*c*d^3*e^5 + 5*a^2*d*e^7
)*x^5 + 8*(35*c^2*d^6*e^2 - 2*a*c*d^4*e^4 + 5*a^2*d^2*e^6)*x^3 - 3*(35*c^2
*d^7 - 2*a*c*d^5*e^2 - 5*a^2*d^3*e^4 + (35*c^2*d^4*e^3 - 2*a*c*d^2*e^5 - 5
*a^2*e^7)*x^6 + 3*(35*c^2*d^5*e^2 - 2*a*c*d^3*e^4 - 5*a^2*d*e^6)*x^4 + 3*(
35*c^2*d^6*e - 2*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*x^2)*sqrt(d*e)*arctan(sqrt(d
*e)*x/d) + 3*(35*c^2*d^7*e - 2*a*c*d^5*e^3 + 11*a^2*d^3*e^5)*x)/(d^4*e^8*x
^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5)]
```

3.135.6 Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.59

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{c^2x}{e^4} - \frac{\sqrt{-\frac{1}{d^7e^9}}\cdot(5a^2e^4+2acd^2e^2-35c^2d^4)\log\left(-d^4e^4\sqrt{-\frac{1}{d^7e^9}}+x\right)}{32}$$
$$+\frac{\sqrt{-\frac{1}{d^7e^9}}\cdot(5a^2e^4+2acd^2e^2-35c^2d^4)\log\left(d^4e^4\sqrt{-\frac{1}{d^7e^9}}+x\right)}{32}$$
$$+\frac{x^5\cdot(15a^2e^6+6acd^2e^4+87c^2d^4e^2)+x^3\cdot(40a^2de^5-16acd^3e^3+136c^2d^5e)+x(33a^2d^2e^4-6acd^4e^2+5}{48d^6e^4+144d^5e^5x^2+144d^4e^6x^4+48d^3e^7x^6}$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**4,x)
```

output
```
c**2*x/e**4 - sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**
2*d**4)*log(-d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + sqrt(-1/(d**7*e**9))
*(5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2*d**4)*log(d**4*e**4*sqrt(-1/(d**
7*e**9)) + x)/32 + (x**5*(15*a**2*e**6 + 6*a*c*d**2*e**4 + 87*c**2*d**4*e*
*2) + x**3*(40*a**2*d*e**5 - 16*a*c*d**3*e**3 + 136*c**2*d**5*e) + x*(33*a
**2*d**2*e**4 - 6*a*c*d**4*e**2 + 57*c**2*d**6))/(48*d**6*e**4 + 144*d**5*
e**5*x**2 + 144*d**4*e**6*x**4 + 48*d**3*e**7*x**6)
```

3.135.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{c^2x}{e^4} - \frac{(35\,c^2d^4 - 2\,acd^2e^2 - 5\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}d^3e^4} + \frac{87\,c^2d^4e^2x^5 + 6\,acd^2e^4x^5 + 15\,a^2e^6x^5 + 136\,c^2d^5ex^3 - 16\,acd^3e^3x^3 + 40\,a^2de^5x^3 + 57\,c^2d^6x - 6\,acd^4e^2x}{48\,(ex^2+d)^3d^3e^4}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="giac")
```

output
```
c^2*x/e^4 - 1/16*(35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*arctan(e*x/sqrt(
d*e))/(sqrt(d*e)*d^3*e^4) + 1/48*(87*c^2*d^4*e^2*x^5 + 6*a*c*d^2*e^4*x^5 +
 15*a^2*e^6*x^5 + 136*c^2*d^5*e*x^3 - 16*a*c*d^3*e^3*x^3 + 40*a^2*d*e^5*x^
3 + 57*c^2*d^6*x - 6*a*c*d^4*e^2*x + 33*a^2*d^2*e^4*x)/((e*x^2 + d)^3*d^3*
e^4)
```

3.135.9 Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{\frac{x^3\,(5\,a^2\,e^5-2\,a\,c\,d^2\,e^3+17\,c^2\,d^4\,e)}{6\,d^2} + \frac{x\,(11\,a^2\,e^4-2\,a\,c\,d^2\,e^2+19\,c^2\,d^4)}{16\,d} + \frac{x^5\,(5\,a^2\,e^6+2\,a\,c\,d^2\,e^4+29\,c^2\,d^4\,e^2)}{16\,d^3}}{d^3\,e^4+3\,d^2\,e^5\,x^2+3\,d\,e^6\,x^4+e^7\,x^6} + \frac{c^2\,x}{e^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(5\,a^2\,e^4+2\,a\,c\,d^2\,e^2-35\,c^2\,d^4)}{16\,d^{7/2}\,e^{9/2}}$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^4,x)
```

output
```
((x^3*(5*a^2*e^5 + 17*c^2*d^4*e - 2*a*c*d^2*e^3))/(6*d^2) + (x*(11*a^2*e^4
 + 19*c^2*d^4 - 2*a*c*d^2*e^2))/(16*d) + (x^5*(5*a^2*e^6 + 29*c^2*d^4*e^2
+ 2*a*c*d^2*e^4))/(16*d^3))/(d^3*e^4 + e^7*x^6 + 3*d*e^6*x^4 + 3*d^2*e^5*x
^2) + (c^2*x)/e^4 + (atan((e^(1/2)*x)/d^(1/2))*(5*a^2*e^4 - 35*c^2*d^4 + 2
*a*c*d^2*e^2))/(16*d^(7/2)*e^(9/2))
```

3.136 $\int \frac{(a+cx^4)^2}{(d+ex^2)^5} dx$

3.136.1 Optimal result

Integrand size = 19, antiderivative size = 223

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5} dx = \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3(d+ex^2)^2} - \frac{(93c^2d^4-6acd^2e^2-35a^2e^4)x}{128d^4e^4(d+ex^2)} + \frac{(35c^2d^4+6acd^2e^2+35a^2e^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}}$$

output
```
1/8*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^4+1/48*(7*a^2-25*c^2*d^4/e^4-18*a*c*
d^2/e^2)*x/d^2/(e*x^2+d)^3+1/192*(35*a^2+163*c^2*d^4/e^4+6*a*c*d^2/e^2)*x/
d^3/(e*x^2+d)^2-1/128*(-35*a^2*e^4-6*a*c*d^2*e^2+93*c^2*d^4)*x/d^4/e^4/(e*
x^2+d)+1/128*(35*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)*arctan(x*e^(1/2)/d^(1/2
))/d^(9/2)/e^(9/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.90

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx$$
$$= \frac{\frac{\sqrt{d}\sqrt{e}x(-6acd^2e^2(3d^3+11d^2ex^2-11de^2x^4-3e^3x^6)+a^2e^4(279d^3+511d^2ex^2+385de^2x^4+105e^3x^6)-c^2d^4(105d^3+385d^2ex^2+511de^2x^4+279e^3x}{(d+ex^2)^4}}{384d^{9/2}e^{9/2}}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

output
```
((Sqrt[d]*Sqrt[e]*x*(-6*a*c*d^2*e^2*(3*d^3 + 11*d^2*e*x^2 - 11*d*e^2*x^4 -
 3*e^3*x^6) + a^2*e^4*(279*d^3 + 511*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x
^6) - c^2*d^4*(105*d^3 + 385*d^2*e*x^2 + 511*d*e^2*x^4 + 279*e^3*x^6)))/(d
 + e*x^2)^4 + 3*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/(384*d^(9/2)*e^(9/2))
```

3.136.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1472, 25, 2345, 25, 1471, 27, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx$$
$$\Big\downarrow \; 1472$$
$$\frac{x(ae^2+cd^2)^2}{8de^4(d+ex^2)^4}-\frac{\int -\frac{\frac{8c^2dx^6}{e}-\frac{8c^2d^2x^4}{e^2}+\frac{8cd(cd^2+2ae^2)x^2}{e^3}+7a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^4}dx}{8d}$$
$$\Big\downarrow \; 25$$
$$\frac{\int \frac{\frac{8c^2dx^6}{e}-\frac{8c^2d^2x^4}{e^2}+\frac{8cd(cd^2+2ae^2)x^2}{e^3}+7a^2-\frac{2acd^2}{e^2}-\frac{c^2d^4}{e^4}}{(ex^2+d)^4}dx}{8d}+\frac{x(ae^2+cd^2)^2}{8de^4(d+ex^2)^4}$$
$$\Big\downarrow \; 2345$$

$$\frac{\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}-\frac{\int -\frac{\frac{19c^2d^4}{e^4}-\frac{96c^2x^2d^3}{e^3}+\frac{48c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+35a^2}{(ex^2+d)^3}dx}{6d}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{\frac{19c^2d^4}{e^4}-\frac{96c^2x^2d^3}{e^3}+\frac{48c^2x^4d^2}{e^2}+\frac{6acd^2}{e^2}+35a^2}{(ex^2+d)^3}dx}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{\frac{\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{4d\left(d+ex^2\right)^2}-\frac{\int -\frac{3\left(64c^2x^2d^3+\left(-\frac{29c^2d^4}{e^4}+\frac{6acd^2}{e^2}+35a^2\right)e^3\right)}{e^3\left(ex^2+d\right)^2}dx}{4d}}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{3\int -\frac{\frac{29c^2d^4}{e}-64c^2x^2d^3-6aced^2-35a^2e^3}{\left(ex^2+d\right)^2}dx}{4de^3}+\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{4d\left(d+ex^2\right)^2}}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{4d\left(d+ex^2\right)^2}-\frac{3\int \frac{\frac{29c^2d^4}{e}-64c^2x^2d^3-6aced^2-35a^2e^3}{\left(ex^2+d\right)^2}dx}{4de^3}}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{4d\left(d+ex^2\right)^2}-\frac{3\left(\frac{x\left(-35a^2e^4-6acd^2e^2+93c^2d^4\right)}{2de\left(d+ex^2\right)}-\frac{\left(35a^2e^4+6acd^2e^2+35c^2d^4\right)\int \frac{1}{ex^2+d}dx}{2de}\right)}{4de^3}}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x\left(ae^2+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}$$

$$\Big\downarrow \text{ 218}$$

3.136. $\int \frac{\left(a+cx^4\right)^2}{\left(d+ex^2\right)^5}\,dx$

$$\frac{\frac{\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{4d(d+ex^2)^2}-\frac{3\left(\frac{x(-35a^2e^4-6acd^2e^2+93c^2d^4)}{2de(d+ex^2)}-\frac{(35a^2e^4+6acd^2e^2+35c^2d^4)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{3/2}}\right)}{4de^3}}{6d}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{6d(d+ex^2)^3}}{8d}+\frac{x(ae^2+cd^2)^2}{8de^4(d+ex^2)^4}$$

input
```
Int[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

output
```
((c*d^2 + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) + (((7*a^2 - (25*c^2*d^4)/e^
4 - (18*a*c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) + (((35*a^2 + (163*c^2*d^4)/e
^4 + (6*a*c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) - (3*(((93*c^2*d^4 - 6*a*c*d^
2*e^2 - 35*a^2*e^4)*x)/(2*d*e*(d + e*x^2)) - ((35*c^2*d^4 + 6*a*c*d^2*e^2
+ 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(3/2))))/(4*d*e^3)
)/(6*d))/(8*d)
```

3.136.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1472
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1
)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d,
e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.136.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{\left(35a^2e^4+6ac\,d^2e^2-93c^2d^4\right)x^7}{128d^4e}+\frac{\left(385a^2e^4+66ac\,d^2e^2-511c^2d^4\right)x^5}{384d^3e^2}+\frac{\left(511a^2e^4-66ac\,d^2e^2-385c^2d^4\right)x^3}{384d^2e^3}+\frac{\left(93a^2e^4-6ac\,d^2e^2-35c^2d^4\right)x}{128e^4d}}{\left(e\,x^2+d\right)^4}+$
risch	$\frac{\frac{\left(35a^2e^4+6ac\,d^2e^2-93c^2d^4\right)x^7}{128d^4e}+\frac{\left(385a^2e^4+66ac\,d^2e^2-511c^2d^4\right)x^5}{384d^3e^2}+\frac{\left(511a^2e^4-66ac\,d^2e^2-385c^2d^4\right)x^3}{384d^2e^3}+\frac{\left(93a^2e^4-6ac\,d^2e^2-35c^2d^4\right)x}{128e^4d}}{\left(e\,x^2+d\right)^4}-$

input
```
int((c*x^4+a)^2/(e*x^2+d)^5,x,method=_RETURNVERBOSE)
```

output
```
(1/128*(35*a^2*e^4+6*a*c*d^2*e^2-93*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+
66*a*c*d^2*e^2-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4-66*a*c*d^2*e^2-
385*c^2*d^4)/d^2/e^3*x^3+1/128*(93*a^2*e^4-6*a*c*d^2*e^2-35*c^2*d^4)/e^4/d
*x)/(e*x^2+d)^4+1/128*(35*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)/d^4/e^4/(e*d)^
(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 806, normalized size of antiderivative = 3.61

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx$$
$$= \left[-\frac{6\,(93\,c^2d^5e^4 - 6\,acd^3e^6 - 35\,a^2de^8)x^7 + 2\,(511\,c^2d^6e^3 - 66\,acd^4e^5 - 385\,a^2d^2e^7)x^5 + 2\,(385\,c^2d^7e^2 + 66}{}\right.$$
$$-\frac{3\,(93\,c^2d^5e^4 - 6\,acd^3e^6 - 35\,a^2de^8)x^7 + (511\,c^2d^6e^3 - 66\,acd^4e^5 - 385\,a^2d^2e^7)x^5 + (385\,c^2d^7e^2 + 66\,acd}{}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="fricas")
```

output
```
[-1/768*(6*(93*c^2*d^5*e^4 - 6*a*c*d^3*e^6 - 35*a^2*d*e^8)*x^7 + 2*(511*c^
2*d^6*e^3 - 66*a*c*d^4*e^5 - 385*a^2*d^2*e^7)*x^5 + 2*(385*c^2*d^7*e^2 + 6
6*a*c*d^5*e^4 - 511*a^2*d^3*e^6)*x^3 + 3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*
a^2*d^4*e^4 + (35*c^2*d^4*e^4 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^
2*d^5*e^3 + 6*a*c*d^3*e^5 + 35*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*
d^4*e^4 + 35*a^2*d^2*e^6)*x^4 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*a^2*d
^3*e^5)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*
(35*c^2*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^5*e^9*x^8 + 4*d^6*e^
8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5), -1/384*(3*(93*c^2*d^5*e^
4 - 6*a*c*d^3*e^6 - 35*a^2*d*e^8)*x^7 + (511*c^2*d^6*e^3 - 66*a*c*d^4*e^5
- 385*a^2*d^2*e^7)*x^5 + (385*c^2*d^7*e^2 + 66*a*c*d^5*e^4 - 511*a^2*d^3*e
^6)*x^3 - 3*(35*c^2*d^8 + 6*a*c*d^6*e^2 + 35*a^2*d^4*e^4 + (35*c^2*d^4*e^4
 + 6*a*c*d^2*e^6 + 35*a^2*e^8)*x^8 + 4*(35*c^2*d^5*e^3 + 6*a*c*d^3*e^5 + 3
5*a^2*d*e^7)*x^6 + 6*(35*c^2*d^6*e^2 + 6*a*c*d^4*e^4 + 35*a^2*d^2*e^6)*x^4
 + 4*(35*c^2*d^7*e + 6*a*c*d^5*e^3 + 35*a^2*d^3*e^5)*x^2)*sqrt(d*e)*arctan
(sqrt(d*e)*x/d) + 3*(35*c^2*d^8*e + 6*a*c*d^6*e^3 - 93*a^2*d^4*e^5)*x)/(d^
5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5)]
```

3.136.6 Sympy [A] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.50

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = -\frac{\sqrt{-\frac{1}{d^9e^9}}\cdot(35a^2e^4+6acd^2e^2+35c^2d^4)\log\left(-d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256} + \frac{\sqrt{-\frac{1}{d^9e^9}}\cdot(35a^2e^4+6acd^2e^2+35c^2d^4)\log\left(d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256} + \frac{x^7\cdot(105a^2e^7+18acd^2e^5-279c^2d^4e^3)+x^5\cdot(385a^2de^6+66acd^3e^4-511c^2d^5e^2)+x^3\cdot(511a^2d^2e^5-66}{384d^8e^4+1536d^7e^5x^2+2304d^6e^6x^4+1536d^5e^7x^6+3}$$

input
```
integrate((c*x**4+a)**2/(e*x**2+d)**5,x)
```

output
```
-sqrt(-1/(d**9*e**9))*(35*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(
-d**5*e**4*sqrt(-1/(d**9*e**9)) + x)/256 + sqrt(-1/(d**9*e**9))*(35*a**2*e
**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(d**5*e**4*sqrt(-1/(d**9*e**9)) +
 x)/256 + (x**7*(105*a**2*e**7 + 18*a*c*d**2*e**5 - 279*c**2*d**4*e**3) +
x**5*(385*a**2*d*e**6 + 66*a*c*d**3*e**4 - 511*c**2*d**5*e**2) + x**3*(511
*a**2*d**2*e**5 - 66*a*c*d**4*e**3 - 385*c**2*d**6*e) + x*(279*a**2*d**3*e
**4 - 18*a*c*d**5*e**2 - 105*c**2*d**7))/(384*d**8*e**4 + 1536*d**7*e**5*x
**2 + 2304*d**6*e**6*x**4 + 1536*d**5*e**7*x**6 + 384*d**4*e**8*x**8)
```

3.136.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.97

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \frac{(35\,c^2d^4+6\,acd^2e^2+35\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\,\sqrt{de}d^4e^4} - \frac{279\,c^2d^4e^3x^7-18\,acd^2e^5x^7-105\,a^2e^7x^7+511\,c^2d^5e^2x^5-66\,acd^3e^4x^5-385\,a^2de^6x^5+385\,c^2d^6ex^3+6}{384\,(ex^2+d)^4d^4e^4}$$

input
```
integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="giac")
```

output
```
1/128*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqr
t(d*e)*d^4*e^4) - 1/384*(279*c^2*d^4*e^3*x^7 - 18*a*c*d^2*e^5*x^7 - 105*a^
2*e^7*x^7 + 511*c^2*d^5*e^2*x^5 - 66*a*c*d^3*e^4*x^5 - 385*a^2*d*e^6*x^5 +
 385*c^2*d^6*e*x^3 + 66*a*c*d^4*e^3*x^3 - 511*a^2*d^2*e^5*x^3 + 105*c^2*d^
7*x + 18*a*c*d^5*e^2*x - 279*a^2*d^3*e^4*x)/((e*x^2 + d)^4*d^4*e^4)
```

3.136.9 Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(35\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4)}{128\,d^{9/2}\,e^{9/2}} - \frac{\frac{x\,(-93\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4)}{128\,d\,e^4} - \frac{x^7\,(35\,a^2\,e^4+6\,a\,c\,d^2\,e^2-93\,c^2\,d^4)}{128\,d^4\,e} + \frac{x^3\,(-511\,a^2\,e^4+66\,a\,c\,d^2\,e^2+385\,c^2\,d^4)}{384\,d^2\,e^3} - \frac{x^5\,(385\,a^2\,e^4+66\,a\,c}{384\,d^3}}{d^4+4\,d^3\,e\,x^2+6\,d^2\,e^2\,x^4+4\,d\,e^3\,x^6+e^4\,x^8}$$

input
```
int((a + c*x^4)^2/(d + e*x^2)^5,x)
```

output
```
(atan((e^(1/2)*x)/d^(1/2))*(35*a^2*e^4 + 35*c^2*d^4 + 6*a*c*d^2*e^2))/(128
*d^(9/2)*e^(9/2)) - ((x*(35*c^2*d^4 - 93*a^2*e^4 + 6*a*c*d^2*e^2))/(128*d*
e^4) - (x^7*(35*a^2*e^4 - 93*c^2*d^4 + 6*a*c*d^2*e^2))/(128*d^4*e) + (x^3*
(385*c^2*d^4 - 511*a^2*e^4 + 66*a*c*d^2*e^2))/(384*d^2*e^3) - (x^5*(385*a^
2*e^4 - 511*c^2*d^4 + 66*a*c*d^2*e^2))/(384*d^3*e^2))/(d^4 + e^4*x^8 + 4*d
^3*e*x^2 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4)
```

3.137 $\int \frac{(d+ex^2)^4}{a+cx^4}\,dx$

3.137.1 Optimal result

Integrand size = 19, antiderivative size = 437

$$
\begin{aligned}
\int \frac{(d+ex^2)^4}{a+cx^4}\,dx
&= \frac{e^2(6cd^2-ae^2)\,x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c} \\
&\quad-\frac{\left(c^2d^4-6acd^2e^2+a^2e^4+4\sqrt{a}\sqrt{c}de(cd^2-ae^2)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad+\frac{\left(c^2d^4-6acd^2e^2+a^2e^4+4\sqrt{a}\sqrt{c}de(cd^2-ae^2)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad-\frac{\left(c^2d^4-6acd^2e^2+a^2e^4-4\sqrt{a}\sqrt{c}de(cd^2-ae^2)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}} \\
&\quad+\frac{\left(c^2d^4-6acd^2e^2+a^2e^4-4\sqrt{a}\sqrt{c}de(cd^2-ae^2)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}
\end{aligned}
$$

output
```
e^2*(-a*e^2+6*c*d^2)*x/c^2+4/3*d*e^3*x^3/c+1/5*e^4*x^5/c-1/8*ln(-a^(1/4)*c
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4-4*d*e
*(-a*e^2+c*d^2)*a^(1/2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/8*ln(a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4-4*d*e*
(-a*e^2+c*d^2)*a^(1/2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/4*arctan(-1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4+4*d*e*(-a*e^2+c*d^2)
*a^(1/2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a
^(1/4))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4+4*d*e*(-a*e^2+c*d^2)*a^(1/2)*c^(1/2
))/a^(3/4)/c^(9/4)*2^(1/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx$$
$$= \frac{-120a^{3/4}\sqrt[4]{c}e^2(-6cd^2+ae^2)\,x+160a^{3/4}c^{5/4}de^3x^3+24a^{3/4}c^{5/4}e^4x^5-30\sqrt{2}\big(c^2d^4+4\sqrt{a}c^{3/2}d^3e-6acd^2e}{}$$

input
```
Integrate[(d + e*x^2)^4/(a + c*x^4),x]
```

output
```
(-120*a^(3/4)*c^(1/4)*e^2*(-6*c*d^2 + a*e^2)*x + 160*a^(3/4)*c^(5/4)*d*e^3
*x^3 + 24*a^(3/4)*c^(5/4)*e^4*x^5 - 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2
)*d^3*e - 6*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*x)/a^(1/4)] + 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2)*d^3*e
 - 6*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*x)/a^(1/4)] - 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*
c*d^2*e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2] + 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*
e - 6*a*c*d^2*e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(120*a^(3/4)*c^(9/4))
```

3.137.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx$$

$$\Big\downarrow \text{1485}$$

$$\int \left(\frac{a^2e^4+4cdex^2(cd^2-ae^2)-6acd^2e^2+c^2d^4}{c^2(a+cx^4)}+\frac{e^2(6cd^2-ae^2)}{c^2}+\frac{4de^3x^2}{c}+\frac{e^4x^4}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(a^2e^4-6acd^2e^2+4\sqrt{a}\sqrt{c}de(cd^2-ae^2)+c^2d^4\right)}{2\sqrt{2}a^{3/4}c^{9/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(a^2e^4-6acd^2e^2+4\sqrt{a}\sqrt{c}de(cd^2-ae^2)+c^2d^4\right)}{2\sqrt{2}a^{3/4}c^{9/4}}-\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de(cd^2-ae^2)+c^2d^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}+\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\sqrt{c}de(cd^2-ae^2)+c^2d^4\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{9/4}}+\frac{e^2x(6cd^2-ae^2)}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}$$

input
```
Int[(d + e*x^2)^4/(a + c*x^4),x]
```

output
```
(e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (4*d*e^3*x^3)/(3*c) + (e^4*x^5)/(5*c) - ((
c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) + ((
c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) - ((
c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4
)*c^(9/4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(
c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
*Sqrt[2]*a^(3/4)*c^(9/4))
```

3.137.3.1 Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.137.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.27

method	result
risch	$\frac{e^4x^5}{5c}+\frac{4d\,e^3x^3}{3c}-\frac{e^4ax}{c^2}+\frac{6e^2d^2x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(4dce\left(-a\,e^2+c\,d^2\right)\_R^2+a^2e^4-6ac\,d^2e^2+c^2d^4\right)\ln\left(x-\_R\right)}{\_R^3}}{4c^3}$
default	$-\frac{e^2\left(-\frac{1}{5}c\,x^5e^2-\frac{4}{3}dc\,x^3e+a\,e^2x-6c\,d^2x\right)}{c^2}+\frac{\frac{\left(a^2e^4-6ac\,d^2e^2+c^2d^4\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arcta}\right.}{8a}}{}$

input
```
int((e*x^2+d)^4/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*e^4*x^5/c+4/3*d*e^3*x^3/c-e^4/c^2*a*x+6*e^2/c*d^2*x+1/4/c^3*sum((4*d*c
*e*(-a*e^2+c*d^2)*_R^2+a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)/_R^3*ln(x-_R),_R=Roo
tOf(_Z^4*c+a))
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2878 vs. 2(352) = 704.

Time = 11.20 (sec) , antiderivative size = 2878, normalized size of antiderivative = 6.59

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/60*(12*c*e^4*x^5 + 80*c*d*e^3*x^3 + 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a*c^2
*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt(-(c^8*d^16 - 56*a*c
^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*
d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14
 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 24*a*c^7*d^14*e^2 - 36*a
^2*c^6*d^12*e^4 + 88*a^3*c^5*d^10*e^6 + 198*a^4*c^4*d^8*e^8 + 88*a^5*c^3*d
^6*e^10 - 36*a^6*c^2*d^4*e^12 - 24*a^7*c*d^2*e^14 + a^8*e^16)*x + (a*c^8*d
^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 23
9*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 + 4*(a^3*c^8*d^3*e
- a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^
4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 +
 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8
*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqr
t(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^1
0*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^1
2 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))) - 15*c^2*sqrt(-(8*
c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - 8*a^3*d*e^7 + a*c^4*sqrt
(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10
*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12
 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))/(a*c^4))*log((c^8*d^16 - 2...
```

3.137.6 Sympy [A] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = x\left(-\frac{ae^4}{c^2}+\frac{6d^2e^2}{c}\right)$$
$$+\mathrm{RootSum}\left(256t^4a^3c^9+t^2\left(-256a^5c^5de^7+1792a^4c^6d^3e^5-1792a^3c^7d^5e^3+256a^2c^8d^7e\right)+a^8e^{16}+8a^7c\right.$$
$$+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}$$

input
```
integrate((e*x**2+d)**4/(c*x**4+a),x)
```

output
```
x*(-a*e**4/c**2 + 6*d**2*e**2/c) + RootSum(256*_t**4*a**3*c**9 + _t**2*(-2
56*a**5*c**5*d*e**7 + 1792*a**4*c**6*d**3*e**5 - 1792*a**3*c**7*d**5*e**3
+ 256*a**2*c**8*d**7*e) + a**8*e**16 + 8*a**7*c*d**2*e**14 + 28*a**6*c**2*
d**4*e**12 + 56*a**5*c**3*d**6*e**10 + 70*a**4*c**4*d**8*e**8 + 56*a**3*c*
*5*d**10*e**6 + 28*a**2*c**6*d**12*e**4 + 8*a*c**7*d**14*e**2 + c**8*d**16
, Lambda(_t, _t*log(x + (256*_t**3*a**4*c**7*d*e**3 - 256*_t**3*a**3*c**8*
d**3*e + 4*_t*a**7*c**2*e**12 - 264*_t*a**6*c**3*d**2*e**10 + 1980*_t*a**5
*c**4*d**4*e**8 - 3696*_t*a**4*c**5*d**6*e**6 + 1980*_t*a**3*c**6*d**8*e**
4 - 264*_t*a**2*c**7*d**10*e**2 + 4*_t*a*c**8*d**12)/(a**8*e**16 - 24*a**7
*c*d**2*e**14 - 36*a**6*c**2*d**4*e**12 + 88*a**5*c**3*d**6*e**10 + 198*a*
*4*c**4*d**8*e**8 + 88*a**3*c**5*d**10*e**6 - 36*a**2*c**6*d**12*e**4 - 24
*a*c**7*d**14*e**2 + c**8*d**16))) + 4*d*e**3*x**3/(3*c) + e**4*x**5/(5*c
)
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = \frac{3\,ce^4x^5 + 20\,cde^3x^3 + 15\,(6\,cd^2e^2 - ae^4)x}{15\,c^2} + \frac{2\,\sqrt{2}\left(c^{\frac{5}{2}}d^4 + 4\,\sqrt{a}c^2d^3e - 6\,ac^{\frac{3}{2}}d^2e^2 - 4\,a^{\frac{3}{2}}cde^3 + a^2\sqrt{c}e^4\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\left(c^{\frac{5}{2}}d^4 + 4\,\sqrt{a}c^2d^3e - 6\,ac^{\frac{3}{2}}d^2e^2 - 4\,a^{\frac{3}{2}}cde^3 \cdots\right.}{\sqrt{a}\sqrt{\sqrt{a}\cdots}}$$

input
```
integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/15*(3*c*e^4*x^5 + 20*c*d*e^3*x^3 + 15*(6*c*d^2*e^2 - a*e^4)*x)/c^2 + 1/8
*(2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 - 4*a
^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2
)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*s
qrt(c)) + 2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e
^2 - 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
- sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(c))*sqrt(c)) + sqrt(2)*(c^(5/2)*d^4 - 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)
*d^2*e^2 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*log(sqrt(c)*x^2 + sqrt(2)*
a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(5/2)*d^4 - 4*
sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*
e^4)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/
4)))/c^2
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx$$

$$= \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4 - 6(ac^3)^{\frac{1}{4}}ac^2d^2e^2 + (ac^3)^{\frac{1}{4}}a^2ce^4 + 4(ac^3)^{\frac{3}{4}}cd^3e - 4(ac^3)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right.}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{4ac^4}$$

$$+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4 - 6(ac^3)^{\frac{1}{4}}ac^2d^2e^2 + (ac^3)^{\frac{1}{4}}a^2ce^4 + 4(ac^3)^{\frac{3}{4}}cd^3e - 4(ac^3)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right.\right.}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{4ac^4}$$

$$+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4 - 6(ac^3)^{\frac{1}{4}}ac^2d^2e^2 + (ac^3)^{\frac{1}{4}}a^2ce^4 - 4(ac^3)^{\frac{3}{4}}cd^3e + 4(ac^3)^{\frac{3}{4}}ade^3\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}\right.}{8ac^4}$$

$$- \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4 - 6(ac^3)^{\frac{1}{4}}ac^2d^2e^2 + (ac^3)^{\frac{1}{4}}a^2ce^4 - 4(ac^3)^{\frac{3}{4}}cd^3e + 4(ac^3)^{\frac{3}{4}}ade^3\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}\right.}{8ac^4}$$

$$+ \frac{3c^4e^4x^5 + 20c^4de^3x^3 + 90c^4d^2e^2x - 15ac^3e^4x}{15c^5}$$

input
```
integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^
3)^(1/4)*a^2*c*e^4 + 4*(a*c^3)^(3/4)*c*d^3*e - 4*(a*c^3)^(3/4)*a*d*e^3)*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/4*sq
rt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^3)^(1/
4)*a^2*c*e^4 + 4*(a*c^3)^(3/4)*c*d^3*e - 4*(a*c^3)^(3/4)*a*d*e^3)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*
((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^3)^(1/4)*a^2
*c*e^4 - 4*(a*c^3)^(3/4)*c*d^3*e + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 + sqrt
(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^
4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3
/4)*c*d^3*e + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(a*c^4) + 1/15*(3*c^4*e^4*x^5 + 20*c^4*d*e^3*x^3 + 90*c^4*d^2*e^
2*x - 15*a*c^3*e^4*x)/c^5
```

3.137.9 Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 4022, normalized size of antiderivative = 9.20

$$\int \frac{(d+ex^2)^4}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^4/(a + c*x^4),x)
```

output
```
atan((((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*
a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2
)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e
+ 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6
*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4
*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2
) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*
c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*
a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*
a^3*c^9))^(1/2)*1i + ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*
c*d^2*e^6 + 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*
a^2*c^5*d^2*e^2)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8
*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5
 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) +
 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*
(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*
d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c
^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c
^9)^(1/2))/(16*a^3*c^9))^(1/2)*1i)/(((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^
6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4...
```

3.138 $\int \frac{(d+ex^2)^3}{a+cx^4} dx$

3.138.1 Optimal result

Integrand size = 19, antiderivative size = 370

$$\int \frac{(d+ex^2)^3}{a+cx^4} dx = \frac{3de^2x}{c} + \frac{e^3x^3}{3c} - \frac{\left(\sqrt{c}d(cd^2-3ae^2)+\sqrt{a}e(3cd^2-ae^2)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{7/4}} + \frac{\left(\sqrt{c}d(cd^2-3ae^2)+\sqrt{a}e(3cd^2-ae^2)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{7/4}} - \frac{\left(\sqrt{c}d(cd^2-3ae^2)-\sqrt{a}e(3cd^2-ae^2)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}} + \frac{\left(\sqrt{c}d(cd^2-3ae^2)-\sqrt{a}e(3cd^2-ae^2)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}$$

output
```
3*d*e^2*x/c+1/3*e^3*x^3/c-1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^
(1/2))*(-e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^
(7/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*(-
a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^(7/4)*2^(1/2)
+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-
3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^(7/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(
1/2)/a^(1/4))*(e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3
/4)/c^(7/4)*2^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$$
$$= \frac{72a^{3/4}c^{3/4}de^2x+8a^{3/4}c^{3/4}e^3x^3+6\sqrt{2}\left(-c^{3/2}d^3-3\sqrt{a}cd^2e+3a\sqrt{c}de^2+a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+}{}$$

input
```
Integrate[(d + e*x^2)^3/(a + c*x^4),x]
```

output
```
(72*a^(3/4)*c^(3/4)*d*e^2*x + 8*a^(3/4)*c^(3/4)*e^3*x^3 + 6*Sqrt[2]*(-(c^(
3/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*
e - 3*a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4
)] - 3*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3
/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2
]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^(3/4)*c^(7/4))
```

3.138.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$$
$$\Big\downarrow \text{1485}$$
$$\int \left(\frac{ex^2(3cd^2-ae^2)-3ade^2+cd^3}{c(a+cx^4)}+\frac{3de^2}{c}+\frac{e^3x^2}{c}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d\left(cd^2-3ae^2\right)+\sqrt{a}e\left(3cd^2-ae^2\right)\right)}{2\sqrt{2}a^{3/4}c^{7/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d\left(cd^2-3ae^2\right)+\sqrt{a}e\left(3cd^2-ae^2\right)\right)}{2\sqrt{2}a^{3/4}c^{7/4}}-\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}d\left(cd^2-3ae^2\right)-\sqrt{a}e\left(3cd^2-ae^2\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{7/4}}+\frac{3de^2x}{c}+\frac{e^3x^3}{3c}$$

input
```
Int[(d + e*x^2)^3/(a + c*x^4),x]
```

output
```
(3*d*e^2*x)/c + (e^3*x^3)/(3*c) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*
e*(3*c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a
^(3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e*(3*c*d^2 - a*e
^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(7/4))
- ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4)) +
 ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4))
```

3.138.3.1 Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.138.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.22

method	result
risch	$\frac{e^3x^3}{3c}+\frac{3d\,e^2x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(e\left(-a\,e^2+3c\,d^2\right)\_R^2-3d\,e^2a+d^3c\right)\ln\left(x-\_R\right)}{\_R^3}}{4c^2}$
default	$\frac{e^2\left(\frac{1}{3}e\,x^3+3dx\right)}{c}+\frac{\frac{\left(-3d\,e^2a+d^3c\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\left(-a\,e^3+3c\,d^2e\right)\cdots}{}}{c}$

input
```
int((e*x^2+d)^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*e^3*x^3/c+3*d*e^2*x/c+1/4/c^2*sum((e*(-a*e^2+3*c*d^2)*_R^2-3*d*e^2*a+d
^3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.138.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. 2(287) = 574.

Time = 2.39 (sec) , antiderivative size = 2133, normalized size of antiderivative = 5.76

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/12*(4*e^3*x^3 + 36*d*e^2*x - 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6
*a^2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e
^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e
^12)/(a^3*c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*c^4*
d^8*e^4 + 27*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x + (a*c^6*d^
9 - 18*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5*c
^2*d*e^8 + (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10
*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 3
0*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e
^3 + 6*a^2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4
*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 +
 a^6*e^12)/(a^3*c^7)))/(a*c^3))) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3
 + 6*a^2*d*e^5 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d
^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a
^6*e^12)/(a^3*c^7)))/(a*c^3))*log(-(c^6*d^12 - 12*a*c^5*d^10*e^2 - 27*a^2*
c^4*d^8*e^4 + 27*a^4*c^2*d^4*e^8 + 12*a^5*c*d^2*e^10 - a^6*e^12)*x - (a*c^
6*d^9 - 18*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a
^5*c^2*d*e^8 + (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*
d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8
 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*...
```

3.138.6 Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3c^7 + t^2\cdot\left(192a^4c^4de^5 - 640a^3c^5d^3e^3 + 192a^2c^6d^5e\right) + a^6e^{12} + 6a^5cd^2e^{10} + 15a^4c^2d^4e^8 \right.$$
$$+ \frac{3de^2x}{c} + \frac{e^3x^3}{3c}$$

input
```
integrate((e*x**2+d)**3/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**7 + _t**2*(192*a**4*c**4*d*e**5 - 640*a**3*c**5*
d**3*e**3 + 192*a**2*c**6*d**5*e) + a**6*e**12 + 6*a**5*c*d**2*e**10 + 15*
a**4*c**2*d**4*e**8 + 20*a**3*c**3*d**6*e**6 + 15*a**2*c**4*d**8*e**4 + 6*
a*c**5*d**10*e**2 + c**6*d**12, Lambda(_t, _t*log(x + (-64*_t**3*a**4*c**5
*e**3 + 192*_t**3*a**3*c**6*d**2*e - 36*_t*a**5*c**2*d*e**8 + 336*_t*a**4*
c**3*d**3*e**6 - 504*_t*a**3*c**4*d**5*e**4 + 144*_t*a**2*c**5*d**7*e**2 -
 4*_t*a*c**6*d**9)/(a**6*e**12 - 12*a**5*c*d**2*e**10 - 27*a**4*c**2*d**4*
e**8 + 27*a**2*c**4*d**8*e**4 + 12*a*c**5*d**10*e**2 - c**6*d**12)))) + 3*
d*e**2*x/c + e**3*x**3/(3*c)
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx = \frac{e^3x^3+9\,de^2x}{3\,c} + \frac{\frac{2\,\sqrt{2}\left(c^{\frac{3}{2}}d^3+3\,\sqrt{a}cd^2e-3\,a\sqrt{c}de^2-a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\left(c^{\frac{3}{2}}d^3+3\,\sqrt{a}cd^2e-3\,a\sqrt{c}de^2-a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\cdots\right.}{2\,\sqrt{\cdots}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}}{\cdots}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/3*(e^3*x^3 + 9*d*e^2*x)/c + 1/8*(2*sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^
2*e - 3*a*sqrt(c)*d*e^2 - a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + s
qrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
c))*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*
e^2 - a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/
4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(
2)*(c^(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*log
(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sq
rt(2)*(c^(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*
log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/
c
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx = \frac{c^2e^3x^3+9\,c^2de^2x}{3\,c^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 - 3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2 + 3\,(ac^3)^{\frac{3}{4}}\,cd^2e - (ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 - 3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2 + 3\,(ac^3)^{\frac{3}{4}}\,cd^2e - (ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 - 3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2 - 3\,(ac^3)^{\frac{3}{4}}\,cd^2e + (ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^4} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 - 3\,(ac^3)^{\frac{1}{4}}\,ac^2de^2 - 3\,(ac^3)^{\frac{3}{4}}\,cd^2e + (ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^4}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="giac")
```

output
```
1/3*(c^2*e^3*x^3 + 9*c^2*d*e^2*x)/c^3 + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3
 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a
*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4)
+ 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*
c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 -
 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e
^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a
*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*
e + (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c
^4)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 2712, normalized size of antiderivative = 7.33

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^3/(a + c*x^4),x)
```

output
```
(e^3*x^3)/(3*c) - atan((a^3*e^6*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (5*d^
3*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c^7)
^(1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^4*
e^2*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c
^2 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^3*
c^7)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*(
-a^3*c^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-
a^3*c^7)^(1/2))/(a*c^2)) - (c^3*d^6*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (
5*d^3*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*
c^7)^(1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*
d^4*e^2*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^
9)/c^2 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-
a^3*c^7)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e
^6*(-a^3*c^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^
2*(-a^3*c^7)^(1/2))/(a*c^2)) + (a*c^2*d^4*e^2*x*((e^6*(-a^3*c^7)^(1/2))/(1
6*c^7) + (5*d^3*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (
d^6*(-a^3*c^7)^(1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c
^6) + (15*d^4*e^2*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e
 + (2*a^4*e^9)/c^2 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3
 + (2*d^9*(-a^3*c^7)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^...
```

3.139 $\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$

3.139.1 Optimal result

Integrand size = 19, antiderivative size = 297

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx = \frac{e^2x}{c} - \frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}} + \frac{\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}} - \frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}} + \frac{\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}$$

output
```
e^2*x/c-1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c*d^2-a*e^
2-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*
2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c*d^2-a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c
^(5/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*d^2-a*e^2+2*d*e
*a^(1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a
^(1/4))*(c*d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$$
$$= \frac{8a^{3/4}\sqrt[4]{c}e^2x - 2\sqrt{2}(cd^2+2\sqrt{a}\sqrt{c}de-ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt{2}(cd^2+2\sqrt{a}\sqrt{c}de-ae^2)\arctan\left(\right.}{}$$

input
```
Integrate[(d + e*x^2)^2/(a + c*x^4),x]
```

output
```
(8*a^(3/4)*c^(1/4)*e^2*x - 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^
2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*
Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(
c*d^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2] + Sqrt[2]*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*a^(3/4)*c^(5/4))
```

3.139.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$$
$$\Big\downarrow \text{1485}$$
$$\int \left(\frac{-ae^2+cd^2+2cdex^2}{c(a+cx^4)}+\frac{e^2}{c}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}c^{5/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}c^{5/4}}-\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{5/4}}+\frac{e^2x}{c}$$

input
```
Int[(d + e*x^2)^2/(a + c*x^4),x]
```

output
```
(e^2*x)/c - ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 + 2*Sqrt[a]*Sqrt
[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*c^(5/4)) - ((c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2
 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4))
```

3.139.3.1 Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.139.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.19

method	result
risch	$\frac{e^2 x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(2\_R^2 cde-a e^2+c d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{4c^2}$
default	$\frac{e^2 x}{c}+\frac{\frac{\left(-a e^2+c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{de\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right.\right.}{}}{c}$

input
```
int((e*x^2+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
e^2*x/c+1/4/c^2*sum((2*_R^2*c*d*e-a*e^2+c*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^
4*c+a))
```

3.139.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1480 vs. 2(216) = 432.

Time = 0.71 (sec) , antiderivative size = 1480, normalized size of antiderivative = 4.98

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(4*e^2*x + c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*
a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)
))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*
d^2*e^6 + a^4*e^8)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4
- a^4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2
*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*
d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*
a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))) - c*sqrt(-(4*c*d^3*e - 4*a*
d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*
a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e
^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 + a^4*e^8)*x - (a*c^4*d^6 - 7*a^
2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*a^3*c^4*d*e*sqrt(-(c^4*d
^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(
a^3*c^5)))*sqrt(-(4*c*d^3*e - 4*a*d*e^3 + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*
d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*
c^2))) + c*sqrt(-(4*c*d^3*e - 4*a*d*e^3 - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*
d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)))/(a*
c^2))*log((c^4*d^8 - 4*a*c^3*d^6*e^2 - 10*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^
6 + a^4*e^8)*x + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*
c*e^6 - 2*a^3*c^4*d*e*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^...
```

3.139.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$$

$$= \operatorname{RootSum}\left(256t^4a^3c^5 + t^2(-128a^3c^3de^3 + 128a^2c^4d^3e) + a^4e^8 + 4a^3cd^2e^6 + 6a^2c^2d^4e^4 + 4ac^3d^6e^2 + c^4d^8,\right.$$

$$+ \frac{e^2x}{c}$$

input
```
integrate((e*x**2+d)**2/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**5 + _t**2*(-128*a**3*c**3*d*e**3 + 128*a**2*c**4
*d**3*e) + a**4*e**8 + 4*a**3*c*d**2*e**6 + 6*a**2*c**2*d**4*e**4 + 4*a*c*
*3*d**6*e**2 + c**4*d**8, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c**4*d*e
- 4*_t*a**4*c*e**6 + 60*_t*a**3*c**2*d**2*e**4 - 60*_t*a**2*c**3*d**4*e**2
 + 4*_t*a*c**4*d**6)/(a**4*e**8 - 4*a**3*c*d**2*e**6 - 10*a**2*c**2*d**4*e
**4 - 4*a*c**3*d**6*e**2 + c**4*d**8))) + e**2*x/c
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx = \frac{e^2x}{c} + \frac{\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^2+2\sqrt{a}cde-a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^2+2\sqrt{a}cde-a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}(c\cdots}{\cdots}}{8c}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

output
```
e^2*x/c + 1/8*(2*sqrt(2)*(c^(3/2)*d^2 + 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*a
rctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqr
t(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^2 +
2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)
*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sq
rt(c)) + sqrt(2)*(c^(3/2)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sqrt(
c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*
(c^(3/2)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*
a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.07

$$\begin{aligned}
\int \frac{(d+ex^2)^2}{a+cx^4}\,dx &= \frac{e^2x}{c} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2+2\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} \\
&+ \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2+2\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} \\
&+ \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2-2\left(ac^3\right)^{\frac{3}{4}}de\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac^3} \\
&- \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2-2\left(ac^3\right)^{\frac{3}{4}}de\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac^3}
\end{aligned}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

output
```
e^2*x/c + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(
a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/
4))/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 +
 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e
^2 - 2*(a*c^3)^(3/4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*
c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c
^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 1479, normalized size of antiderivative = 4.98

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx = \frac{e^2 x}{c} - 2\,\mathrm{atanh}\left(\frac{8c^3 d^4 x\sqrt{\frac{d e^3}{4c^2} - \frac{d^3 e}{4ac} + \frac{d^4\sqrt{-a^3c^5}}{16a^3c^3} + \frac{e^4\sqrt{-a^3c^5}}{16ac^5} - \frac{3d^2e^2\sqrt{-a^3c^5}}{8a^2c^4}}}{4a^2de^5 - \frac{2d^6\sqrt{-a^3c^5}}{a^2} + 4c^2d^5e + \frac{2ae^6\sqrt{-a^3c^5}}{c^3} - 24acd^3e^3 - \frac{14d^2e^4\sqrt{-a^3c^5}}{c^2} + \frac{14d^4e^2\sqrt{-a^3c^5}}{ac}} + \frac{8a^2ce^4x\sqrt{\frac{d e^3}{4c^2} - \frac{d^3 e}{4ac} + \frac{d^4\sqrt{-a^3c^5}}{16a^3c^3} + \frac{e^4\sqrt{-a^3c^5}}{16ac^5} - \frac{3d^2e^2\sqrt{-a^3c^5}}{8a^2c^4}}}{4a^2de^5 - \frac{2d^6\sqrt{-a^3c^5}}{a^2} + 4c^2d^5e + \frac{2ae^6\sqrt{-a^3c^5}}{c^3} - 24acd^3e^3 - \frac{14d^2e^4\sqrt{-a^3c^5}}{c^2} + \frac{14d^4e^2\sqrt{-a^3c^5}}{ac}} - \frac{48ac^2d^2e^2x\sqrt{\frac{d e^3}{4c^2} - \frac{d^3 e}{4ac} + \frac{d^4\sqrt{-a^3c^5}}{16a^3c^3} + \frac{e^4\sqrt{-a^3c^5}}{16ac^5} - \frac{3d^2e^2\sqrt{-a^3c^5}}{8a^2c^4}}}{4a^2de^5 - \frac{2d^6\sqrt{-a^3c^5}}{a^2} + 4c^2d^5e + \frac{2ae^6\sqrt{-a^3c^5}}{c^3} - 24acd^3e^3 - \frac{14d^2e^4\sqrt{-a^3c^5}}{c^2} + \frac{14d^4e^2\sqrt{-a^3c^5}}{ac}}\right)\sqrt{\frac{a^2 e^4 \sqrt{\cdots}}{\cdots}} - 2\,\mathrm{atanh}\left(\frac{8c^3d^4x\sqrt{\frac{d e^3}{4c^2} - \frac{d^3 e}{4ac} - \frac{d^4\sqrt{-a^3c^5}}{16a^3c^3} - \frac{e^4\sqrt{-a^3c^5}}{16ac^5} + \frac{3d^2e^2\sqrt{-a^3c^5}}{8a^2c^4}}}{\frac{2d^6\sqrt{-a^3c^5}}{a^2} + 4a^2de^5 + 4c^2d^5e - \frac{2ae^6\sqrt{-a^3c^5}}{c^3} - 24acd^3e^3 + \frac{14d^2e^4\sqrt{-a^3c^5}}{c^2} - \frac{14d^4e^2\sqrt{-a^3c^5}}{ac}} + \frac{8a^2ce^4x\sqrt{\frac{d e^3}{4c^2} - \frac{d^3 e}{4ac} - \frac{d^4\sqrt{-a^3c^5}}{16a^3c^3} - \frac{e^4\sqrt{-a^3c^5}}{16ac^5} + \frac{3d^2e^2\sqrt{-a^3c^5}}{8a^2c^4}}}{\frac{2d^6\sqrt{-a^3c^5}}{a^2} + 4a^2de^5 + 4c^2d^5e - \frac{2ae^6\sqrt{-a^3c^5}}{c^3} - 24acd^3e^3 + \frac{14d^2e^4\sqrt{-a^3c^5}}{c^2} - \frac{14d^4e^2\sqrt{-a^3c^5}}{ac}} - \frac{48ac^2d^2e^2x\sqrt{\frac{d e^3}{4c^2} - \frac{d^3 e}{4ac} - \frac{d^4\sqrt{-a^3c^5}}{16a^3c^3} - \frac{e^4\sqrt{-a^3c^5}}{16ac^5} + \frac{3d^2e^2\sqrt{-a^3c^5}}{8a^2c^4}}}{\frac{2d^6\sqrt{-a^3c^5}}{a^2} + 4a^2de^5 + 4c^2d^5e - \frac{2ae^6\sqrt{-a^3c^5}}{c^3} - 24acd^3e^3 + \frac{14d^2e^4\sqrt{-a^3c^5}}{c^2} - \frac{14d^4e^2\sqrt{-a^3c^5}}{ac}}\right)\sqrt{-\frac{a^2 e^4 \cdots}{\cdots}}$$

input
```
int((d + e*x^2)^2/(a + c*x^4),x)
```

output
```
(e^2*x)/c - 2*atanh((8*c^3*d^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4
*(-a^3*c^5)^(1/2))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d
^2*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c
^5)^(1/2))/a^2 + 4*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3
*e^3 - (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(
a*c)) + (8*a^2*c*e^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*(-a^3*c^5
)^(1/2))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2*e^2*(-a
^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)^(1/2))
/a^2 + 4*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 - (14
*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)) - (4
8*a*c^2*d^2*e^2*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*(-a^3*c^5)^(1/
2))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2*e^2*(-a^3*c^
5)^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)^(1/2))/a^2
+ 4*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 - (14*d^2*
e^4*(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)))*((a^2*e^
4*(-a^3*c^5)^(1/2) + c^2*d^4*(-a^3*c^5)^(1/2) - 4*a^2*c^4*d^3*e + 4*a^3*c^
3*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^5)^(1/2))/(16*a^3*c^5))^(1/2) - 2*atanh((8
*c^3*d^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) - (d^4*(-a^3*c^5)^(1/2))/(16
*a^3*c^3) - (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) + (3*d^2*e^2*(-a^3*c^5)^(1/2
))/(8*a^2*c^4))^(1/2))/((2*d^6*(-a^3*c^5)^(1/2))/a^2 + 4*a^2*d*e^5 + 4*...
```

3.139. $\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$

3.140 $\int \frac{d+ex^2}{a+cx^4} dx$

3.140.1 Optimal result

Integrand size = 17, antiderivative size = 247

$$\int \frac{d+ex^2}{a+cx^4} dx = -\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$

output
```
-1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1
/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(-1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*a
rctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2
^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.74

$$\int \frac{d+ex^2}{a+cx^4}\,dx$$
$$= \frac{-2\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-\left(\sqrt{c}d-\sqrt{a}e\right)\left(\log\left(\sqrt{a}\right.\right.}{4\sqrt{2}a^{3/4}c^{3/4}}$$

input
```
Integrate[(d + e*x^2)/(a + c*x^4),x]
```

output
```
(-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(S
qrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d
 - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(4*Sqrt[2]*a^(3/4)*
c^(3/4))
```

3.140.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{a+cx^4}\,dx$$
$$\Big\downarrow \text{1482}$$
$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{2c}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{2c}$$
$$\Big\downarrow \text{27}$$
$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}$$
$$\Big\downarrow \text{1476}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+$$
$$\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}$$

$$\Big\downarrow \text{25}$$

3.140. $\int \frac{d+ex^2}{a+cx^4}\,dx$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}$$

input
```
Int[(d + e*x^2)/(a + c*x^4),x]
```

3.140. $\int \frac{d+ex^2}{a+cx^4}\,dx$

output
```
(((Sqrt[c]*d)/Sqrt[a] + e)*(-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqr
t[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*
a^(1/4)*c^(1/4))))/(2*Sqrt[c]) + (((Sqrt[c]*d)/Sqrt[a] - e)*(-1/2*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)
*c^(1/4))))/(2*Sqrt[c])
```

3.140.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

3.140.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.14

method	result
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(\_R^{2}e+d\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4c}$
default	$\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\operatorname{arct}\right.}{8c(}$

input
```
int((e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum((_R^2*e+d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.140.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(166) = 332$.

Time = 0.30 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.11

$$\int \frac{d+ex^2}{a+cx^4}\,dx = -\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\log\left(-(c^2d^4-a^2e^4)x + \left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\right) + \frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\log\left(-(c^2d^4-a^2e^4)x - \left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}+2\,de}{ac}}\right) + \frac{1}{4}\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\log\left(-(c^2d^4-a^2e^4)x + \left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-ac^2d^3+a^2cde^2\right)\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\right) - \frac{1}{4}\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\log\left(-(c^2d^4-a^2e^4)x - \left(a^3c^2e\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-ac^2d^3+a^2cde^2\right)\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2\,acd^2e^2+a^2e^4}{a^3c^3}}-2\,de}{ac}}\right)$$

input
```
integrate((e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d
*e)/(a*c))*log(-(c^2*d^4 - a^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*
d^2*e^2 + a^2*e^4)/(a^3*c^3)) + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt
(-(a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c)
)*log(-(c^2*d^4 - a^2*e^4)*x - (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 +
 a^2*e^4)/(a^3*c^3)) + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt((a*c*sqr
t(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-(c^
2*d^4 - a^2*e^4)*x + (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a^3*c^3)) - a*c^2*d^3 + a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))) - 1/4*sqrt((a*c*sqrt(-(c^2*d^4
 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-(c^2*d^4 - a^2
*e^4)*x - (a^3*c^2*e*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3))
- a*c^2*d^3 + a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*
e^4)/(a^3*c^3)) - 2*d*e)/(a*c)))
```

3.140.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int \frac{d+ex^2}{a+cx^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3c^3 + 64t^2a^2c^2de + a^2e^4 + 2acd^2e^2 + c^2d^4, \left(t \mapsto t\log\left(x + \frac{64t^3a^3c^2e + 12ta^2cde^2 - 4t}{a^2e^4 - c^2d^4}\right.\right.\right.$$

input
```
integrate((e*x**2+d)/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**3 + 64*_t**2*a**2*c**2*d*e + a**2*e**4 + 2*a*c*d
**2*e**2 + c**2*d**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*c**2*e + 12*_t*
a**2*c*d*e**2 - 4*_t*a*c**2*d**3)/(a**2*e**4 - c**2*d**4))))
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.89

$$\int \frac{d+ex^2}{a+cx^4}\,dx = \frac{\sqrt{2}(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}}$$

input
```
integrate((e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))
*sqrt(c)) + 1/4*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt
(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(c))*sqrt(c)) + 1/8*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^
2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(
sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(
a))/(a^(3/4)*c^(3/4))
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

$$\int \frac{d+ex^2}{a+cx^4}\,dx = \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3} - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

input
```
integrate((e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)
*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*
log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3
)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a*c^3)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.43

$$\int \frac{d+ex^2}{a+cx^4}\,dx = -2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}-\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}-\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}} - \frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}-\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}-\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}\right)\sqrt{-\frac{c\,d^2\,\sqrt{-a^3\,c^3}-a\,e^2\,\sqrt{-a^3\,c^3}+2\,a^2\,c^2\,d\,e}{16\,a^3\,c^3}} - 2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}-\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}+\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}} - \frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}-\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}+\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}\right)\sqrt{-\frac{a\,e^2\,\sqrt{-a^3\,c^3}-c\,d^2\,\sqrt{-a^3\,c^3}+2\,a^2\,c^2\,d\,e}{16\,a^3\,c^3}}$$

input
```
int((d + e*x^2)/(a + c*x^4),x)
```

output
```
- 2*atanh((8*c^3*d^2*x*((e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3) - (d^2*(-a^3*c
^3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 +
 (2*c*d^3*(-a^3*c^3)^(1/2))/a^2 - (2*d*e^2*(-a^3*c^3)^(1/2))/a) - (8*a*c^2
*e^2*x*((e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3) - (d^2*(-a^3*c^3)^(1/2))/(16*a
^3*c^2) - (d*e)/(8*a*c))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 + (2*c*d^3*(-a^3*
c^3)^(1/2))/a^2 - (2*d*e^2*(-a^3*c^3)^(1/2))/a))*(-(c*d^2*(-a^3*c^3)^(1/2)
 - a*e^2*(-a^3*c^3)^(1/2) + 2*a^2*c^2*d*e)/(16*a^3*c^3))^(1/2) - 2*atanh((
8*c^3*d^2*x*((d^2*(-a^3*c^3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c) - (e^2*(-
a^3*c^3)^(1/2))/(16*a^2*c^3))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 - (2*c*d^3*(
-a^3*c^3)^(1/2))/a^2 + (2*d*e^2*(-a^3*c^3)^(1/2))/a) - (8*a*c^2*e^2*x*((d^
2*(-a^3*c^3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c) - (e^2*(-a^3*c^3)^(1/2))/
(16*a^2*c^3))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 - (2*c*d^3*(-a^3*c^3)^(1/2))
/a^2 + (2*d*e^2*(-a^3*c^3)^(1/2))/a))*(-(a*e^2*(-a^3*c^3)^(1/2) - c*d^2*(-
a^3*c^3)^(1/2) + 2*a^2*c^2*d*e)/(16*a^3*c^3))^(1/2)
```

3.141 $\int \frac{1}{a+cx^4}\,dx$

3.141.1 Optimal result

Integrand size = 9, antiderivative size = 185

$$\int \frac{1}{a+cx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

output
```
1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*arcta
n(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*ln(-a^(1/4)*c^(
1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*ln(a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.72

$$\int \frac{1}{a+cx^4}\,dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

input
```
Integrate[(a + c*x^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

3.141.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+cx^4}dx$$

$$\Big\downarrow \text{755}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

input
```
Int[(a + c*x^4)^(-1),x]
```

output
```
(-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + Ar
cTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a
]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]
*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])
```

3.141. $\int \frac{1}{a+cx^4}\,dx$

3.141.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.141.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.15

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}}{4c}$	27
default	$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}$	102

input
```
int(1/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.141.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.61

$$\int \frac{1}{a+cx^4}\,dx = \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)+\frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)-\frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)-\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)$$

input
```
integrate(1/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/(a^3*c))^(1/4) + x) + 1/4*I*(-1/(a^3*c))^
(1/4)*log(I*a*(-1/(a^3*c))^(1/4) + x) - 1/4*I*(-1/(a^3*c))^(1/4)*log(-I*a*
(-1/(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4)
 + x)
```

3.141.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.11

$$\int \frac{1}{a+cx^4}\,dx = \text{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

input
```
integrate(1/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2
 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}$$

input
```
integrate(1/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
+ sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

3.141.9 Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.18

$$\int \frac{1}{a+cx^4}\,dx = -\frac{\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)+\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4),x)
```

output
```
-(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3
/4)*c^(1/4))
```

3.142 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

3.142.1 Optimal result

Integrand size = 19, antiderivative size = 336

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx &= \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)} \\ &+ \frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^2+ae^2)} \\ &- \frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)} \\ &+ \frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^2+ae^2)}\end{aligned}$$

output
```
1/4*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^
(3/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4)
)*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*c^(1/4)*ln(-a^(
1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/
(a*e^2+c*d^2)*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)+e^(3/2)*arct
an(x*e^(1/2)/d^(1/2))/(a*e^2+c*d^2)/d^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \frac{8a^{3/4}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\sqrt[4]{c}\sqrt{d}\left(\left(-2\sqrt{c}d+2\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(1\right.\right.}{8a^{3/4}\sqrt{d}\,(cd^2}$$

input
```
Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*(
(-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*
d + Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d
]*(c*d^2 + a*e^2))
```

3.142.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)(d+ex^2)}\,dx$$

$$\Big\downarrow \text{1485}$$

$$\int \left(\frac{e^2}{(d+ex^2)(ae^2+cd^2)}+\frac{c(d-ex^2)}{(a+cx^4)(ae^2+cd^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}d-\sqrt{a}e\right)}{2\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2+cd^2\right)}$$

input
```
Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

output
```
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)
*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/
4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[
a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a
^(3/4)*(c*d^2 + a*e^2))
```

3.142.3.1 Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.142.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.75

method	result
default	$\frac{c\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.\right.}{8c\left(\frac{a}{c}\right)^{\frac{1}{4}}}}{a\,e^2+c\,d^2}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^5e^4+2a^4c\,d^2e^2+a^3c^2d^4\right)\_Z^4-4a^2cde\_Z^2+c\right)}\_R\ln\left(\left(\left(-2a^5e^7-2a^4c\,d^2e^5+2a^3c^2d^4e^3+2a^2c^3d^6e\right)\_R^4+\left(15a^2cd\right.\right.\right.\right.}{4}$

input
```
int(1/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
c/(a*e^2+c*d^2)*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(
a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/c)^(1/4)*2^(
1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(
1/4)*x-1)))+e^2/(a*e^2+c*d^2)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.142.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. 2(247) = 494.

Time = 0.70 (sec) , antiderivative size = 4084, normalized size of antiderivative = 12.15

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
[-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^
3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 +
2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*
c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a
*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4
*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d
^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d
^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e
+ (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2
 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6
*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2
*d^2 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^
2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^
8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*s
qrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*
a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^
4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^
4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a...
```

3.142.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.142.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx &= \frac{e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(cd^2+ae^2)\sqrt{de}} \\ &+ \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ &+ \frac{\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ &+ \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)} \\ &- \frac{\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}\end{aligned}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
e^2*arctan(e*x/sqrt(d*e))/((c*d^2 + a*e^2)*sqrt(d*e)) + 1/2*((a*c^3)^(1/4)
*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/4)
*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)
*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sq
rt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3
)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2
 + sqrt(2)*a^2*c^2*e^2)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 15.59 (sec) , antiderivative size = 4802, normalized size of antiderivative = 14.29

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)*(d + e*x^2)),x)
```

output
```
atan(((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a
^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*
(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^
2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/
2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*
d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 +
448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*
d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5
*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i - (((a*e^2*(-a^3*c)^(1
/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a
^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))
^(1/2)*(256*a^4*c^4*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2)
+ 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*
a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*
e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(
16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c...
```

3.143 $\int \frac{1}{(d+ex^2)^2(a+cx^4)}\,dx$

3.143.1 Optimal result

Integrand size = 19, antiderivative size = 453

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2(a+cx^4)}\,dx &= \frac{e^2x}{2d(cd^2+ae^2)(d+ex^2)} \\ &+ \frac{2c\sqrt{d}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} \\ &- \frac{c^{3/4}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\ &+ \frac{c^{3/4}\left(cd^2-2\sqrt{a}\sqrt{c}de-ae^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\ &- \frac{c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\ &+ \frac{c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2}\end{aligned}$$

output
```
1/2*e^2*x/d/(a*e^2+c*d^2)/(e*x^2+d)+1/2*e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/
d^(3/2)/(a*e^2+c*d^2)+1/4*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*
d^2-a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*c^(3/
4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*d^2-a*e^2-2*d*e*a^(1/2)*c^(1/2))
/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)
+a^(1/2)+x^2*c^(1/2))*(c*d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c
*d^2)^2*2^(1/2)+1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/
2))*(c*d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+2*
c*e^(3/2)*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(a*e^2+c*d^2)^2
```

3.143.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.80

$$\int \frac{1}{(d+ex^2)^2(a+cx^4)}\,dx$$

$$= \frac{\frac{4e^2(cd^2+ae^2)x}{d(d+ex^2)} + \frac{4e^{3/2}(5cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2\sqrt{2}c^{3/4}(-cd^2+2\sqrt{a}\sqrt{c}de+ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}} - \frac{2\sqrt{2}c^{3/4}(-cd^2+2\sqrt{a}\sqrt{c}de+}{a^{3}}}{}$$

input
```
Integrate[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

output
```
((4*e^2*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (4*e^(3/2)*(5*c*d^2 + a*e^2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sq
rt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4
) - (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*c^(3/4)*(-(c*d^2) - 2*
Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/a^(3/4) + (Sqrt[2]*c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e
^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4))/(8*(c
*d^2 + a*e^2)^2)
```

3.143.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)(d+ex^2)^2}dx$$

$$\Big\downarrow \text{ 1485}$$

$$\int \left(\frac{2cde^2}{(d+ex^2)(ae^2+cd^2)^2}+\frac{e^2}{(d+ex^2)^2(ae^2+cd^2)}+\frac{c(-ae^2+cd^2-2cdex^2)}{(a+cx^4)(ae^2+cd^2)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{c^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(-2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2}-\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{c^{3/4}\left(2\sqrt{a}\sqrt{c}de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{2c\sqrt{d}e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(ae^2+cd^2)^2}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(ae^2+cd^2)}+\frac{e^2x}{2d(d+ex^2)(ae^2+cd^2)}$$

input
```
Int[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

output
```
(e^2*x)/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (2*c*Sqrt[d]*e^(3/2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(2*d^(3/2)*(c*d^2 + a*e^2)) - (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e
- a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^
2 + a*e^2)^2) + (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c
^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3
/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
```

3.143.3.1 Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.143.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.68

method	result
default	$-\frac{c\left(\frac{\left(a e^2-c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{de\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\right.}{}\right.}{\left(a e^2+c d^2\right)^2}$
risch	Expression too large to display

input
```
int(1/(e*x^2+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^2+c*d^2)^2*(1/8*(a*e^2-c*d^2)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)
^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*a
rctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/4*d*e/
(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)
^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(
2^(1/2)/(a/c)^(1/4)*x-1)))+e^2/(a*e^2+c*d^2)^2*(1/2*(a*e^2+c*d^2)/d*x/(e*x
^2+d)+1/2*(a*e^2+5*c*d^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.143.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4193 vs. $2(352) = 704$.

Time = 9.78 (sec) , antiderivative size = 8409, normalized size of antiderivative = 18.56

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.143.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(c*x**4+a),x)
```

output
```
Timed out
```

3.143.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)}\,dx &= \frac{e^2x}{2\,(cd^3+ade^2)(ex^2+d)} \\ &+ \frac{\left((ac^3)^{\frac{1}{4}}c^2d^2 - (ac^3)^{\frac{1}{4}}ace^2 - 2\,(ac^3)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^4 + 2\,\sqrt{2}a^2c^2d^2e^2 + \sqrt{2}a^3ce^4\right)} \\ &+ \frac{\left((ac^3)^{\frac{1}{4}}c^2d^2 - (ac^3)^{\frac{1}{4}}ace^2 - 2\,(ac^3)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^4 + 2\,\sqrt{2}a^2c^2d^2e^2 + \sqrt{2}a^3ce^4\right)} \\ &+ \frac{\left((ac^3)^{\frac{1}{4}}c^2d^2 - (ac^3)^{\frac{1}{4}}ace^2 + 2\,(ac^3)^{\frac{3}{4}}de\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4 + 2\,\sqrt{2}a^2c^2d^2e^2 + \sqrt{2}a^3ce^4\right)} \\ &- \frac{\left((ac^3)^{\frac{1}{4}}c^2d^2 - (ac^3)^{\frac{1}{4}}ace^2 + 2\,(ac^3)^{\frac{3}{4}}de\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4 + 2\,\sqrt{2}a^2c^2d^2e^2 + \sqrt{2}a^3ce^4\right)} \\ &+ \frac{(5\,cd^2e^2 + ae^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,(c^2d^5 + 2\,acd^3e^2 + a^2de^4)\sqrt{de}}\end{aligned}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

output
```
1/2*e^2*x/((c*d^3 + a*d*e^2)*(e*x^2 + d)) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (
a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e
^2 + sqrt(2)*a^3*c*e^4) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e
^2 - 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c
*e^4) + 1/4*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/
4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 +
2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) - 1/4*((a*c^3)^(1/4)*c^2*d^
2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)
^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(
2)*a^3*c*e^4) + 1/2*(5*c*d^2*e^2 + a*e^4)*arctan(e*x/sqrt(d*e))/((c^2*d^5
+ 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d*e))
```

3.143.9 Mupad [B] (verification not implemented)

Time = 16.80 (sec) , antiderivative size = 16369, normalized size of antiderivative = 36.13

$$\int \frac{1}{(d+ex^2)^2(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)*(d + e*x^2)^2),x)
```

output
```
(e^2*x)/(2*d*(d + e*x^2)*(a*e^2 + c*d^2)) - atan(((((((256*a^8*c^4*d*e^16
- 128*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 89
60*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176
*a^7*c^5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*
d^4*e^6 + 6*a^2*c^2*d^6*e^4)) + (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-
a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c
^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^
2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14
*e^5 + 4608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^
11 - 4608*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17)
)/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*
d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^
3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 +
 a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^
(1/2) + (x*(32*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 10
24*a^2*c^9*d^9*e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^
5*c^6*d^3*e^12))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e
^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(
1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))
/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a...
```

3.143. $\int \frac{1}{(d+ex^2)^2(a+cx^4)}\,dx$

3.144 $\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$

3.144.1 Optimal result

Integrand size = 19, antiderivative size = 363

$$\begin{aligned}\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx =& -\frac{e^3x^3}{c(a+cx^4)}+\frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)\,x^2)}{4ac\,(a+cx^4)}\\&-\frac{3\left(\sqrt{c}d+\sqrt{a}e\right)(cd^2+ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}}\\&+\frac{3\left(\sqrt{c}d+\sqrt{a}e\right)(cd^2+ae^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{7/4}}\\&-\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}\\&+\frac{3\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{7/4}}\end{aligned}$$

output
```
-e^3*x^3/c/(c*x^4+a)+1/4*x*(d*(-3*a*e^2+c*d^2)+3*e*(a*e^2+c*d^2)*x^2)/a/c/
(c*x^4+a)-3/32*(a*e^2+c*d^2)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2)+3/32*(a*e^2+c*d^2)*ln
(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(
7/4)/c^(7/4)*2^(1/2)+3/16*(a*e^2+c*d^2)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2)+3/16*(a*e^2+c*d^2)*arctan
(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2
)
```

3.144.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$$
$$= \frac{-\frac{8a^{3/4}c^{3/4}(ae^2x(3d+ex^2)-cd^2x(d+3ex^2))}{a+cx^4} - 6\sqrt{2}\left(c^{3/2}d^3+\sqrt{a}cd^2e+a\sqrt{c}de^2+a^{3/2}e^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+6}{}$$

input
```
Integrate[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

output
```
((-8*a^(3/4)*c^(3/4)*(a*e^2*x*(3*d + e*x^2) - c*d^2*x*(d + 3*e*x^2)))/(a +
 c*x^4) - 6*Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(
3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3
 + Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*(-(c^(3/2)*d^3) + Sqrt[a]*c*d^2*e - a*Sqrt[c
]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2] + 3*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - a^(3/2)
*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^(7/4)*
c^(7/4))
```

3.144.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1519, 25, 2397, 27, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \text{1519}$$
$$-\frac{\int -\frac{3cde^2x^4+3e(cd^2+ae^2)x^2+cd^3}{(cx^4+a)^2}dx}{c} - \frac{e^3x^3}{c(a+cx^4)}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3cde^2x^4+3e(cd^2+ae^2)x^2+cd^3}{(cx^4+a)^2}dx}{c}-\frac{e^3x^3}{c(a+cx^4)}$$

$\Big\downarrow$ 2397

$$\frac{\frac{x(3ex^2(ae^2+cd^2)+d(cd^2-3ae^2))}{4a(a+cx^4)}-\frac{\int -\frac{3c(cd^2+ae^2)(ex^2+d)}{cx^4+a}dx}{4ac}}{c}-\frac{e^3x^3}{c(a+cx^4)}$$

$\Big\downarrow$ 27

$$\frac{\frac{3(ae^2+cd^2)\int \frac{ex^2+d}{cx^4+a}dx}{4a}+\frac{x(3ex^2(ae^2+cd^2)+d(cd^2-3ae^2))}{4a(a+cx^4)}}{c}-\frac{e^3x^3}{c(a+cx^4)}$$

$\Big\downarrow$ 1482

$$\frac{\frac{3(ae^2+cd^2)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{c}(\sqrt{a}-\sqrt{c}x^2)}{cx^4+a}dx}{2c}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}(\sqrt{c}x^2+\sqrt{a})}{cx^4+a}dx}{2c}\right)}{4a}+\frac{x(3ex^2(ae^2+cd^2)+d(cd^2-3ae^2))}{4a(a+cx^4)}}{c}-\frac{e^3x^3}{c(a+cx^4)}$$

$\Big\downarrow$ 27

$$\frac{\frac{3(ae^2+cd^2)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}\right)}{4a}+\frac{x(3ex^2(ae^2+cd^2)+d(cd^2-3ae^2))}{4a(a+cx^4)}}{c}-\frac{e^3x^3}{c(a+cx^4)}$$

$\Big\downarrow$ 1476

$$\frac{\frac{3(ae^2+cd^2)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{4a}+\frac{x(3ex^2(ae^2+cd^2)+d(cd^2-3ae^2))}{4a(a+cx^4)}}{c}-\frac{e^3x^3}{c(a+cx^4)}$$

$\Big\downarrow$ 1082

3.144. $\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$

$$\frac{\frac{3(ae^2+cd^2)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}\right)}{4a}+\frac{x(3ex}{}}{c}$$

$$\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$\downarrow$ 217

$$\frac{\frac{3(ae^2+cd^2)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}\right)}{4a}+\frac{x\left(3ex^2\left(ae^2+cd^2\right)+d\left(cd^2-3ae^2\right)\right)}{4a(a+cx^4)}}{c}-$$

$$\frac{e^3x^3}{c\left(a+cx^4\right)}$$

$\downarrow$ 1479

3.144. $\int\frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$

$$\frac{\frac{3(ae^2+cd^2)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\cdots\right.}{2\sqrt{c}}\cdots}{4a}\cdots}{c}$$

$$\frac{e^3x^3}{c\,(a+cx^4)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3(ae^2+cd^2)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}\cdots\right.}{2\sqrt{c}}\cdots}{4a}\cdots}{c}$$

$$\frac{e^3x^3}{c\,(a+cx^4)}$$

$$\Big\downarrow \; 27$$

3.144. $\int \frac{\left(d+ex^2\right)^3}{\left(a+cx^4\right)^2}\,dx$

$$\frac{\frac{3(ae^2+cd^2)\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}\right)}{4a}+\frac{x(3e}{}}{c}$$

$$\frac{e^3x^3}{c(a+cx^4)}$$

$\downarrow$ 1103

$$\frac{\frac{3(ae^2+cd^2)\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right.}{2\sqrt{c}}}{4a}}{c}$$

$$\frac{e^3x^3}{c(a+cx^4)}$$

input
```
Int[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

output
```
-((e^3*x^3)/(c*(a + c*x^4))) + ((x*(d*(c*d^2 - 3*a*e^2) + 3*e*(c*d^2 + a*e
^2)*x^2))/(4*a*(a + c*x^4)) + (3*(c*d^2 + a*e^2)*((((Sqrt[c]*d)/Sqrt[a] +
e)*(-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) +
 ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sq
rt[c]) + (((Sqrt[c]*d)/Sqrt[a] - e)*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[c]))
)/(4*a))/c
```

3.144. $\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$

3.144.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

3.144.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.33

method	result
risch	$\frac{-\frac{e(ae^2-3cd^2)x^3}{4ac}-\frac{d(3ae^2-cd^2)x}{4ac}}{cx^4+a}+\frac{3\left(\sum_{\_R=\text{RootOf}(c\_Z^4+a)}\frac{\left(e(ae^2+cd^2)\_R^2+d(ae^2+cd^2)\right)\ln(x-\_R)}{\_R^3}\right)}{16ac^2}$
default	$\frac{-\frac{e(ae^2-3cd^2)x^3}{4ac}-\frac{d(3ae^2-cd^2)x}{4ac}}{cx^4+a}+\frac{3(ae^2+cd^2)\left(\frac{d\left(\frac{a}{c}\right)^{\frac14}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac14}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac14}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac14}}+1\right)+2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac14}}-1\cdots\right.\right.}{8a}\cdots\right)}{4ac}$

input
```
int((e*x^2+d)^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*e*(a*e^2-3*c*d^2)/a/c*x^3-1/4*d*(3*a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+3/1
6/a/c^2*sum((e*(a*e^2+c*d^2)*_R^2+d*(a*e^2+c*d^2))/_R^3*ln(x-_R),_R=RootOf
(_Z^4*c+a))
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. 2(280) = 560.

Time = 0.72 (sec) , antiderivative size = 2116, normalized size of antiderivative = 5.83

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(4*(3*c*d^2*e - a*e^3)*x^3 - 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2*d^5*e
 + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^
2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^
10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 + 3*a*c^4*d^8*e^2
+ 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8 - a^5*e^10)*x +
27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6 + a^6*
c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6
*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))*sqrt(-(2
*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c
^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^
5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))) + 3*(a*c^2*x^4 + a^2*c)*s
qrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-(c^6*d^12
+ 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8
 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)))/(a^3*c^3))*log(-27*(c^5*d^10 +
 3*a*c^4*d^8*e^2 + 2*a^2*c^3*d^6*e^4 - 2*a^3*c^2*d^4*e^6 - 3*a^4*c*d^2*e^8
 - a^5*e^10)*x - 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^4*c^3*d^3*e^4 - a^5
*c^2*d*e^6 + a^6*c^5*e*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^
4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^
7*c^7)))*sqrt(-(2*c^2*d^5*e + 4*a*c*d^3*e^3 + 2*a^2*d*e^5 + a^3*c^3*sqrt(-
(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^...
```

3.144.6 Sympy [A] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$$
$$= \text{RootSum}\Big(65536t^4a^7c^7 + t^2\cdot\left(9216a^6c^4de^5 + 18432a^5c^5d^3e^3 + 9216a^4c^6d^5e\right) + 81a^6e^{12} + 486a^5cd^2e^{10} + \cdots$$
$$+ \frac{x^3(-ae^3+3cd^2e) + x(-3ade^2+cd^3)}{4a^2c+4ac^2x^4}$$

input
```
integrate((e*x**2+d)**3/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**7 + _t**2*(9216*a**6*c**4*d*e**5 + 18432*a**5*
c**5*d**3*e**3 + 9216*a**4*c**6*d**5*e) + 81*a**6*e**12 + 486*a**5*c*d**2*
e**10 + 1215*a**4*c**2*d**4*e**8 + 1620*a**3*c**3*d**6*e**6 + 1215*a**2*c*
*4*d**8*e**4 + 486*a*c**5*d**10*e**2 + 81*c**6*d**12, Lambda(_t, _t*log(x
+ (4096*_t**3*a**6*c**5*e + 432*_t*a**5*c**2*d*e**6 + 720*_t*a**4*c**3*d**
3*e**4 + 144*_t*a**3*c**4*d**5*e**2 - 144*_t*a**2*c**5*d**7)/(27*a**5*e**1
0 + 81*a**4*c*d**2*e**8 + 54*a**3*c**2*d**4*e**6 - 54*a**2*c**3*d**6*e**4
- 81*a*c**4*d**8*e**2 - 27*c**5*d**10)))) + (x**3*(-a*e**3 + 3*c*d**2*e) +
 x*(-3*a*d*e**2 + c*d**3))/(4*a**2*c + 4*a*c**2*x**4)
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx = \frac{(3\,cd^2e-ae^3)x^3+(cd^3-3\,ade^2)x}{4\,(ac^2x^4+a^2c)} + \frac{3\,(cd^2+ae^2)\left(\frac{2\,\sqrt{2}(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}(\sqrt{c}d-\sqrt{}}{}\right.}{32\,ac}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*((3*c*d^2*e - a*e^3)*x^3 + (c*d^3 - 3*a*d*e^2)*x)/(a*c^2*x^4 + a^2*c)
+ 3/32*(c*d^2 + a*e^2)*(2*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(
2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan
(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(sqrt(c)*d - sqrt(a)*e
)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4))
 - sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a*c)
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx = \frac{3\,cd^2ex^3 - ae^3x^3 + cd^3x - 3\,ade^2x}{4\,(cx^4+a)ac}$$
$$+ \frac{3\,\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 + (ac^3)^{\frac{3}{4}}\,cd^2e + (ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^4}$$
$$+ \frac{3\,\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 + (ac^3)^{\frac{3}{4}}\,cd^2e + (ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^4}$$
$$+ \frac{3\,\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 - (ac^3)^{\frac{3}{4}}\,cd^2e - (ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^4}$$
$$- \frac{3\,\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 - (ac^3)^{\frac{3}{4}}\,cd^2e - (ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^4}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(3*c*d^2*e*x^3 - a*e^3*x^3 + c*d^3*x - 3*a*d*e^2*x)/((c*x^4 + a)*a*c)
+ 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3
)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 +
 (a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/
32*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3
/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a^2*c^4) - 3/32*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^
2*d*e^2 - (a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x
*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 14.97 (sec) , antiderivative size = 2560, normalized size of antiderivative = 7.05

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^3/(a + c*x^4)^2,x)
```

output
```
- ((d*x*(3*a*e^2 - c*d^2))/(4*a*c) + (e*x^3*(a*e^2 - 3*c*d^2))/(4*a*c))/(a
 + c*x^4) - 2*atanh((9*c^3*d^6*x*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) -
 (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^7)^(1/2
))/(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/2))/(2
56*a^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*c
*d^6*e^3)/16 - (27*a^3*e^9)/(32*c^2) + (27*c^2*d^8*e)/(32*a) - (27*a^2*d^2
*e^7)/(16*c) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^5*c) - (27*d*e^8*(-a^7*c^7)
^(1/2))/(32*a*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^2*c^4) + (27*d^7*
e^2*(-a^7*c^7)^(1/2))/(16*a^4*c^2))) + (9*a*e^6*x*((9*e^6*(-a^7*c^7)^(1/2)
)/(256*a^4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d^
6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*e^4*(-a
^7*c^7)^(1/2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))
^(1/2))/(2*((27*a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16
*a^2) - (27*c^2*d^8*e)/(32*a^3) - (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c) + (
27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^3*c^5) + (27*d^3*e^6*(-a^7*c^7)^(1/2))/(1
6*a^4*c^4) - (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) + (9*c*d^2*e^4*x
*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d^3*
e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(12
8*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*c
^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*a*e^9)/(32*c^2) + (27*d^2*e^7)...
```

3.145 $\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$

3.145.1 Optimal result

Integrand size = 19, antiderivative size = 349

$$\begin{aligned}\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = &-\frac{e^2x}{3c(a+cx^4)}+\frac{x(3cd^2+ae^2+6cdex^2)}{12ac(a+cx^4)}\\ &-\frac{(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}}\\ &+\frac{(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}}\\ &-\frac{(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}\\ &+\frac{(3cd^2-2\sqrt{a}\sqrt{c}de+ae^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{5/4}}\end{aligned}$$

output
```
-1/3*e^2*x/c/(c*x^4+a)+1/12*x*(6*c*d*e*x^2+a*e^2+3*c*d^2)/a/c/(c*x^4+a)-1/
32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2+a*e^2-2*d*e
*a^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)+1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2+a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/c^(5/
4)*2^(1/2)+1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2+a*e^2+2*d*e*
a^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a
^(1/4))*(3*c*d^2+a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$$
$$= \frac{-\frac{8a^{3/4}\sqrt[4]{c}(ae^2x-cdx(d+2ex^2))}{a+cx^4} - 2\sqrt{2}(3cd^2+2\sqrt{a}\sqrt{c}de+ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}(3cd^2+2\sqrt{a}\sqrt{cd}}{}$$

input
```
Integrate[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

output
```
((-8*a^(3/4)*c^(1/4)*(a*e^2*x - c*d*x*(d + 2*e*x^2)))/(a + c*x^4) - 2*Sqrt
[2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*
x)/a^(1/4)] + 2*Sqrt[2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e
 + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]
*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(32*a^(7/4)*c^(5/4))
```

3.145.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1519, 25, 1493, 27, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \ 1519$$
$$-\frac{\int -\frac{3cd^2+6cex^2d+ae^2}{(cx^4+a)^2}dx}{3c} - \frac{e^2x}{3c(a+cx^4)}$$
$$\Big\downarrow \ 25$$
$$\frac{\int \frac{3cd^2+6cex^2d+ae^2}{(cx^4+a)^2}dx}{3c} - \frac{e^2x}{3c(a+cx^4)}$$

$$\Big\downarrow \text{1493}$$

$$\frac{\frac{x(ae^2+3cd^2+6cdex^2)}{4a(a+cx^4)}-\frac{\int -\frac{3\left(3cd^2+2cex^2d+ae^2\right)}{cx^4+a}dx}{4a}}{3c}-\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{3cd^2+2cex^2d+ae^2}{cx^4+a}dx}{4a}+\frac{x(ae^2+3cd^2+6cdex^2)}{4a(a+cx^4)}}{3c}-\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1482}$$

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{2\sqrt{a}\sqrt{c}}+\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{2\sqrt{a}\sqrt{c}}\right)}{4a}+\frac{x(ae^2+3cd^2+6cdex^2)}{4a(a+cx^4)}}{3c}-\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x(ae^2+3cd^2+6cdex^2)}{4a(a+cx^4)}}{3c}-\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{3\left(\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{a}}+\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x(ae^2+3cd^2+6cdex^2)}{4a(a+cx^4)}}{3c}-\frac{e^2x}{3c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1082}$$

3.145. $\displaystyle\int \frac{\left(d+ex^2\right)^2}{\left(a+cx^4\right)^2}\,dx$

$$\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right.}{2\sqrt{a}}}{4a}}{3c}$$

$$\frac{e^2x}{3c\,(a+cx^4)}$$

$\downarrow$ 217

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)}{2\sqrt{a}}\right)}{4a}+\frac{x\left(ae^2+3cd^2+6cdex^2\right)}{4a(a+cx^4)}}{3c}-$$

$$\frac{e^2x}{3c\,(a+cx^4)}$$

$\downarrow$ 1479

3.145. $\int\frac{\left(d+ex^2\right)^2}{\left(a+cx^4\right)^2}\,dx$

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{a}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\cdots}{2\sqrt{a}}\cdots\right)}{4a}\cdots}{3c}$$

$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

$\big\downarrow$ 25

$$\frac{\frac{3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{a}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(2\sqrt{\cdots}\right.}{2\sqrt{a}}\cdots\right)}{4a}\cdots}{3c}$$

$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

$\big\downarrow$ 27

3.145. $\int \frac{\left(d+ex^2\right)^2}{\left(a+cx^4\right)^2}\,dx$

$$3\left(\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{2\sqrt{a}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd\right.}{2\sqrt{a}}\right.$$
$$\overline{\qquad 4a \qquad}$$
$$\overline{\qquad 3c \qquad}$$
$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1103

$$3\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)}{2\sqrt{a}}+\frac{\left(-2\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{\right.}}{2\sqrt{2}\sqrt[4]{}}\right.}{2\sqrt{a}}\right.$$
$$\overline{\qquad 4a \qquad}$$
$$\overline{\qquad 3c \qquad}$$
$$\frac{e^2x}{3c\left(a+cx^4\right)}$$

input
```
Int[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

output
```
-1/3*(e^2*x)/(c*(a + c*x^4)) + ((x*(3*c*d^2 + a*e^2 + 6*c*d*e*x^2))/(4*a*(
a + c*x^4)) + (3*(((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*(-(ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[a]) + ((3*c*d
^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[a]))
)/(4*a))/(3*c)
```

3.145. $\int \frac{\left(d+ex^2\right)^2}{\left(a+cx^4\right)^2}\,dx$

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

3.145.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.28

method	result
risch	$\frac{\frac{e d x^3}{2a}-\frac{\left(a e^2-c d^2\right)x}{4ac}}{c x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+a\right)}\frac{\left(2ed\_R^2+\frac{a e^2+3c d^2}{c}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$
default	$\frac{\frac{e d x^3}{2a}-\frac{\left(a e^2-c d^2\right)x}{4ac}}{c x^4+a}+\frac{\frac{\left(a e^2+3c d^2\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{de\sqrt{2}\left(\ln\left(\frac{x^2 \cdots}{x^2 \cdots}\right.\right.}{\cdots}}{4ac}$

input
```
int((e*x^2+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*e*d/a*x^3-1/4*(a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+1/16/a/c*sum((2*e*d*_R^2
+1/c*(a*e^2+3*c*d^2))/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1596 vs. $2(264) = 528$.

Time = 0.72 (sec) , antiderivative size = 1596, normalized size of antiderivative = 4.57

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(8*c*d*e*x^3 + (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 +
 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c
^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^3*d^6*
e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (2*a^6*c^4*d*e*
sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^
6 + a^4*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^4*c^2*
d^2*e^4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 +
 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e +
 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(81*c^4
*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/
(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108*a*c^
3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x - (2*a^6*c^
4*d*e*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*
d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 27*a^2*c^4*d^6 + 15*a^3*c^3*d^4*e^2 + 5*a^
4*c^2*d^2*e^4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6
*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d
^3*e + 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt((a^3*c^2*sqrt(-(8
1*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*
e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log((81*c^4*d^8 + 108
*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 + a^4*e^8)*x + (...
```

3.145.6 Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$$
$$= \text{RootSum}\Big(65536t^4a^7c^5 + t^2\cdot\left(2048a^5c^3de^3 + 6144a^4c^4d^3e\right) + a^4e^8 + 20a^3cd^2e^6 + 118a^2c^2d^4e^4 + 180ac^3d$$
$$+ \frac{2cdex^3 + x(-ae^2 + cd^2)}{4a^2c + 4ac^2x^4}$$

input
```
integrate((e*x**2+d)**2/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**5 + _t**2*(2048*a**5*c**3*d*e**3 + 6144*a**4*c
**4*d**3*e) + a**4*e**8 + 20*a**3*c*d**2*e**6 + 118*a**2*c**2*d**4*e**4 +
180*a*c**3*d**6*e**2 + 81*c**4*d**8, Lambda(_t, _t*log(x + (-8192*_t**3*a*
*6*c**4*d*e + 16*_t*a**5*c*e**6 - 48*_t*a**4*c**2*d**2*e**4 - 144*_t*a**3*
c**3*d**4*e**2 + 432*_t*a**2*c**4*d**6)/(a**4*e**8 + 12*a**3*c*d**2*e**6 +
 38*a**2*c**2*d**4*e**4 + 108*a*c**3*d**6*e**2 + 81*c**4*d**8)))) + (2*c*d
*e*x**3 + x*(-a*e**2 + c*d**2))/(4*a**2*c + 4*a*c**2*x**4)
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = \frac{2\,cdex^3+(cd^2-ae^2)x}{4\,(ac^2x^4+a^2c)} + \frac{\frac{2\sqrt{2}\left(3\,c^{\frac{3}{2}}d^2+2\sqrt{a}cde+a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\left(3\,c^{\frac{3}{2}}d^2+2\sqrt{a}cde+a\sqrt{c}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{}}{}}{32\,ac}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(2*c*d*e*x^3 + (c*d^2 - a*e^2)*x)/(a*c^2*x^4 + a^2*c) + 1/32*(2*sqrt(2
)*(3*c^(3/2)*d^2 + 2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*
sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*c^(3/2)*d^2 + 2*sqrt(a)*c*d*e + a
*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/s
qrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3
*c^(3/2)*d^2 - 2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*
a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*c^(3/2)*d^2 -
2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)
*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a*c)
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$$
$$= \frac{2\,cdex^3+cd^2x-ae^2x}{4\,(cx^4+a)ac}$$
$$+ \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2+2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2+2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2-2\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$
$$- \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{1}{4}}\,ace^2-2\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(2*c*d*e*x^3 + c*d^2*x - a*e^2*x)/((c*x^4 + a)*a*c) + 1/16*sqrt(2)*(3*
(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*arcta
n(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sq
rt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3)
+ 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3
)^(3/4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32
*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4
)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

3.145.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 1565, normalized size of antiderivative = 4.48

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^2/(a + c*x^4)^2,x)
```

output
```
2*atanh((9*c^3*d^4*x*((9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (3*d^3*e)/(
64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) +
(d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/(2*((27*d^6*(-a^7*c^5)^(1
/2))/(32*a^5) - (c*d^3*e^3)/8 - (a*d*e^5)/16 - (9*c^2*d^5*e)/(16*a) + (e^6
*(-a^7*c^5)^(1/2))/(32*a^2*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^3*c^2
) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^4*c))) + (c*e^4*x*((9*d^4*(-a^7*c^
5)^(1/2))/(256*a^7*c^3) - (3*d^3*e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e
^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) + (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c
^4))^(1/2))/(2*((27*d^6*(-a^7*c^5)^(1/2))/(32*a^7) - (d*e^5)/(16*a) - (c*d
^3*e^3)/(8*a^2) - (9*c^2*d^5*e)/(16*a^3) + (e^6*(-a^7*c^5)^(1/2))/(32*a^4*
c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^5*c^2) + (15*d^4*e^2*(-a^7*c^5)^
(1/2))/(32*a^6*c))) + (c^2*d^2*e^2*x*((9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^
3) - (3*d^3*e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^(1/2))/
(256*a^5*c^5) + (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/((27*d^6*
(-a^7*c^5)^(1/2))/(32*a^6) - (d*e^5)/16 - (c*d^3*e^3)/(8*a) - (9*c^2*d^5*e
)/(16*a^2) + (e^6*(-a^7*c^5)^(1/2))/(32*a^3*c^3) + (5*d^2*e^4*(-a^7*c^5)^(
1/2))/(32*a^4*c^2) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^5*c)))*((a^2*e^4*
(-a^7*c^5)^(1/2) + 9*c^2*d^4*(-a^7*c^5)^(1/2) - 12*a^4*c^4*d^3*e - 4*a^5*c
^3*d*e^3 + 2*a*c*d^2*e^2*(-a^7*c^5)^(1/2))/(256*a^7*c^5))^(1/2) - 2*atanh(
(9*c^3*d^4*x*(- (d*e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c) - (9*d^4*(-...
```

3.146 $\int \frac{d+ex^2}{(a+cx^4)^2}\,dx$

3.146.1 Optimal result

Integrand size = 17, antiderivative size = 275

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = \frac{x(d+ex^2)}{4a(a+cx^4)} - \frac{(3\sqrt{c}d+\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d+\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} - \frac{(3\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}$$

output
```
1/4*x*(e*x^2+d)/a/(c*x^4+a)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*c^(1/2))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/32*ln(a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/c
^(3/4)*2^(1/2)+1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*d*c^
(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e
*a^(1/2)+3*d*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

3.146.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.97

$$\int \frac{d+ex^2}{(a+cx^4)^2} dx$$
$$= \frac{\frac{8ax(d+ex^2)}{a+cx^4} - \frac{2\sqrt{2}\sqrt[4]{a}(3\sqrt{c}d+\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt{2}\sqrt[4]{a}(3\sqrt{c}d+\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{\sqrt{2}\left(-3\sqrt[4]{a}\sqrt{c}d+a^{3/4}e\right)}{}}{32a^2}$$

input
```
Integrate[(d + e*x^2)/(a + c*x^4)^2,x]
```

output
```
((8*a*x*(d + e*x^2))/(a + c*x^4) - (2*Sqrt[2]*a^(1/4)*(3*Sqrt[c]*d + Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (2*Sqrt[2]*a^(1/4
)*(3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/
4) + (Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d - a
^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))
/(32*a^2)
```

3.146.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1493, 25, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a+cx^4)^2} dx$$
$$\Big\downarrow \; 1493$$
$$\frac{x(d+ex^2)}{4a(a+cx^4)} - \frac{\int -\frac{ex^2+3d}{cx^4+a}dx}{4a}$$
$$\Big\downarrow \; 25$$
$$\frac{\int \frac{ex^2+3d}{cx^4+a}dx}{4a} + \frac{x(d+ex^2)}{4a(a+cx^4)}$$

$$\Big\downarrow \text{1482}$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{2c}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{2c}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1479}$$

3.146. $\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\frac{3\sqrt{c}d}{\sqrt{a}}+e\right)}{2\sqrt{c}}+\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{c}}}{4a}+\frac{x\left(d+ex^2\right)}{4a\left(a+cx^4\right)}$$

3.146. $\int \frac{d+ex^2}{\left(a+cx^4\right)^2}\,dx$

input
```
Int[(d + e*x^2)/(a + c*x^4)^2,x]
```

output
```
(x*(d + e*x^2))/(4*a*(a + c*x^4)) + ((((3*Sqrt[c]*d)/Sqrt[a] + e)*(-(ArcTa
n[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[c]) + ((
(3*Sqrt[c]*d)/Sqrt[a] - e)*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[c]))/(4*a)
```

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

3.146.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\frac{e x^3}{4a}+\frac{dx}{4a}}{c x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_R^2 e+3d\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$
default	$d\left(\frac{x}{4a(c x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+e\left(\frac{x^3}{4a(c x^4+a)}\right.$

input
```
int((e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*e/a*x^3+1/4*d/a*x)/(c*x^4+a)+1/16/a/c*sum((_R^2*e+3*d)/_R^3*ln(x-_R),
_R=RootOf(_Z^4*c+a))
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(192) = 384$.

Time = 0.29 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.17

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx$$
$$= \frac{4\,ex^3 - (acx^4+a^2)\sqrt{-\frac{a^3c\sqrt{-\frac{81\,c^2d^4-18\,acd^2e^2+a^2e^4}{a^7c^3}}+6\,de}{a^3c}}\log\left(-(81\,c^2d^4-a^2e^4)x+\left(a^6c^2e\sqrt{-\frac{81\,c^2d^4-18\,acd^2e^2+}{a^7c^3}}\right.\right.}{}$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(4*e*x^3 - (a*c*x^4 + a^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^
2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^2*e^4)*
x + (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) +
27*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^
2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (a*c*x^4 + a^2)*sqrt(-(a^
3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3
*c))*log(-(81*c^2*d^4 - a^2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c
*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 27*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^
3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3
*c))) + (a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a
^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))*log(-(81*c^2*d^4 - a^2*e^4)*x + (a^6*
c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 27*a^2*c^
2*d^3 + 3*a^3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^
2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))) - (a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(
81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))*log(-(
81*c^2*d^4 - a^2*e^4)*x - (a^6*c^2*e*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 +
a^2*e^4)/(a^7*c^3)) - 27*a^2*c^2*d^3 + 3*a^3*c*d*e^2)*sqrt((a^3*c*sqrt(-(8
1*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))) + 4*d*
x)/(a*c*x^4 + a^2)
```

3.146.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.49

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx$$
$$= \text{RootSum}\left(65536t^4a^7c^3 + 3072t^2a^4c^2de + a^2e^4 + 18acd^2e^2 + 81c^2d^4, \left(t \mapsto t\log\left(x + \frac{4096t^3a^6c^2e + 144t}{a^2e^4 -}\right.\right.\right.$$
$$+ \frac{dx+ex^3}{4a^2+4acx^4}$$

input
```
integrate((e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**3 + 3072*_t**2*a**4*c**2*d*e + a**2*e**4 + 18*
a*c*d**2*e**2 + 81*c**2*d**4, Lambda(_t, _t*log(x + (4096*_t**3*a**6*c**2*
e + 144*_t*a**3*c*d*e**2 - 432*_t*a**2*c**2*d**3)/(a**2*e**4 - 81*c**2*d**
4)))) + (d*x + e*x**3)/(4*a**2 + 4*a*c*x**4)
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.92

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = \frac{ex^3+dx}{4(acx^4+a^2)}$$
$$+ \frac{\frac{2\sqrt{2}(3\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}(3\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}(3\sqrt{c}d-\sqrt{a}e)\log\left(\sqrt{c}x^2+\right.}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{32\,a}$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(e*x^3 + d*x)/(a*c*x^4 + a^2) + 1/32*(2*sqrt(2)*(3*sqrt(c)*d + sqrt(a)
*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*sqrt(c)
*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))
/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*
(3*sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sq
rt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x
^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/a
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.97

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = \frac{ex^3+dx}{4\,(cx^4+a)a} + \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d+(ac^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d+(ac^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d-(ac^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} - \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d-(ac^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(e*x^3 + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d +
(a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4
))/(a^2*c^3) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arct
an(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/32*s
qrt(2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)
^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c
^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 637, normalized size of antiderivative = 2.32

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = \frac{\frac{e\,x^3}{4\,a}+\frac{d\,x}{4\,a}}{c\,x^4+a} - 2\,\mathrm{atanh}\left(\frac{c^2\,e^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}-\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}}}{2\,\left(\frac{c\,e^3}{32\,a}-\frac{9\,c^2\,d^2\,e}{32\,a^2}-\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^6}+\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^5}\right)} - \frac{9\,c^3\,d^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}-\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}}}{2\,\left(\frac{c\,e^3}{32}-\frac{9\,c^2\,d^2\,e}{32\,a}-\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^5}+\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^4}\right)}\right)\sqrt{-\frac{9\,c\,d^2\,\sqrt{-a^7\,c^3}-a\,e^2\,\sqrt{-a^7\,c^3}+6\,a^4\,c^2\,d\,e}{256\,a^7\,c^3}} - 2\,\mathrm{atanh}\left(\frac{c^2\,e^2\,x\,\sqrt{\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}-\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}}}{2\,\left(\frac{c\,e^3}{32\,a}-\frac{9\,c^2\,d^2\,e}{32\,a^2}+\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^6}-\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^5}\right)} - \frac{9\,c^3\,d^2\,x\,\sqrt{\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}-\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}}}{2\,\left(\frac{c\,e^3}{32}-\frac{9\,c^2\,d^2\,e}{32\,a}+\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^5}-\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^4}\right)}\right)\sqrt{-\frac{a\,e^2\,\sqrt{-a^7\,c^3}-9\,c\,d^2\,\sqrt{-a^7\,c^3}+6\,a^4\,c^2\,d\,e}{256\,a^7\,c^3}}$$

input
```
int((d + e*x^2)/(a + c*x^4)^2,x)
```

output
```
((e*x^3)/(4*a) + (d*x)/(4*a))/(a + c*x^4) - 2*atanh((c^2*e^2*x*((e^2*(-a^7
*c^3)^(1/2))/(256*a^6*c^3) - (9*d^2*(-a^7*c^3)^(1/2))/(256*a^7*c^2) - (3*d
*e)/(128*a^3*c))^(1/2))/(2*((c*e^3)/(32*a) - (9*c^2*d^2*e)/(32*a^2) - (27*
c*d^3*(-a^7*c^3)^(1/2))/(32*a^6) + (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^5))) -
 (9*c^3*d^2*x*((e^2*(-a^7*c^3)^(1/2))/(256*a^6*c^3) - (9*d^2*(-a^7*c^3)^(1
/2))/(256*a^7*c^2) - (3*d*e)/(128*a^3*c))^(1/2))/(2*((c*e^3)/32 - (9*c^2*d
^2*e)/(32*a) - (27*c*d^3*(-a^7*c^3)^(1/2))/(32*a^5) + (3*d*e^2*(-a^7*c^3)^
(1/2))/(32*a^4))))*(-(9*c*d^2*(-a^7*c^3)^(1/2) - a*e^2*(-a^7*c^3)^(1/2) +
6*a^4*c^2*d*e)/(256*a^7*c^3))^(1/2) - 2*atanh((c^2*e^2*x*((9*d^2*(-a^7*c^3
)^(1/2))/(256*a^7*c^2) - (3*d*e)/(128*a^3*c) - (e^2*(-a^7*c^3)^(1/2))/(256
*a^6*c^3))^(1/2))/(2*((c*e^3)/(32*a) - (9*c^2*d^2*e)/(32*a^2) + (27*c*d^3*
(-a^7*c^3)^(1/2))/(32*a^6) - (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^5))) - (9*c^
3*d^2*x*((9*d^2*(-a^7*c^3)^(1/2))/(256*a^7*c^2) - (3*d*e)/(128*a^3*c) - (e
^2*(-a^7*c^3)^(1/2))/(256*a^6*c^3))^(1/2))/(2*((c*e^3)/32 - (9*c^2*d^2*e)/
(32*a) + (27*c*d^3*(-a^7*c^3)^(1/2))/(32*a^5) - (3*d*e^2*(-a^7*c^3)^(1/2))
/(32*a^4))))*(-(a*e^2*(-a^7*c^3)^(1/2) - 9*c*d^2*(-a^7*c^3)^(1/2) + 6*a^4*
c^2*d*e)/(256*a^7*c^3))^(1/2)
```

3.147 $\int \frac{1}{(a+cx^4)^2} dx$

3.147.1 Optimal result

Integrand size = 9, antiderivative size = 202

$$\int \frac{1}{(a+cx^4)^2} dx = \frac{x}{4a(a+cx^4)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

output
```
1/4*x/a/(c*x^4+a)+3/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4
)*2^(1/2)+3/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)*2^(1/2)
-3/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*2
^(1/2)+3/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1
/4)*2^(1/2)
```

3.147.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+cx^4)^2} dx = \frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{6\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} - \frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\right.}{\sqrt[4]{c}}}{32a^{7/4}}$$

input
```
Integrate[(a + c*x^4)^(-2),x]
```

output
```
((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/
4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^
(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/c^(1/4))/(32*a^(7/4))
```

3.147.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+cx^4\right)^2}dx$$

$$\Big\downarrow \text{749}$$

$$\frac{3\int \frac{1}{cx^4+a}dx}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\downarrow$ 217

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\downarrow$ 1479

$$\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\downarrow$ 25

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\downarrow$ 27

3.147. $\int \frac{1}{\left(a+cx^4\right)^2}\,dx$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}$$

$$\Big\downarrow \; 1103$$

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}$$

input
```
Int[(a + c*x^4)^(-2),x]
```

output
```
x/(4*a*(a + c*x^4)) + (3*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt
[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a
^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])))/(4*a
)
```

3.147.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.147. $\int \frac{1}{(a+cx^4)^2} \, dx$

3.147.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{x}{4a(c x^4+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16ac}$	46
default	$\frac{x}{4a(c x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}$	118

input

```
int(1/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/4*x/a/(c*x^4+a)+3/16/a/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.147.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+cx^4)^2}\,dx$$
$$= \frac{3\,(acx^4+a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\,(-i\,acx^4-i\,a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(i\,a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\,(i\,acx^4}{16\,(acx^4+a}$$

input

```
integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

output

```
1/16*(3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x)
 - 3*(-I*a*c*x^4 - I*a^2)*(-1/(a^7*c))^(1/4)*log(I*a^2*(-1/(a^7*c))^(1/4)
+ x) - 3*(I*a*c*x^4 + I*a^2)*(-1/(a^7*c))^(1/4)*log(-I*a^2*(-1/(a^7*c))^(1
/4) + x) - 3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4
) + x) + 4*x)/(a*c*x^4 + a^2)
```

3.147.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.19

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a^2+4acx^4} + \operatorname{RootSum}\left(65536t^4a^7c+81, \left(t \mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a)**2,x)
```

output
```
x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*l
og(16*_t*a**2/3 + x)))
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,(acx^4+a^2)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2-\right.}{a^{\frac{3}{4}}}\right.}{32\,a}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sq
rt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(
2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)
))/a
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,(cx^4+a)a} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32
*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
) - 3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(a^2*c)
```

3.147.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,a\,(c\,x^4+a)} + \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}} + \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4)^2,x)
```

output
```
x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/
4)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

3.148 $\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$

3.148.1 Optimal result

Integrand size = 19, antiderivative size = 689

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx &= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2+ae^2)^2} \\
&-\frac{\sqrt[4]{c}e^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&-\frac{\sqrt[4]{c}(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)} \\
&+\frac{\sqrt[4]{c}e^2(\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&+\frac{\sqrt[4]{c}(3\sqrt{c}d-\sqrt{a}e)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)} \\
&-\frac{\sqrt[4]{c}e^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&-\frac{\sqrt[4]{c}(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)} \\
&+\frac{\sqrt[4]{c}e^2(\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^2} \\
&+\frac{\sqrt[4]{c}(3\sqrt{c}d+\sqrt{a}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)}\end{aligned}$$

output
```
1/4*c*x*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/4*c^(1/4)*e^2*arctan(-1+c^(
1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(
1/2)+1/4*c^(1/4)*e^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(
1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*e^2*ln(-a^(1/4)*c^(1/4)*
x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)
^2*2^(1/2)+1/8*c^(1/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2
))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/16*c^(1/4)*arct
an(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c
*d^2)*2^(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)
+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c
*d^2)*2^(1/2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2
))*(e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+e^(7/2)*arctan(x*
e^(1/2)/d^(1/2))/(a*e^2+c*d^2)^2/d^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.62

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$$
$$= \frac{\frac{8c(cd^2+ae^2)x(d-ex^2)}{a(a+cx^4)} + \frac{32e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\sqrt[4]{c}(-3c^{3/2}d^3+\sqrt{a}cd^2e-7a\sqrt{c}de^2+5a^{3/2}e^3)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}} - \frac{2\sqrt{2}\sqrt[4]{c}(-3}{}}{}$$

input
```
Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[
a]*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^
2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*
Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + S
qrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e +
 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

3.148.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2(d+ex^2)}dx$$

$$\Big\downarrow \text{1568}$$

$$\int \left(-\frac{ce^2(ex^2-d)}{(a+cx^4)(ae^2+cd^2)^2}+\frac{c(d-ex^2)}{(a+cx^4)^2(ae^2+cd^2)}+\frac{e^4}{(d+ex^2)(ae^2+cd^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{c}e^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}e^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(3\sqrt{c}d-\sqrt{a}e)}{8\sqrt{2}a^{7/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(3\sqrt{c}d-\sqrt{a}e)}{8\sqrt{2}a^{7/4}(ae^2+cd^2)}-\frac{\sqrt[4]{c}e^2(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}e^2(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}(\sqrt{a}e+3\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}(\sqrt{a}e+3\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(ae^2+cd^2)}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(ae^2+cd^2)^2}+\frac{cx(d-ex^2)}{4a(a+cx^4)(ae^2+cd^2)}$$

input
```
Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

output
```
(c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d -
Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*
d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a
^(3/4)*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/
4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + S
qrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d
 + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*
Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(
c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

3.148.3.1 Defintions of rubi rules used

rule 1568
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.148.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.48

method	result
default	$\frac{c\left(\frac{-\frac{e\left(a e^{2}+c d^{2}\right)x^{3}}{4a}+\frac{d\left(a e^{2}+c d^{2}\right)x}{4a}}{c x^{4}+a}+\frac{\frac{\left(7d e^{2}a+3d^{3}c\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-}{4a}\right)}{\left(a e^{2}+c d^{2}\right)^{2}}$
risch	Expression too large to display

input
```
int(1/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c/(a*e^2+c*d^2)^2*((-1/4*e*(a*e^2+c*d^2)/a*x^3+1/4*d*(a*e^2+c*d^2)/a*x)/(c
*x^4+a)+1/4/a*(1/8*(7*a*d*e^2+3*c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c
)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*
arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(-5
*a*e^3-c*d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)
^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))+e^4/(a*e^2+c*d^2)^2/(e*d)^(1
/2)*arctan(e*x/(e*d)^(1/2))
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. 2(518) = 1036.

Time = 12.22 (sec) , antiderivative size = 9892, normalized size of antiderivative = 14.36

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.148.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.148.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 621, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{e^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{de}} + \frac{\left(3\,(ac^3)^{\frac{1}{4}}c^3d^3+7\,(ac^3)^{\frac{1}{4}}ac^2de^2-(ac^3)^{\frac{3}{4}}cd^2e-5\,(ac^3)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} + \frac{\left(3\,(ac^3)^{\frac{1}{4}}c^3d^3+7\,(ac^3)^{\frac{1}{4}}ac^2de^2-(ac^3)^{\frac{3}{4}}cd^2e-5\,(ac^3)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} + \frac{\left(3\,(ac^3)^{\frac{1}{4}}c^3d^3+7\,(ac^3)^{\frac{1}{4}}ac^2de^2+(ac^3)^{\frac{3}{4}}cd^2e+5\,(ac^3)^{\frac{3}{4}}ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} - \frac{\left(3\,(ac^3)^{\frac{1}{4}}c^3d^3+7\,(ac^3)^{\frac{1}{4}}ac^2de^2+(ac^3)^{\frac{3}{4}}cd^2e+5\,(ac^3)^{\frac{3}{4}}ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\,\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)} - \frac{cex^3-cdx}{4\,(cx^4+a)(acd^2+a^2e^2)}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d*e))
+ 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/
4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + s
qrt(2)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2
*d*e^2 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)
*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 +
7*(a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^
3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*s
qrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*
d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4
)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4
 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/4*(c*e*x^3 - c*d*x
)/((c*x^4 + a)*(a*c*d^2 + a^2*e^2))
```

3.148.9 Mupad [B] (verification not implemented)

Time = 16.36 (sec) , antiderivative size = 17945, normalized size of antiderivative = 26.04

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^2*(d + e*x^2)),x)
```

output
```
((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x
^4) - atan((((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5
*c^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*
a^8*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(25
6*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2
*d^4*e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6
*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-
a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d
^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65
536*a^13*c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 5
89824*a^8*c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11
 + 589824*a^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a
^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((
9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 4
4*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*
a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*
e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d
^13*e^2 - 49024*a^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^
9*e^6 + 8704*a^5*c^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3
*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e...
```

3.148. $\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$

3.149 $\int \frac{1}{(d+ex^2)^2(a+cx^4)^2}\,dx$

3.149.1 Optimal result

Integrand size = 19, antiderivative size = 864

$$
\begin{aligned}
&\int \frac{1}{(d+ex^2)^2(a+cx^4)^2}\,dx \\
&= \frac{e^4x}{2d(cd^2+ae^2)^2(d+ex^2)} + \frac{cx(cd^2-ae^2-2cdex^2)}{4a(cd^2+ae^2)^2(a+cx^4)} + \frac{4c\sqrt{d}e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^3} \\
&\quad + \frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)^2} - \frac{c^{3/4}e^2(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^3} \\
&\quad - \frac{c^{3/4}(3cd^2-2\sqrt{a}\sqrt{c}de-3ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)^2} \\
&\quad + \frac{c^{3/4}e^2(3cd^2-4\sqrt{a}\sqrt{c}de-ae^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^3} \\
&\quad + \frac{c^{3/4}(3cd^2-2\sqrt{a}\sqrt{c}de-3ae^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)^2} \\
&\quad - \frac{c^{3/4}e^2(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^3} \\
&\quad - \frac{c^{3/4}(3cd^2+2\sqrt{a}\sqrt{c}de-3ae^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)^2} \\
&\quad + \frac{c^{3/4}e^2(3cd^2+4\sqrt{a}\sqrt{c}de-ae^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^3} \\
&\quad + \frac{c^{3/4}(3cd^2+2\sqrt{a}\sqrt{c}de-3ae^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)^2}
\end{aligned}
$$

output
```
1/2*e^4*x/d/(a*e^2+c*d^2)^2/(e*x^2+d)+1/4*c*x*(-2*c*d*e*x^2-a*e^2+c*d^2)/a
/(a*e^2+c*d^2)^2/(c*x^4+a)+1/2*e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(
a*e^2+c*d^2)^2+1/4*c^(3/4)*e^2*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d
^2-a*e^2-4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^3*2^(1/2)+1/4*c^(3/4
)*e^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2-a*e^2-4*d*e*a^(1/2)*c^(
1/2))/a^(3/4)/(a*e^2+c*d^2)^3*2^(1/2)+1/16*c^(3/4)*arctan(-1+c^(1/4)*x*2^(
1/2)/a^(1/4))*(3*c*d^2-3*a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^2+c*d^2
)^2*2^(1/2)+1/16*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2-3*a*
e^2-2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/32*c^(3/4)*ln
(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2-3*a*e^2+2*d*e*a^
(1/2)*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/32*c^(3/4)*ln(a^(1/4)*c^(
1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2-3*a*e^2+2*d*e*a^(1/2)*c^(1/2)
)/a^(7/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*e^2*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2-a*e^2+4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(
a*e^2+c*d^2)^3*2^(1/2)+1/8*c^(3/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*c^(1/2))*(3*c*d^2-a*e^2+4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)
^3*2^(1/2)+4*c*e^(7/2)*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(a*e^2+c*d^2)^3
```

3.149.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.62

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)^2} \, dx$$

$$= \frac{\frac{16e^4(cd^2+ae^2)x}{d(d+ex^2)} + \frac{8c(cd^2+ae^2)x(-ae^2+cd(d-2ex^2))}{a(a+cx^4)} + \frac{16e^{7/2}(9cd^2+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2\sqrt{2}c^{3/4}(-3c^2d^4+2\sqrt{a}c^{3/2}d^3e-12acd^2e^2+1}{a^{7/}}}{}$$

input
```
Integrate[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

output
```
((16*e^4*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (8*c*(c*d^2 + a*e^2)*x*(-(a*
e^2) + c*d*(d - 2*e*x^2)))/(a*(a + c*x^4)) + (16*e^(7/2)*(9*c*d^2 + a*e^2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-3*c^2*d^4 + 2
*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 + 7*a^2
*e^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(3/4
)*(-3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt
[c]*d*e^3 + 7*a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) -
(Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*e^2 + 1
8*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/
2)*d^3*e + 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^
2)^3)
```

3.149.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2 (d+ex^2)^2} \, dx$$

$$\Big\downarrow \text{1568}$$

$$\int \left(-\frac{ce^2 (ae^2-3cd^2+4cdex^2)}{(a+cx^4)(ae^2+cd^2)^3}+\frac{c(-ae^2+cd^2-2cdex^2)}{(a+cx^4)^2 (ae^2+cd^2)^2}+\frac{4cde^4}{(d+ex^2)(ae^2+cd^2)^3}+\frac{e^4}{(d+ex^2)^2 (ae^2+cd^2)^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{xe^4}{2d(cd^2+ae^2)^2(ex^2+d)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}(cd^2+ae^2)^2}+\frac{4c\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{(cd^2+ae^2)^3}-\frac{c^{3/4}(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^2}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^3}+\frac{c^{3/4}(3cd^2-4\sqrt{a}\sqrt{c}ed-ae^2)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^2}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^3}-\frac{c^{3/4}(3cd^2+4\sqrt{a}\sqrt{c}ed-ae^2)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^2}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^3}+\frac{c^{3/4}(3cd^2+4\sqrt{a}\sqrt{c}ed-ae^2)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^2}{4\sqrt{2}a^{3/4}(cd^2+ae^2)^3}-\frac{c^{3/4}(3cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)^2}+\frac{c^{3/4}(3cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}(cd^2+ae^2)^2}-\frac{c^{3/4}(3cd^2+2\sqrt{a}\sqrt{c}ed-3ae^2)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)^2}+\frac{c^{3/4}(3cd^2+2\sqrt{a}\sqrt{c}ed-3ae^2)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^2+ae^2)^2}+\frac{cx(cd^2-2cex^2d-ae^2)}{4a(cd^2+ae^2)^2(cx^4+a)}$$

input
```
Int[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

3.149. $\int \frac{1}{(d+ex^2)^2(a+cx^4)^2}\,dx$

output
```
(e^4*x)/(2*d*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (c*x*(c*d^2 - a*e^2 - 2*c*d*
e*x^2))/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^4)) + (4*c*Sqrt[d]*e^(7/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^3 + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(2*d^(3/2)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sq
rt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*
e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 +
a*e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[
1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) +
(c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(
3*c*d^2 + 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3
*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*
e^2*(3*c*d^2 + 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/
4)*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2)
```

3.149.3.1 Defintions of rubi rules used

rule 1568
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.149. $\int \frac{1}{(d+ex^2)^2(a+cx^4)^2} \, dx$

3.149.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.47

method	result
default	$-\frac{c\left(\frac{\frac{dce\left(ae^2+cd^2\right)x^3}{2a}+\frac{\left(a^2e^4-c^2d^4\right)x}{4a}}{cx^4+a}+\frac{\frac{\left(7a^2e^4-12acd^2e^2-3c^2d^4\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{8a}}{}\right.}{\left(ae^2+cd^2\right)^3}$
risch	Expression too large to display

input
```
int(1/(e*x^2+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^2+c*d^2)^3*((1/2*d*c*e*(a*e^2+c*d^2)/a*x^3+1/4*(a^2*e^4-c^2*d^4)/a
*x)/(c*x^4+a)+1/4/a*(1/8*(7*a^2*e^4-12*a*c*d^2*e^2-3*c^2*d^4)*(a/c)^(1/4)/
a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2
^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a
/c)^(1/4)*x-1))+1/8*(18*a*c*d*e^3+2*c^2*d^3*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((
x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))
))+e^4/(a*e^2+c*d^2)^3*(1/2*(a*e^2+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+9*c*d^2
)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7634 vs. 2(680) = 1360.

Time = 109.95 (sec) , antiderivative size = 15292, normalized size of antiderivative = 17.70

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.149.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.149.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 879, normalized size of antiderivative = 1.02

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)^2}\,dx$$

$$= \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^4 + 12\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e^2 - 7\,(ac^3)^{\frac{1}{4}}\,a^2ce^4 - 2\,(ac^3)^{\frac{3}{4}}\,cd^3e - 18\,(ac^3)^{\frac{3}{4}}\,ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right.\right.}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{8\left(\sqrt{2}a^2c^4d^6 + 3\,\sqrt{2}a^3c^3d^4e^2 + 3\,\sqrt{2}a^4c^2d^2e^4 + \sqrt{2}a^5ce^6\right)}$$

$$+ \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^4 + 12\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e^2 - 7\,(ac^3)^{\frac{1}{4}}\,a^2ce^4 - 2\,(ac^3)^{\frac{3}{4}}\,cd^3e - 18\,(ac^3)^{\frac{3}{4}}\,ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\right.}{2\left(\frac{a}{c}\right)}\right.}{8\left(\sqrt{2}a^2c^4d^6 + 3\,\sqrt{2}a^3c^3d^4e^2 + 3\,\sqrt{2}a^4c^2d^2e^4 + \sqrt{2}a^5ce^6\right)}$$

$$+ \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^4 + 12\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e^2 - 7\,(ac^3)^{\frac{1}{4}}\,a^2ce^4 + 2\,(ac^3)^{\frac{3}{4}}\,cd^3e + 18\,(ac^3)^{\frac{3}{4}}\,ade^3\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)\right.}{16\left(\sqrt{2}a^2c^4d^6 + 3\,\sqrt{2}a^3c^3d^4e^2 + 3\,\sqrt{2}a^4c^2d^2e^4 + \sqrt{2}a^5ce^6\right)}$$

$$- \frac{\left(3\,(ac^3)^{\frac{1}{4}}\,c^3d^4 + 12\,(ac^3)^{\frac{1}{4}}\,ac^2d^2e^2 - 7\,(ac^3)^{\frac{1}{4}}\,a^2ce^4 + 2\,(ac^3)^{\frac{3}{4}}\,cd^3e + 18\,(ac^3)^{\frac{3}{4}}\,ade^3\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)\right.}{16\left(\sqrt{2}a^2c^4d^6 + 3\,\sqrt{2}a^3c^3d^4e^2 + 3\,\sqrt{2}a^4c^2d^2e^4 + \sqrt{2}a^5ce^6\right)}$$

$$+ \frac{(9\,cd^2e^4 + ae^6)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,(c^3d^7 + 3\,ac^2d^5e^2 + 3\,a^2cd^3e^4 + a^3de^6)\sqrt{de}}$$

$$- \frac{2\,c^2d^2e^2x^5 - 2\,ace^4x^5 + c^2d^3ex^3 + acde^3x^3 - c^2d^4x + acd^2e^2x - 2\,a^2e^4x}{4\,(ac^2d^5 + 2\,a^2cd^3e^2 + a^3de^4)(cex^6 + cdx^4 + aex^2 + ad)}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/8*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 7*(a*c^3)^
(1/4)*a^2*c*e^4 - 2*(a*c^3)^(3/4)*c*d^3*e - 18*(a*c^3)^(3/4)*a*d*e^3)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d
^6 + 3*sqrt(2)*a^3*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c
*e^6) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 7*
(a*c^3)^(1/4)*a^2*c*e^4 - 2*(a*c^3)^(3/4)*c*d^3*e - 18*(a*c^3)^(3/4)*a*d*e
^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a
^2*c^4*d^6 + 3*sqrt(2)*a^3*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(
2)*a^5*c*e^6) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2
*e^2 - 7*(a*c^3)^(1/4)*a^2*c*e^4 + 2*(a*c^3)^(3/4)*c*d^3*e + 18*(a*c^3)^(3
/4)*a*d*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4
*d^6 + 3*sqrt(2)*a^3*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5
*c*e^6) - 1/16*(3*(a*c^3)^(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 -
 7*(a*c^3)^(1/4)*a^2*c*e^4 + 2*(a*c^3)^(3/4)*c*d^3*e + 18*(a*c^3)^(3/4)*a*
d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^6 +
 3*sqrt(2)*a^3*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6
) + 1/2*(9*c*d^2*e^4 + a*e^6)*arctan(e*x/sqrt(d*e))/((c^3*d^7 + 3*a*c^2*d^
5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*sqrt(d*e)) - 1/4*(2*c^2*d^2*e^2*x^5 -
 2*a*c*e^4*x^5 + c^2*d^3*e*x^3 + a*c*d*e^3*x^3 - c^2*d^4*x + a*c*d^2*e^2*x
 - 2*a^2*e^4*x)/((a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4)*(c*e*x^6 + c...
```

3.149.9 Mupad [B] (verification not implemented)

Time = 17.35 (sec) , antiderivative size = 28923, normalized size of antiderivative = 33.48

$$\int \frac{1}{(d+ex^2)^2 (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^2*(d + e*x^2)^2),x)
```

output
```
((x*(2*a^2*e^4 + c^2*d^4 - a*c*d^2*e^2))/(4*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c
*d^2*e^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)) + (c*e^2*x^5*(a*e^2 - c*d^2))
/(2*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a*d + a*e*x^2 + c*d*x^4 + c
*e*x^6) + atan(((((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c
^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 128243
2*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15
- 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 +
a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e
^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*
a^10*c^2*d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 -
 212992*a^5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^
17*e^8 + 10960896*a^8*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750
464*a^10*c^9*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^
7*e^18 + 5554176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c
^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*
c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*
e^10 + 28*a^10*c^2*d^6*e^12)) - (x*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*
d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*
d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^
3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(25...
```

3.149. $\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx$

3.150 $\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx$

3.150.1 Optimal result

Integrand size = 21, antiderivative size = 388

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \frac{e^2(42cd^2-5ae^2)\,x\sqrt{a+cx^4}}{21c^2} + \frac{4de^3x^3\sqrt{a+cx^4}}{5c} + \frac{e^4x^5\sqrt{a+cx^4}}{7c} + \frac{4de(5cd^2-3ae^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{4\sqrt[4]{a}de(5cd^2-3ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}} + \frac{\left(105c^2d^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2-252a^{3/2}\sqrt{c}de^3+25a^2e^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}}{210\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}}$$

output
```
1/21*e^2*(-5*a*e^2+42*c*d^2)*x*(c*x^4+a)^(1/2)/c^2+4/5*d*e^3*x^3*(c*x^4+a)
^(1/2)/c+1/7*e^4*x^5*(c*x^4+a)^(1/2)/c+4/5*d*e*(-3*a*e^2+5*c*d^2)*x*(c*x^4
+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-4/5*a^(1/4)*d*e*(-3*a*e^2+5*c*d^2)
*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)
))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^
(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)+1
/210*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(
1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(105*c^2*d^
4-210*a*c*d^2*e^2+25*a^2*e^4+420*c^(3/2)*d^3*e*a^(1/2)-252*a^(3/2)*d*e^3*c
^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^
(1/4)/c^(9/4)/(c*x^4+a)^(1/2)
```

3.150.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.46

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{5(21c^2d^4-42acd^2e^2+5a^2e^4)\,x\sqrt{1+\frac{cx^4}{a}}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex\Big(-e(a+cx^4)\,(25ae^2-}{105c^2\sqrt{a-}}$$

input
```
Integrate[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

output
```
(5*(21*c^2*d^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[1 + (c*x^4)/a]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + e*x*(-(e*(a + c*x^4)*(25*a*e^2 -
 3*c*(70*d^2 + 28*d*e*x^2 + 5*e^2*x^4))) + 28*c*d*(5*c*d^2 - 3*a*e^2)*x^2*
Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(105*
c^2*Sqrt[a + c*x^4])
```

3.150.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1519, 2427, 2427, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx$$
$$\Big\downarrow \text{1519}$$
$$\frac{\int \frac{28cde^3x^6+e^2(42cd^2-5ae^2)x^4+28cd^3ex^2+7cd^4}{\sqrt{cx^4+a}}dx}{7c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$
$$\Big\downarrow \text{2427}$$
$$\frac{\frac{\int \frac{35c^2d^4+28ce(5cd^2-3ae^2)x^2d+5ce^2(42cd^2-5ae^2)x^4}{\sqrt{cx^4+a}}dx}{5c}+\frac{28}{5}de^3x^3\sqrt{a+cx^4}}{7c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \text{2427}$$

$$\frac{\frac{\frac{\int \frac{c\left(84cde\left(5cd^2-3ae^2\right)x^2+5\left(21c^2d^4-42ace^2d^2+5a^2e^4\right)\right)}{\sqrt{cx^4+a}}dx}{3c}+\frac{5}{3}e^2x\sqrt{a+cx^4}\left(42cd^2-5ae^2\right)}{5c}+\frac{28}{5}de^3x^3\sqrt{a+cx^4}}{7c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{1}{3}\int \frac{84cde\left(5cd^2-3ae^2\right)x^2+5\left(21c^2d^4-42ace^2d^2+5a^2e^4\right)}{\sqrt{cx^4+a}}dx+\frac{5}{3}e^2x\sqrt{a+cx^4}\left(42cd^2-5ae^2\right)}{5c}+\frac{28}{5}de^3x^3\sqrt{a+cx^4}}{7c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \text{1512}$$

$$\frac{\frac{\frac{1}{3}\left(\left(-252a^{3/2}\sqrt{c}de^3+25a^2e^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2+105c^2d^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx-84\sqrt{a}\sqrt{c}de\left(5cd^2-3ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx\right)+\frac{5}{3}e^2x\sqrt{a+cx^4}\left(42c\right.}{5c}}{7c}$$
$$\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{1}{3}\left(\left(-252a^{3/2}\sqrt{c}de^3+25a^2e^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2+105c^2d^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx-84\sqrt{c}de\left(5cd^2-3ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx\right)+\frac{5}{3}e^2x\sqrt{a+cx^4}\left(42cd^2-\right.}{5c}}{7c}$$
$$\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\frac{1}{3}\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-252a^{3/2}\sqrt{c}de^3+25a^2e^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2+105c^2d^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-84\sqrt{c}de\left(5cd^2-3ae^2\right)\int\right.}{5c}}{7c}$$
$$\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

$$\Big\downarrow \text{1510}$$

3.150. $\displaystyle\int \frac{\left(d+ex^2\right)^4}{\sqrt{a+cx^4}}\,dx$

$$\frac{\frac{\frac{1}{3}\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-252a^{3/2}\sqrt{c}de^3+25a^2e^4+420\sqrt{a}c^{3/2}d^3e-210acd^2e^2+105c^2d^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-84\sqrt{c}de\left(5cd^2-3ae^2\right)\left(\cdots\right.\right.}{5c}}{7c}$$

$$\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

input
```
Int[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

output
```
(e^4*x^5*Sqrt[a + c*x^4])/(7*c) + ((28*d*e^3*x^3*Sqrt[a + c*x^4])/5 + ((5*
e^2*(42*c*d^2 - 5*a*e^2)*x*Sqrt[a + c*x^4])/3 + (-84*Sqrt[c]*d*e*(5*c*d^2
- 3*a*e^2)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])) + ((105*c^
2*d^4 + 420*Sqrt[a]*c^(3/2)*d^3*e - 210*a*c*d^2*e^2 - 252*a^(3/2)*Sqrt[c]*
d*e^3 + 25*a^2*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^
(1/4)*Sqrt[a + c*x^4]))/3)/(5*c))/(7*c)
```

3.150.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.150.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.74

method	result
elliptic	$\frac{e^{4} x^{5} \sqrt{c\,x^{4}+a}}{7 c}+\frac{4 d\,e^{3} x^{3} \sqrt{c\,x^{4}+a}}{5 c}+\frac{\left(6 e^{2} d^{2}-\frac{5 e^{4} a}{7 c}\right) x \sqrt{c\,x^{4}+a}}{3 c}+\frac{\left(d^{4}-\frac{\left(6 e^{2} d^{2}-\frac{5 e^{4} a}{7 c}\right) a}{3 c}\right) \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right.}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$
risch	$-\frac{e^{2} x \left(-15 e^{2} x^{4} c-84 d e\,x^{2} c+25 a\,e^{2}-210 c\,d^{2}\right) \sqrt{c\,x^{4}+a}}{105 c^{2}}+\frac{\frac{25 a^{2} e^{4} \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{105 c^{2} d^{4} \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{c}}{}}}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+}}}{}$
default	$\frac{d^{4} \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+e^{4} \left(\frac{x^{5} \sqrt{c\,x^{4}+a}}{7 c}-\frac{5 a x \sqrt{c\,x^{4}+a}}{21 c^{2}}+\frac{5 a^{2} \sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{21 c^{2} \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)$

input
```
int((e*x^2+d)^4/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*e^4*x^5*(c*x^4+a)^(1/2)/c+4/5*d*e^3*x^3*(c*x^4+a)^(1/2)/c+1/3*(6*e^2*d
^2-5/7*e^4/c*a)/c*x*(c*x^4+a)^(1/2)+(d^4-1/3*(6*e^2*d^2-5/7*e^4/c*a)/c*a)/
(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I
*(4*d^3*e-12/5*d*e^3/c*a)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*
(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^
(1/2),I))
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{84\,(5\,acd^3e-3\,a^2de^3)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+(105\,c^2d^4-420\,acd^3e-210\,acd^2e^2+252\,a^2d}{}$$

input
```
integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(84*(5*a*c*d^3*e - 3*a^2*d*e^3)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(ar
csin((-a/c)^(1/4)/x), -1) + (105*c^2*d^4 - 420*a*c*d^3*e - 210*a*c*d^2*e^2
 + 252*a^2*d*e^3 + 25*a^2*e^4)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-
a/c)^(1/4)/x), -1) + (15*a*c*e^4*x^6 + 84*a*c*d*e^3*x^4 + 420*a*c*d^3*e -
252*a^2*d*e^3 + 5*(42*a*c*d^2*e^2 - 5*a^2*e^4)*x^2)*sqrt(c*x^4 + a))/(a*c^
2*x)
```

3.150.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \frac{d^4x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{d^3ex^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3d^2e^2x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{de^3x^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\Gamma\left(\frac{11}{4}\right)} + \frac{e^4x^9\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((e*x**2+d)**4/(c*x**4+a)**(1/2),x)
```

output
```
d**4*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(5/4)) + d**3*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x*
*4*exp_polar(I*pi)/a)/(sqrt(a)*gamma(7/4)) + 3*d**2*e**2*x**5*gamma(5/4)*h
yper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4))
+ d*e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)
/a)/(sqrt(a)*gamma(11/4)) + e**4*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,)
, c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4))
```

3.150.7 Maxima [F]

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^4}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)
```

3.150.8 Giac [F]

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^4}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^4/sqrt(c*x^4 + a), x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \int \frac{(e\,x^2+d)^4}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x^2)^4/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^4/(a + c*x^4)^(1/2), x)
```

3.151 $\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$

3.151.1 Optimal result

Integrand size = 21, antiderivative size = 326

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{de^2x\sqrt{a+cx^4}}{c} + \frac{e^3x^3\sqrt{a+cx^4}}{5c} + \frac{3e(5cd^2-ae^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{3\sqrt[4]{a}e(5cd^2-ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}} + \frac{\sqrt[4]{a}\left(15cd^2e-3ae^3+\frac{5\sqrt{c}d(cd^2-ae^2)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}}$$

output
```
d*e^2*x*(c*x^4+a)^(1/2)/c+1/5*e^3*x^3*(c*x^4+a)^(1/2)/c+3/5*e*(-a*e^2+5*c*
d^2)*x*(c*x^4+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-3/5*a^(1/4)*e*(-a*e^2
+5*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a
)^(1/2)+1/10*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1
/2))*(a^(1/2)+x^2*c^(1/2))*(15*c*d^2*e-3*a*e^3+5*d*(-a*e^2+c*d^2)*c^(1/2)/
a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)
```

3.151.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.43

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{5d(cd^2-ae^2)\,x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex\Big(e(5d+ex^2)\,(a+cx^4)+(5cd^2-ae^2)}{5c\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

output
```
(5*d*(c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4
, -((c*x^4)/a)] + e*x*(e*(5*d + e*x^2)*(a + c*x^4) + (5*c*d^2 - a*e^2)*x^2
*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(5*c
*Sqrt[a + c*x^4])
```

3.151.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1519, 2427, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$$
$$\Big\downarrow \text{1519}$$
$$\frac{\int \frac{15cde^2x^4+3e(5cd^2-ae^2)x^2+5cd^3}{\sqrt{cx^4+a}}dx}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$
$$\Big\downarrow \text{2427}$$
$$\frac{\frac{\int \frac{3c\left(3e\left(5cd^2-ae^2\right)x^2+5d\left(cd^2-ae^2\right)\right)}{\sqrt{cx^4+a}}dx}{3c}+5de^2x\sqrt{a+cx^4}}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3e(5cd^2-ae^2)x^2+5d(cd^2-ae^2)}{\sqrt{cx^4+a}}dx+5de^2x\sqrt{a+cx^4}}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

$$\Big\downarrow \; 1512$$

$$\frac{\frac{\left(-3a^{3/2}e^3+15\sqrt{a}cd^2e-5a\sqrt{c}de^2+5c^{3/2}d^3\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{3\sqrt{a}e\left(5cd^2-ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}+5de^2x\sqrt{a+cx^4}}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(-3a^{3/2}e^3+15\sqrt{a}cd^2e-5a\sqrt{c}de^2+5c^{3/2}d^3\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{3e\left(5cd^2-ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+5de^2x\sqrt{a+cx^4}}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

$$\Big\downarrow \; 761$$

$$\frac{-\frac{3e\left(5cd^2-ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-3a^{3/2}e^3+15\sqrt{a}cd^2e-5a\sqrt{c}de^2+5c^{3/2}d^3\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}+}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

$$\Big\downarrow \; 1510$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-3a^{3/2}e^3+15\sqrt{a}cd^2e-5a\sqrt{c}de^2+5c^{3/2}d^3\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}-3e\left(5cd^2-ae^2\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\ }}{}\right.}{5c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

input
```
Int[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

3.151. $\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$

output
```
(e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (5*d*e^2*x*Sqrt[a + c*x^4] - (3*e*(5*c*d
^2 - a*e^2)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c]
+ ((5*c^(3/2)*d^3 + 15*Sqrt[a]*c*d^2*e - 5*a*Sqrt[c]*d*e^2 - 3*a^(3/2)*e^3
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(3/4)*Sqrt[a + c*x
^4]))/(5*c)
```

3.151.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

3.151. $\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.151.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.72

method	result
elliptic	$\frac{e^3x^3\sqrt{c\,x^4+a}}{5c}+\frac{d\,e^2x\sqrt{c\,x^4+a}}{c}+\frac{\left(d^3-\frac{d\,e^2a}{c}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{i\left(3d^2e-\frac{3e^3a}{5c}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{}}{}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c}}$
risch	$\frac{e^2x(e\,x^2+5d)\sqrt{c\,x^4+a}}{5c}-\frac{-\frac{5d^3c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{5d\,e^2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{i\left(3a\,e^3-15c\,d^2e\right)}{}}{5c}$
default	$\frac{d^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+e^3\left(\frac{x^3\sqrt{c\,x^4+a}}{5c}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5c^{\frac{3}{2}}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right.$

input
```
int((e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*e^3*x^3*(c*x^4+a)^(1/2)/c+d*e^2*x*(c*x^4+a)^(1/2)/c+(d^3-d*e^2/c*a)/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*(
3*d^2*e-3/5*e^3/c*a)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(Elli
pticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2)
,I))
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.51

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{3\,(5\,acd^2e-a^2e^3)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)+(5\,c^2d^3-15\,acd^2e-5\,acde^2+3\,a^2e^3)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}}{5\,ac^2x}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(3*(5*a*c*d^2*e - a^2*e^3)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-
a/c)^(1/4)/x), -1) + (5*c^2*d^3 - 15*a*c*d^2*e - 5*a*c*d*e^2 + 3*a^2*e^3)*
sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + (a*c*e^3*x
^4 + 5*a*c*d*e^2*x^2 + 15*a*c*d^2*e - 3*a^2*e^3)*sqrt(c*x^4 + a))/(a*c^2*x
)
```

3.151.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

output
```
d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*
x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)*
hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
 + e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/
a)/(4*sqrt(a)*gamma(11/4))
```

3.151.7 Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

3.151.8 Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(e\,x^2+d)^3}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x^2)^3/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^3/(a + c*x^4)^(1/2), x)
```

3.152 $\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$

3.152.1 Optimal result

Integrand size = 21, antiderivative size = 264

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \frac{e^2x\sqrt{a+cx^4}}{3c}+\frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{2\sqrt[4]{a}de\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\left(3cd^2+6\sqrt{a}\sqrt{c}de-ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}$$

output
```
1/3*e^2*x*(c*x^4+a)^(1/2)/c+2*d*e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c
^(1/2))-2*a^(1/4)*d*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(
1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3
/4)/(c*x^4+a)^(1/2)+1/6*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*c^(1/2))*(3*c*d^2-a*e^2+6*d*e*a^(1/2)*c^(1/2))*((c*x^
4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(5/4)/(c*x^4+a)^(1/2)
```

3.152.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.45

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{(3cd^2-ae^2)\,x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex\Big(e(a+cx^4)+2cdx^2\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeo}}{3c\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

output
```
((3*c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
-((c*x^4)/a)] + e*x*(e*(a + c*x^4) + 2*c*d*x^2*Sqrt[1 + (c*x^4)/a]*Hyperge
ometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*c*Sqrt[a + c*x^4])
```

3.152.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1519, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$$
$$\Big\downarrow \text{1519}$$
$$\frac{\int \frac{3cd^2+6cex^2d-ae^2}{\sqrt{cx^4+a}}dx}{3c}+\frac{e^2x\sqrt{a+cx^4}}{3c}$$
$$\Big\downarrow \text{1512}$$
$$\frac{\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)\int \frac{1}{\sqrt{cx^4+a}}dx-6\sqrt{a}\sqrt{c}de\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{3c}+\frac{e^2x\sqrt{a+cx^4}}{3c}$$
$$\Big\downarrow \text{27}$$
$$\frac{\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)\int \frac{1}{\sqrt{cx^4+a}}dx-6\sqrt{c}de\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{3c}+\frac{e^2x\sqrt{a+cx^4}}{3c}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-6\sqrt{c}de\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{3c}+\frac{e^2x\sqrt{a+cx^4}}{3c}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6\sqrt{a}\sqrt{c}de-ae^2+3cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-6\sqrt{c}de\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{3c}$$

$$\frac{e^2x\sqrt{a+cx^4}}{3c}$$

input
```
Int[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

output
```
(e^2*x*Sqrt[a + c*x^4])/(3*c) + (-6*Sqrt[c]*d*e*(-((x*Sqrt[a + c*x^4])/(Sq
rt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(c^(1/4)*Sqrt[a + c*x^4])) + ((3*c*d^2 + 6*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])
)/(3*c)
```

3.152.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

3.152.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.76

method	result
elliptic	$\frac{e^2 x\sqrt{c x^4+a}}{3c}+\frac{\left(d^2-\frac{a e^2}{3c}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}}+\frac{2ied\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\cdots}\right.\right.\cdots}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}\,\sqrt{c}}$
default	$\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}}+e^2\left(\frac{x\sqrt{c x^4+a}}{3c}-\frac{a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}}\right)+\frac{2ied\sqrt{a}\,\sqrt{1-\cdots}}{\cdots}$
risch	$\frac{e^2 x\sqrt{c x^4+a}}{3c}-\frac{\frac{a e^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}}-\frac{3c\,d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}}-\frac{6id\sqrt{c}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\cdots}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\cdots}}{3c}$

input
```
int((e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*e^2*x*(c*x^4+a)^(1/2)/c+(d^2-1/3*a/c*e^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1
-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+2*I*e*d*a^(1/2)/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-Ellip
ticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.43

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{6\,a\sqrt{c}dex\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\left(3\,cd^2-6\,ade-ae^2\right)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+(}{3\,acx}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(6*a*sqrt(c)*d*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1)
 + (3*c*d^2 - 6*a*d*e - a*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-
a/c)^(1/4)/x), -1) + (a*e^2*x^2 + 6*a*d*e)*sqrt(c*x^4 + a))/(a*c*x)
```

3.152.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.47

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

output
```
d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*
exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1/
2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

3.152.7 Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

3.152.8 Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(e\,x^2+d)^2}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x^2)^2/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^2/(a + c*x^4)^(1/2), x)
```

3.153 $\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$

3.153.1 Optimal result

Integrand size = 19, antiderivative size = 226

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{ex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}$$

output
```
e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*arctan(
c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+
a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(cos
(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2)
)*(e+d*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/
(c*x^4+a)^(1/2)
```

3.153.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.34

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{\sqrt{1+\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)\right)}{3\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
 + e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[a + c*x^
4])
```

3.153.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \text{1512}$$

$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{e\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{e\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

input
```
Int[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

output
```
-((e*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c]) + ((d
+ (Sqrt[a]*e)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)
*c^(1/4)*Sqrt[a + c*x^4])
```

3.153.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

3.153.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$	169
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$	169

input
```
int((e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+I*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.39

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{a\sqrt{c}ex\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)+(cd-ae)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)+\sqrt{cx^4+a}\,ae}{acx}$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
(a*sqrt(c)*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) + (c*d
- a*e)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + sqr
t(c*x^4 + a)*a*e)/(a*c*x)
```

3.153.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.35

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(c*x**4+a)**(1/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_p
olar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

3.153.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

3.153.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 + a), x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x^2)/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(a + c*x^4)^(1/2), x)
```

3.154 $\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

3.154.1 Optimal result

Integrand size = 21, antiderivative size = 334

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \frac{\sqrt{e}\arctan\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2+ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}} - \frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}d\left(cd^2-ae^2\right)\sqrt{a+cx^4}}$$

output
```
1/2*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))*e^(1/2)/
d^(1/2)/(a*e^2+c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1
/2))^2)^(1/2)/a^(1/4)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)-1/4*a^(3/4)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))
*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2
/d/e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/
2))^2*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(
c*x^4+a)^(1/2)
```

3.154.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.28

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\frac{i\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d}, i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right), -1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}d\sqrt{a+cx^4}}$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[a
+ c*x^4])
```

3.154.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1541, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)}\,dx$$

$$\Big\downarrow \ 1541$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e} - \frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e} - \frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \ 761$$

$$\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right), \frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\,(\sqrt{c}d-\sqrt{a}e)} - \frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\downarrow \text{2221}$$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{e\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}\right)}{\sqrt{c}d-\sqrt{a}e}$$

input
```
Int[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d -
Sqrt[a]*e)*Sqrt[a + c*x^4]) - (e*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sq
rt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(Sqrt[d]*Sqrt[e]*
Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[
c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + c*x^4])))/(Sqrt[c]*d - Sqrt[a]*e)
```

3.154.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

3.154.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.32

method	result	size
default	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	107
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	107

input
```
int(1/(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2)
,I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2
))
```

3.154.5 Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)
```

3.154.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)), x)
```

3.154.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

3.154.8 Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)), x)
```

3.155 $\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$

3.155.1 Optimal result

Integrand size = 21, antiderivative size = 581

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx &= -\frac{\sqrt{c}ex\sqrt{a+cx^4}}{2d(cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)}\\
&+\frac{e^2x\sqrt{a+cx^4}}{2d(cd^2+ae^2)(d+ex^2)}+\frac{\sqrt{e}(3cd^2+ae^2)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}(cd^2+ae^2)^{3/2}}\\
&+\frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d(cd^2+ae^2)\sqrt{a+cx^4}}\\
&+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}}\\
&-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)(3cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)\sqrt{a+cx^4}}\end{aligned}$$

output
```
1/4*(a*e^2+3*c*d^2)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)
^(1/2))*e^(1/2)/d^(3/2)/(a*e^2+c*d^2)^(3/2)+1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a
*e^2+c*d^2)/(e*x^2+d)-1/2*e*x*c^(1/2)*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(a^(
1/2)+x^2*c^(1/2))+1/2*a^(1/4)*c^(1/4)*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1
/2))^2)^(1/2)/d/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)
/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)
^(1/2)-1/8*(a*e^2+3*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-
1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(e*a^(1/2)+d
*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
a^(1/4)/c^(1/4)/d^2/(a*e^2+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)
```

3.155.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.59 (sec) , antiderivative size = 522, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$$
$$= \frac{a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}de^2x+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cde^2x^5-\sqrt{a}\sqrt{c}de(d+ex^2)\sqrt{1+\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{c}d(i\sqrt{c}d+\sqrt{a}}{}$$

input
```
Integrate[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

output
```
(a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e^2*x + Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d*e^2*x
^5 - Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*d*(I*Sqrt[c]*d + Sqrt[a]*e)
*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt
[a]]*x], -1] - (3*I)*c*d^3*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)
/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a*d*e^2*Sqrt
[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[a]]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 + (c*x^4)/a]*Ellipti
cPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
-1] - I*a*e^3*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]
*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(2*Sqrt[(I*Sqrt[c])/Sqrt
[a]]*d^2*(c*d^2 + a*e^2)*(d + e*x^2)*Sqrt[a + c*x^4])
```

3.155.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 569, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1552, 25, 2233, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)^2}\,dx$$

$$\Big\downarrow \ 1552$$

$$\frac{e^2x\sqrt{a+cx^4}}{2d\,(d+ex^2)\,(ae^2+cd^2)}-\frac{\int -\frac{-ce^2x^4-2cdex^2+2cd^2+ae^2}{(ex^2+d)\sqrt{cx^4+a}}dx}{2d\,(ae^2+cd^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-ce^2x^4-2cdex^2+2cd^2+ae^2}{(ex^2+d)\sqrt{cx^4+a}}dx}{2d\,(ae^2+cd^2)}+\frac{e^2x\sqrt{a+cx^4}}{2d\,(d+ex^2)\,(ae^2+cd^2)}$$

$$\Big\downarrow \ 2233$$

$$\frac{\frac{\int \frac{ce\left(2cd^2-\sqrt{a}\sqrt{c}ed+ae^2-\sqrt{c}e(\sqrt{c}d+\sqrt{a}e)x^2\right)}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{ce}+\sqrt{a}\sqrt{c}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{2d\,(ae^2+cd^2)}+\frac{e^2x\sqrt{a+cx^4}}{2d\,(d+ex^2)\,(ae^2+cd^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2cd^2-\sqrt{a}\sqrt{c}ed+ae^2-\sqrt{c}e(\sqrt{c}d+\sqrt{a}e)x^2}{(ex^2+d)\sqrt{cx^4+a}}dx+\sqrt{c}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{2d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$\big\downarrow$ 1510

$$\frac{\int \frac{2cd^2-\sqrt{a}\sqrt{c}ed+ae^2-\sqrt{c}e(\sqrt{c}d+\sqrt{a}e)x^2}{(ex^2+d)\sqrt{cx^4+a}}dx+\sqrt{c}e\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{2d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$\big\downarrow$ 2227

$$\frac{\frac{2\sqrt{c}(ae^2+cd^2)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\sqrt{a}e(ae^2+3cd^2)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\sqrt{c}e\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{2d\left(ae^2+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$\big\downarrow$ 27

$$\frac{\frac{2\sqrt{c}(ae^2+cd^2)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{e(ae^2+3cd^2)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\sqrt{c}e\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{}}{\sqrt{a}}\right.}{2d\left(ae^2+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$\big\downarrow$ 761

$$\frac{-\frac{e(ae^2+3cd^2)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(ae^2+cd^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}(\sqrt{c}d-\sqrt{a}e)}+\sqrt{c}e\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2}{}\right.}{2d\left(ae^2+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$\big\downarrow$ 2221

3.155. $\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$

$$\frac{\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(ae^2+cd^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}(\sqrt{c}d-\sqrt{a}e)} - \frac{e(ae^2+3cd^2)\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(\sqrt{a}e+\sqrt{c}d)\,\text{EllipticPi}\left(-\ldots\right.}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+\ldots}}\right.}{\ldots}}{2\ldots}$$

$$\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

input
```
Int[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

output
```
(e^2*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (Sqrt[c]*e*(-(
(x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])) + (c^(1/4)*(c*d^2 + a*e^
2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*(Sqrt[c]*d - Sqrt[a]*
e)*Sqrt[a + c*x^4]) - (e*(3*c*d^2 + a*e^2)*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*
ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(Sqrt[d
]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[
a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + c*x^4])))/(Sqrt[c]*d - Sqrt[a]*e
))/(2*d*(c*d^2 + a*e^2))
```

3.155.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1552
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sq
rt[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
 + c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && ILtQ[q, -1]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.155. $\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$

3.155.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 556, normalized size of antiderivative = 0.96

method	result
default	$\frac{e^{2} x \sqrt{c\,x^{4}+a}}{2 d \left(a\,e^{2}+c\,d^{2}\right) \left(e\,x^{2}+d\right)}-\frac{c \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{2 \left(a\,e^{2}+c\,d^{2}\right) \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}-\frac{i \sqrt{c}\, e \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{2 d \left(a\,e^{2}+c\,d^{2}\right) \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+\frac{i \sqrt{c}\, e \sqrt{}}{}$
elliptic	$\frac{e^{2} x \sqrt{c\,x^{4}+a}}{2 d \left(a\,e^{2}+c\,d^{2}\right) \left(e\,x^{2}+d\right)}-\frac{c \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{2 \left(a\,e^{2}+c\,d^{2}\right) \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}-\frac{i \sqrt{c}\, e \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{2 d \left(a\,e^{2}+c\,d^{2}\right) \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+\frac{i \sqrt{c}\, e \sqrt{}}{}$

input
```
int(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)-1/2*c/(a*e^2+c*d^2)/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2
*I*c^(1/2)*e/d/(a*e^2+c*d^2)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I*c^(1/2)*e/d/(a*e^2+c*d^2)*a^(1/2
)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+1/2/d^2/(a*e^2+c*d^2)*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I
/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(
I/a^(1/2)*c^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1
-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1
/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/
2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c
```

3.155.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.155.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**2), x)
```

3.155.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

3.155.8 Giac [F]

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

3.156 $\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx$

3.156.1 Optimal result

Integrand size = 21, antiderivative size = 729

$$\begin{aligned}\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx &= -\frac{3\sqrt{c}e(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{4d(cd^2+ae^2)(d+ex^2)^2}\\
&+\frac{3e^2(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^2(d+ex^2)}+\frac{3\sqrt{e}(5c^2d^4+2acd^2e^2+a^2e^4)\arctan\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}(cd^2+ae^2)^{5/2}}\\
&+\frac{3\sqrt[4]{a}\sqrt[4]{c}e(3cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2(cd^2+ae^2)^2\sqrt{a+cx^4}}\\
&+\frac{\sqrt[4]{c}\left(4cd^2-\sqrt{a}\sqrt{c}de+3ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}d^2\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)\sqrt{a+cx^4}}\\
&-\frac{3\left(\sqrt{c}d+\sqrt{a}e\right)(5c^2d^4+2acd^2e^2+a^2e^4)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\right.\right.}{32\sqrt[4]{a}\sqrt[4]{c}d^3\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2\sqrt{a+cx^4}}\end{aligned}$$

output
```
3/16*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2
)/e^(1/2)/(c*x^4+a)^(1/2))*e^(1/2)/d^(5/2)/(a*e^2+c*d^2)^(5/2)+1/4*e^2*x*(
c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^2)*x*(c*x^
4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)-3/8*e*(a*e^2+3*c*d^2)*x*c^(1/2)*(
c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(a^(1/2)+x^2*c^(1/2))+3/8*a^(1/4)*c^(1/
4)*e*(a*e^2+3*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1
/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/d^2/(
a*e^2+c*d^2)^2/(c*x^4+a)^(1/2)-3/32*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*(cos
(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/
e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^(1/2)+x^2*c^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^3/(a*e^2+c*d^
2)^2/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)+1/8*c^(1/4)*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*
arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(4*c*d^2+3*a
*e^2-d*e*a^(1/2)*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4
)/d^2/(a*e^2+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)
```

3.156.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.85 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.46

$$\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx$$
$$= \frac{\frac{de^2x(a+cx^4)(ae^2(5d+3ex^2)+cd^2(11d+9ex^2))}{(d+ex^2)^2} + \frac{\sqrt{1+\frac{cx^4}{a}}\left(-3\sqrt{a}\sqrt{c}de(3cd^2+ae^2)E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+i\left(\sqrt{c}d(7c^{3/2}d^3-9i\sqrt{a}c\cdots\right.\right.}{\cdots}}{8d^3(cd^2+ae^2)\cdots}$$

input
```
Integrate[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

output
```
((d*e^2*x*(a + c*x^4)*(a*e^2*(5*d + 3*e*x^2) + c*d^2*(11*d + 9*e*x^2)))/(d
 + e*x^2)^2 + (Sqrt[1 + (c*x^4)/a]*(-3*Sqrt[a]*Sqrt[c]*d*e*(3*c*d^2 + a*e^
2)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*(Sqrt[c]*d*(7
*c^(3/2)*d^3 - (9*I)*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - (3*I)*a^(3/2)*e^3
)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - 3*(5*c^2*d^4 + 2
*a*c*d^2*e^2 + a^2*e^4)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh
[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])))/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(8*d^3*(c
*d^2 + a*e^2)^2*Sqrt[a + c*x^4])
```

3.156.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 700, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {1552, 25, 2211, 25, 2233, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{ 1552}$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\,(d+ex^2)^2\,(ae^2+cd^2)}-\frac{\int -\frac{ce^2x^4-4cdex^2+4cd^2+3ae^2}{(ex^2+d)^2\sqrt{cx^4+a}}dx}{4d\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{ce^2x^4-4cdex^2+4cd^2+3ae^2}{(ex^2+d)^2\sqrt{cx^4+a}}dx}{4d\,(ae^2+cd^2)}+\frac{e^2x\sqrt{a+cx^4}}{4d\,(d+ex^2)^2\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 2211}$$

$$\frac{\frac{3e^2x\sqrt{a+cx^4}(ae^2+3cd^2)}{2d(d+ex^2)(ae^2+cd^2)}-\frac{\int -\frac{8c^2d^4+5ace^2d^2-4ce\left(4cd^2+ae^2\right)x^2d+3a^2e^4-3ce^2\left(3cd^2+ae^2\right)x^4}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{2d(ae^2+cd^2)}}{4d\,(ae^2+cd^2)}+\frac{e^2x\sqrt{a+cx^4}}{4d\,(d+ex^2)^2\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{8c^2d^4+5ace^2d^2-4ce\left(4cd^2+ae^2\right)x^2d+3a^2e^4-3ce^2\left(3cd^2+ae^2\right)x^4}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{2d(ae^2+cd^2)}+\frac{3e^2x\sqrt{a+cx^4}(ae^2+3cd^2)}{2d(d+ex^2)(ae^2+cd^2)}}{4d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 2233

$$\frac{\frac{\frac{\int \frac{ce\left(8c^2d^4+5ace^2d^2-3\sqrt{a}\sqrt{c}e\left(3cd^2+ae^2\right)d+3a^2e^4-\sqrt{c}e\left(7c^{3/2}d^3+9\sqrt{a}ced^2+a\sqrt{c}e^2d+3a^{3/2}e^3\right)x^2\right)}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{ce}+3\sqrt{a}\sqrt{c}e(ae^2+3cd^2)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{2d(ae^2+cd^2)}+\frac{3e^2x\sqrt{\cdots}}{2d(d\cdots)}}{4d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{8c^2d^4+5ace^2d^2-3\sqrt{a}\sqrt{c}e\left(3cd^2+ae^2\right)d+3a^2e^4-\sqrt{c}e\left(7c^{3/2}d^3+9\sqrt{a}ced^2+a\sqrt{c}e^2d+3a^{3/2}e^3\right)x^2}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx+3\sqrt{c}e(ae^2+3cd^2)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{2d(ae^2+cd^2)}+\frac{3e^2x\sqrt{a+cx^4}(a\cdots}{2d(d+ex^2)(ae\cdots}}{4d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 1510

$$\frac{\frac{\int \frac{8c^2d^4+5ace^2d^2-3\sqrt{a}\sqrt{c}e\left(3cd^2+ae^2\right)d+3a^2e^4-\sqrt{c}e\left(7c^{3/2}d^3+9\sqrt{a}ced^2+a\sqrt{c}e^2d+3a^{3/2}e^3\right)x^2}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx+3\sqrt{c}e(ae^2+3cd^2)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\cdots}{\sqrt[4]{c}\sqrt{a+cx\cdots}}\cdots\right)}{2d(ae^2+cd^2)}}{4d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 2227

$$\frac{\frac{-\frac{3\sqrt{a}e\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2\sqrt{c}\left(ae^2+cd^2\right)\left(-\sqrt{a}\sqrt{c}de+3ae^2+4cd^2\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+3\sqrt{c}e(ae^2+3cd^2)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\cdots}}{\cdots}\cdots\right)}{2d(ae^2+cd^2)}}{4d\left(ae^2+cd^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

3.156. $\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx$

$\downarrow$ 27

$$\frac{\frac{-\frac{3e\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2\sqrt{c}\left(ae^2+cd^2\right)\left(-\sqrt{a}\sqrt{c}de+3ae^2+4cd^2\right)\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+3\sqrt{c}e\left(ae^2+3cd^2\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+\cdots}{(\sqrt{a}+\cdots}}}{\sqrt[4]{\cdots}}\right.\cdots}{2d(ae^2+cd^2)}}{4d\left(ae^2+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 761

$$\frac{\frac{-\frac{3e\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^2+cd^2\right)\left(-\sqrt{a}\sqrt{c}de+3ae^2+4cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}\cdots}{2d(ae^2+cd^2)}}{4d\left(ae^2+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 2221

$$-\frac{3e\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{x\sqrt{ae^2+cd\cdots}}{\sqrt{d}\sqrt{e}\sqrt{a+c\cdots}}\right.}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}\cdots\right.}{\sqrt{c}d-\sqrt{a}e}\cdots$$

$$\frac{e^2x\sqrt{a+cx^4}}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

input
```
Int[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

3.156. $\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx$

output
```
(e^2*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d + e*x^2)^2) + ((3*e^2*(3*c
*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (3*Sq
rt[c]*e*(3*c*d^2 + a*e^2)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2))
+ (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^
4])) + (c^(1/4)*(c*d^2 + a*e^2)*(4*c*d^2 - Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*
Sqrt[a + c*x^4]) - (3*e*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(-1/2*((Sqrt
[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a
+ c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*
e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticPi[-1/4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + c*x^4])))/(Sq
rt[c]*d - Sqrt[a]*e))/(2*d*(c*d^2 + a*e^2)))/(4*d*(c*d^2 + a*e^2))
```

3.156.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.156. $\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx$

rule 1552
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sq
rt[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
 + c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && ILtQ[q, -1]
```

rule 2211
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))
Int[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(
2*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(
C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.156. $\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx$

3.156.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 1018, normalized size of antiderivative = 1.40

method	result	size
default	Expression too large to display	1018
elliptic	Expression too large to display	1018

input
```
int(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d
^2)*x*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)-1/8*c/d/(a*e^2+c*d^2)^
2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
*a*e^2-7/8*c^2*d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+9/8
*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellip
ticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/8*I*c^(1/2)*e^3/d^2/(a*e^2+c*d^2)^2*
a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1
/2),I)-3/8*I*c^(1/2)*e^3/d^2/(a*e^2+c*d^2)^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(
1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/8/d^3/(a*e^2+c*d^2)^
2*e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/
2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))...
```

3.156.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.156.6 Sympy [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\left(d+ex^2\right)^3}\,dx$$

input
```
integrate(1/(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**3), x)
```

3.156.7 Maxima [F]

$$\int \frac{1}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

3.156.8 Giac [F]

$$\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(e\,x^2+d)^3}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^3),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

3.157 $\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$

3.157.1 Optimal result

Integrand size = 22, antiderivative size = 213

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = -\frac{de^2x\sqrt{a-cx^4}}{c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{3a^{3/4}e(5cd^2+ae^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}} + \frac{a^{3/4}\left(\frac{5\sqrt{c}d(cd^2+ae^2)}{\sqrt{a}}-3e(5cd^2+ae^2)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{7/4}\sqrt{a-cx^4}}$$

output
```
-d*e^2*x*(-c*x^4+a)^(1/2)/c-1/5*e^3*x^3*(-c*x^4+a)^(1/2)/c+3/5*a^(3/4)*e*(
a*e^2+5*c*d^2)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(7/4)/(-
c*x^4+a)^(1/2)+1/5*a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-3*e*(a*e^2+5*c
*d^2)+5*d*(a*e^2+c*d^2)*c^(1/2)/a^(1/2))*(1-c*x^4/a)^(1/2)/c^(7/4)/(-c*x^4
+a)^(1/2)
```

3.157.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$$
$$= \frac{5d(cd^2+ae^2)\,x\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex\Big(e(5d+ex^2)\,(-a+cx^4)+(5cd^2+ae^2)}{5c\sqrt{a-cx^4}}$$

input
```
Integrate[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

output
```
(5*d*(c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4
, (c*x^4)/a] + e*x*(e*(5*d + e*x^2)*(-a + c*x^4) + (5*c*d^2 + a*e^2)*x^2*S
qrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(5*c*Sqrt
[a - c*x^4])
```

3.157.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1519, 25, 2427, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$$

$$\Big\downarrow \text{1519}$$

$$-\frac{\int -\frac{15cde^2x^4+3e(5cd^2+ae^2)x^2+5cd^3}{\sqrt{a-cx^4}}dx}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{15cde^2x^4+3e(5cd^2+ae^2)x^2+5cd^3}{\sqrt{a-cx^4}}dx}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \text{2427}$$

$$-\frac{-\frac{\int -\frac{3c\left(3e\left(5cd^2+ae^2\right)x^2+5d\left(cd^2+ae^2\right)\right)}{\sqrt{a-cx^4}}dx}{3c}-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\downarrow 27$$

$$\frac{\int \frac{3e(5cd^2+ae^2)x^2+5d(cd^2+ae^2)}{\sqrt{a-cx^4}}dx-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\downarrow 1513$$

$$\frac{\left(5d(ae^2+cd^2)-\frac{3\sqrt{a}e(ae^2+5cd^2)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{3\sqrt{a}e(ae^2+5cd^2)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\downarrow 27$$

$$\frac{\left(5d(ae^2+cd^2)-\frac{3\sqrt{a}e(ae^2+5cd^2)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{3e(ae^2+5cd^2)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\downarrow 765$$

$$\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(5d(ae^2+cd^2)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{3e(ae^2+5cd^2)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\downarrow 762$$

$$\frac{\frac{3e(ae^2+5cd^2)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d(ae^2+cd^2)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-5de^2x\sqrt{a-cx^4}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\downarrow 1390$$

$$\frac{\frac{3e\sqrt{1-\frac{cx^4}{a}}(ae^2+5cd^2)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d(ae^2+cd^2)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-5de^2x\sqrt{a-c}}{5c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

3.157. $\int \frac{\left(d+ex^2\right)^3}{\sqrt{a-cx^4}}\,dx$

$$\Big\downarrow \text{1389}$$

$$\frac{\frac{3\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}(ae^2+5cd^2)\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d(ae^2+cd^2)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-5de^2x\sqrt{a-}}{5c}$$

$$\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}(ae^2+5cd^2)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5d(ae^2+cd^2)-\frac{3\sqrt{a}e\left(ae^2+5cd^2\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-5}{5c}$$

$$\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

input
```
Int[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

output
```
-1/5*(e^3*x^3*Sqrt[a - c*x^4])/c + (-5*d*e^2*x*Sqrt[a - c*x^4] + (3*a^(3/4
)*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(
1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(5*d*(c*d^2 + a*e^2) - (3
*Sqrt[a]*e*(5*c*d^2 + a*e^2))/Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]))/(5*c)
```

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.157. $\int\frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

3.157. $\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx$

3.157.4 Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.04

method	result
elliptic	$-\frac{e^3x^3\sqrt{-cx^4+a}}{5c}-\frac{de^2x\sqrt{-cx^4+a}}{c}+\frac{\left(d^3+\frac{de^2a}{c}\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}-\frac{\left(3d^2e+\frac{3e^3a}{5c}\right)\sqrt{a}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{}}$
risch	$-\frac{e^2x(ex^2+5d)\sqrt{-cx^4+a}}{5c}+\frac{\frac{5d^3c\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}+\frac{5de^2a\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}-\frac{(3ae^3+15cd^2e)\sqrt{a}}{}}{5c}$
default	$\frac{d^3\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}+e^3\left(-\frac{x^3\sqrt{-cx^4+a}}{5c}-\frac{3a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5c^{\frac{3}{2}}\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}\right.$

input
```
int((e*x^2+d)^3/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*e^3*x^3*(-c*x^4+a)^(1/2)/c-d*e^2*x*(-c*x^4+a)^(1/2)/c+(d^3+d*e^2/c*a)
/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
-(3*d^2*e+3/5*e^3/c*a)*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(E
llipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1
/2),I))
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx =$$
$$-\frac{3\,(5\,acd^2e+a^2e^3)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-(5\,c^2d^3+15\,acd^2e+5\,acde^2+3\,a^2e^3)\sqrt{-c}x\left(\frac{a}{c}\right)}{5\,ac^2x}$$

input
```
integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(3*(5*a*c*d^2*e + a^2*e^3)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_e(arcsin((
a/c)^(1/4)/x), -1) - (5*c^2*d^3 + 15*a*c*d^2*e + 5*a*c*d*e^2 + 3*a^2*e^3)*
sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + (a*c*e^3*x^
4 + 5*a*c*d*e^2*x^2 + 15*a*c*d^2*e + 3*a^2*e^3)*sqrt(-c*x^4 + a))/(a*c^2*x
)
```

3.157.6 Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = \frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

output
```
d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4
*sqrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,),
c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5
/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma
(9/4)) + e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(
2*I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

3.157.7 Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

3.157.8 Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = \int \frac{(e\,x^2+d)^3}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int((d + e*x^2)^3/(a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^3/(a - c*x^4)^(1/2), x)
```

3.158 $\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx$

3.158.1 Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{3c} + \frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\sqrt{c}de+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}}$$

output
```
-1/3*e^2*x*(-c*x^4+a)^(1/2)/c+2*a^(3/4)*d*e*EllipticE(c^(1/4)*x/a^(1/4),I)
*(1-c*x^4/a)^(1/2)/c^(3/4)/(-c*x^4+a)^(1/2)+1/3*a^(1/4)*EllipticF(c^(1/4)*
x/a^(1/4),I)*(3*c*d^2+a*e^2-6*d*e*a^(1/2)*c^(1/2))*(1-c*x^4/a)^(1/2)/c^(5/
4)/(-c*x^4+a)^(1/2)
```

3.158.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = \frac{(3cd^2+ae^2)\,x\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex\left(-ae+cex^4+2cdx^2\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeo}\right.}{3c\sqrt{a-cx^4}}$$

input
```
Integrate[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

output
```
((3*c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
(c*x^4)/a] + e*x*(-(a*e) + c*e*x^4 + 2*c*d*x^2*Sqrt[1 - (c*x^4)/a]*Hyperge
ometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*c*Sqrt[a - c*x^4])
```

3.158.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1519, 25, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}dx$$

$$\Big\downarrow \text{ 1519}$$

$$-\frac{\int -\frac{3cd^2+6cex^2d+ae^2}{\sqrt{a-cx^4}}dx}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3cd^2+6cex^2d+ae^2}{\sqrt{a-cx^4}}dx}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \text{ 1513}$$

$$\frac{\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{1}{\sqrt{a-cx^4}}dx+6\sqrt{a}\sqrt{c}de\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{1}{\sqrt{a-cx^4}}dx+6\sqrt{c}de\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+6\sqrt{c}de\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{6\sqrt{c}de\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\frac{6\sqrt{c}de\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\frac{6\sqrt{a}\sqrt{c}de\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{6a^{3/4}\sqrt[4]{c}de\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(-6\sqrt{a}\sqrt{c}de+ae^2+3cd^2)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

input
```
Int[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

output
```
-1/3*(e^2*x*Sqrt[a - c*x^4])/c + ((6*a^(3/4)*c^(1/4)*d*e*Sqrt[1 - (c*x^4)/
a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^4] + (a^(1/4)*
(3*c*d^2 - 6*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[Ar
cSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]))/(3*c)
```

3.158.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.158. $\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx$

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

3.158. $\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}} dx$

3.158.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.15

method	result
elliptic	$-\frac{e^2x\sqrt{-cx^4+a}}{3c}+\frac{\left(d^2+\frac{ae^2}{3c}\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}-\frac{2ed\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\right.\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}\,\sqrt{c}}$
default	$\frac{d^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}+e^2\left(-\frac{x\sqrt{-cx^4+a}}{3c}+\frac{a\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}\right)-\frac{2ed\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{}$
risch	$-\frac{e^2x\sqrt{-cx^4+a}}{3c}+\frac{\frac{ae^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}+\frac{3cd^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}-\frac{6d\sqrt{c}\,e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}}{3c}$

input
```
int((e*x^2+d)^2/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*e^2*x*(-c*x^4+a)^(1/2)/c+(d^2+1/3*a/c*e^2)/(1/a^(1/2)*c^(1/2))^(1/2)*
(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)
^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-2*e*d*a^(1/2)/(1/a^(1/2)*c
^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-Ell
ipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx =$$
$$-\frac{6\,a\sqrt{-c}dex\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-(3\,cd^2+6\,ade+ae^2)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+(}{3\,acx}$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(6*a*sqrt(-c)*d*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1)
 - (3*c*d^2 + 6*a*d*e + a*e^2)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a
/c)^(1/4)/x), -1) + (a*e^2*x^2 + 6*a*d*e)*sqrt(-c*x^4 + a))/(a*c*x)
```

3.158.6 Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

output
```
d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4
*sqrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**
4*exp_polar(2*I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper
((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

3.158.7 Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

3.158.8 Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = \int \frac{(e\,x^2+d)^2}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int((d + e*x^2)^2/(a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^2/(a - c*x^4)^(1/2), x)
```

3.159 $\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx$

3.159.1 Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$

output
```
a^(3/4)*e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(3/4)/(-c*x^4
+a)^(1/2)+a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-e+d*c^(1/2)/a^(1/2))*(1
-c*x^4/a)^(1/2)/c^(3/4)/(-c*x^4+a)^(1/2)
```

3.159.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{a-cx^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

output
```
(Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] +
e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[a - c*x^4])
```

3.159.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}dx$$

$$\Big\downarrow \text{1513}$$

$$\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{762}$$

$$\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}$$

input
```
Int[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

output
```
(a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(d - (Sqrt[a]*e)/Sqrt[c])*Sqrt[1 - (
c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*
x^4])
```

3.159.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

3.159.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{d\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,\sqrt{c}}$	154
elliptic	$\frac{d\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,\sqrt{c}}$	154

input
```
int((e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I
)-e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1
/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/
2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = -\frac{a\sqrt{-c}ex\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-(cd+ae)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\sqrt{-cx^4+a}ae}{acx}$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-(a*sqrt(-c)*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - (c*d
+ a*e)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + sqrt
(-c*x^4 + a)*a*e)/(a*c*x)
```

3.159.6 Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp
_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

3.159.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

3.159.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int((d + e*x^2)/(a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(a - c*x^4)^(1/2), x)
```

3.160 $\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx$

3.160.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

output
```
a^(1/4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(
1/2)/c^(1/4)/d/(-c*x^4+a)^(1/2)
```

3.160.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = -\frac{i\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d\sqrt{a-cx^4}}$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh
[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[a - c*
x^4])
```

3.160.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)}\,dx$$

$$\Big\downarrow \; 1543$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{(ex^2+d)\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \; 1542$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}$$

input
```
Int[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin
[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x^4])
```

3.160.3.1 Defintions of rubi rules used

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.160.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{e\sqrt{a}}{d\sqrt{c}},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}$	97
elliptic	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{e\sqrt{a}}{d\sqrt{c}},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}$	97

input
```
int(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2
),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2
))
```

3.160.5 Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

3.160.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)), x)
```

3.160.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

3.160.8 Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

3.161 $\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx$

3.161.1 Optimal result

Integrand size = 22, antiderivative size = 299

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx &= -\frac{e^2x\sqrt{a-cx^4}}{2d(cd^2-ae^2)(d+ex^2)} - \frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d(cd^2-ae^2)\sqrt{a-cx^4}} \\ &\quad - \frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2d\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{a-cx^4}} \\ &\quad + \frac{\sqrt[4]{a}(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}d^2(cd^2-ae^2)\sqrt{a-cx^4}}\end{aligned}$$

output
```
-1/2*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)-1/2*a^(3/4)*c^(1/4)
*e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d/(-a*e^2+c*d^2)/(-c*x
^4+a)^(1/2)+1/2*a^(1/4)*(-a*e^2+3*c*d^2)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a
^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/d^2/(-a*e^2+c*d^2)/(-c*x^4+a
)^(1/2)-1/2*a^(1/4)*c^(1/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/
2)/d/(e*a^(1/2)+d*c^(1/2))/(-c*x^4+a)^(1/2)
```

3.161.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.81 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.70

$$\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx$$
$$= \frac{-a\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}de^2x+\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cde^2x^5+i\sqrt{a}\sqrt{c}de(d+ex^2)\sqrt{1-\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-i\sqrt{c}d(-\sqrt{}}{}$$

input
```
Integrate[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

output
```
(-(a*Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e^2*x) + Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d*e^2*
x^5 + I*Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*Ar
cSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*Sqrt[c]*d*(-(Sqrt[c]*d) + Sqrt[
a]*e)*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/S
qrt[a])]*x], -1] - (3*I)*c*d^3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e
)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*d*e^2*Sqr
t[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(S
qrt[c]/Sqrt[a])]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 - (c*x^4)/a]*EllipticP
i[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] +
 I*a*e^3*x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*
ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(2*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*
(c*d^2 - a*e^2)*(d + e*x^2)*Sqrt[a - c*x^4])
```

3.161.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1552, 2235, 27, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^2}\,dx$$
$$\Big\downarrow \; 1552$$

$$\frac{\int \frac{-ce^2x^4-2cdex^2+2cd^2-ae^2}{(ex^2+d)\sqrt{a-cx^4}}dx}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 2235$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-\frac{\int \frac{ce^2\left(ex^2+d\right)}{\sqrt{a-cx^4}}dx}{e^2}}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\int \frac{ex^2+d}{\sqrt{a-cx^4}}dx}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1513$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\left(\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\left(\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}dx+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 765$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\left(\frac{\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 762$$

3.161. $\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)}{2d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1543}$$

3.161. $\int \frac{1}{\left(d+ex^2\right)^2\sqrt{a-cx^4}}\,dx$

$$\frac{\frac{\sqrt{1-\frac{cx^4}{a}}(3cd^2-ae^2)\int \frac{1}{(ex^2+d)\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}} - c\left(\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{\sqrt[4]{c}\sqrt{a-cx^4}}\right.}{2d(cd^2-ae^2)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{2d(d+ex^2)(cd^2-ae^2)}$$

$\Big\downarrow$ 1542

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(3cd^2-ae^2)\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}} - c\left(\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{El}}{\sqrt[4]{c}}\right.}{2d(cd^2-ae^2)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{2d(d+ex^2)(cd^2-ae^2)}$$

input
```
Int[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

output
```
-1/2*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)) + (-(c*((a^(3
/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(
3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(d - (Sqrt[a]*e)/Sqrt[c])*Sqrt[1 - (c*x^4
)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4])
)) + (a^(1/4)*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*
e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x
^4]))/(2*d*(c*d^2 - a*e^2))
```

3.161.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.161. $\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx$

rule 1552
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sq
rt[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
 + c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && ILtQ[q, -1]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.161.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(245) = 490$.

Time = 0.77 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.75

method	result
default	$\frac{e^2 x\sqrt{-c x^4+a}}{2(a e^2-c d^2)d(e x^2+d)}+\frac{c\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2(a e^2-c d^2)\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}-\frac{\sqrt{c}\,e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2(a e^2-c d^2)d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}+\frac{\sqrt{c}\,e\sqrt{a}\,\sqrt{1}}{2(a\,e}$
elliptic	$\frac{e^2 x\sqrt{-c x^4+a}}{2(a e^2-c d^2)d(e x^2+d)}+\frac{c\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2(a e^2-c d^2)\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}-\frac{\sqrt{c}\,e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2(a e^2-c d^2)d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}+\frac{\sqrt{c}\,e\sqrt{a}\,\sqrt{1}}{2(a\,e}$

input
```
int(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/2*c/(a*e^2-c*d^2)/(
1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-1
/2*c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elli
pticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2*c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)
/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
+1/2/(a*e^2-c*d^2)/d^2*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(
1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(
1/a^(1/2)*c^(1/2))^(1/2))*a-3/2/(a*e^2-c*d^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1
-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(
1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/
2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c
```

3.161.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.161.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**2), x)
```

3.161.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

3.161.8 Giac [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

3.162 $\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx$

3.162.1 Optimal result

Integrand size = 22, antiderivative size = 425

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{4d(cd^2-ae^2)(d+ex^2)^2}-\frac{3e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{8d^2(cd^2-ae^2)^2(d+ex^2)} - \frac{3a^{3/4}\sqrt[4]{c}e(3cd^2-ae^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2(cd^2-ae^2)^2\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\sqrt[4]{c}(7cd^2-2\sqrt{a}\sqrt{c}de-3ae^2)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8d^2(\sqrt{c}d+\sqrt{a}e)(cd^2-ae^2)\sqrt{a-cx^4}} + \frac{3\sqrt[4]{a}(5c^2d^4-2acd^2e^2+a^2e^4)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{c}d^3(cd^2-ae^2)^2\sqrt{a-cx^4}}$$

output
```
-1/4*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^2-3/8*e^2*(-a*e^2+3
*c*d^2)*x*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)-3/8*a^(3/4)*c^(1
/4)*e*(-a*e^2+3*c*d^2)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d^
2/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)+3/8*a^(1/4)*(a^2*e^4-2*a*c*d^2*e^2+5*c
^2*d^4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(
1/2)/c^(1/4)/d^3/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)-1/8*a^(1/4)*c^(1/4)*Ell
ipticF(c^(1/4)*x/a^(1/4),I)*(7*c*d^2-3*a*e^2-2*d*e*a^(1/2)*c^(1/2))*(1-c*x
^4/a)^(1/2)/d^2/(-a*e^2+c*d^2)/(e*a^(1/2)+d*c^(1/2))/(-c*x^4+a)^(1/2)
```

3.162.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.98 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.76

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx$$
$$= \frac{\frac{de^2x(a-cx^4)(ae^2(5d+3ex^2)-cd^2(11d+9ex^2))}{(d+ex^2)^2} - \frac{i\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}de(-3cd^2+ae^2)E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+(-7c^2d^4+9\sqrt{a}c^{3/2}d^3e\cdots}{\cdots}}{8d^3(cd^2-ae^2\cdots}$$

input
```
Integrate[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

output
```
((d*e^2*x*(a - c*x^4)*(a*e^2*(5*d + 3*e*x^2) - c*d^2*(11*d + 9*e*x^2)))/(d
 + e*x^2)^2 - (I*Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*Sqrt[c]*d*e*(-3*c*d^2 + a*
e^2)*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (-7*c^2*d^4 +
9*Sqrt[a]*c^(3/2)*d^3*e + a*c*d^2*e^2 - 3*a^(3/2)*Sqrt[c]*d*e^3)*EllipticF
[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + 3*(5*c^2*d^4 - 2*a*c*d^2*e^2
 + a^2*e^4)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c
]/Sqrt[a])]*x], -1]))/Sqrt[-(Sqrt[c]/Sqrt[a])])/(8*d^3*(c*d^2 - a*e^2)^2*S
qrt[a - c*x^4])
```

3.162.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {1552, 2211, 2235, 27, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^3}\,dx$$
$$\Big\downarrow \; 1552$$
$$\frac{\int \frac{ce^2x^4-4cdex^2+4cd^2-3ae^2}{(ex^2+d)^2\sqrt{a-cx^4}}dx}{4d(cd^2-ae^2)} - \frac{e^2x\sqrt{a-cx^4}}{4d(d+ex^2)^2(cd^2-ae^2)}$$
$$\Big\downarrow \; 2211$$

$$\frac{\frac{\int \frac{8c^2d^4-5ace^2d^2-4ce\left(4cd^2-ae^2\right)x^2d+3a^2e^4-3ce^2\left(3cd^2-ae^2\right)x^4}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{2d(cd^2-ae^2)}-\frac{3e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{2d(d+ex^2)(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 2235

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-\frac{\int \frac{ce^2\left(3e\left(3cd^2-ae^2\right)x^2+d\left(7cd^2-ae^2\right)\right)}{\sqrt{a-cx^4}}dx}{e^2}}{2d(cd^2-ae^2)}-\frac{3e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{2d(d+ex^2)(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\int \frac{3e\left(3cd^2-ae^2\right)x^2+d\left(7cd^2-ae^2\right)}{\sqrt{a-cx^4}}dx}{2d(cd^2-ae^2)}-\frac{3e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{2d(d+ex^2)(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 1513

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{(\sqrt{c}d-\sqrt{a}e)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\int \frac{1}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{3\sqrt{a}e\left(3cd^2-ae^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d(cd^2-ae^2)}-\frac{3e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{2d(d+ex^2)(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{(\sqrt{c}d-\sqrt{a}e)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\int \frac{1}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{3e\left(3cd^2-ae^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d(cd^2-ae^2)}-\frac{3e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{2d(d+ex^2)(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}-\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 765

3.162. $\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx$

$$\frac{\frac{3(a^2e^4-2acd^2e^2+5c^2d^4)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\left(\frac{\sqrt{1-\frac{cx^4}{a}}(\sqrt{c}d-\sqrt{a}e)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{3e\left(3cd^2-ae^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{2d(cd^2-ae^2)}-\cdots}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 762

$$\frac{\frac{3(a^2e^4-2acd^2e^2+5c^2d^4)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\left(\frac{3e\left(3cd^2-ae^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(\sqrt{c}d-\sqrt{a}e)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{c^{3/4}\sqrt{a-cx^4}}\cdots\right.}{2d(cd^2-ae^2)}\cdots}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 1390

$$\frac{\frac{3(a^2e^4-2acd^2e^2+5c^2d^4)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\left(\frac{3e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(\sqrt{c}d-\sqrt{a}e)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\text{EllipticF}\cdots}{c^{3/4}\sqrt{a-cx^4}}\cdots\right.}{2d(cd^2-ae^2)}\cdots}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 1389

$$\frac{\frac{3(a^2e^4-2acd^2e^2+5c^2d^4)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx-c\left(\frac{3\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(\sqrt{c}d-\sqrt{a}e)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+7cd^2\right)\text{Elli}\cdots}{c^{3/4}\sqrt{a-cx^4}}\cdots\right.}{2d(cd^2-ae^2)}\cdots}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 327

3.162. $\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx$

$$\frac{\frac{3\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\left(-2\sqrt{a}\sqrt{c}de-3ae^2+\right.}{c^{3/4}\sqrt{a-c}}\right.}{2d(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 1543

$$\frac{\frac{\frac{3\sqrt{1-\frac{cx^4}{a}}\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-c\left(\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\left(-2\sqrt{a}\sqrt{c}de\right.}{c^3}\right.}{2d(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 1542

$$\frac{\frac{\frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}-c\left(\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\right.}{}\right.}{2d(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

output
```
-1/4*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)^2) + ((-3*e^2*
(3*c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(2*d*(c*d^2 - a*e^2)*(d + e*x^2)) + (
-(c*((3*a^(3/4)*e*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(
c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(Sqrt[c]*d
- Sqrt[a]*e)*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Sqrt[1 - (c*x^4)/
a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4])))
 + (3*a^(1/4)*(5*c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*Sqrt[1 - (c*x^4)/a]*El
lipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^
(1/4)*d*Sqrt[a - c*x^4]))/(2*d*(c*d^2 - a*e^2)))/(4*d*(c*d^2 - a*e^2))
```

3.162. $\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx$

3.162.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.162. $\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx$

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 1552
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sq
rt[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
 + c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && ILtQ[q, -1]
```

rule 2211
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))
Int[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(
2*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(
C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.162.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(363) = 726$.

Time = 1.82 (sec) , antiderivative size = 961, normalized size of antiderivative = 2.26

method	result
default	$\frac{e^2 x \sqrt{-c x^4+a}}{4(a e^2-c d^2) d (e x^2+d)^2}+\frac{3 e^2 (a e^2-3 c d^2) x \sqrt{-c x^4+a}}{8(a e^2-c d^2)^2 d^2 (e x^2+d)}+\frac{c \sqrt{1-\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\, F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right) a e^2}{8 d (a e^2-c d^2)^2 \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c x^4+a}}-\frac{7 c^2 d \sqrt{1-\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\, \sqrt{1+\cdots}}{8(a e^2-c d^2)^2 \sqrt{\cdots}}$
elliptic	$\frac{e^2 x \sqrt{-c x^4+a}}{4(a e^2-c d^2) d (e x^2+d)^2}+\frac{3 e^2 (a e^2-3 c d^2) x \sqrt{-c x^4+a}}{8(a e^2-c d^2)^2 d^2 (e x^2+d)}+\frac{c \sqrt{1-\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\, F\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right) a e^2}{8 d (a e^2-c d^2)^2 \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c x^4+a}}-\frac{7 c^2 d \sqrt{1-\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\, \sqrt{1+\cdots}}{8(a e^2-c d^2)^2 \sqrt{\cdots}}$

input
```
int(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+3/8*e^2*(a*e^2-3*c*
d^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/8*c/d/(a*e^2-c*d^2
)^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2)
,I)*a*e^2-7/8*c^2*d/(a*e^2-c*d^2)^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)
*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^
(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/
2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/
2),I)+9/8*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1
/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/
2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^
2/d^2*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1
/2))^(1/2),I)-9/8*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1
/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^
4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8/(a*e^2-c*d^2)^2/d^
3*e^4/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/
2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1
/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))...
```

3.162. $\int \frac{1}{(d+ex^2)^3 \sqrt{a-cx^4}}\, dx$

3.162.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.162.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^3}\,dx$$

input
```
integrate(1/(e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**3), x)
```

3.162.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

3.162.8 Giac [F]

$$\int \frac{1}{(d+ex^2)^3 \sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^3 \sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^3}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^3),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

3.163 $\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx$

3.163.1 Optimal result

Integrand size = 22, antiderivative size = 563

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{6d(cd^2-ae^2)(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)x\sqrt{a-cx^4}}{24d^2(cd^2-ae^2)^2(d+ex^2)^2} - \frac{e^2(29c^2d^4-14acd^2e^2+5a^2e^4)x\sqrt{a-cx^4}}{16d^3(cd^2-ae^2)^3(d+ex^2)} - \frac{a^{3/4}\sqrt[4]{c}e(29c^2d^4-14acd^2e^2+5a^2e^4)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{16d^3(cd^2-ae^2)^3\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\sqrt[4]{c}\left(57c^2d^4-30\sqrt{a}c^{3/2}d^3e-32acd^2e^2+10a^{3/2}\sqrt{c}de^3+15a^2e^4\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{48d^3\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(\sqrt{c}d+\sqrt{a}e\right)^3\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}(35c^3d^6-7ac^2d^4e^2+17a^2cd^2e^4-5a^3e^6)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{16\sqrt[4]{c}d^4(cd^2-ae^2)^3\sqrt{a-cx^4}}$$

output
```
-1/6*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^3-5/24*e^2*(-a*e^2+
3*c*d^2)*x*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)^2-1/16*e^2*(5*a
^2*e^4-14*a*c*d^2*e^2+29*c^2*d^4)*x*(-c*x^4+a)^(1/2)/d^3/(-a*e^2+c*d^2)^3/
(e*x^2+d)-1/16*a^(3/4)*c^(1/4)*e*(5*a^2*e^4-14*a*c*d^2*e^2+29*c^2*d^4)*Ell
ipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d^3/(-a*e^2+c*d^2)^3/(-c*x^4
+a)^(1/2)+1/16*a^(1/4)*(-5*a^3*e^6+17*a^2*c*d^2*e^4-7*a*c^2*d^4*e^2+35*c^3
*d^6)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1/
2)/c^(1/4)/d^4/(-a*e^2+c*d^2)^3/(-c*x^4+a)^(1/2)-1/48*a^(1/4)*c^(1/4)*Elli
pticF(c^(1/4)*x/a^(1/4),I)*(57*c^2*d^4-32*a*c*d^2*e^2+15*a^2*e^4-30*c^(3/2
)*d^3*e*a^(1/2)+10*a^(3/2)*d*e^3*c^(1/2))*(1-c*x^4/a)^(1/2)/d^3/(-e*a^(1/2
)+d*c^(1/2))^2/(e*a^(1/2)+d*c^(1/2))^3/(-c*x^4+a)^(1/2)
```

3.163.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.38 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.81

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \frac{-\frac{de^2x(a-cx^4)\left(8(cd^3-ade^2)^2+10d(cd^2-ae^2)(3cd^2-ae^2)(d+ex^2)+3(29c^2d^4-14acd^2e^2+5a^2e^4)(d+ex^2)^2\right)}{(cd^2-ae^2)^3(d+ex^2)^3} - \frac{i\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}de(29c^2d\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```

output
```
(-((d*e^2*x*(a - c*x^4)*(8*(c*d^3 - a*d*e^2)^2 + 10*d*(c*d^2 - a*e^2)*(3*c
*d^2 - a*e^2)*(d + e*x^2) + 3*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*(d
 + e*x^2)^2))/((c*d^2 - a*e^2)^3*(d + e*x^2)^3)) - (I*Sqrt[1 - (c*x^4)/a]*
(3*Sqrt[a]*Sqrt[c]*d*e*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*EllipticE
[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*d*(57*c^(5/2)*d^5 -
87*Sqrt[a]*c^2*d^4*e - 2*a*c^(3/2)*d^3*e^2 + 42*a^(3/2)*c*d^2*e^3 + 5*a^2*
Sqrt[c]*d*e^4 - 15*a^(5/2)*e^5)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a]
)]*x], -1] + 3*(-35*c^3*d^6 + 7*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 + 5*a^3*e
^6)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a
])]*x], -1]))/(Sqrt[-(Sqrt[c]/Sqrt[a])]*(-(c*d^2) + a*e^2)^3))/(48*d^4*Sqr
t[a - c*x^4])
```

3.163.3 Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {1552, 2211, 2211, 2235, 27, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^4}\,dx$$

$$\Big\downarrow \text{1552}$$

$$\frac{\int \frac{3ce^2x^4-6cdex^2+6cd^2-5ae^2}{(ex^2+d)^3\sqrt{a-cx^4}}dx}{6d\,(cd^2-ae^2)}-\frac{e^2x\sqrt{a-cx^4}}{6d\,(d+ex^2)^3\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{2211}$$

$$\frac{\frac{\int \frac{24c^2d^4-29ace^2d^2-8ce\left(6cd^2-ae^2\right)x^2d+15a^2e^4+5ce^2\left(3cd^2-ae^2\right)x^4}{\left(ex^2+d\right)^2\sqrt{a-cx^4}}dx}{4d(cd^2-ae^2)}-\frac{5e^2x\sqrt{a-cx^4}(3cd^2-ae^2)}{4d(d+ex^2)^2(cd^2-ae^2)}}{6d\,(cd^2-ae^2)}-\frac{e^2x\sqrt{a-cx^4}}{6d\,(d+ex^2)^3\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{2211}$$

$$\frac{\frac{\frac{\int \frac{48c^3d^6-19ac^2e^2d^4+46a^2ce^4d^2-4ce\left(36c^2d^4-11ace^2d^2+5a^2e^4\right)x^2d-15a^3e^6-3ce^2\left(29c^2d^4-14ace^2d^2+5a^2e^4\right)x^4}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{2d\left(cd^2-ae^2\right)}-\frac{3e^2x\sqrt{a-cx^4}\left(5a^2e^4-14acd^2e^2+29c^2\cdots\right.}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}\cdots}{4d(cd^2-ae^2)}\cdots}{6d\,(cd^2-ae^2)}-\frac{e^2x\sqrt{a-cx^4}}{6d\,(d+ex^2)^3\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{2235}$$

$$\frac{\frac{\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-\frac{\int \frac{ce^2\left(3e\left(29c^2d^4-14ace^2d^2+5a^2e^4\right)x^2+d\left(57c^2d^4-2ace^2d^2+5a^2e^4\right)\right)}{\sqrt{a-cx^4}}dx}{e^2}}{2d\left(cd^2-ae^2\right)}-\frac{3e^2x\sqrt{a-cx^4}\left(\cdots\right.}{2d\left(d\cdots\right.}\cdots}{4d(cd^2-ae^2)}\cdots}{6d\,(cd^2-ae^2)}-\frac{e^2x\sqrt{a-cx^4}}{6d\,(d+ex^2)^3\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\int\frac{3e\left(29c^2d^4-14ace^2d^2+5a^2e^4\right)x^2+d\left(57c^2d^4-2ace^2d^2+5a^2e^4\right)}{\sqrt{a-cx^4}}dx}{2d\left(cd^2-ae^2\right)}-\frac{3e^2x\sqrt{a-cx^4}\left(5a^2e\ldots\right.}{2d\left(d+ex^2\ldots\right.}}{4d(cd^2-ae^2)}}{6d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$\downarrow$ 1513

$$\frac{\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}+\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2d^4\ldots\right.}{\sqrt{c}}\ldots\right.\right.}{2d\left(cd^2-ae^2\right)}}{4d(cd^2-ae^2)}}{6d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{\sqrt{c}}+5\ldots\right.\right.}{2d\left(cd^2-ae^2\right)}}{4d(cd^2-ae^2)}}{6d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$\downarrow$ 765

$$\frac{\frac{3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29\ldots\right.}{\sqrt{c}}\ldots\right.}{\ldots}\right.}{2d\left(cd^2-ae^2\right)}}{4d(cd^2-ae^2)}}{6d\left(cd^2-ae^2\right)}$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$\downarrow$ 762

3.163. $\int\frac{1}{\left(d+ex^2\right)^4\sqrt{a-cx^4}}\,dx$

$$3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e\right.}{\sqrt{c}}\right.}{}\right.$$

$$2d\left(cd^2-ae^2\right)$$

$$4d(cd^2-ae^2)$$

$$6d\left(cd^2-\right.$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 1390

$$3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3e\sqrt{1-\frac{cx^4}{a}}\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2e^4\right.}{}\right.}{}\right.$$

$$2d\left(cd^2-ae^2\right)$$

$$4d(cd^2-ae^2)$$

$$6d\left(cd^2\right.$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 1389

$$3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{3\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a\right.}{}\right.}{}\right.$$

$$2d\left(cd^2-ae^2\right)$$

$$4d(cd^2-ae^2)$$

$$6d\left(c\right.$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 327

$$3\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx-c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{\sqrt{c}}+5a^2de^4-2acd^3e^2+57c^2d^5\right)\text{Elliptic}}{\sqrt[4]{c}\sqrt{a-cx^4}}\right.$$

$$2d\left(cd^2-ae^2\right)$$

$$4d(cd^2-ae^2)$$

$$6$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

3.163. $\int \frac{1}{\left(d+ex^2\right)^4\sqrt{a-cx^4}}\,dx$

$$\Big\downarrow \text{1543}$$

$$\frac{\frac{\frac{3\sqrt{1-\frac{cx^4}{a}}\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{\sqrt{c}}+5a^2de^4-2acd^3e^2+57c^2d\ldots\right.}{\sqrt[4]{c}\sqrt{a-cx^4}}\ldots\right.}{2d\left(cd^2-ae^2\right)}\ldots}{4d(cd^2-\ldots}\ldots$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\frac{\frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}-c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-\frac{3\sqrt{a}e\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{\sqrt{c}}+5a\ldots\right.}{\sqrt[4]{c}\sqrt{\ldots}}\ldots\right.}{2d\left(cd^2-ae^2\right)}\ldots}{\ldots}\ldots$$

$$\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

input
```
Int[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```

output
```
-1/6*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)^3) + ((-5*e^2*
(3*c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(4*d*(c*d^2 - a*e^2)*(d + e*x^2)^2) +
 ((-3*e^2*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[a - c*x^4])/(2*
d*(c*d^2 - a*e^2)*(d + e*x^2)) + (-(c*((3*a^(3/4)*e*(29*c^2*d^4 - 14*a*c*d
^2*e^2 + 5*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/
4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(57*c^2*d^5 - 2*a*c*d^3*e^2
 + 5*a^2*d*e^4 - (3*Sqrt[a]*e*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4))/S
qrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c
^(1/4)*Sqrt[a - c*x^4]))) + (3*a^(1/4)*(35*c^3*d^6 - 7*a*c^2*d^4*e^2 + 17*
a^2*c*d^2*e^4 - 5*a^3*e^6)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(S
qrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x^4]))
/(2*d*(c*d^2 - a*e^2)))/(4*d*(c*d^2 - a*e^2)))/(6*d*(c*d^2 - a*e^2))
```

3.163. $\int\frac{1}{\left(d+ex^2\right)^4\sqrt{a-cx^4}}\,dx$

3.163.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.163. $\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx$

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 1552
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sq
rt[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
 + c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && ILtQ[q, -1]
```

rule 2211
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))
Int[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(
2*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(
C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.163.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1419 vs. 2(489) = 978.

Time = 2.47 (sec) , antiderivative size = 1420, normalized size of antiderivative = 2.52

method	result	size
default	Expression too large to display	1420
elliptic	Expression too large to display	1420

input
```
int(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^3+5/24*e^2*(a*e^2-3*c
*d^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+1/16*e^2*(5*a^2*e
^4-14*a*c*d^2*e^2+29*c^2*d^4)/(a*e^2-c*d^2)^3/d^3*x*(-c*x^4+a)^(1/2)/(e*x^
2+d)-35/16/(a*e^2-c*d^2)^3*d^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi
(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/
2)/(1/a^(1/2)*c^(1/2))^(1/2))*c^3+19/16*c^3*d^2/(a*e^2-c*d^2)^3/(1/a^(1/2)
*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^
(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+5/16/(a*e^
2-c*d^2)^3/d^4*e^6/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/
2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2
)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2
)*c^(1/2))^(1/2))*a^3+7/16/(a*e^2-c*d^2)^3*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(
1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^
(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1
/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a*c^2+5/48*c/d^2/(a*e^2-c*d^
2)^3/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2
),I)*a^2*e^4-1/24*c^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*...
```

3.163.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.163.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)^4}\,dx$$

input
```
integrate(1/(e*x**2+d)**4/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**4), x)
```

3.163.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^4}\,dx$$

input
```
integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)
```

3.163.8 Giac [F]

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+a}(ex^2+d)^4}\,dx$$

input
```
integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + a)*(e*x^2 + d)^4), x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = \int \frac{1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^4}\,dx$$

input
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^4),x)
```

output
```
int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^4), x)
```

3.164 $\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx$

3.164.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx=\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{-a+cx^4}}+\frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{-a+cx^4}}$$

output
```
a^(3/4)*e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(3/4)/(c*x^4-
a)^(1/2)+a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-e+d*c^(1/2)/a^(1/2))*(1-
c*x^4/a)^(1/2)/c^(3/4)/(c*x^4-a)^(1/2)
```

3.164.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx=\frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

output
```
(Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] +
e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4])
```

3.164.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{cx^4-a}}dx$$

$$\Big\downarrow \text{1513}$$

$$\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4-a}}dx+\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4-a}}dx+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{cx^4-a}}+\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{762}$$

$$\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{e\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{cx^4-a}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{a}e\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{cx^4-a}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

$$\Big\downarrow \; 327$$

$$\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

input
```
Int[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

output
```
(a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(c^(3/4)*Sqrt[-a + c*x^4]) + (a^(1/4)*(d - (Sqrt[a]*e)/Sqrt[c])*Sqrt[1 -
(c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[-a +
c*x^4])
```

3.164.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

3.164.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{d\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}+\frac{e\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}\,\sqrt{c}}$	160
elliptic	$\frac{d\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}+\frac{e\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}\,\sqrt{c}}$	160

input
```
int((e*x^2+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),
I)+e*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-1/a^(
1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = \frac{a\sqrt{c}ex\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-(cd+ae)\sqrt{c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\sqrt{cx^4-a}ae}{acx}$$

input
```
integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
(a*sqrt(c)*e*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - (c*d +
a*e)*sqrt(c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + sqrt(c*
x^4 - a)*a*e)/(a*c*x)
```

3.164.6 Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = -\frac{idx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}-\frac{iex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(c*x**4-a)**(1/2),x)
```

output
```
-I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a)*gamma(5/4
)) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*ga
mma(7/4))
```

3.164.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

3.164.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 - a), x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{c\,x^4-a}}\,dx$$

input
```
int((d + e*x^2)/(c*x^4 - a)^(1/2),x)
```

output
```
int((d + e*x^2)/(c*x^4 - a)^(1/2), x)
```

3.165 $\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx$

3.165.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx=\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{-a+cx^4}}$$

output
```
a^(1/4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(
1/2)/c^(1/4)/d/(c*x^4-a)^(1/2)
```

3.165.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx=-\frac{i\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d\sqrt{-a+cx^4}}$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

output
```
((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh
[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[-a + c
*x^4])
```

3.165.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx^4-a}\,(d+ex^2)}\,dx$$

$$\Big\downarrow \text{1543}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{(ex^2+d)\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{cx^4-a}}$$

input
```
Int[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

output
```
(a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin
[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[-a + c*x^4])
```

3.165.3.1 Defintions of rubi rules used

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.165.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

method	result	size
default	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},\frac{e\sqrt{a}}{d\sqrt{c}},\frac{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}$	99
elliptic	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},\frac{e\sqrt{a}}{d\sqrt{c}},\frac{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}$	99

input
```
int(1/(e*x^2+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticPi(x*(-1/a^(1/2)*c^(1/2))^(1/
2),e*a^(1/2)/d/c^(1/2),(1/a^(1/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2
))
```

3.165.5 Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 - a)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

3.165.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{-a+cx^4}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(c*x**4-a)**(1/2),x)
```

output
```
Integral(1/(sqrt(-a + c*x**4)*(d + e*x**2)), x)
```

3.165.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

3.165.8 Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 - a)*(e*x^2 + d)), x)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4-a}\,(e\,x^2+d)}\,dx$$

input
```
int(1/((c*x^4 - a)^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((c*x^4 - a)^(1/2)*(d + e*x^2)), x)
```

3.166 $\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx$

3.166.1 Optimal result

Integrand size = 29, antiderivative size = 54

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{-a+cx^4}}$$

output
```
a^(3/4)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/(c*x^4-a)
^(1/2)
```

3.166.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+\sqrt{c}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}$$

input
```
Integrate[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

output
```
(Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)
/a] + Sqrt[c]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a
 + c*x^4])
```

3.166.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{cx^4-a}}dx$$

$$\Big\downarrow \text{1390}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{327}$$

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\sqrt{cx^4-a}}$$

input
```
Int[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

output
```
(a^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(
c^(1/4)*Sqrt[-a + c*x^4])
```

3.166.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

3.166.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(42) = 84$.

Time = 0.62 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.93

method	result
default	$\frac{\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}+\frac{\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}$
elliptic	$\frac{\left(\sqrt{a}+x^2\sqrt{c}\right)\sqrt{-(-c\,x^4+a)\,c}\,\sqrt{-(-c\,x^4+a)\,a}\left(\frac{\sqrt{c}\,\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c^2x^4-ac}}+\frac{a\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\cdots}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{ac\,x\cdots}}\right.}{\sqrt{c\,x^4-a}\left(c\,x^2\sqrt{-(-c\,x^4+a)\,a}+a\sqrt{-(-c\,x^4+a)\,c}\right)}$

input
```
int((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^
(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^
(1/2),I)+a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*(EllipticF(x*(-1/a^(1/2)
*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

3.166.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(41) = 82.

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx$$
$$= \frac{2\,acx\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-\sqrt{2}\left(\sqrt{2}acx\sqrt{\frac{a}{c}}+\sqrt{2}\sqrt{a}c^{\frac{3}{2}}x\sqrt{\frac{a}{c}}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+2\sqrt{}}{2\,acx}$$

input
```
integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*a*c*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - sqrt(2)*(
sqrt(2)*a*c*x*sqrt(a/c) + sqrt(2)*sqrt(a)*c^(3/2)*x*sqrt(a/c))*(a/c)^(1/4)
*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + 2*sqrt(c*x^4 - a)*a*sqrt(c))/(a*c
*x)
```

3.166.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = -\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{i\sqrt{c}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((a**(1/2)+x**2*c**(1/2))/(c*x**4-a)**(1/2),x)
```

output
```
-I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*gamma(5/4)) - I*sqr
t(c)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamma(
7/4))
```

3.166.7 Maxima [F]

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

3.166.8 Giac [F]

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{c\,x^4-a}}\,dx$$

input
```
int((a^(1/2) + c^(1/2)*x^2)/(c*x^4 - a)^(1/2),x)
```

output
```
int((a^(1/2) + c^(1/2)*x^2)/(c*x^4 - a)^(1/2), x)
```

3.167 $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx$

3.167.1 Optimal result

Integrand size = 29, antiderivative size = 52

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx=\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{-a+cx^4}}$$

output
```
EllipticE((c/a)^(1/4)*x,I)*(1-c*x^4/a)^(1/2)/(c/a)^(1/4)/(c*x^4-a)^(1/2)
```

3.167.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx$$
$$=\frac{\sqrt{1-\frac{cx^4}{a}}\left(3x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+\sqrt{\frac{c}{a}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}$$

input
```
Integrate[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

output
```
(Sqrt[1 - (c*x^4)/a]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + Sq
rt[c/a]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x
^4])
```

3.167.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1390, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}dx$$

$$\Big\downarrow \text{1390}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{c}{a}}x^2+1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{1388}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\sqrt{\frac{c}{a}}x^2+1}}{\sqrt{1-\frac{cx^2}{a\sqrt{\frac{c}{a}}}}}dx}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\sqrt[4]{\frac{c}{a}}x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\sqrt{cx^4-a}}$$

input
```
Int[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

output
```
(Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c/a)^(1/4)*x], -1])/((c/a)^(1/4)*Sq
rt[-a + c*x^4])
```

3.167.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

3.167.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(44) = 88$.

Time = 1.85 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.17

method	result
default	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}+\frac{\sqrt{\frac{c}{a}}\,\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}\,\sqrt{c}}$
elliptic	$\frac{\left(1+x^{2}\sqrt{\frac{c}{a}}\right)a\sqrt{-\frac{\left(-c\,x^{4}+a\right)c}{a}}\left(\frac{\sqrt{c}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{\frac{c^{2}x^{4}}{a}-c}}+\frac{\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}\right)}{c\,x^{2}\sqrt{c\,x^{4}-a}+a\sqrt{-\frac{\left(-c\,x^{4}+a\right)c}{a}}}$

input
```
int((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),
I)+(c/a)^(1/2)*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2
)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF
(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)
)
```

3.167.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.52

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = \frac{2\,a\sqrt{c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}\sqrt{\frac{c}{a}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1) - \sqrt{2}\left(\sqrt{2}ax\sqrt{\frac{a}{c}}\sqrt{\frac{c}{a}} + \sqrt{2}cx\sqrt{\frac{a}{c}}\right)\sqrt{c}\left(\frac{a}{c}\right)^{\frac{1}{4}}F(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)}{2\,acx}$$

input
```
integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*a*sqrt(c)*x*(a/c)^(3/4)*sqrt(c/a)*elliptic_e(arcsin((a/c)^(1/4)/x),
 -1) - sqrt(2)*(sqrt(2)*a*x*sqrt(a/c)*sqrt(c/a) + sqrt(2)*c*x*sqrt(a/c))*s
qrt(c)*(a/c)^(1/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + 2*sqrt(c*x^4 -
a)*a*sqrt(c/a))/(a*c*x)
```

3.167.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(37) = 74$.

Time = 0.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = -\frac{ix^3\sqrt{\frac{c}{a}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} - \frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((1+x**2*(c/a)**(1/2))/(c*x**4-a)**(1/2),x)
```

output
```
-I*x**3*sqrt(c/a)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a
)*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt
(a)*gamma(5/4))
```

3.167.7 Maxima [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

3.167.8 Giac [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4-a}}\,dx$$

input
```
integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{\sqrt{-a+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{c\,x^4-a}}\,dx$$

input
```
int((x^2*(c/a)^(1/2) + 1)/(c*x^4 - a)^(1/2),x)
```

output
```
int((x^2*(c/a)^(1/2) + 1)/(c*x^4 - a)^(1/2), x)
```

3.168 $\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx$

3.168.1 Optimal result

Integrand size = 22, antiderivative size = 236

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx$$
$$= -\frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}}$$
$$+ \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}}$$

output
```
-e*x*(-c*x^4-a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*arcta
n(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^
4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-c*x^4-a)^(1/2)+1/2*a^(1/4)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))
*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1
/2))*(e+d*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/
4)/(-c*x^4-a)^(1/2)
```

3.168.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.34

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = \frac{\sqrt{1+\frac{cx^4}{a}}\left(3dx\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)\right)}{3\sqrt{-a-cx^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```

output
```
(Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
 + e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[-a - c*x
^4])
```

3.168.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx$$

$$\Big\downarrow \text{1512}$$

$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{-cx^4-a}}dx-\frac{\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{-cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{-cx^4-a}}dx-\frac{e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{-cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{-a-cx^4}}-\frac{e\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{-cx^4-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{-a-cx^4}}-\frac{e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{-a-cx^4}}+\frac{x\sqrt{-a-cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

input
```
Int[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```

output
```
-((e*((x*Sqrt[-a - c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) + (a^(1/4)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[-a - c*x^4])))/Sqrt[c]) + ((d +
 (Sqrt[a]*e)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*
c^(1/4)*Sqrt[-a - c*x^4])
```

3.168.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

3.168.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{d\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}}-\frac{ie\sqrt{a}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}\,\sqrt{c}}$	175
elliptic	$\frac{d\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}}-\frac{ie\sqrt{a}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}\,\sqrt{c}}$	175

input
```
int((e*x^2+d)/(-c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*c^(1/2))^(1/2)
,I)-I*e*a^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)
*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-I
/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-I/a^(1/2)*c^(1/2))^(1/2),I))
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.41

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx =$$
$$-\frac{a\sqrt{-c}ex\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+(cd-ae)\sqrt{-c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+\sqrt{-cx^4-}}{acx}$$

input
```
integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
-(a*sqrt(-c)*e*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) + (c*
d - a*e)*sqrt(-c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) +
sqrt(-c*x^4 - a)*a*e)/(a*c*x)
```

3.168.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.35

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = -\frac{idx\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{iex^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

output
```
-I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(5/4)) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*
exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

3.168.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-cx^4-a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

3.168.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx=\int \frac{ex^2+d}{\sqrt{-cx^4-a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 - a), x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{-c\,x^4-a}}\,dx$$

input
```
int((d + e*x^2)/(- a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(- a - c*x^4)^(1/2), x)
```

3.169 $\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx$

3.169.1 Optimal result

Integrand size = 24, antiderivative size = 347

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \frac{\sqrt{e}\arctan\left(\frac{\sqrt{-cd^2-ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2-ae^2}} + \frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-a-cx^4}} - \frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}d\,(cd^2-ae^2)\sqrt{-a-cx^4}}$$

output
```
1/2*arctan(x*(-a*e^2-c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(-c*x^4-a)^(1/2))*e^(1/2
)/d^(1/2)/(-a*e^2-c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c
^(1/2))^2)^(1/2)/a^(1/4)/(-e*a^(1/2)+d*c^(1/2))/(-c*x^4-a)^(1/2)-1/4*a^(3/
4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2
))^2/d/e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a
^(1/2))^2*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^
2)/(-c*x^4-a)^(1/2)
```

3.169.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.28

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d}, i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right), -1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}d\sqrt{-a-cx^4}}$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

output
```
((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[-a
 - c*x^4])
```

3.169.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1541, 27, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-a-cx^4}\,(d+ex^2)}dx$$

$$\Big\downarrow \; 1541$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{-cx^4-a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{-cx^4-a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{-cx^4-a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{-cx^4-a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \; 761$$

$$\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{-a-cx^4}\,(\sqrt{c}d-\sqrt{a}e)}-\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{-cx^4-a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$\Big\downarrow$ 2223

$$\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{-a-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{e\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{a}e+\sqrt{c}d)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{-a-cx^4}}-\frac{(\sqrt{c}d-\sqrt{a}e)\text{arctanh}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}\right)}{\sqrt{c}d-\sqrt{a}e}$$

input
```
Int[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

output
```
(c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d -
Sqrt[a]*e)*Sqrt[-a - c*x^4]) - (e*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTanh[(
Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a - c*x^4])])/(Sqrt[d]*Sqrt[
e]*Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sq
rt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[-a - c*x^4])))/(Sqrt[c]*d - Sqrt[a]*e)
```

3.169.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

3.169.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.32

method	result	size
default	$\frac{\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}}$	110
elliptic	$\frac{\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}-a}}$	110

input
```
int(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticPi(x*(-I/a^(1/2)*c^(1/2))^(1
/2),-I*a^(1/2)/c^(1/2)*e/d,(I/a^(1/2)*c^(1/2))^(1/2)/(-I/a^(1/2)*c^(1/2))^
(1/2))
```

3.169.5 Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 - a)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)
```

3.169.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-a-cx^4}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

output
```
Integral(1/(sqrt(-a - c*x**4)*(d + e*x**2)), x)
```

3.169.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

3.169.8 Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 - a)*(e*x^2 + d)), x)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \int \frac{1}{\sqrt{-c\,x^4-a}\,(e\,x^2+d)}\,dx$$

input
```
int(1/((- a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((- a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

3.170 $\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}} dx$

3.170.1 Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}} dx = \frac{\text{EllipticPi}\left(-\frac{2b}{\sqrt{5}a}, \arcsin\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right), -1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

output
```
1/10*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b/a*5^(1/2),I)*5^(3/4)/a*2^(1/2
)
```

3.170.2 Mathematica [A] (verified)

Time = 10.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}} dx = \frac{\text{EllipticPi}\left(-\frac{2b}{\sqrt{5}a}, \arcsin\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right), -1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

output
```
EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5
^(1/4)*a)
```

3.170.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4-5x^4}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \text{1537}$$

$$\sqrt{5}\int \frac{1}{\sqrt{2\sqrt{5}-5x^2}\sqrt{5x^2+2\sqrt{5}}\,(bx^2+a)}dx$$

$$\Big\downarrow \text{412}$$

$$\frac{\text{EllipticPi}\left(-\frac{2b}{\sqrt{5}a},\arcsin\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

input
```
Int[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

output
```
EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5
^(1/4)*a)
```

3.170.3.1 Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

3.170.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(32) = 64.

Time = 1.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

method	result	size
default	$\frac{\sqrt{2}\,5^{\frac{3}{4}}\sqrt{1-\frac{x^{2}\sqrt{5}}{2}}\,\sqrt{1+\frac{x^{2}\sqrt{5}}{2}}\,\Pi\left(\frac{5^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b\sqrt{5}}{5a},\frac{\sqrt{-\frac{\sqrt{5}}{2}}\,\sqrt{2}\,5^{\frac{3}{4}}}{5}\right)}{5a\sqrt{-5x^{4}+4}}$	79
elliptic	$\frac{\sqrt{2}\,5^{\frac{3}{4}}\sqrt{1-\frac{x^{2}\sqrt{5}}{2}}\,\sqrt{1+\frac{x^{2}\sqrt{5}}{2}}\,\Pi\left(\frac{5^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b\sqrt{5}}{5a},\frac{\sqrt{-\frac{\sqrt{5}}{2}}\,\sqrt{2}\,5^{\frac{3}{4}}}{5}\right)}{5a\sqrt{-5x^{4}+4}}$	79

input
```
int(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/a*2^(1/2)*5^(3/4)*(1-1/2*x^2*5^(1/2))^(1/2)*(1+1/2*x^2*5^(1/2))^(1/2)/
(-5*x^4+4)^(1/2)*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b/a*5^(1/2),1/5*(-1
/2*5^(1/2))^(1/2)*2^(1/2)*5^(3/4))
```

3.170.5 Fricas [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 - 4*b*x^2 - 4*a), x)
```

3.170.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{4-5x^4}\,(a+bx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)/(-5*x**4+4)**(1/2),x)
```

output
```
Integral(1/(sqrt(4 - 5*x**4)*(a + b*x**2)), x)
```

3.170.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

3.170.8 Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{\sqrt{-5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{4-5\,x^4}}\,dx$$

input
```
int(1/((a + b*x^2)*(4 - 5*x^4)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(4 - 5*x^4)^(1/2)), x)
```

3.171 $\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx$

3.171.1 Optimal result

Integrand size = 21, antiderivative size = 310

$$\begin{aligned}
&\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx \\
&= \frac{\sqrt{b}\arctan\left(\frac{\sqrt{5a^2+4b^2}x}{\sqrt{a}\sqrt{b}\sqrt{4+5x^4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}} \\
&+ \frac{\sqrt[4]{5}(\sqrt{5}a+2b)\,(2+\sqrt{5}x^2)\sqrt{\frac{4+5x^4}{(2+\sqrt{5}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\,(5a^2-4b^2)\sqrt{4+5x^4}} \\
&- \frac{(\sqrt{5}a+2b)^2\,(2+\sqrt{5}x^2)\sqrt{\frac{4+5x^4}{(2+\sqrt{5}x^2)^2}}\operatorname{EllipticPi}\left(-\frac{(\sqrt{5}a-2b)^2}{8\sqrt{5}ab},2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt[4]{5}a\,(5a^2-4b^2)\sqrt{4+5x^4}}
\end{aligned}$$

output
```
1/2*arctan(x*(5*a^2+4*b^2)^(1/2)/a^(1/2)/b^(1/2)/(5*x^4+4)^(1/2))*b^(1/2)/
a^(1/2)/(5*a^2+4*b^2)^(1/2)+1/4*5^(1/4)*(cos(2*arctan(1/2*5^(1/4)*x*2^(1/2
)))^2)^(1/2)/cos(2*arctan(1/2*5^(1/4)*x*2^(1/2)))*EllipticF(sin(2*arctan(1
/2*5^(1/4)*x*2^(1/2))),1/2*2^(1/2))*(2*b+a*5^(1/2))*(2+x^2*5^(1/2))*((5*x^
4+4)/(2+x^2*5^(1/2))^2)^(1/2)/(5*a^2-4*b^2)*2^(1/2)/(5*x^4+4)^(1/2)-1/40*(
cos(2*arctan(1/2*5^(1/4)*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*5^(1/4)*x*2
^(1/2)))*EllipticPi(sin(2*arctan(1/2*5^(1/4)*x*2^(1/2))),-1/40*(-2*b+a*5^(
1/2))^2/a/b*5^(1/2),1/2*2^(1/2))*(2*b+a*5^(1/2))^2*(2+x^2*5^(1/2))*((5*x^4
+4)/(2+x^2*5^(1/2))^2)^(1/2)*5^(3/4)/a/(5*a^2-4*b^2)*2^(1/2)/(5*x^4+4)^(1/
2)
```

3.171.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.16

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\text{EllipticPi}\left(-\frac{2ib}{\sqrt{5}a},i\text{arcsinh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{5}x\right),-1\right)}{\sqrt[4]{5}a}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

output
```
((-1/2 - I/2)*EllipticPi[((-2*I)*b)/(Sqrt[5]*a), I*ArcSinh[(1/2 + I/2)*5^(
1/4)*x], -1])/(5^(1/4)*a)
```

3.171.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1541, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{5x^4+4}\,(a+bx^2)}dx$$

$$\Big\downarrow \ 1541$$

$$\frac{\left(5a+2\sqrt{5}b\right)\int \frac{1}{\sqrt{5x^4+4}}dx}{5a^2-4b^2}-\frac{2b\left(\sqrt{5}a+2b\right)\int \frac{\sqrt{5}x^2+2}{2(bx^2+a)\sqrt{5x^4+4}}dx}{5a^2-4b^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(5a+2\sqrt{5}b\right)\int \frac{1}{\sqrt{5x^4+4}}dx}{5a^2-4b^2}-\frac{b\left(\sqrt{5}a+2b\right)\int \frac{\sqrt{5}x^2+2}{(bx^2+a)\sqrt{5x^4+4}}dx}{5a^2-4b^2}$$

$$\Big\downarrow \ 761$$

$$\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(5a+2\sqrt{5}b\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt[4]{5}\sqrt{5x^4+4}\,(5a^2-4b^2)}-\frac{b\left(\sqrt{5}a+2b\right)\int \frac{\sqrt{5}x^2+2}{(bx^2+a)\sqrt{5x^4+4}}dx}{5a^2-4b^2}$$

$$\Big\downarrow \text{2221}$$

$$\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(5a+2\sqrt{5}b\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt[4]{5}\sqrt{5x^4+4}\left(5a^2-4b^2\right)}-\frac{b\left(\sqrt{5}a+2b\right)\left(\frac{\left(\sqrt{5}x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}x^2+2\right)^2}}\left(\sqrt{5}a+2b\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{5}a-2b\right)^2}{8\sqrt{5}ab},2\arctan\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}\sqrt[4]{5}ab\sqrt{5x^4+4}}-\frac{\left(\sqrt{5}a-2b\right)\arctan\left(\frac{x\sqrt{5a^2+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{5x^4+4}}\right)}{2\sqrt{a}\sqrt{b}\sqrt{5a^2+4b^2}}\right)}{5a^2-4b^2}$$

input
```
Int[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

output
```
((5*a + 2*Sqrt[5]*b)*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^
2]*EllipticF[2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2])/(2*Sqrt[2]*5^(1/4)*(5*a^
2 - 4*b^2)*Sqrt[4 + 5*x^4]) - (b*(Sqrt[5]*a + 2*b)*(-1/2*((Sqrt[5]*a - 2*b
)*ArcTan[(Sqrt[5*a^2 + 4*b^2]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + 5*x^4])])/(Sqrt
[a]*Sqrt[b]*Sqrt[5*a^2 + 4*b^2]) + ((Sqrt[5]*a + 2*b)*(2 + Sqrt[5]*x^2)*Sq
rt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticPi[-1/8*(Sqrt[5]*a - 2*b)^2/(S
qrt[5]*a*b), 2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2])/(4*Sqrt[2]*5^(1/4)*a*b*S
qrt[4 + 5*x^4])))/(5*a^2 - 4*b^2)
```

3.171.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

3.171.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.28

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{5}\,x^{2}}{2}}\,\sqrt{1+\frac{i\sqrt{5}\,x^{2}}{2}}\,\Pi\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{5}}\,x}{2},\frac{2i\sqrt{5}\,b}{5a},\frac{\sqrt{-\frac{i\sqrt{5}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{5}}}\right)}{a\sqrt{i\sqrt{5}}\,\sqrt{5x^{4}+4}}$	86
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{5}\,x^{2}}{2}}\,\sqrt{1+\frac{i\sqrt{5}\,x^{2}}{2}}\,\Pi\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{5}}\,x}{2},\frac{2i\sqrt{5}\,b}{5a},\frac{\sqrt{-\frac{i\sqrt{5}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{5}}}\right)}{a\sqrt{i\sqrt{5}}\,\sqrt{5x^{4}+4}}$	86

input
```
int(1/(b*x^2+a)/(5*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/(1/2*I*5^(1/2))^(1/2)*(1-1/2*I*5^(1/2)*x^2)^(1/2)*(1+1/2*I*5^(1/2)*x^2
)^(1/2)/(5*x^4+4)^(1/2)*EllipticPi((1/2*I*5^(1/2))^(1/2)*x,2/5*I*5^(1/2)*b
/a,(-1/2*I*5^(1/2))^(1/2)/(1/2*I*5^(1/2))^(1/2))
```

3.171.5 Fricas [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx=\int \frac{1}{\sqrt{5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 + 4*b*x^2 + 4*a), x)
```

3.171.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{5x^4+4}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(5*x**4+4)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*sqrt(5*x**4 + 4)), x)
```

3.171.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{\sqrt{5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

3.171.8 Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{\sqrt{5\,x^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{5\,x^4+4}}\,dx$$

input
```
int(1/((a + b*x^2)*(5*x^4 + 4)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(5*x^4 + 4)^(1/2)), x)
```

3.172 $\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx$

3.172.1 Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \frac{\text{EllipticPi}\left(-\frac{2b}{a\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

output
```
1/2*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d^(1/2),I)/a/d^(1/4)*2^(1/2)
```

3.172.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = -\frac{i\,\text{EllipticPi}\left(-\frac{2b}{a\sqrt{d}},i\text{arcsinh}\left(\frac{\sqrt{-\sqrt{d}}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}a\sqrt{-\sqrt{d}}}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

output
```
((-I)*EllipticPi[(-2*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[-Sqrt[d]]*x)/Sqrt[2]]
, -1])/(Sqrt[2]*a*Sqrt[-Sqrt[d]])
```

3.172.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4-dx^4}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \text{ 1542}$$

$$\frac{\text{EllipticPi}\left(-\frac{2b}{a\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),-1\right)}{\sqrt{2}a\sqrt[4]{d}}$$

input
```
Int[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

output
```
EllipticPi[(-2*b)/(a*Sqrt[d]), ArcSin[(d^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*a
*d^(1/4))
```

3.172.3.1 Defintions of rubi rules used

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.172.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.

Time = 1.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{x^{2}\sqrt{d}}{2}}\,\sqrt{1+\frac{x^{2}\sqrt{d}}{2}}\,\Pi\left(\frac{d^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b}{a\sqrt{d}},\frac{\sqrt{-\frac{\sqrt{d}}{2}}\,\sqrt{2}}{d^{\frac{1}{4}}}\right)}{a\,d^{\frac{1}{4}}\sqrt{-d\,x^{4}+4}}$	78
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{x^{2}\sqrt{d}}{2}}\,\sqrt{1+\frac{x^{2}\sqrt{d}}{2}}\,\Pi\left(\frac{d^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b}{a\sqrt{d}},\frac{\sqrt{-\frac{\sqrt{d}}{2}}\,\sqrt{2}}{d^{\frac{1}{4}}}\right)}{a\,d^{\frac{1}{4}}\sqrt{-d\,x^{4}+4}}$	78

input
```
int(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*2^(1/2)/d^(1/4)*(1-1/2*x^2*d^(1/2))^(1/2)*(1+1/2*x^2*d^(1/2))^(1/2)/(-
d*x^4+4)^(1/2)*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d^(1/2),(-1/2*d^(1/
2))^(1/2)*2^(1/2)/d^(1/4))
```

3.172.5 Fricas [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \int \frac{1}{\sqrt{-dx^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-d*x^4 + 4)/(b*d*x^6 + a*d*x^4 - 4*b*x^2 - 4*a), x)
```

3.172.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{-dx^4+4}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(-d*x**4+4)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*sqrt(-d*x**4 + 4)), x)
```

3.172.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \int \frac{1}{\sqrt{-dx^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

3.172.8 Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \int \frac{1}{\sqrt{-dx^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{4-d\,x^4}}\,dx$$

input
```
int(1/((a + b*x^2)*(4 - d*x^4)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(4 - d*x^4)^(1/2)), x)
```

3.173 $\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx$

3.173.1 Optimal result

Integrand size = 21, antiderivative size = 300

$$\begin{aligned}\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx &= \frac{\sqrt{b}\arctan\left(\frac{\sqrt{4b^2+a^2d}\,x}{\sqrt{a}\sqrt{b}\sqrt{4+dx^4}}\right)}{2\sqrt{a}\sqrt{4b^2+a^2d}} \\ &\quad - \frac{\sqrt[4]{d}\left(2+\sqrt{d}x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\left(2b-a\sqrt{d}\right)\sqrt{4+dx^4}} \\ &\quad + \frac{\left(2b+a\sqrt{d}\right)\left(2+\sqrt{d}x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(2b-a\sqrt{d}\right)^2}{8ab\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}a\left(2b-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{4+dx^4}}\end{aligned}$$

output
```
1/2*arctan(x*(a^2*d+4*b^2)^(1/2)/a^(1/2)/b^(1/2)/(d*x^4+4)^(1/2))*b^(1/2)/
a^(1/2)/(a^2*d+4*b^2)^(1/2)-1/4*d^(1/4)*(cos(2*arctan(1/2*d^(1/4)*x*2^(1/2
)))^2)^(1/2)/cos(2*arctan(1/2*d^(1/4)*x*2^(1/2)))*EllipticF(sin(2*arctan(1
/2*d^(1/4)*x*2^(1/2))),1/2*2^(1/2))*(2+x^2*d^(1/2))*((d*x^4+4)/(2+x^2*d^(1
/2))^2)^(1/2)*2^(1/2)/(2*b-a*d^(1/2))/(d*x^4+4)^(1/2)+1/8*(cos(2*arctan(1/
2*d^(1/4)*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*d^(1/4)*x*2^(1/2)))*Ellipt
icPi(sin(2*arctan(1/2*d^(1/4)*x*2^(1/2))),-1/8*(2*b-a*d^(1/2))^2/a/b/d^(1/
2),1/2*2^(1/2))*(2*b+a*d^(1/2))*(2+x^2*d^(1/2))*((d*x^4+4)/(2+x^2*d^(1/2))
^2)^(1/2)/a/d^(1/4)*2^(1/2)/(2*b-a*d^(1/2))/(d*x^4+4)^(1/2)
```

3.173.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.22

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = -\frac{i\,\text{EllipticPi}\left(-\frac{2ib}{a\sqrt{d}}, i\text{arcsinh}\left(\frac{\sqrt{i\sqrt{d}}x}{\sqrt{2}}\right), -1\right)}{\sqrt{2}a\sqrt{i\sqrt{d}}}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```

output
```
((-I)*EllipticPi[((-2*I)*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[I*Sqrt[d]]*x)/Sqr
t[2]], -1])/(Sqrt[2]*a*Sqrt[I*Sqrt[d]])
```

3.173.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1541, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx^4+4}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \text{ 1541}$$

$$\frac{2b\int \frac{\sqrt{d}x^2+2}{2(bx^2+a)\sqrt{dx^4+4}}dx}{2b-a\sqrt{d}} - \frac{\sqrt{d}\int \frac{1}{\sqrt{dx^4+4}}dx}{2b-a\sqrt{d}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\int \frac{\sqrt{d}x^2+2}{(bx^2+a)\sqrt{dx^4+4}}dx}{2b-a\sqrt{d}} - \frac{\sqrt{d}\int \frac{1}{\sqrt{dx^4+4}}dx}{2b-a\sqrt{d}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{b\int \frac{\sqrt{d}x^2+2}{(bx^2+a)\sqrt{dx^4+4}}dx}{2b-a\sqrt{d}} - \frac{\sqrt[4]{d}\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right), \frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}$$

$$\Big\downarrow \text{ 2221}$$

$$\frac{b\left(\frac{\left(2b-a\sqrt{d}\right)\arctan\left(\frac{x\sqrt{a^2d+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{dx^4+4}}\right)}{2\sqrt{a}\sqrt{b}\sqrt{a^2d+4b^2}}+\frac{\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\left(a\sqrt{d}+2b\right)\text{EllipticPi}\left(-\frac{\left(2b-a\sqrt{d}\right)^2}{8ab\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{4\sqrt{2}ab\sqrt[4]{d}\sqrt{dx^4+4}}\right)}{2b-a\sqrt{d}}-\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt{2}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}$$

input
```
Int[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```

output
```
-1/2*(d^(1/4)*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(Sqrt[2]*(2*b - a*Sqrt[d])*Sqrt
[4 + d*x^4]) + (b*(((2*b - a*Sqrt[d])*ArcTan[(Sqrt[4*b^2 + a^2*d]*x)/(Sqrt
[a]*Sqrt[b]*Sqrt[4 + d*x^4])])/(2*Sqrt[a]*Sqrt[b]*Sqrt[4*b^2 + a^2*d]) + (
(2*b + a*Sqrt[d])*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*
EllipticPi[-1/8*(2*b - a*Sqrt[d])^2/(a*b*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/Sq
rt[2]], 1/2])/(4*Sqrt[2]*a*b*d^(1/4)*Sqrt[4 + d*x^4])))/(2*b - a*Sqrt[d])
```

3.173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

3.173.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{2}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{2}}\,\Pi\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{d}}\,x}{2},\frac{2ib}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{d}}}\right)}{a\sqrt{i\sqrt{d}}\,\sqrt{d\,x^4+4}}$	86
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{2}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{2}}\,\Pi\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{d}}\,x}{2},\frac{2ib}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{d}}}\right)}{a\sqrt{i\sqrt{d}}\,\sqrt{d\,x^4+4}}$	86

input
```
int(1/(b*x^2+a)/(d*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/(1/2*I*d^(1/2))^(1/2)*(1-1/2*I*d^(1/2)*x^2)^(1/2)*(1+1/2*I*d^(1/2)*x^2
)^(1/2)/(d*x^4+4)^(1/2)*EllipticPi((1/2*I*d^(1/2))^(1/2)*x,2*I/d^(1/2)*b/a
,(-1/2*I*d^(1/2))^(1/2)/(1/2*I*d^(1/2))^(1/2))
```

3.173.5 Fricas [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \int \frac{1}{\sqrt{dx^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + 4)/(b*d*x^6 + a*d*x^4 + 4*b*x^2 + 4*a), x)
```

3.173.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{dx^4+4}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**4+4)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*sqrt(d*x**4 + 4)), x)
```

3.173.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \int \frac{1}{\sqrt{dx^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

3.173.8 Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \int \frac{1}{\sqrt{dx^4+4}(bx^2+a)}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^4+4}}\,dx$$

input
```
int(1/((a + b*x^2)*(d*x^4 + 4)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)*(d*x^4 + 4)^(1/2)), x)
```

3.174 $\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$

3.174.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx=\frac{a\sqrt{1-x^2}\sqrt{\frac{a(1+x^2)}{a+bx^2}}\,\text{EllipticPi}\left(\frac{b}{a+b},\arcsin\left(\frac{\sqrt{a+b}x}{\sqrt{a+bx^2}}\right),-\frac{a-b}{a+b}\right)}{\sqrt{a+b}\sqrt{1+x^2}\sqrt{\frac{a(1-x^2)}{a+bx^2}}}$$

output
```
a*EllipticPi(x*(a+b)^(1/2)/(b*x^2+a)^(1/2),b/(a+b),((-a+b)/(a+b))^(1/2))*(
-x^2+1)^(1/2)*(a*(x^2+1)/(b*x^2+a))^(1/2)/(a+b)^(1/2)/(x^2+1)^(1/2)/(a*(-x
^2+1)/(b*x^2+a))^(1/2)
```

3.174.2 Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx=\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
```

output
```
Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

3.174.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

$$\Big\downarrow \text{1571}$$

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}dx$$

input
```
Int[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
```

output
```
$Aborted
```

3.174.3.1 Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

3.174.4 Maple [F]

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{-x^4+1}}dx$$

input
```
int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

3.174.5 Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 1)*sqrt(b*x^2 + a)/(x^4 - 1), x)
```

3.174.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(-x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.174.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

3.174.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{1-x^4}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(1 - x^4)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(1 - x^4)^(1/2), x)
```

3.175 $\int (c+ex^2)^q (a+bx^4)^p \, dx$

3.175.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int (c+ex^2)^q (a+bx^4)^p \, dx = \operatorname{Int}\left((c+ex^2)^q (a+bx^4)^p, x\right)$$

output
```
Unintegrable((e*x^2+c)^q*(b*x^4+a)^p,x)
```

3.175.2 Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (c+ex^2)^q (a+bx^4)^p \, dx = \int (c+ex^2)^q (a+bx^4)^p \, dx$$

input
```
Integrate[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

output
```
Integrate[(c + e*x^2)^q*(a + b*x^4)^p, x]
```

3.175.3 Rubi [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1571}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^p \left(c+ex^2\right)^q \, dx$$

$$\Big\downarrow \text{1571}$$

$$\int \left(a+bx^4\right)^p \left(c+ex^2\right)^q dx$$

input
```
Int[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

output
```
$Aborted
```

3.175.3.1 Defintions of rubi rules used

rule 1571
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> U
nintegrable[(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e, p, q}, x]
```

3.175.4 Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(e\,x^2+c\right)^q \left(b\,x^4+a\right)^p dx$$

input
```
int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

output
```
int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

3.175.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (c+ex^2)^q (a+bx^4)^p \, dx = \int (bx^4+a)^p (ex^2+c)^q \, dx$$

input
```
integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

3.175.6 Sympy [F(-1)]

Timed out.

$$\int (c+ex^2)^q (a+bx^4)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x**2+c)**q*(b*x**4+a)**p,x)
```

output
```
Timed out
```

3.175.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (c+ex^2)^q (a+bx^4)^p \, dx = \int (bx^4+a)^p (ex^2+c)^q \, dx$$

input
```
integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

3.175.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (c+ex^2)^q (a+bx^4)^p \, dx = \int (bx^4+a)^p (ex^2+c)^q \, dx$$

input
```
integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p*(e*x^2 + c)^q, x)
```

3.175.9 Mupad [N/A]

Not integrable

Time = 13.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (c+ex^2)^q (a+bx^4)^p \, dx = \int (b\,x^4+a)^p \,(e\,x^2+c)^q \, dx$$

input
```
int((a + b*x^4)^p*(c + e*x^2)^q,x)
```

output
```
int((a + b*x^4)^p*(c + e*x^2)^q, x)
```

3.176 $\int (c+ex^2)^3 (a+bx^4)^p \, dx$

3.176.1 Optimal result

Integrand size = 19, antiderivative size = 204

$$\int (c+ex^2)^3 (a+bx^4)^p \, dx = \frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)} + c^3x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) - \frac{e(ae^2-bc^2(7+4p))\, x^3(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)}{b(7+4p)} + \frac{3}{5}ce^2x^5(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)$$

output
```
e^3*x^3*(b*x^4+a)^(p+1)/b/(7+4*p)+c^3*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5
/4],-b*x^4/a)/((1+b*x^4/a)^p)-e*(a*e^2-b*c^2*(7+4*p))*x^3*(b*x^4+a)^p*hype
rgeom([3/4, -p],[7/4],-b*x^4/a)/b/(7+4*p)/((1+b*x^4/a)^p)+3/5*c*e^2*x^5*(b
*x^4+a)^p*hypergeom([5/4, -p],[9/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

3.176.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int (c+ex^2)^3 (a+bx^4)^p \, dx$$
$$= \frac{1}{35}x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(35c^3 \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) + ex^2\left(35c^2 \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right) + ex^2\left(21c\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right) + 5ex^2 \text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^4}{a}\right)\right)\right)\right)$$

input
```
Integrate[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*(35*c^3*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e
*x^2*(35*c^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + e*x^2*(21*c*H
ypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 5*e*x^2*Hypergeometric2F1[7
/4, -p, 11/4, -((b*x^4)/a)]))))/(35*(1 + (b*x^4)/a)^p)
```

3.176.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1519, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+ex^2)^3 (a+bx^4)^p \, dx$$
$$\Big\downarrow \; 1519$$
$$\frac{\int (bx^4+a)^p \left(3bce^2(4p+7)x^4-3e\left(ae^2-bc^2(4p+7)\right)x^2+bc^3(4p+7)\right)dx}{b(4p+7)}+\frac{e^3x^3(a+bx^4)^{p+1}}{b(4p+7)}$$
$$\Big\downarrow \; 2432$$
$$\frac{\int \left(3bce^2(4p+7)x^4(bx^4+a)^p+3e\left(bc^2(4p+7)-ae^2\right)x^2(bx^4+a)^p+bc^3(4p+7)(bx^4+a)^p\right)dx}{b(4p+7)}+\frac{e^3x^3(a+bx^4)^{p+1}}{b(4p+7)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc^3(4p+7)x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)-ex^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}(ae^2-bc}{}$$

$$\frac{e^3x^3(a+bx^4)^{p+1}}{b(4p+7)}$$

input
```
Int[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

output
```
(e^3*x^3*(a + b*x^4)^(1 + p))/(b*(7 + 4*p)) + ((b*c^3*(7 + 4*p)*x*(a + b*x
^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p - (
e*(a*e^2 - b*c^2*(7 + 4*p))*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7
/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (3*b*c*e^2*(7 + 4*p)*x^5*(a + b*x^4
)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p))/
(b*(7 + 4*p))
```

3.176.3.1 Defintions of rubi rules used

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.176.4 Maple [F]

$$\int \left(e\,x^{2}+c\right)^{3} \left(b\,x^{4}+a\right)^{p}dx$$

input
```
int((e*x^2+c)^3*(b*x^4+a)^p,x)
```

output
```
int((e*x^2+c)^3*(b*x^4+a)^p,x)
```

3.176.5 Fricas [F]

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^3 \left(bx^4+a\right)^p \, dx$$

input
```
integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + a)^p, x)
```

3.176.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 60.59 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.82

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = \frac{a^p c^3 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3a^p c^2 e x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3a^p c e^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{a^p e^3 x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{7}{4}, -p \\ \frac{11}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+c)**3*(b*x**4+a)**p,x)
```

output
```
a**p*c**3*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4)) + 3*a**p*c**2*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*
x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + 3*a**p*c*e**2*x**5*gamma(5/4)*hyp
er((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + a**p*e**3
*x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*ga
mma(11/4))
```

3.176.7 Maxima [F]

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^3 \left(bx^4+a\right)^p \, dx$$

input
```
integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)
```

3.176.8 Giac [F]

$$\int \left(c+ex^2\right)^3 \left(a+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^3 \left(bx^4+a\right)^p \, dx$$

input
```
integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^2 + c)^3*(b*x^4 + a)^p, x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int (c+ex^2)^3 (a+bx^4)^p \, dx = \int (b\,x^4+a)^p \,(e\,x^2+c)^3 \, dx$$

input
```
int((a + b*x^4)^p*(c + e*x^2)^3,x)
```

output
```
int((a + b*x^4)^p*(c + e*x^2)^3, x)
```

3.177 $\int (c+ex^2)^2 (a+bx^4)^p \, dx$

3.177.1 Optimal result

Integrand size = 19, antiderivative size = 150

$$\begin{aligned}\int (c+ex^2)^2 (a+bx^4)^p \, dx &= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} \\ &\quad - \frac{(ae^2-bc^2(5+4p))\, x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)}{b(5+4p)} \\ &\quad + \frac{2}{3}cex^3(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)\end{aligned}$$

output
```
e^2*x*(b*x^4+a)^(p+1)/b/(5+4*p)-(a*e^2-b*c^2*(5+4*p))*x*(b*x^4+a)^p*hyperg
eom([1/4, -p],[5/4],-b*x^4/a)/b/(5+4*p)/((1+b*x^4/a)^p)+2/3*c*e*x^3*(b*x^4
+a)^p*hypergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

3.177.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

$$\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx = \frac{1}{15}x\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(15c^2 \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+ex^2\left(10c\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)+3ex^2\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)\right)\right)$$

input
```
Integrate[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*(15*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e
*x^2*(10*c*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + 3*e*x^2*Hyperge
ometric2F1[5/4, -p, 9/4, -((b*x^4)/a)])))/(15*(1 + (b*x^4)/a)^p)
```

3.177.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1519, 25, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+ex^2\right)^2 \left(a+bx^4\right)^p \, dx$$

$$\Big\downarrow \text{1519}$$

$$\frac{\int -\left(\left(-b(4p+5)c^2-2be(4p+5)x^2c+ae^2\right)\left(bx^4+a\right)^p\right)dx}{b(4p+5)}+\frac{e^2x\left(a+bx^4\right)^{p+1}}{b(4p+5)}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^2x\left(a+bx^4\right)^{p+1}}{b(4p+5)}-\frac{\int \left(-b(4p+5)c^2-2be(4p+5)x^2c+ae^2\right)\left(bx^4+a\right)^p dx}{b(4p+5)}$$

$$\Big\downarrow \text{1516}$$

$$\frac{e^2x\left(a+bx^4\right)^{p+1}}{b(4p+5)}-\frac{\int\left(ae^2\left(1-\frac{bc^2(4p+5)}{ae^2}\right)\left(bx^4+a\right)^p-2bce(4p+5)x^2\left(bx^4+a\right)^p\right)dx}{b(4p+5)}$$

$\downarrow$ 2009

$$\frac{e^2x\left(a+bx^4\right)^{p+1}}{b(4p+5)}-$$
$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(ae^2-bc^2(4p+5)\right)\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)-\frac{2}{3}bce(4p+5)x^3\left(a+bx^4\right)^p\left(\right.}{b(4p+5)}$$

input
```
Int[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

output
```
(e^2*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) - (((a*e^2 - b*c^2*(5 + 4*p))*x*
(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/
a)^p - (2*b*c*e*(5 + 4*p)*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4
, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^p))/(b*(5 + 4*p))
```

3.177.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.177.4 Maple [F]

$$\int \left(e\,x^{2}+c\right)^{2} \left(b\,x^{4}+a\right)^{p}d x$$

input
```
int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

output
```
int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

3.177.5 Fricas [F]

$$\int (c+ex^2)^2 (a+bx^4)^p \, dx = \int (ex^2+c)^2 (bx^4+a)^p \, dx$$

input
```
integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + a)^p, x)
```

3.177.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 32.63 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.79

$$\int (c+ex^2)^2 (a+bx^4)^p \, dx = \frac{a^p c^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^p c e x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{a^p e^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((e*x**2+c)**2*(b*x**4+a)**p,x)
```

output
```
a**p*c**2*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4)) + a**p*c*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*
exp_polar(I*pi)/a)/(2*gamma(7/4)) + a**p*e**2*x**5*gamma(5/4)*hyper((5/4,
-p), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

3.177.7 Maxima [F]

$$\int (c+ex^2)^2 (a+bx^4)^p \, dx = \int (ex^2+c)^2 (bx^4+a)^p \, dx$$

input
```
integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + c)^2*(b*x^4 + a)^p, x)
```

3.177.8 Giac [F]

$$\int (c+ex^2)^2 (a+bx^4)^p \, dx = \int (ex^2+c)^2 (bx^4+a)^p \, dx$$

input
```
integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^2 + c)^2*(b*x^4 + a)^p, x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int (c+ex^2)^2 (a+bx^4)^p \, dx = \int (b\,x^4+a)^p \, (e\,x^2+c)^2 \, dx$$

input
```
int((a + b*x^4)^p*(c + e*x^2)^2,x)
```

output
```
int((a + b*x^4)^p*(c + e*x^2)^2, x)
```

3.178 $\int (c+ex^2)(a+bx^4)^p\,dx$

3.178.1 Optimal result

Integrand size = 17, antiderivative size = 96

$$\int (c+ex^2)(a+bx^4)^p\,dx = cx(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

output
```
c*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)+1/3*e*
x^3*(b*x^4+a)^p*hypergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

3.178.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int (c+ex^2)(a+bx^4)^p\,dx = \frac{1}{3}x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(3c\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+ex^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)\right)$$

input
```
Integrate[(c + e*x^2)*(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*(3*c*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*x^
2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)]))/(3*(1 + (b*x^4)/a)^p)
```

3.178.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+ex^2\right)\left(a+bx^4\right)^p\,dx$$

$$\Big\downarrow \text{ 1516}$$

$$\int \left(c\left(a+bx^4\right)^p+ex^2\left(a+bx^4\right)^p\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

input
```
Int[(c + e*x^2)*(a + b*x^4)^p,x]
```

output
```
(c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*
x^4)/a)^p + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)
/a)])/(3*(1 + (b*x^4)/a)^p)
```

3.178.3.1 Defintions of rubi rules used

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.178.4 Maple [F]

$$\int \left(e\,x^{2}+c\right) \left(b\,x^{4}+a\right)^{p}d x$$

input
```
int((e*x^2+c)*(b*x^4+a)^p,x)
```

output
```
int((e*x^2+c)*(b*x^4+a)^p,x)
```

3.178.5 Fricas [F]

$$\int (c+ex^2)\left(a+bx^4\right)^p\,dx=\int (ex^2+c)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^2 + c)*(b*x^4 + a)^p, x)
```

3.178.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int (c+ex^2)\left(a+bx^4\right)^p\,dx=\frac{a^pcx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^pex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+c)*(b*x**4+a)**p,x)
```

output
```
a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(5/4)) + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(7/4))
```

3.178.7 Maxima [F]

$$\int (c+ex^2)\left(a+bx^4\right)^p\,dx=\int (ex^2+c)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + c)*(b*x^4 + a)^p, x)
```

3.178.8 Giac [F]

$$\int (c+ex^2)\left(a+bx^4\right)^p\,dx=\int (ex^2+c)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^2 + c)*(b*x^4 + a)^p, x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int (c+ex^2)\left(a+bx^4\right)^p\,dx=\int \left(b\,x^4+a\right)^p\,\left(e\,x^2+c\right)\,dx$$

input
```
int((a + b*x^4)^p*(c + e*x^2),x)
```

output
```
int((a + b*x^4)^p*(c + e*x^2), x)
```

3.179 $\int (a+bx^4)^p \, dx$

3.179.1 Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^4)^p \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

output
```
x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

3.179.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^p \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

input
```
Integrate[(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^
4)/a)^p
```

3.179.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^p \, dx$$

$$\Big\downarrow \text{779}$$

$$(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \int \left(\frac{bx^4}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

input
```
Int[(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^
4)/a)^p
```

3.179.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.179.4 Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p}dx$$

input
```
int((b*x^4+a)^p,x)
```

output
```
int((b*x^4+a)^p,x)
```

3.179.5 Fricas [F]

$$\int \left(a+bx^4\right)^p\,dx=\int \left(bx^{4}+a\right)^{p}dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p, x)
```

3.179.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+bx^4\right)^p\,dx=\frac{a^{p}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**p,x)
```

output
```
a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*ga
mma(5/4))
```

3.179.7 Maxima [F]

$$\int \left(a+bx^4\right)^p\,dx=\int \left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p, x)
```

3.179.8 Giac [F]

$$\int \left(a+bx^4\right)^p\,dx=\int \left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p, x)
```

3.179.9 Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(a+bx^4\right)^p\,dx=\frac{x\left(b\,x^4+a\right)^p\,{}_2\mathrm{F}_1\!\left(\frac{1}{4},-p;\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(\frac{b\,x^4}{a}+1\right)^p}$$

input
```
int((a + b*x^4)^p,x)
```

output
```
(x*(a + b*x^4)^p*hypergeom([1/4, -p], 5/4, -(b*x^4)/a))/((b*x^4)/a + 1)^p
```

3.180 $\int \frac{(a+bx^4)^p}{c+ex^2} dx$

3.180.1 Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{(a+bx^4)^p}{c+ex^2} dx = \frac{x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c} - \frac{ex^3(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

output
```
x*(b*x^4+a)^p*AppellF1(1/4,1,-p,5/4,e^2*x^4/c^2,-b*x^4/a)/c/((1+b*x^4/a)^p
)-1/3*e*x^3*(b*x^4+a)^p*AppellF1(3/4,1,-p,7/4,e^2*x^4/c^2,-b*x^4/a)/c^2/((
1+b*x^4/a)^p)
```

3.180.2 Mathematica [F]

$$\int \frac{(a+bx^4)^p}{c+ex^2} dx = \int \frac{(a+bx^4)^p}{c+ex^2} dx$$

input
```
Integrate[(a + b*x^4)^p/(c + e*x^2),x]
```

output
```
Integrate[(a + b*x^4)^p/(c + e*x^2), x]
```

3.180.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx$$

$$\Big\downarrow \text{1569}$$

$$\int \left(\frac{c(a+bx^4)^p}{c^2-e^2x^4}+\frac{ex^2(a+bx^4)^p}{e^2x^4-c^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

input
```
Int[(a + b*x^4)^p/(c + e*x^2),x]
```

output
```
(x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(
c*(1 + (b*x^4)/a)^p) - (e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 1, 7/4, -((b
*x^4)/a), (e^2*x^4)/c^2])/(3*c^2*(1 + (b*x^4)/a)^p)
```

3.180.3.1 Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.180.4 Maple [F]

$$\int \frac{(b\,x^4+a)^p}{e\,x^2+c}dx$$

input
```
int((b*x^4+a)^p/(e*x^2+c),x)
```

output
```
int((b*x^4+a)^p/(e*x^2+c),x)
```

3.180.5 Fricas [F]

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx = \int \frac{(bx^4+a)^p}{ex^2+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p/(e*x^2 + c), x)
```

3.180.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**p/(e*x**2+c),x)
```

output
```
Timed out
```

3.180.7 Maxima [F]

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx = \int \frac{(bx^4+a)^p}{ex^2+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p/(e*x^2 + c), x)
```

3.180.8 Giac [F]

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx = \int \frac{(bx^4+a)^p}{ex^2+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p/(e*x^2 + c), x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx = \int \frac{(b\,x^4+a)^p}{e\,x^2+c}\,dx$$

input
```
int((a + b*x^4)^p/(c + e*x^2),x)
```

output
```
int((a + b*x^4)^p/(c + e*x^2), x)
```

3.181 $\int \frac{(a+bx^4)^p}{(c+ex^2)^2} dx$

3.181.1 Optimal result

Integrand size = 19, antiderivative size = 189

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2} dx = \frac{x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2} - \frac{2ex^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-p,2,\frac{7}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3} + \frac{e^2x^5(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{5}{4},-p,2,\frac{9}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{5c^4}$$

output
```
x*(b*x^4+a)^p*AppellF1(1/4,2,-p,5/4,e^2*x^4/c^2,-b*x^4/a)/c^2/((1+b*x^4/a)
^p)-2/3*e*x^3*(b*x^4+a)^p*AppellF1(3/4,2,-p,7/4,e^2*x^4/c^2,-b*x^4/a)/c^3/
((1+b*x^4/a)^p)+1/5*e^2*x^5*(b*x^4+a)^p*AppellF1(5/4,2,-p,9/4,e^2*x^4/c^2,
-b*x^4/a)/c^4/((1+b*x^4/a)^p)
```

3.181.2 Mathematica [F]

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx$$

input
```
Integrate[(a + b*x^4)^p/(c + e*x^2)^2,x]
```

output
```
Integrate[(a + b*x^4)^p/(c + e*x^2)^2, x]
```

3.181.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx$$

$$\Big\downarrow \text{1569}$$

$$\int \left(\frac{c^2(a+bx^4)^p}{(c^2-e^2x^4)^2}+\frac{e^2x^4(a+bx^4)^p}{(e^2x^4-c^2)^2}-\frac{2cex^2(a+bx^4)^p}{(c^2-e^2x^4)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2}+\frac{e^2x^5(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{4},-p,2,\frac{9}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{5c^4}-\frac{2ex^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-p,2,\frac{7}{4},-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3}$$

input
```
Int[(a + b*x^4)^p/(c + e*x^2)^2,x]
```

output
```
(x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(
c^2*(1 + (b*x^4)/a)^p) - (2*e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 2, 7/4,
-((b*x^4)/a), (e^2*x^4)/c^2])/(3*c^3*(1 + (b*x^4)/a)^p) + (e^2*x^5*(a + b*
x^4)^p*AppellF1[5/4, -p, 2, 9/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(5*c^4*(1 +
 (b*x^4)/a)^p)
```

3.181.3.1 Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.181.4 Maple [F]

$$\int \frac{(b\,x^4+a)^p}{(e\,x^2+c)^2}dx$$

input
```
int((b*x^4+a)^p/(e*x^2+c)^2,x)
```

output
```
int((b*x^4+a)^p/(e*x^2+c)^2,x)
```

3.181.5 Fricas [F]

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx = \int \frac{(bx^4+a)^p}{(ex^2+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)
```

3.181.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**p/(e*x**2+c)**2,x)
```

output
```
Timed out
```

3.181.7 Maxima [F]

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx = \int \frac{(bx^4+a)^p}{(ex^2+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)
```

3.181.8 Giac [F]

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx = \int \frac{(bx^4+a)^p}{(ex^2+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p/(e*x^2 + c)^2, x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(b\,x^4+a)^p}{(e\,x^2+c)^2}\,dx$$

input
```
int((a + b*x^4)^p/(c + e*x^2)^2,x)
```

output
```
int((a + b*x^4)^p/(c + e*x^2)^2, x)
```

3.182 $\int (1-x^2)^3 (1+bx^4)^p \, dx$

3.182.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\int (1-x^2)^3 (1+bx^4)^p \, dx = -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + x \, \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) + \frac{(1-b(7+4p))x^3 \, \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)}{b(7+4p)} + \frac{3}{5}x^5 \, \text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-bx^4\right)$$

output
```
-x^3*(b*x^4+1)^(p+1)/b/(7+4*p)+x*hypergeom([1/4, -p],[5/4],-b*x^4)+(1-b*(7
+4*p))*x^3*hypergeom([3/4, -p],[7/4],-b*x^4)/b/(7+4*p)+3/5*x^5*hypergeom([
5/4, -p],[9/4],-b*x^4)
```

3.182.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int (1-x^2)^3 (1+bx^4)^p \, dx = x \, \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) - x^3 \, \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right) + \frac{3}{5}x^5 \, \text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-bx^4\right) - \frac{1}{7}x^7 \, \text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-bx^4\right)$$

input
```
Integrate[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

output
```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5 -
(x^7*Hypergeometric2F1[7/4, -p, 11/4, -(b*x^4)])/7
```

3.182.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1519, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x^2)^3 (bx^4+1)^p \, dx$$

$$\Big\downarrow \text{ 1519}$$

$$\frac{\int (bx^4+1)^p (3b(4p+7)x^4+3(1-b(4p+7))x^2+b(4p+7)) \, dx}{b(4p+7)}-\frac{x^3 (bx^4+1)^{p+1}}{b(4p+7)}$$

$$\Big\downarrow \text{ 2432}$$

$$\frac{\int (3b(4p+7)x^4(bx^4+1)^p+3(1-b(4p+7))x^2(bx^4+1)^p+b(4p+7)(bx^4+1)^p) \, dx}{b(4p+7)}-\frac{x^3 (bx^4+1)^{p+1}}{b(4p+7)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b(4p+7)x \text{ Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)+\frac{3}{5}b(4p+7)x^5 \text{ Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-bx^4\right)+x^3(1-b(}{b(4p+7)} - \frac{x^3 (bx^4+1)^{p+1}}{b(4p+7)}$$

input
```
Int[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

output
```
-((x^3*(1 + b*x^4)^(1 + p))/(b*(7 + 4*p))) + (b*(7 + 4*p)*x*Hypergeometric
2F1[1/4, -p, 5/4, -(b*x^4)] + (1 - b*(7 + 4*p))*x^3*Hypergeometric2F1[3/4,
 -p, 7/4, -(b*x^4)] + (3*b*(7 + 4*p)*x^5*Hypergeometric2F1[5/4, -p, 9/4, -
(b*x^4)])/5)/(b*(7 + 4*p))
```

3.182.3.1 Defintions of rubi rules used

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.182.4 Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

method	result	size
meijerg	$-\frac{x^{7} {}_{2}F_{1}\left(\frac{7}{4},-p;\frac{11}{4};-b\,x^{4}\right)}{7}+\frac{3 x^{5} {}_{2}F_{1}\left(\frac{5}{4},-p;\frac{9}{4};-b\,x^{4}\right)}{5}-x^{3} {}_{2}F_{1}\left(\frac{3}{4},-p;\frac{7}{4};-b\,x^{4}\right)+x\, {}_{2}F_{1}\left(\frac{1}{4},-p;\frac{5}{4};-b\,x^{4}\right)$	75

input
```
int((-x^2+1)^3*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/7*x^7*hypergeom([7/4,-p],[11/4],-b*x^4)+3/5*x^5*hypergeom([5/4,-p],[9/4
],-b*x^4)-x^3*hypergeom([3/4,-p],[7/4],-b*x^4)+x*hypergeom([1/4,-p],[5/4],
-b*x^4)
```

3.182.5 Fricas [F]

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = \int -\left(x^2-1\right)^3 \left(bx^4+1\right)^p \, dx$$

input
```
integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="fricas")
```

output
```
integral(-(x^6 - 3*x^4 + 3*x^2 - 1)*(b*x^4 + 1)^p, x)
```

3.182.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 50.67 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = -\frac{x^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix}\frac{7}{4}, -p \\ \frac{11}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{3x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix}\frac{5}{4}, -p \\ \frac{9}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{3x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{3}{4}, -p \\ \frac{7}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4}, -p \\ \frac{5}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-x**2+1)**3*(b*x**4+1)**p,x)
```

output
```
-x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi))/(4*gamm
a(11/4)) + 3*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*p
i))/(4*gamma(9/4)) - 3*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp
_polar(I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**
4*exp_polar(I*pi))/(4*gamma(5/4))
```

3.182.7 Maxima [F]

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = \int -\left(x^2-1\right)^3 \left(bx^4+1\right)^p \, dx$$

input
```
integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

3.182.8 Giac [F]

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = \int -\left(x^2-1\right)^3 \left(bx^4+1\right)^p \, dx$$

input
```
integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \left(1-x^2\right)^3 \left(1+bx^4\right)^p \, dx = -\int \left(x^2-1\right)^3 \left(b\,x^4+1\right)^p \, dx$$

input
```
int(-(x^2 - 1)^3*(b*x^4 + 1)^p,x)
```

output
```
-int((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

3.183 $\int (1-x^2)^2 (1+bx^4)^p \, dx$

3.183.1 Optimal result

Integrand size = 19, antiderivative size = 86

$$\int (1-x^2)^2 (1+bx^4)^p \, dx = \frac{x(1+bx^4)^{1+p}}{b(5+4p)} - \frac{(1-b(5+4p))x \,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)}{b(5+4p)} - \frac{2}{3}x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)$$

output
```
x*(b*x^4+1)^(p+1)/b/(5+4*p)-(1-b*(5+4*p))*x*hypergeom([1/4, -p],[5/4],-b*x
^4)/b/(5+4*p)-2/3*x^3*hypergeom([3/4, -p],[7/4],-b*x^4)
```

3.183.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int (1-x^2)^2 (1+bx^4)^p \, dx = x \,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) - \frac{2}{3}x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right) + \frac{1}{5}x^5 \,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-bx^4\right)$$

input
```
Integrate[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

output
```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4
, -p, 7/4, -(b*x^4)])/3 + (x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/
5
```

3.183.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1519, 25, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-x^2\right)^2 \left(bx^4+1\right)^p \, dx$$

$$\Big\downarrow \text{ 1519}$$

$$\frac{\int -\left(\left(2b(4p+5)x^2-b(4p+5)+1\right)\left(bx^4+1\right)^p\right)dx}{b(4p+5)}+\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}-\frac{\int \left(2b(4p+5)x^2-5b-4bp+1\right)\left(bx^4+1\right)^p dx}{b(4p+5)}$$

$$\Big\downarrow \text{ 1516}$$

$$\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}-\frac{\int \left(2b(4p+5)x^2\left(bx^4+1\right)^p+(1-b(4p+5))\left(bx^4+1\right)^p\right)dx}{b(4p+5)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(bx^4+1\right)^{p+1}}{b(4p+5)}-\frac{x(1-b(4p+5))\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)+\frac{2}{3}b(4p+5)x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)}{b(4p+5)}$$

input
```
Int[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

output
```
(x*(1 + b*x^4)^(1 + p))/(b*(5 + 4*p)) - ((1 - b*(5 + 4*p))*x*Hypergeometri
c2F1[1/4, -p, 5/4, -(b*x^4)] + (2*b*(5 + 4*p)*x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)])/3)/(b*(5 + 4*p))
```

3.183.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 1519
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Simp[1/(c
*(4*p + 2*q + 1))   Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*
x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x
], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.183.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.65

method	result	size
meijerg	$\frac{x^{5}{}_{2}F_{1}\left(\frac{5}{4},-p;\frac{9}{4};-b\,x^{4}\right)}{5}-\frac{2x^{3}{}_{2}F_{1}\left(\frac{3}{4},-p;\frac{7}{4};-b\,x^{4}\right)}{3}+x\,{}_{2}F_{1}\left(\frac{1}{4},-p;\frac{5}{4};-b\,x^{4}\right)$	56

input
```
int((-x^2+1)^2*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*hypergeom([5/4,-p],[9/4],-b*x^4)-2/3*x^3*hypergeom([3/4,-p],[7/4],
-b*x^4)+x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

3.183.5 Fricas [F]

$$\int (1-x^2)^2 (1+bx^4)^p \, dx = \int (x^2-1)^2 (bx^4+1)^p \, dx$$

input
```
integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="fricas")
```

output
```
integral((x^4 - 2*x^2 + 1)*(b*x^4 + 1)^p, x)
```

3.183.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.98 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int (1-x^2)^2 (1+bx^4)^p \, dx = \frac{x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix}\middle| bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix}\middle| bx^4 e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-x**2+1)**2*(b*x**4+1)**p,x)
```

output
```
x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi))/(4*gamma(
9/4)) - x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(
2*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*p
i))/(4*gamma(5/4))
```

3.183.7 Maxima [F]

$$\int (1-x^2)^2 (1+bx^4)^p \, dx = \int (x^2-1)^2 (bx^4+1)^p \, dx$$

input
```
integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

3.183.8 Giac [F]

$$\int (1-x^2)^2 (1+bx^4)^p \, dx = \int (x^2-1)^2 (bx^4+1)^p \, dx$$

input
```
integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="giac")
```

output
```
integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int (1-x^2)^2 (1+bx^4)^p \, dx = \int (x^2-1)^2 (b\,x^4+1)^p \, dx$$

input
```
int((x^2 - 1)^2*(b*x^4 + 1)^p,x)
```

output
```
int((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

3.184 $\int (1-x^2)(1+bx^4)^p \, dx$

3.184.1 Optimal result

Integrand size = 17, antiderivative size = 42

$$\int (1-x^2)(1+bx^4)^p \, dx = x \text{ Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) - \frac{1}{3}x^3 \text{ Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)$$

output
```
x*hypergeom([1/4, -p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4, -p],[7/4],-b*x
^4)
```

3.184.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (1-x^2)(1+bx^4)^p \, dx = x \text{ Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right) - \frac{1}{3}x^3 \text{ Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)$$

input
```
Integrate[(1 - x^2)*(1 + b*x^4)^p,x]
```

output
```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4,
-p, 7/4, -(b*x^4)])/3
```

3.184.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-x^2\right) \left(bx^4+1\right)^p \, dx$$

$$\Big\downarrow \text{ 1516}$$

$$\int \left(\left(bx^4+1\right)^p-x^2 \left(bx^4+1\right)^p\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$x \,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)-\frac{1}{3} x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-bx^4\right)$$

input
```
Int[(1 - x^2)*(1 + b*x^4)^p,x]
```

output
```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4,
-p, 7/4, -(b*x^4)])/3
```

3.184.3.1 Defintions of rubi rules used

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.184.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
meijerg	$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-b\,x^4\right)-\frac{x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-b\,x^4\right)}{3}$	37

input
```
int((-x^2+1)*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,-p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4,-p],[7/4],-b*x^4
)
```

3.184.5 Fricas [F]

$$\int (1-x^2)\left(1+bx^4\right)^p\,dx=\int -\left(x^2-1\right)\left(bx^4+1\right)^p\,dx$$

input
```
integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="fricas")
```

output
```
integral(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

3.184.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.87 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int (1-x^2)\left(1+bx^4\right)^p\,dx=-\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-x**2+1)*(b*x**4+1)**p,x)
```

output
```
-x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(4*gamma
(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*
gamma(5/4))
```

3.184.7 Maxima [F]

$$\int (1-x^2)(1+bx^4)^p \, dx = \int -(x^2-1)(bx^4+1)^p \, dx$$

input
```
integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)*(b*x^4 + 1)^p, x)
```

3.184.8 Giac [F]

$$\int (1-x^2)(1+bx^4)^p \, dx = \int -(x^2-1)(bx^4+1)^p \, dx$$

input
```
integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int (1-x^2)(1+bx^4)^p \, dx = -\int (x^2-1)\,(b\,x^4+1)^p \, dx$$

input
```
int(-(x^2 - 1)*(b*x^4 + 1)^p,x)
```

output
```
-int((x^2 - 1)*(b*x^4 + 1)^p, x)
```

3.185 $\int (1+bx^4)^p \, dx$

3.185.1 Optimal result

Integrand size = 9, antiderivative size = 18

$$\int (1+bx^4)^p \, dx = x \, \text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -bx^4\right)$$

output
```
x*hypergeom([1/4, -p],[5/4],-b*x^4)
```

3.185.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (1+bx^4)^p \, dx = x \, \text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -bx^4\right)$$

input
```
Integrate[(1 + b*x^4)^p,x]
```

output
```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

3.185.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^4+1)^p \, dx$$

$$\Big\downarrow \text{778}$$

$$x \,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-bx^4\right)$$

input
```
Int[(1 + b*x^4)^p,x]
```

output
```
x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

3.185.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

3.185.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
meijerg	$x {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-b\,x^4\right)$	17

input
```
int((b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

3.185.5 Fricas [F]

$$\int \left(1+bx^4\right)^p \, dx = \int \left(bx^4+1\right)^p \, dx$$

input
```
integrate((b*x^4+1)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + 1)^p, x)
```

3.185.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \left(1+bx^4\right)^p \, dx = \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| bx^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+1)**p,x)
```

output
```
x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*gamma(5/4
))
```

3.185.7 Maxima [F]

$$\int \left(1+bx^4\right)^p \, dx = \int \left(bx^4+1\right)^p \, dx$$

input
```
integrate((b*x^4+1)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + 1)^p, x)
```

3.185.8 Giac [F]

$$\int \left(1+bx^4\right)^p\, dx = \int \left(bx^4+1\right)^p dx$$

input
```
integrate((b*x^4+1)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + 1)^p, x)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \left(1+bx^4\right)^p\, dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -b\,x^4\right)$$

input
```
int((b*x^4 + 1)^p,x)
```

output
```
x*hypergeom([1/4, -p], 5/4, -b*x^4)
```

3.186 $\int \frac{(1+bx^4)^p}{1-x^2} dx$

3.186.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{(1+bx^4)^p}{1-x^2} dx = x \operatorname{AppellF1}\left(\frac{1}{4},1,-p,\frac{5}{4},x^4,-bx^4\right) + \frac{1}{3}x^3 \operatorname{AppellF1}\left(\frac{3}{4},1,-p,\frac{7}{4},x^4,-bx^4\right)$$

output
```
x*AppellF1(1/4,1,-p,5/4,x^4,-b*x^4)+1/3*x^3*AppellF1(3/4,1,-p,7/4,x^4,-b*x
^4)
```

3.186.2 Mathematica [F]

$$\int \frac{(1+bx^4)^p}{1-x^2} dx = \int \frac{(1+bx^4)^p}{1-x^2} dx$$

input
```
Integrate[(1 + b*x^4)^p/(1 - x^2),x]
```

output
```
Integrate[(1 + b*x^4)^p/(1 - x^2), x]
```

3.186.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^4+1)^p}{1-x^2} \, dx$$

$$\Big\downarrow \text{1569}$$

$$\int \left(\frac{(bx^4+1)^p}{1-x^4}-\frac{x^2(bx^4+1)^p}{x^4-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x \operatorname{AppellF1}\left(\frac{1}{4},1,-p,\frac{5}{4},x^4,-bx^4\right)+\frac{1}{3}x^3 \operatorname{AppellF1}\left(\frac{3}{4},1,-p,\frac{7}{4},x^4,-bx^4\right)$$

input
```
Int[(1 + b*x^4)^p/(1 - x^2),x]
```

output
```
x*AppellF1[1/4, 1, -p, 5/4, x^4, -(b*x^4)] + (x^3*AppellF1[3/4, 1, -p, 7/4
, x^4, -(b*x^4)])/3
```

3.186.3.1 Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.186.4 Maple [F]

$$\int \frac{(b\,x^4+1)^p}{-x^2+1}dx$$

input
```
int((b*x^4+1)^p/(-x^2+1),x)
```

output
```
int((b*x^4+1)^p/(-x^2+1),x)
```

3.186.5 Fricas [F]

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = \int -\frac{(bx^4+1)^p}{x^2-1}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="fricas")
```

output
```
integral(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

3.186.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+1)**p/(-x**2+1),x)
```

output
```
Timed out
```

3.186.7 Maxima [F]

$$\int \frac{(1+bx^4)^p}{1-x^2} dx = \int -\frac{(bx^4+1)^p}{x^2-1} dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="maxima")
```

output
```
-integrate((b*x^4 + 1)^p/(x^2 - 1), x)
```

3.186.8 Giac [F]

$$\int \frac{(1+bx^4)^p}{1-x^2} dx = \int -\frac{(bx^4+1)^p}{x^2-1} dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="giac")
```

output
```
integrate(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{1-x^2} dx = -\int \frac{(b\,x^4+1)^p}{x^2-1} dx$$

input
```
int(-(b*x^4 + 1)^p/(x^2 - 1),x)
```

output
```
-int((b*x^4 + 1)^p/(x^2 - 1), x)
```

3.187 $\int \frac{(1+bx^4)^p}{(1-x^2)^2} dx$

3.187.1 Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2} dx = x \operatorname{AppellF1}\left(\frac{1}{4},2,-p,\frac{5}{4},x^4,-bx^4\right) + \frac{2}{3}x^3 \operatorname{AppellF1}\left(\frac{3}{4},2,-p,\frac{7}{4},x^4,-bx^4\right) + \frac{1}{5}x^5 \operatorname{AppellF1}\left(\frac{5}{4},2,-p,\frac{9}{4},x^4,-bx^4\right)$$

output
```
x*AppellF1(1/4,2,-p,5/4,x^4,-b*x^4)+2/3*x^3*AppellF1(3/4,2,-p,7/4,x^4,-b*x
^4)+1/5*x^5*AppellF1(5/4,2,-p,9/4,x^4,-b*x^4)
```

3.187.2 Mathematica [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2} dx = \int \frac{(1+bx^4)^p}{(1-x^2)^2} dx$$

input
```
Integrate[(1 + b*x^4)^p/(1 - x^2)^2,x]
```

output
```
Integrate[(1 + b*x^4)^p/(1 - x^2)^2, x]
```

3.187.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^4+1)^p}{(1-x^2)^2} \, dx$$

$$\Big\downarrow \ 1569$$

$$\int \left(\frac{x^4(bx^4+1)^p}{(x^4-1)^2}+\frac{(bx^4+1)^p}{(x^4-1)^2}+\frac{2x^2(bx^4+1)^p}{(x^4-1)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$x \operatorname{AppellF1}\left(\frac{1}{4},2,-p,\frac{5}{4},x^4,-bx^4\right)+\frac{1}{5}x^5 \operatorname{AppellF1}\left(\frac{5}{4},2,-p,\frac{9}{4},x^4,-bx^4\right)+\frac{2}{3}x^3 \operatorname{AppellF1}\left(\frac{3}{4},2,-p,\frac{7}{4},x^4,-bx^4\right)$$

input
```
Int[(1 + b*x^4)^p/(1 - x^2)^2,x]
```

output
```
x*AppellF1[1/4, 2, -p, 5/4, x^4, -(b*x^4)] + (2*x^3*AppellF1[3/4, 2, -p, 7
/4, x^4, -(b*x^4)])/3 + (x^5*AppellF1[5/4, 2, -p, 9/4, x^4, -(b*x^4)])/5
```

3.187.3.1 Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.187.4 Maple [F]

$$\int \frac{(b\,x^4+1)^p}{(-x^2+1)^2}dx$$

input
```
int((b*x^4+1)^p/(-x^2+1)^2,x)
```

output
```
int((b*x^4+1)^p/(-x^2+1)^2,x)
```

3.187.5 Fricas [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \int \frac{(bx^4+1)^p}{(x^2-1)^2}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4 + 1)^p/(x^4 - 2*x^2 + 1), x)
```

3.187.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+1)**p/(-x**2+1)**2,x)
```

output
```
Timed out
```

3.187.7 Maxima [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2} \, dx = \int \frac{(bx^4+1)^p}{(x^2-1)^2} \, dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

3.187.8 Giac [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2} \, dx = \int \frac{(bx^4+1)^p}{(x^2-1)^2} \, dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2} \, dx = \int \frac{(b\,x^4+1)^p}{(x^2-1)^2} \, dx$$

input
```
int((b*x^4 + 1)^p/(x^2 - 1)^2,x)
```

output
```
int((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

3.188 $\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx$

3.188.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\begin{aligned}\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx &= x\,\text{AppellF1}\left(\frac{1}{4},3,-p,\frac{5}{4},x^4,-bx^4\right)\\ &\quad+x^3\,\text{AppellF1}\left(\frac{3}{4},3,-p,\frac{7}{4},x^4,-bx^4\right)\\ &\quad+\frac{3}{5}x^5\,\text{AppellF1}\left(\frac{5}{4},3,-p,\frac{9}{4},x^4,-bx^4\right)\\ &\quad+\frac{1}{7}x^7\,\text{AppellF1}\left(\frac{7}{4},3,-p,\frac{11}{4},x^4,-bx^4\right)\end{aligned}$$

output
```
x*AppellF1(1/4,3,-p,5/4,x^4,-b*x^4)+x^3*AppellF1(3/4,3,-p,7/4,x^4,-b*x^4)+
3/5*x^5*AppellF1(5/4,3,-p,9/4,x^4,-b*x^4)+1/7*x^7*AppellF1(7/4,3,-p,11/4,x
^4,-b*x^4)
```

3.188.2 Mathematica [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx$$

input
```
Integrate[(1 + b*x^4)^p/(1 - x^2)^3,x]
```

output
```
Integrate[(1 + b*x^4)^p/(1 - x^2)^3, x]
```

3.188.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^4+1)^p}{(1-x^2)^3}\,dx$$
$$\Big\downarrow \ 1569$$
$$\int \left(-\frac{3x^4(bx^4+1)^p}{(x^4-1)^3}-\frac{(bx^4+1)^p}{(x^4-1)^3}-\frac{x^6(bx^4+1)^p}{(x^4-1)^3}-\frac{3x^2(bx^4+1)^p}{(x^4-1)^3}\right)dx$$
$$\Big\downarrow \ 2009$$
$$x\operatorname{AppellF1}\left(\frac{1}{4},3,-p,\frac{5}{4},x^4,-bx^4\right)+\frac{1}{7}x^7\operatorname{AppellF1}\left(\frac{7}{4},3,-p,\frac{11}{4},x^4,-bx^4\right)+\frac{3}{5}x^5\operatorname{AppellF1}\left(\frac{5}{4},3,-p,\frac{9}{4},x^4,-bx^4\right)+x^3\operatorname{AppellF1}\left(\frac{3}{4},3,-p,\frac{7}{4},x^4,-bx^4\right)$$

input
```
Int[(1 + b*x^4)^p/(1 - x^2)^3,x]
```

output
```
x*AppellF1[1/4, 3, -p, 5/4, x^4, -(b*x^4)] + x^3*AppellF1[3/4, 3, -p, 7/4,
 x^4, -(b*x^4)] + (3*x^5*AppellF1[5/4, 3, -p, 9/4, x^4, -(b*x^4)])/5 + (x^
7*AppellF1[7/4, 3, -p, 11/4, x^4, -(b*x^4)])/7
```

3.188.3.1 Defintions of rubi rules used

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.188.4 Maple [F]

$$\int \frac{(b\,x^4+1)^p}{(-x^2+1)^3}dx$$

input
```
int((b*x^4+1)^p/(-x^2+1)^3,x)
```

output
```
int((b*x^4+1)^p/(-x^2+1)^3,x)
```

3.188.5 Fricas [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int -\frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="fricas")
```

output
```
integral(-(b*x^4 + 1)^p/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

3.188.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+1)**p/(-x**2+1)**3,x)
```

output
```
Timed out
```

3.188.7 Maxima [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int -\frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="maxima")
```

output
```
-integrate((b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

3.188.8 Giac [F]

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int -\frac{(bx^4+1)^p}{(x^2-1)^3}\,dx$$

input
```
integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx = \int -\frac{(b\,x^4+1)^p}{(x^2-1)^3}\,dx$$

input
```
int(-(b*x^4 + 1)^p/(x^2 - 1)^3,x)
```

output
```
int(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

3.189 $\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx$

3.189.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\frac{8d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

output
```
-7*d^2*x-4/3*d*e*x^3-1/5*e^2*x^5+8*d^(5/2)*arctanh(x*e^(1/2)/d^(1/2))/e^(1
/2)
```

3.189.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\frac{8d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

output
```
-7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sq
rt[d]])/Sqrt[e]
```

3.189.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1388, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{(d+ex^2)^3}{d-ex^2}dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{8d^3}{d-ex^2}-7d^2-4dex^2-e^2x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{8d^{5/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4}{3}dex^3-\frac{1}{5}e^2x^5$$

input
```
Int[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

output
```
-7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sq
rt[d]])/Sqrt[e]
```

3.189.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.189.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{e^{2} x^{5}}{5}-\frac{4 d e \,x^{3}}{3}-7 d^{2} x +\frac{8 d^{3} \operatorname{arctanh}\left(\frac{e x}{\sqrt{e d}}\right)}{\sqrt{e d}}$	42
risch	$-\frac{e^{2} x^{5}}{5}-\frac{4 d e \,x^{3}}{3}-7 d^{2} x -\frac{4 \sqrt{e d}\, d^{2} \ln \left(\sqrt{e d}\, x -d \right)}{e}+\frac{4 \sqrt{e d}\, d^{2} \ln \left(-\sqrt{e d}\, x -d \right)}{e}$	74

input
```
int((e*x^2+d)^4/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*e^2*x^5-4/3*d*e*x^3-7*d^2*x+8*d^3/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2)
)
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.27

$$\int \frac{\left(d+e x^2\right)^4}{d^2-e^2 x^4}\, dx = \left[-\frac{1}{5}\, e^{2} x^{5} - \frac{4}{3}\, d e x^{3} + 4\, d^{2} \sqrt{\frac{d}{e}} \log\left(\frac{e x^{2} + 2\, e x \sqrt{\frac{d}{e}} + d}{e x^{2} - d}\right) - 7\, d^{2} x,\ -\frac{1}{5}\, e^{2} x^{5} - \frac{4}{3}\, d e x^{3} - 8\, d^{2} \sqrt{-\frac{d}{e}} \arctan\left(\frac{e x \sqrt{-\frac{d}{e}}}{d}\right) - 7\, d^{2} x\right]$$

input
```
integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[-1/5*e^2*x^5 - 4/3*d*e*x^3 + 4*d^2*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e)
 + d)/(e*x^2 - d)) - 7*d^2*x, -1/5*e^2*x^5 - 4/3*d*e*x^3 - 8*d^2*sqrt(-d/e
)*arctan(e*x*sqrt(-d/e)/d) - 7*d^2*x]
```

3.189.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -7d^2x - \frac{4dex^3}{3} - \frac{e^2x^5}{5} - 4\sqrt{\frac{d^5}{e}}\log\left(x-\frac{\sqrt{\frac{d^5}{e}}}{d^2}\right) + 4\sqrt{\frac{d^5}{e}}\log\left(x+\frac{\sqrt{\frac{d^5}{e}}}{d^2}\right)$$

input
```
integrate((e*x**2+d)**4/(-e**2*x**4+d**2),x)
```

output
```
-7*d**2*x - 4*d*e*x**3/3 - e**2*x**5/5 - 4*sqrt(d**5/e)*log(x - sqrt(d**5/
e)/d**2) + 4*sqrt(d**5/e)*log(x + sqrt(d**5/e)/d**2)
```

3.189.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -\frac{8\,d^3\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}} - \frac{3\,e^7x^5+20\,de^6x^3+105\,d^2e^5x}{15\,e^5}$$

input
```
integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
-8*d^3*arctan(e*x/sqrt(-d*e))/sqrt(-d*e) - 1/15*(3*e^7*x^5 + 20*d*e^6*x^3
+ 105*d^2*e^5*x)/e^5
```

3.189.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = -7\,d^2\,x - \frac{e^2\,x^5}{5} - \frac{4\,d\,e\,x^3}{3} - \frac{d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x\,1\mathrm{i}}{\sqrt{d}}\right)\,8\mathrm{i}}{\sqrt{e}}$$

input
```
int((d + e*x^2)^4/(d^2 - e^2*x^4),x)
```

output
```
- 7*d^2*x - (e^2*x^5)/5 - (d^(5/2)*atan((e^(1/2)*x*1i)/d^(1/2))*8i)/e^(1/2
) - (4*d*e*x^3)/3
```

3.190 $\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx$

3.190.1 Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = -3dx-\frac{ex^3}{3}+\frac{4d^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

output
```
-3*d*x-1/3*e*x^3+4*d^(3/2)*arctanh(x*e^(1/2)/d^(1/2))/e^(1/2)
```

3.190.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = -3dx-\frac{ex^3}{3}+\frac{4d^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
```

output
```
-3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

3.190.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1388, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{(d+ex^2)^2}{d-ex^2}dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{4d^2}{d-ex^2}-3d-ex^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4d^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-3dx-\frac{ex^3}{3}$$

input
```
Int[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
```

output
```
-3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

3.190.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.190.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{e\,x^{3}}{3}-3dx+\frac{4d^{2}\operatorname{arctanh}\left(\frac{ex}{\sqrt{ed}}\right)}{\sqrt{ed}}$	31
risch	$-\frac{e\,x^{3}}{3}-3dx+\frac{2\sqrt{ed}\,d\ln\left(\sqrt{ed}\,x+d\right)}{e}-\frac{2\sqrt{ed}\,d\ln\left(-\sqrt{ed}\,x+d\right)}{e}$	55

input
```
int((e*x^2+d)^3/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*e*x^3-3*d*x+4*d^2/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2))
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.37

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = \left[-\frac{1}{3}\,ex^3+2\,d\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-3\,dx,-\frac{1}{3}\,ex^3 -4\,d\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right)-3\,dx\right]$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[-1/3*e*x^3 + 2*d*sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d))
 - 3*d*x, -1/3*e*x^3 - 4*d*sqrt(-d/e)*arctan(e*x*sqrt(-d/e)/d) - 3*d*x]
```

3.190.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = -3dx - \frac{ex^3}{3} - 2\sqrt{\frac{d^3}{e}}\log\left(x - \frac{\sqrt{\frac{d^3}{e}}}{d}\right) + 2\sqrt{\frac{d^3}{e}}\log\left(x + \frac{\sqrt{\frac{d^3}{e}}}{d}\right)$$

input
```
integrate((e*x**2+d)**3/(-e**2*x**4+d**2),x)
```

output
```
-3*d*x - e*x**3/3 - 2*sqrt(d**3/e)*log(x - sqrt(d**3/e)/d) + 2*sqrt(d**3/e
)*log(x + sqrt(d**3/e)/d)
```

3.190.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = -\frac{4\,d^2\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}} - \frac{e^4x^3+9\,de^3x}{3\,e^3}$$

input
```
integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
-4*d^2*arctan(e*x/sqrt(-d*e))/sqrt(-d*e) - 1/3*(e^4*x^3 + 9*d*e^3*x)/e^3
```

3.190.9 Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = \frac{4\,d^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{e\,x^3}{3} - 3\,d\,x$$

input
```
int((d + e*x^2)^3/(d^2 - e^2*x^4),x)
```

output
```
(4*d^(3/2)*atanh((e^(1/2)*x)/d^(1/2)))/e^(1/2) - (e*x^3)/3 - 3*d*x
```

3.191 $\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx$

3.191.1 Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx=-x+\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

output
```
-x+2*arctanh(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

3.191.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx=-x+\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^2/(d^2 - e^2*x^4),x]
```

output
```
-x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

3.191.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1388, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{d+ex^2}{d-ex^2}dx$$

$$\Big\downarrow \text{299}$$

$$2d\int \frac{1}{d-ex^2}dx-x$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

input
```
Int[(d + e*x^2)^2/(d^2 - e^2*x^4),x]
```

output
```
-x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

3.191.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.191.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
default	$-x+\frac{2d\,\mathrm{arctanh}\left(\frac{ex}{\sqrt{ed}}\right)}{\sqrt{ed}}$	22
risch	$-x-\frac{\sqrt{ed}\,\ln\left(\sqrt{ed}\,x-d\right)}{e}+\frac{\sqrt{ed}\,\ln\left(-\sqrt{ed}\,x-d\right)}{e}$	49

input
```
int((e*x^2+d)^2/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
-x+2*d/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2))
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx=\left[\sqrt{\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{\frac{d}{e}}+d}{ex^2-d}\right)-x,-2\sqrt{-\frac{d}{e}}\arctan\left(\frac{ex\sqrt{-\frac{d}{e}}}{d}\right)-x\right]$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[sqrt(d/e)*log((e*x^2 + 2*e*x*sqrt(d/e) + d)/(e*x^2 - d)) - x, -2*sqrt(-d/
e)*arctan(e*x*sqrt(-d/e)/d) - x]
```

3.191.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = -x-\sqrt{\frac{d}{e}}\log\left(x-\sqrt{\frac{d}{e}}\right)+\sqrt{\frac{d}{e}}\log\left(x+\sqrt{\frac{d}{e}}\right)$$

input
```
integrate((e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

output
```
-x - sqrt(d/e)*log(x - sqrt(d/e)) + sqrt(d/e)*log(x + sqrt(d/e))
```

3.191.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = -\frac{2\,d\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}}-x$$

input
```
integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
-2*d*arctan(e*x/sqrt(-d*e))/sqrt(-d*e) - x
```

3.191.9 Mupad [B] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx = \frac{2\sqrt{d}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

input
```
int((d + e*x^2)^2/(d^2 - e^2*x^4),x)
```

output
```
(2*d^(1/2)*atanh((e^(1/2)*x)/d^(1/2)))/e^(1/2) - x
```

3.192 $\int \frac{d+ex^2}{d^2-e^2x^4}\,dx$

3.192.1 Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

output
```
arctanh(x*e^(1/2)/d^(1/2))/d^(1/2)/e^(1/2)
```

3.192.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)/(d^2 - e^2*x^4),x]
```

output
```
ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

3.192.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1388, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx$$

$$\Big\downarrow \ 1388$$

$$\int \frac{1}{d-ex^2}dx$$

$$\Big\downarrow \ 221$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

input
```
Int[(d + e*x^2)/(d^2 - e^2*x^4),x]
```

output
```
ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

3.192.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.192.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{ed}}\right)}{\sqrt{ed}}$	16
risch	$\frac{\ln\left(ex+\sqrt{ed}\right)}{2\sqrt{ed}}-\frac{\ln\left(-ex+\sqrt{ed}\right)}{2\sqrt{ed}}$	37

input
```
int((e*x^2+d)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2))
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = \left[\frac{\sqrt{de}\log\left(\frac{ex^2+2\sqrt{de}x+d}{ex^2-d}\right)}{2\,de}, -\frac{\sqrt{-de}\arctan\left(\frac{\sqrt{-de}x}{d}\right)}{de}\right]$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d))/(d*e), -sqrt(-
d*e)*arctan(sqrt(-d*e)*x/d)/(d*e)]
```

3.192.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{d+ex^2}{d^2-e^2x^4}\,dx = -\frac{\sqrt{\frac{1}{de}}\log\left(-d\sqrt{\frac{1}{de}}+x\right)}{2}+\frac{\sqrt{\frac{1}{de}}\log\left(d\sqrt{\frac{1}{de}}+x\right)}{2}$$

input
```
integrate((e*x**2+d)/(-e**2*x**4+d**2),x)
```

output
```
-sqrt(1/(d*e))*log(-d*sqrt(1/(d*e)) + x)/2 + sqrt(1/(d*e))*log(d*sqrt(1/(d
*e)) + x)/2
```

3.192.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex^2}{d^2 - e^2x^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{d + ex^2}{d^2 - e^2x^4} \, dx = -\frac{\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{\sqrt{-de}}$$

input
```
integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
-arctan(e*x/sqrt(-d*e))/sqrt(-d*e)
```

3.192.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{d + ex^2}{d^2 - e^2x^4} \, dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}$$

input
```
int((d + e*x^2)/(d^2 - e^2*x^4),x)
```

output
```
atanh((e^(1/2)*x)/d^(1/2))/(d^(1/2)*e^(1/2))
```

3.192. $\int \frac{d+ex^2}{d^2-e^2x^4}\,dx$

3.193 $\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx$

3.193.1 Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx=\frac{x}{4d^2(d+ex^2)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}$$

output
```
1/4*x/d^2/(e*x^2+d)+1/2*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)+1/4*arct
anh(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)
```

3.193.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx=\frac{\frac{\sqrt{d}x}{d+ex^2}+\frac{2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{4d^{5/2}}$$

input
```
Integrate[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

output
```
((Sqrt[d]*x)/(d + e*x^2) + (2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ArcTa
nh[(Sqrt[e]*x)/Sqrt[d]]/Sqrt[e])/(4*d^(5/2))
```

3.193.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1388, 316, 25, 27, 397, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}dx$$
$$\Big\downarrow \text{1388}$$
$$\int \frac{1}{(d-ex^2)(d+ex^2)^2}dx$$
$$\Big\downarrow \text{316}$$
$$\frac{x}{4d^2(d+ex^2)}-\frac{\int -\frac{e(3d-ex^2)}{(d-ex^2)(ex^2+d)}dx}{4d^2e}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{e(3d-ex^2)}{(d-ex^2)(ex^2+d)}dx}{4d^2e}+\frac{x}{4d^2(d+ex^2)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{3d-ex^2}{(d-ex^2)(ex^2+d)}dx}{4d^2}+\frac{x}{4d^2(d+ex^2)}$$
$$\Big\downarrow \text{397}$$
$$\frac{\int \frac{1}{d-ex^2}dx+2\int \frac{1}{ex^2+d}dx}{4d^2}+\frac{x}{4d^2(d+ex^2)}$$
$$\Big\downarrow \text{218}$$
$$\frac{\int \frac{1}{d-ex^2}dx+\frac{2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}}{4d^2}+\frac{x}{4d^2(d+ex^2)}$$
$$\Big\downarrow \text{221}$$
$$\frac{\frac{2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}}{4d^2}+\frac{x}{4d^2(d+ex^2)}$$

input
```
Int[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

output
```
x/(4*d^2*(d + e*x^2)) + ((2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])
 + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e]))/(4*d^2)
```

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.193.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{ed}}\right)}{4d^2\sqrt{ed}}+\frac{\frac{x}{e\,x^2+d}+\frac{2\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{\sqrt{ed}}}{4d^2}$	54
risch	$\frac{x}{4d^2(e\,x^2+d)}-\frac{\ln(-ex-\sqrt{-ed})}{4\sqrt{-ed}\,d^2}+\frac{\ln(ex-\sqrt{-ed})}{4\sqrt{-ed}\,d^2}+\frac{\ln\left(ex+\sqrt{ed}\right)}{8\sqrt{ed}\,d^2}-\frac{\ln\left(-ex+\sqrt{ed}\right)}{8\sqrt{ed}\,d^2}$	107

input
```
int(1/(e*x^2+d)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d^2/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2))+1/4/d^2*(x/(e*x^2+d)+2/(e*d)^
(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.62

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx$$
$$=\left[\frac{2\,dex+4\,(ex^2+d)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)+(ex^2+d)\sqrt{de}\log\left(\frac{ex^2+2\,\sqrt{de}x+d}{ex^2-d}\right)}{8\,(d^3e^2x^2+d^4e)},\frac{dex-(ex^2+d)\sqrt{-de}\arctan\right.$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/8*(2*d*e*x + 4*(e*x^2 + d)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (e*x^2 + d
)*sqrt(d*e)*log((e*x^2 + 2*sqrt(d*e)*x + d)/(e*x^2 - d)))/(d^3*e^2*x^2 + d
^4*e), 1/4*(d*e*x - (e*x^2 + d)*sqrt(-d*e)*arctan(sqrt(-d*e)*x/d) - (e*x^2
 + d)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^2*x
^2 + d^4*e)]
```

3.193.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(63) = 126.

Time = 0.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.14

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{x}{4d^3+4d^2ex^2} - \frac{\sqrt{\frac{1}{d^5e}}\log\left(-\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} - \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} + \frac{\sqrt{\frac{1}{d^5e}}\log\left(\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} + \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} - \frac{\sqrt{-\frac{1}{d^5e}}\log\left(-\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5} - \frac{9d^3\sqrt{-\frac{1}{d^5e}}}{5} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^5e}}\log\left(\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5} + \frac{9d^3\sqrt{-\frac{1}{d^5e}}}{5} + x\right)}{4}$$

input
```
integrate(1/(e*x**2+d)/(-e**2*x**4+d**2),x)
```

output
```
x/(4*d**3 + 4*d**2*e*x**2) - sqrt(1/(d**5*e))*log(-d**8*e*(1/(d**5*e))**(3
/2)/10 - 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 + sqrt(1/(d**5*e))*log(d**8*e*(
1/(d**5*e))**(3/2)/10 + 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 - sqrt(-1/(d**5*
e))*log(-4*d**8*e*(-1/(d**5*e))**(3/2)/5 - 9*d**3*sqrt(-1/(d**5*e))/5 + x)
/4 + sqrt(-1/(d**5*e))*log(4*d**8*e*(-1/(d**5*e))**(3/2)/5 + 9*d**3*sqrt(-
1/(d**5*e))/5 + x)/4
```

3.193.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^2} - \frac{\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{4\sqrt{-de}d^2} + \frac{x}{4(ex^2+d)d^2}$$

input
```
integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/2*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2) - 1/4*arctan(e*x/sqrt(-d*e))/(sq
rt(-d*e)*d^2) + 1/4*x/((e*x^2 + d)*d^2)
```

3.193.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx = \frac{x}{4\,d^2\,(e\,x^2+d)} + \frac{\operatorname{atanh}\left(\frac{x\,\sqrt{d^5\,e}}{d^3}\right)\,\sqrt{d^5\,e}}{4\,d^5\,e} - \frac{\operatorname{atanh}\left(\frac{x\,\sqrt{-d^5\,e}}{d^3}\right)\,\sqrt{-d^5\,e}}{2\,d^5\,e}$$

input
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)),x)
```

output
```
x/(4*d^2*(d + e*x^2)) + (atanh((x*(d^5*e)^(1/2))/d^3)*(d^5*e)^(1/2))/(4*d^
5*e) - (atanh((x*(-d^5*e)^(1/2))/d^3)*(-d^5*e)^(1/2))/(2*d^5*e)
```

3.194 $\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx$

3.194.1 Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = \frac{x}{8d^2(d+ex^2)^2}+\frac{5x}{16d^3(d+ex^2)} + \frac{7\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}$$

output
```
1/8*x/d^2/(e*x^2+d)^2+5/16*x/d^3/(e*x^2+d)+7/16*arctan(x*e^(1/2)/d^(1/2))/
d^(7/2)/e^(1/2)+1/8*arctanh(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(1/2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = \frac{\frac{\sqrt{d}x(7d+5ex^2)}{(d+ex^2)^2}+\frac{7\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}}{16d^{7/2}}$$

input
```
Integrate[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

output
```
((Sqrt[d]*x*(7*d + 5*e*x^2))/(d + e*x^2)^2 + (7*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/Sqrt[e] + (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e])/(16*d^(7/2))
```

3.194.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1388, 316, 25, 27, 402, 27, 397, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (d^2-e^2x^4)}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{(d-ex^2)(d+ex^2)^3}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{x}{8d^2(d+ex^2)^2}-\frac{\int -\frac{e(7d-3ex^2)}{(d-ex^2)(ex^2+d)^2}dx}{8d^2e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e(7d-3ex^2)}{(d-ex^2)(ex^2+d)^2}dx}{8d^2e}+\frac{x}{8d^2(d+ex^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7d-3ex^2}{(d-ex^2)(ex^2+d)^2}dx}{8d^2}+\frac{x}{8d^2(d+ex^2)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{5x}{2d(d+ex^2)}-\frac{\int -\frac{2de(9d-5ex^2)}{(d-ex^2)(ex^2+d)}dx}{4d^2e}}{8d^2}+\frac{x}{8d^2(d+ex^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{9d-5ex^2}{(d-ex^2)(ex^2+d)}dx}{2d}+\frac{5x}{2d(d+ex^2)}}{8d^2}+\frac{x}{8d^2(d+ex^2)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{2\int \frac{1}{d-ex^2}dx+7\int \frac{1}{ex^2+d}dx}{2d}+\frac{5x}{2d(d+ex^2)}}{8d^2}+\frac{x}{8d^2(d+ex^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2\int \frac{1}{d-ex^2}dx+\frac{7\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}}{2d}+\frac{5x}{2d(d+ex^2)}}{8d^2}+\frac{x}{8d^2\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{7\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+\frac{2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}}{2d}+\frac{5x}{2d(d+ex^2)}}{8d^2}+\frac{x}{8d^2\left(d+ex^2\right)^2}$$

input
```
Int[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

output
```
x/(8*d^2*(d + e*x^2)^2) + ((5*x)/(2*d*(d + e*x^2)) + ((7*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) + (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*
Sqrt[e]))/(2*d))/(8*d^2)
```

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.194.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{ed}}\right)}{8d^{3}\sqrt{ed}}+\frac{\frac{\frac{5}{2}e\,x^{3}+\frac{7}{2}dx}{\left(e\,x^{2}+d\right)^{2}}+\frac{7\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2\sqrt{ed}}}{8d^{3}}$	64
risch	$\frac{\frac{5e\,x^{3}}{16d^{3}}+\frac{7x}{16d^{2}}}{\left(e\,x^{2}+d\right)^{2}}-\frac{7\ln\left(-ex-\sqrt{-ed}\right)}{32\sqrt{-ed}\,d^{3}}+\frac{7\ln\left(ex-\sqrt{-ed}\right)}{32\sqrt{-ed}\,d^{3}}+\frac{\ln\left(ex+\sqrt{ed}\right)}{16\sqrt{ed}\,d^{3}}-\frac{\ln\left(-ex+\sqrt{ed}\right)}{16\sqrt{ed}\,d^{3}}$	118

input
```
int(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d^3/(e*d)^(1/2)*arctanh(e*x/(e*d)^(1/2))+1/8/d^3*((5/2*e*x^3+7/2*d*x)/
(e*x^2+d)^2+7/2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(65) = 130$.

Time = 0.27 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.12

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx$$
$$= \left[\frac{5\,de^2x^3 + 7\,d^2ex + 7\,(e^2x^4 + 2\,dex^2 + d^2)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right) + (e^2x^4 + 2\,dex^2 + d^2)\sqrt{de}\log\left(\frac{ex^2+2\sqrt{de}x+}{ex^2-d}\right.}{16\,(d^4e^3x^4 + 2\,d^5e^2x^2 + d^6e)}\right.$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/16*(5*d*e^2*x^3 + 7*d^2*e*x + 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(d*e)*a
rctan(sqrt(d*e)*x/d) + (e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(d*e)*log((e*x^2 +
2*sqrt(d*e)*x + d)/(e*x^2 - d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e), 1/
32*(10*d*e^2*x^3 + 14*d^2*e*x - 4*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-d*e)*a
rctan(sqrt(-d*e)*x/d) - 7*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(-d*e)*log((e*x^
2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^4*e^3*x^4 + 2*d^5*e^2*x^2 + d^6*e
)]
```

3.194.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(82) = 164.

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.89

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = -\frac{\sqrt{\frac{1}{d^7e}}\log\left(-\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371}-\frac{351d^4\sqrt{\frac{1}{d^7e}}}{371}+x\right)}{16} + \frac{\sqrt{\frac{1}{d^7e}}\log\left(\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371}+\frac{351d^4\sqrt{\frac{1}{d^7e}}}{371}+x\right)}{16} - \frac{7\sqrt{-\frac{1}{d^7e}}\log\left(-\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106}-\frac{351d^4\sqrt{-\frac{1}{d^7e}}}{106}+x\right)}{32} + \frac{7\sqrt{-\frac{1}{d^7e}}\log\left(\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106}+\frac{351d^4\sqrt{-\frac{1}{d^7e}}}{106}+x\right)}{32} - \frac{-7dx-5ex^3}{16d^5+32d^4ex^2+16d^3e^2x^4}$$

input
```
integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

output
```
-sqrt(1/(d**7*e))*log(-20*d**11*e*(1/(d**7*e))**(3/2)/371 - 351*d**4*sqrt(
1/(d**7*e))/371 + x)/16 + sqrt(1/(d**7*e))*log(20*d**11*e*(1/(d**7*e))**(3
/2)/371 + 351*d**4*sqrt(1/(d**7*e))/371 + x)/16 - 7*sqrt(-1/(d**7*e))*log(
-245*d**11*e*(-1/(d**7*e))**(3/2)/106 - 351*d**4*sqrt(-1/(d**7*e))/106 + x
)/32 + 7*sqrt(-1/(d**7*e))*log(245*d**11*e*(-1/(d**7*e))**(3/2)/106 + 351*
d**4*sqrt(-1/(d**7*e))/106 + x)/32 - (-7*d*x - 5*e*x**3)/(16*d**5 + 32*d**
4*e*x**2 + 16*d**3*e**2*x**4)
```

3.194.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = \frac{7\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\sqrt{de}d^3} - \frac{\arctan\left(\frac{ex}{\sqrt{-de}}\right)}{8\sqrt{-de}d^3} + \frac{5ex^3+7dx}{16(ex^2+d)^2d^3}$$

input
```
integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
7/16*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3) - 1/8*arctan(e*x/sqrt(-d*e))/(s
qrt(-d*e)*d^3) + 1/16*(5*e*x^3 + 7*d*x)/((e*x^2 + d)^2*d^3)
```

3.194.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx = \frac{\frac{7\,x}{16\,d^2}+\frac{5\,e\,x^3}{16\,d^3}}{d^2+2\,d\,e\,x^2+e^2\,x^4} + \frac{\mathrm{atanh}\left(\frac{x\,\sqrt{d^7\,e}}{d^4}\right)\,\sqrt{d^7\,e}}{8\,d^7\,e} - \frac{7\,\mathrm{atanh}\left(\frac{x\,\sqrt{-d^7\,e}}{d^4}\right)\,\sqrt{-d^7\,e}}{16\,d^7\,e}$$

input
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)^2),x)
```

output
```
((7*x)/(16*d^2) + (5*e*x^3)/(16*d^3))/(d^2 + e^2*x^4 + 2*d*e*x^2) + (atanh
((x*(d^7*e)^(1/2))/d^4)*(d^7*e)^(1/2))/(8*d^7*e) - (7*atanh((x*(-d^7*e)^(1
/2))/d^4)*(-d^7*e)^(1/2))/(16*d^7*e)
```

3.194. $\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx$

3.195 $\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx$

3.195.1 Optimal result

Integrand size = 26, antiderivative size = 62

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

output
```
-arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(1/2)+arctanh(x*2^(1/2)*e^(1/2)/(e*x
^2+d)^(1/2))*2^(1/2)/e^(1/2)
```

3.195.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx=\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right)+\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{\sqrt{e}}$$

input
```
Integrate[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

output
```
(Sqrt[2]*ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2])/(Sqrt[2]*d)] + Lo
g[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/Sqrt[e]
```

3.195.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1388, 301, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx$$

$$\Big\downarrow \; 1388$$

$$\int \frac{\sqrt{d+ex^2}}{d-ex^2}dx$$

$$\Big\downarrow \; 301$$

$$2d\int \frac{1}{(d-ex^2)\sqrt{ex^2+d}}dx-\int \frac{1}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \; 224$$

$$2d\int \frac{1}{(d-ex^2)\sqrt{ex^2+d}}dx-\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}$$

$$\Big\downarrow \; 219$$

$$2d\int \frac{1}{(d-ex^2)\sqrt{ex^2+d}}dx-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

$$\Big\downarrow \; 291$$

$$2d\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

input
```
Int[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

output
```
-(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[e]) + (Sqrt[2]*ArcTanh[(Sqrt[2
]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e]
```

3.195.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.195.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}\,\sqrt{2}}{2x\sqrt{e}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)}{\sqrt{e}}$	50
default	Expression too large to display	1356

input
```
int((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
(2^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/2))-arctanh((e*x^2+d)^
(1/2)/x/e^(1/2)))/e^(1/2)
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.21

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \left[\frac{\sqrt{2}\sqrt{e}\log\left(\frac{17\,e^2x^4+14\,dex^2+d^2+\frac{4\,\sqrt{2}\left(3\,e^2x^3+dex\right)\sqrt{ex^2+d}}{\sqrt{e}}}{e^2x^4-2\,dex^2+d^2}\right)+2\,\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{\right.}}{4\,e}\right.$$

$$\left.-\frac{\sqrt{2}e\sqrt{-\frac{1}{e}}\arctan\left(\frac{\sqrt{2}(3\,ex^2+d)\sqrt{ex^2+d}\sqrt{-\frac{1}{e}}}{4\,(ex^3+dx)}\right)-2\,\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2\,e}\right]$$

input
```
integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + d^2 + 4*sqrt(2)*(3*e^
2*x^3 + d*e*x)*sqrt(e*x^2 + d)/sqrt(e))/(e^2*x^4 - 2*d*e*x^2 + d^2)) + 2*s
qrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/e, -1/2*(sqrt(2)*e
*sqrt(-1/e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-1/e)/(e
*x^3 + d*x)) - 2*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e]
```

3.195.6 Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = -\int \frac{\sqrt{d+ex^2}}{-d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

output
```
-Integral(sqrt(d + e*x**2)/(-d + e*x**2), x)
```

3.195.7 Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \int -\frac{(ex^2+d)^{\frac{3}{2}}}{e^2x^4-d^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
-integrate((e*x^2 + d)^(3/2)/(e^2*x^4 - d^2), x)
```

3.195.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(46) = 92$.

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \frac{\sqrt{2}d\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{2\sqrt{e}|d|} + \frac{\log\left(\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2\right)}{2\sqrt{e}}$$

input
```
integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*d*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d)
 - 6*d)/abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(
sqrt(e)*abs(d)) + 1/2*log((sqrt(e)*x - sqrt(e*x^2 + d))^2)/sqrt(e)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{d^2-e^2\,x^4}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x)
```

output
```
int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4), x)
```

3.196 $\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx$

3.196.1 Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

output
```
1/2*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d*2^(1/2)/e^(1/2)
```

3.196.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \frac{\operatorname{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right)}{\sqrt{2}d\sqrt{e}}$$

input
```
Integrate[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
```

output
```
ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2])/(Sqrt[2]*d)]/(Sqrt[2]*d*Sq
rt[e])
```

3.196.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1388, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{(d-ex^2)\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{d-\frac{2dex^2}{d+ex^2}}d\frac{x}{\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}d\sqrt{e}}$$

input
```
Int[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
```

output
```
ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

3.196.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.196.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}\,\sqrt{2}}{2x\sqrt{e}}\right)}{2d\sqrt{e}}$
default	$-\frac{e\left(\sqrt{\left(x-\frac{\sqrt{ed}}{e}\right)^{2}e+2\sqrt{ed}\left(x-\frac{\sqrt{ed}}{e}\right)+2d}+\frac{\sqrt{ed}\,\ln\left(\frac{\sqrt{ed}+e\left(x-\frac{\sqrt{ed}}{e}\right)}{\sqrt{e}}+\sqrt{\left(x-\frac{\sqrt{ed}}{e}\right)^{2}e+2\sqrt{ed}\left(x-\frac{\sqrt{ed}}{e}\right)+2d}\right)}{\sqrt{e}}-\sqrt{d}\,\sqrt{2}\,\ln\left(\frac{4d}{}\right.\right.}{2\left(\sqrt{ed}-\sqrt{-ed}\right)\left(\sqrt{ed}+\sqrt{-ed}\right)\sqrt{ed}}$

input
```
int((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*2^(1/2)/e^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/2))
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.63

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \left[\frac{\sqrt{2}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}(3\,ex^3+dx)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right)}{8\,d\sqrt{e}}, -\frac{\sqrt{2}\sqrt{-e}\arctan\left(\frac{\sqrt{2}(3\,ex^2+d)\sqrt{ex^2+d}\sqrt{-e}}{4\,(e^2x^3+dex)}\right)}{4\,de}\right]$$

input
```
integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/8*sqrt(2)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt
(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2))/(d*sqrt(e)), -1/4*
sqrt(2)*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)
/(e^2*x^3 + d*e*x))/(d*e)]
```

3.196.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = -\int \frac{1}{-d\sqrt{d+ex^2}+ex^2\sqrt{d+ex^2}}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

output
```
-Integral(1/(-d*sqrt(d + e*x**2) + e*x**2*sqrt(d + e*x**2)), x)
```

3.196.7 Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \int -\frac{\sqrt{ex^2+d}}{e^2x^4-d^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(e*x^2 + d)/(e^2*x^4 - d^2), x)
```

3.196.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(29) = 58$.

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{4\sqrt{e}|d|}$$

input
```
integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d) -
 6*d)/abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(sq
rt(e)*abs(d))
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx=\int \frac{\sqrt{e\,x^2+d}}{d^2-e^2\,x^4}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4),x)
```

output
```
int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4), x)
```

3.197 $\int \frac{1}{\sqrt{d+ex^2}(d^2-e^2x^4)}\,dx$

3.197.1 Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx=\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}$$

output
```
1/4*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d^2*2^(1/2)/e^(1/2)+1/2*x/d
^2/(e*x^2+d)^(1/2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx=\frac{\frac{2x}{\sqrt{d+ex^2}}+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right)}{\sqrt{e}}}{4d^2}$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

output
```
((2*x)/Sqrt[d + e*x^2] + (Sqrt[2]*ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d +
e*x^2])/(Sqrt[2]*d)])/Sqrt[e])/(4*d^2)
```

3.197.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1388, 296, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}\left(d^2-e^2x^4\right)}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{(d-ex^2)(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{296}$$

$$\frac{\int \frac{1}{(d-ex^2)\sqrt{ex^2+d}}dx}{2d}+\frac{x}{2d^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2d}+\frac{x}{2d^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d^2\sqrt{e}}+\frac{x}{2d^2\sqrt{d+ex^2}}$$

input
```
Int[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

output
```
x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(
2*Sqrt[2]*d^2*Sqrt[e])
```

3.197.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.197.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}\,\sqrt{2}}{2x\sqrt{e}}\right)\sqrt{e\,x^{2}+d}+2x\sqrt{e}}{4\sqrt{e\,x^{2}+d}\,\sqrt{e}\,d^{2}}$
default	$\frac{e\sqrt{2}\,\ln\left(\frac{4d+2\sqrt{ed}\left(x-\frac{\sqrt{ed}}{e}\right)+2\sqrt{2}\,\sqrt{d}\,\sqrt{\left(x-\frac{\sqrt{ed}}{e}\right)^{2}e+2\sqrt{ed}\left(x-\frac{\sqrt{ed}}{e}\right)+2d}}{x-\frac{\sqrt{ed}}{e}}\right)}{4\left(\sqrt{ed}-\sqrt{-ed}\right)\left(\sqrt{ed}+\sqrt{-ed}\right)\sqrt{ed}\,\sqrt{d}}-\frac{e\sqrt{2}\,\ln\left(\frac{4d-2\sqrt{ed}\left(x+\frac{\sqrt{ed}}{e}\right)+2\sqrt{2}\,\sqrt{d}\,\sqrt{\left(x+\frac{\sqrt{e}}{e}\right.}}{x+\frac{\sqrt{ed}}{e}}\right.}{4\left(\sqrt{ed}-\sqrt{-ed}\right)\left(\sqrt{ed}+\sqrt{-e}\right.}$

input
```
int(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/2))*(e*x^2+d)^(1/2
)+2*x*e^(1/2))/(e*x^2+d)^(1/2)/e^(1/2)/d^2
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.43

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = \left[\frac{\sqrt{2}(ex^2+d)\sqrt{e}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}(3\,ex^3+dx)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right)+8\,\sqrt{ex^2+d}ex}{16\,(d^2e^2x^2+d^3e)}, -\frac{\sqrt{2}(ex^2+d)\sqrt{-e}\arctan\left(\frac{\sqrt{2}(3\,ex^2+d)\sqrt{ex^2+d}\sqrt{-e}}{4\,(e^2x^3+dex)}\right)-4\,\sqrt{ex^2+d}ex}{8\,(d^2e^2x^2+d^3e)}\right]$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/16*(sqrt(2)*(e*x^2 + d)*sqrt(e)*log((17*e^2*x^4 + 14*d*e*x^2 + 4*sqrt(2
)*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^4 - 2*d*e*x^2 + d^
2)) + 8*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e), -1/8*(sqrt(2)*(e*x^2 +
 d)*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^
2*x^3 + d*e*x)) - 4*sqrt(e*x^2 + d)*e*x)/(d^2*e^2*x^2 + d^3*e)]
```

3.197.6 Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = -\int \frac{1}{-d^2\sqrt{d+ex^2}+e^2x^4\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

output
```
-Integral(1/(-d**2*sqrt(d + e*x**2) + e**2*x**4*sqrt(d + e*x**2)), x)
```

3.197.7 Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = \int -\frac{1}{(e^2x^4-d^2)\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
-integrate(1/((e^2*x^4 - d^2)*sqrt(e*x^2 + d)), x)
```

3.197.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(45) = 90$.

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.66

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = \frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{8d\sqrt{e}|d|} + \frac{x}{2\sqrt{ex^2+d}d^2}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d) -
 6*d)/abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(d*
sqrt(e)*abs(d)) + 1/2*x/(sqrt(e*x^2 + d)*d^2)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx = \int \frac{1}{(d^2-e^2\,x^4)\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(1/2)), x)
```

3.198 $\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx$

3.198.1 Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}d^3\sqrt{e}}$$

output
```
1/6*x/d^2/(e*x^2+d)^(3/2)+1/8*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d
^3*2^(1/2)/e^(1/2)+7/12*x/d^3/(e*x^2+d)^(1/2)
```

3.198.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = \frac{\frac{2(9dx+7ex^3)}{(d+ex^2)^{3/2}} + \frac{3\sqrt{2}\operatorname{arctanh}\left(\frac{d-ex^2+\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{2}d}\right)}{\sqrt{e}}}{24d^3}$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

output
```
((2*(9*d*x + 7*e*x^3))/(d + e*x^2)^(3/2) + (3*Sqrt[2]*ArcTanh[(d - e*x^2 +
 Sqrt[e]*x*Sqrt[d + e*x^2])/(Sqrt[2]*d)])/Sqrt[e])/(24*d^3)
```

3.198.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1388, 316, 25, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{(d-ex^2)(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{x}{6d^2(d+ex^2)^{3/2}}-\frac{\int -\frac{e(5d-2ex^2)}{(d-ex^2)(ex^2+d)^{3/2}}dx}{6d^2e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e(5d-2ex^2)}{(d-ex^2)(ex^2+d)^{3/2}}dx}{6d^2e}+\frac{x}{6d^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5d-2ex^2}{(d-ex^2)(ex^2+d)^{3/2}}dx}{6d^2}+\frac{x}{6d^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{7x}{2d\sqrt{d+ex^2}}-\frac{\int -\frac{3d^2e}{(d-ex^2)\sqrt{ex^2+d}}dx}{2d^2e}}{6d^2}+\frac{x}{6d^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3}{2}\int \frac{1}{(d-ex^2)\sqrt{ex^2+d}}dx+\frac{7x}{2d\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{3}{2}\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{7x}{2d\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow 221$$

$$\frac{\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}d\sqrt{e}}+\frac{7x}{2d\sqrt{d+ex^2}}}{6d^2}+\frac{x}{6d^2\left(d+ex^2\right)^{3/2}}$$

input
```
Int[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

output
```
x/(6*d^2*(d + e*x^2)^(3/2)) + ((7*x)/(2*d*Sqrt[d + e*x^2]) + (3*ArcTanh[(S
qrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[2]*d*Sqrt[e]))/(6*d^2)
```

3.198.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.198.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{14e^{\frac{3}{2}}x^3+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}\,\sqrt{2}}{2x\sqrt{e}}\right)\left(e\,x^2+d\right)^{\frac{3}{2}}+18\sqrt{e}\,dx}{24\sqrt{e}\left(e\,x^2+d\right)^{\frac{3}{2}}d^3}$
default	$-\frac{e\left(\frac{1}{2d\sqrt{\left(x-\frac{\sqrt{ed}}{e}\right)^2 e+2\sqrt{ed}\left(x-\frac{\sqrt{ed}}{e}\right)+2d}}-\frac{\sqrt{ed}\left(2e\left(x-\frac{\sqrt{ed}}{e}\right)+2\sqrt{ed}\right)}{4d^2e\sqrt{\left(x-\frac{\sqrt{ed}}{e}\right)^2 e+2\sqrt{ed}\left(x-\frac{\sqrt{ed}}{e}\right)+2d}}-\frac{\sqrt{2}\ln\left(\frac{4d+2\sqrt{ed}\left(x-\frac{\sqrt{ed}}{e}\right)+2\sqrt{2}\sqrt{d}\sqrt{(\cdots}}{x-\cdots}\right.}{4d^{\frac{3}{2}}\cdots}\cdots\right.}{2\left(\sqrt{ed}-\sqrt{-ed}\right)\left(\sqrt{ed}+\sqrt{-ed}\right)\sqrt{ed}\cdots}$

input
```
int(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(14*e^(3/2)*x^3+3*2^(1/2)*arctanh(1/2*(e*x^2+d)^(1/2)/x*2^(1/2)/e^(1/
2))*(e*x^2+d)^(3/2)+18*e^(1/2)*d*x)/e^(1/2)/(e*x^2+d)^(3/2)/d^3
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(60) = 120.

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.49

$$\int \frac{1}{(d+ex^2)^{3/2}\,(d^2-e^2x^4)}\,dx = \left[\frac{3\,\sqrt{2}(e^2x^4+2\,dex^2+d^2)\sqrt{e}\log\left(\frac{17\,e^2x^4+14\,dex^2+4\,\sqrt{2}(3\,ex^3+dx)\sqrt{ex^2+d}\sqrt{e}+d^2}{e^2x^4-2\,dex^2+d^2}\right)}{96\,(d^3e^3x^4+2\,d^4e^2x^2+d^5e)}\right.$$
$$\left.-\frac{3\,\sqrt{2}(e^2x^4+2\,dex^2+d^2)\sqrt{-e}\arctan\left(\frac{\sqrt{2}(3\,ex^2+d)\sqrt{ex^2+d}\sqrt{-e}}{4\,(e^2x^3+dex)}\right)-4\,(7\,e^2x^3+9\,dex)\sqrt{ex^2+d}}{48\,(d^3e^3x^4+2\,d^4e^2x^2+d^5e)}\right]$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

output
```
[1/96*(3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e)*log((17*e^2*x^4 + 14*
d*e*x^2 + 4*sqrt(2)*(3*e*x^3 + d*x)*sqrt(e*x^2 + d)*sqrt(e) + d^2)/(e^2*x^
4 - 2*d*e*x^2 + d^2)) + 8*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*
x^4 + 2*d^4*e^2*x^2 + d^5*e), -1/48*(3*sqrt(2)*(e^2*x^4 + 2*d*e*x^2 + d^2)
*sqrt(-e)*arctan(1/4*sqrt(2)*(3*e*x^2 + d)*sqrt(e*x^2 + d)*sqrt(-e)/(e^2*x
^3 + d*e*x)) - 4*(7*e^2*x^3 + 9*d*e*x)*sqrt(e*x^2 + d))/(d^3*e^3*x^4 + 2*d
^4*e^2*x^2 + d^5*e)]
```

3.198.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)^{3/2}\,(d^2-e^2x^4)}\,dx = -\int \frac{1}{-d^3\sqrt{d+ex^2}-d^2ex^2\sqrt{d+ex^2}+de^2x^4\sqrt{d+ex^2}+e^3x^6\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

output
```
-Integral(1/(-d**3*sqrt(d + e*x**2) - d**2*e*x**2*sqrt(d + e*x**2) + d*e**
2*x**4*sqrt(d + e*x**2) + e**3*x**6*sqrt(d + e*x**2)), x)
```

3.198.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = \int -\frac{1}{(e^2x^4-d^2)(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

output
```
-integrate(1/((e^2*x^4 - d^2)*(e*x^2 + d)^(3/2)), x)
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = \frac{x\left(\frac{7\,ex^2}{d^3}+\frac{9}{d^2}\right)}{12\,(ex^2+d)^{\frac{3}{2}}} + \frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\,\sqrt{2}|d|-6\,d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\,\sqrt{2}|d|-6\,d\right|}\right)}{16\,d^2\sqrt{e}|d|}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

output
```
1/12*x*(7*e*x^2/d^3 + 9/d^2)/(e*x^2 + d)^(3/2) + 1/16*sqrt(2)*log(abs(2*(s
qrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d) - 6*d)/abs(2*(sqrt(e)*x -
 sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(d^2*sqrt(e)*abs(d))
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx = \int \frac{1}{(d^2-e^2\,x^4)\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(3/2)), x)
```

3.199 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

3.199.1 Optimal result

Integrand size = 28, antiderivative size = 153

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = -\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
-1/4*x*(-b*x^2+a)*(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)-9/8*a*x*(-b*x^2+a)*
(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/8*a^2*arctan(x*b^(1/2)/(-b*x^2+a)^
(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

3.199.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.72 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = -\frac{(11ax+2bx^3)\sqrt{a^2-b^2x^4}}{8\sqrt{a+bx^2}} + \frac{19ia^2\log\left(-2i\sqrt{b}x+\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

output
```
-1/8*((11*a*x + 2*b*x^3)*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2] + (((19*I)/8
)*a^2*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqr
t[b]
```

3.199.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1396, 318, 25, 27, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\sqrt{a^2-b^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{\left(bx^2+a\right)^2}{\sqrt{a-bx^2}}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(-\frac{\int -\frac{ab\left(9bx^2+5a\right)}{\sqrt{a-bx^2}}dx}{4b}-\frac{1}{4}x\sqrt{a-bx^2}\left(a+bx^2\right)\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\int \frac{ab\left(9bx^2+5a\right)}{\sqrt{a-bx^2}}dx}{4b}-\frac{1}{4}x\sqrt{a-bx^2}\left(a+bx^2\right)\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{1}{4}a\int \frac{9bx^2+5a}{\sqrt{a-bx^2}}dx-\frac{1}{4}x\sqrt{a-bx^2}\left(a+bx^2\right)\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{1}{4}a\left(\frac{19}{2}a\int \frac{1}{\sqrt{a-bx^2}}dx-\frac{9}{2}x\sqrt{a-bx^2}\right)-\frac{1}{4}x\sqrt{a-bx^2}\left(a+bx^2\right)\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{1}{4}a\left(\frac{19}{2}a\int\frac{1}{\frac{bx^2}{a-bx^2}+1}d\frac{x}{\sqrt{a-bx^2}}-\frac{9}{2}x\sqrt{a-bx^2}\right)-\frac{1}{4}x\sqrt{a-bx^2}(a+bx^2)\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{1}{4}a\left(\frac{19a\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}}-\frac{9}{2}x\sqrt{a-bx^2}\right)-\frac{1}{4}x\sqrt{a-bx^2}(a+bx^2)\right)}{\sqrt{a^2-b^2x^4}}$$

input
```
Int[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*(-1/4*(x*Sqrt[a - b*x^2]*(a + b*x^2)) + (
a*((-9*x*Sqrt[a - b*x^2])/2 + (19*a*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(
2*Sqrt[b])))/4))/Sqrt[a^2 - b^2*x^4]
```

3.199.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.199. $\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.199.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\sqrt{-b^{2} x^{4}+a^{2}}\, \left(-2 b^{\frac{3}{2}} x^{3} \sqrt{-b \,x^{2}+a}-11 a x \sqrt{-b \,x^{2}+a}\, \sqrt{b}+19 \arctan \left(\frac{\sqrt{b}\, x}{\sqrt{-b \,x^{2}+a}}\right) a^{2}\right)}{8 \sqrt{b \,x^{2}+a}\, \sqrt{-b \,x^{2}+a}\, \sqrt{b}}$	96
risch	$-\frac{x \left(2 b \,x^{2}+11 a \right) \sqrt{-b \,x^{2}+a}\, \sqrt{\frac{-b^{2} x^{4}+a^{2}}{b \,x^{2}+a}}\, \sqrt{b \,x^{2}+a}}{8 \sqrt{-b^{2} x^{4}+a^{2}}}+\frac{19 a^{2} \arctan \left(\frac{\sqrt{b}\, x}{\sqrt{-b \,x^{2}+a}}\right) \sqrt{\frac{-b^{2} x^{4}+a^{2}}{b \,x^{2}+a}}\, \sqrt{b \,x^{2}+a}}{8 \sqrt{b}\, \sqrt{-b^{2} x^{4}+a^{2}}}$	143

input
```
int((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-b^2*x^4+a^2)^(1/2)*(-2*b^(3/2)*x^3*(-b*x^2+a)^(1/2)-11*a*x*(-b*x^2+a
)^(1/2)*b^(1/2)+19*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*a^2)/(b*x^2+a)^(1/2)
/(-b*x^2+a)^(1/2)/b^(1/2)
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \left[-\frac{19\,(a^2bx^2+a^3)\sqrt{-b}\log\left(-\frac{2\,b^2x^4+abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{bx^2+a}\right)+2\,\sqrt{-b^2x^4+a^2}(2}{16\,(b^2x^2+ab)}\right.$$

$$\left.-\frac{19\,(a^2bx^2+a^3)\sqrt{b}\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right)+\sqrt{-b^2x^4+a^2}(2\,b^2x^3+11\,abx)\sqrt{bx^2+a}}{8\,(b^2x^2+ab)}\right]$$

input
```
integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(19*(a^2*b*x^2 + a^3)*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-
b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a)) + 2*sqrt(-b^
2*x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*sqrt(b*x^2 + a))/(b^2*x^2 + a*b), -1/8
*(19*(a^2*b*x^2 + a^3)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)
*sqrt(b)/(b^2*x^3 + a*b*x)) + sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*
sqrt(b*x^2 + a))/(b^2*x^2 + a*b)]
```

3.199.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{\sqrt{-(-a+bx^2)(a+bx^2)}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.199.7 Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.199.8 Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2),x)
```

output
```
int((a + b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.200 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx$

3.200.1 Optimal result

Integrand size = 28, antiderivative size = 110

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
-1/2*x*(-b*x^2+a)*(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/2*a*arctan(x*b^(1
/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a
^2)^(1/2)
```

3.200.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = -\frac{x\sqrt{a^2-b^2x^4}}{2\sqrt{a+bx^2}} + \frac{3ia\log\left(-2i\sqrt{b}x+\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

output
```
-1/2*(x*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2] + (((3*I)/2)*a*Log[(-2*I)*Sqr
t[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]
```

3.200.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1396, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{bx^2+a}{\sqrt{a-bx^2}}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3}{2}a\int \frac{1}{\sqrt{a-bx^2}}dx-\frac{1}{2}x\sqrt{a-bx^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3}{2}a\int \frac{1}{\frac{bx^2}{a-bx^2}+1}d\frac{x}{\sqrt{a-bx^2}}-\frac{1}{2}x\sqrt{a-bx^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3a\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}}-\frac{1}{2}x\sqrt{a-bx^2}\right)}{\sqrt{a^2-b^2x^4}}$$

input
```
Int[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*(-1/2*(x*Sqrt[a - b*x^2]) + (3*a*ArcTan[(
Sqrt[b]*x)/Sqrt[a - b*x^2]])/(2*Sqrt[b])))/Sqrt[a^2 - b^2*x^4]
```

3.200.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] && !IntegerQ[p] && !(EqQ[q, 1] && EqQ[n, 2])
```

3.200.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\sqrt{-b^{2} x^{4}+a^{2}}\,\left(-x \sqrt{b}\,\sqrt{-b\,x^{2}+a}+3 \arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^{2}+a}}\right) a\right)}{2 \sqrt{b\,x^{2}+a}\,\sqrt{-b\,x^{2}+a}\,\sqrt{b}}$	75
risch	$-\frac{x \sqrt{-b\,x^{2}+a}\,\sqrt{\frac{-b^{2} x^{4}+a^{2}}{b\,x^{2}+a}}\,\sqrt{b\,x^{2}+a}}{2 \sqrt{-b^{2} x^{4}+a^{2}}}+\frac{3 a \arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^{2}+a}}\right) \sqrt{\frac{-b^{2} x^{4}+a^{2}}{b\,x^{2}+a}}\,\sqrt{b\,x^{2}+a}}{2 \sqrt{b}\,\sqrt{-b^{2} x^{4}+a^{2}}}$	131

input
```
int((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b^2*x^4+a^2)^(1/2)*(-x*b^(1/2)*(-b*x^2+a)^(1/2)+3*arctan(b^(1/2)*x/(
-b*x^2+a)^(1/2))*a)/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/b^(1/2)
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.03

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \left[-\frac{2\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\,bx+3\,(abx^2+a^2)\sqrt{-b}\log\left(-\frac{2\,b^2x^4+abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{}}{bx^2+a}\right.}{4\,(b^2x^2+ab)}\right.$$
$$\left.-\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\,bx+3\,(abx^2+a^2)\sqrt{b}\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right)}{2\,(b^2x^2+ab)}\right]$$

input
```
integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt
(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sq
rt(-b)*x - a^2)/(b*x^2 + a)))/(b^2*x^2 + a*b), -1/2*(sqrt(-b^2*x^4 + a^2)*
sqrt(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2
)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(b^2*x^2 + a*b)]
```

3.200.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{-(-a+bx^2)\,(a+bx^2)}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.200.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.200.8 Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2),x)
```

output
```
int((a + b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.201 $\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx$

3.201.1 Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2
)/(-b^2*x^4+a^2)^(1/2)
```

3.201.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx=\frac{i\log\left(-2i\sqrt{b}x+\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

output
```
(I*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b
]
```

3.201.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1396, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{\sqrt{a-bx^2}}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{\frac{bx^2}{a-bx^2}+1}d\frac{x}{\sqrt{a-bx^2}}}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

input
```
Int[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqr
t[b]*Sqrt[a^2 - b^2*x^4])
```

3.201.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.201.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{-b^{2} x^{4}+a^{2}}\, \arctan\left(\frac{\sqrt{b}\, x}{\sqrt{-b\, x^{2}+a}}\right)}{\sqrt{b\, x^{2}+a}\, \sqrt{-b\, x^{2}+a}\, \sqrt{b}}$	54

input
```
int((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)/(-b*x^2+a)^(1/2)/b^(1/2)*arctan(b^(
1/2)*x/(-b*x^2+a)^(1/2))
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-\frac{2\,b^2x^4+abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{bx^2+a}\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{b^2x^3+abx}\right)}{\sqrt{b}}\right]$$

input
```
integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x
^2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a))/b, -arctan(sqrt(-b^2*x^4 + a^2)*sqr
t(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x))/sqrt(b)]
```

3.201.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.201.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

3.201.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.202 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx$

3.202.1 Optimal result

Integrand size = 28, antiderivative size = 78

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
1/2*arctan(x*2^(1/2)*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^
(1/2)/a*2^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a^2-b^2x^4}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a^2 - b^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt[2]
*a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

3.202.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1396, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{\sqrt{a-bx^2}(bx^2+a)}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{\frac{2abx^2}{a-bx^2}+a}d\frac{x}{\sqrt{a-bx^2}}}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2
]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

3.202.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.202.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(63) = 126$.

Time = 0.59 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.19

method	result
default	$\frac{\sqrt{-b^{2} x^{4}+a^{2}}\,\sqrt{b}\left(\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2 b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^{2}+a}+\sqrt{-a b}\,x+a\right)}{b x+\sqrt{-a b}}\right)\sqrt{b}-\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2 b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^{2}+a}-\sqrt{-a b}\,x+a\right)}{b x-\sqrt{-a b}}\right)\sqrt{b}+2\arctan\left(\ldots\right.}{2\sqrt{b\,x^{2}+a}\,\sqrt{-b\,x^{2}+a}\,\left(-\sqrt{-a b}+\sqrt{a b}\right)\left(\sqrt{-a b}+\sqrt{a b}\right)\sqrt{-a b}}$

input
```
int(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*
(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*b^(1/2)-a^(1/2)*2^(
1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b
)^(1/2)))*b^(1/2)+2*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1
/2)))^(1/2))*(-a*b)^(1/2)-2*(-a*b)^(1/2)*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2)
))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/(-(-a*b)^(1/2)+(a*b)^(1/2))/((-a*b)^(1
/2)+(a*b)^(1/2))/(-a*b)^(1/2)
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.95

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx = \left[-\frac{\sqrt{2}\sqrt{-b}\log\left(-\frac{3\,b^2x^4+2\,abx^2-2\,\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{-b}x-a^2}{b^2x^4+2\,abx^2+a^2}\right)}{4\,ab}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{2\,(b^2x^3+abx)}\right)}{2\,a\sqrt{b}}\right]$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x
^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2))/(
a*b), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)
*sqrt(b)/(b^2*x^3 + a*b*x))/(a*sqrt(b))]
```

3.202.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\sqrt{a+bx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a + b*x**2)), x)
```

3.202.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)
```

3.202.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{a^2-b^2\,x^4}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(1/2)), x)
```

3.203 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

3.203.1 Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
1/4*x*(-b*x^2+a)/a^2/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/8*arctan(x*2^(
1/2)*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^2*2^(1/2
)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a-bx^2}+3\sqrt{2}(a+bx^2)\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)\right)}{8a^2\sqrt{b}\sqrt{a-bx^2}\,(a+bx^2)^{3/2}}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2] + 3*Sqrt[2]*(a + b*x^2)*
ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(8*a^2*Sqrt[b]*Sqrt[a - b*x^
2]*(a + b*x^2)^(3/2))
```

3.203.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1396, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{\sqrt{a-bx^2}(bx^2+a)^2}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{296}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3\int \frac{1}{\sqrt{a-bx^2}(bx^2+a)}dx}{4a}+\frac{x\sqrt{a-bx^2}}{4a^2(a+bx^2)}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3\int \frac{1}{\frac{2abx^2}{a-bx^2}+a}d\frac{x}{\sqrt{a-bx^2}}}{4a}+\frac{x\sqrt{a-bx^2}}{4a^2(a+bx^2)}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}}+\frac{x\sqrt{a-bx^2}}{4a^2(a+bx^2)}\right)}{\sqrt{a^2-b^2x^4}}$$

input
```
Int[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*((x*Sqrt[a - b*x^2])/(4*a^2*(a + b*x^2))
+ (3*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(4*Sqrt[2]*a^2*Sqrt[b]))
)/Sqrt[a^2 - b^2*x^4]
```

3.203.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.203.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(101) = 202$.

Time = 0.23 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.90

method	result
default	$-\frac{\sqrt{-b^{2} x^{4}+a^{2}}\, b^{\frac{5}{2}} \left(3 \ln \left(\frac{2 b \left(\sqrt{2}\, \sqrt{a}\, \sqrt{-b \,x^{2}+a}-\sqrt{-a b}\, x+a\right)}{b x-\sqrt{-a b}}\right) \sqrt{2}\, b^{\frac{3}{2}} x^{2} \sqrt{a}-3 \ln \left(\frac{2 b \left(\sqrt{2}\, \sqrt{a}\, \sqrt{-b \,x^{2}+a}+\sqrt{-a b}\, x+a\right)}{b x+\sqrt{-a b}}\right) \sqrt{2}\, b^{\frac{3}{2}} x^{2} \sqrt{a}+3 \ln \left(\right.\right.}{}$

input
```
int(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1
/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*b^(3/2)*x^2*a^(1/2)-3*ln
(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)
))*2^(1/2)*b^(3/2)*x^2*a^(1/2)+3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-
(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*a^(3/2)*b^(1/2)-3*ln(2*b*(2^
(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*2^(1/
2)*a^(3/2)*b^(1/2)+4*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*b*x^2*(-a*b)^(1/2)
-4*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2)))^(1/2))*b*x^
2*(-a*b)^(1/2)-4*(-b*x^2+a)^(1/2)*b^(1/2)*(-a*b)^(1/2)*x+4*arctan(b^(1/2)*
x/(-b*x^2+a)^(1/2))*a*(-a*b)^(1/2)-4*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/
2))*(b*x+(a*b)^(1/2)))^(1/2))*a*(-a*b)^(1/2))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(
1/2)/(-(-a*b)^(1/2)+(a*b)^(1/2))^2/((-a*b)^(1/2)+(a*b)^(1/2))^2/(b*x+(-a*b
)^(1/2))/(b*x-(-a*b)^(1/2))/(-a*b)^(1/2)
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.38

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \left[\frac{4\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}bx - 3\sqrt{2}(b^2x^4+2abx^2+a^2)\sqrt{-b}\log\left(-\frac{3b^2x^4+2}{}\right.}{16\,(a^2b^3x^4+2a^3b^2x^2+a^4b)}\right.$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2
*a*b*x^2 + a^2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2
*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2))
)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(
b*x^2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b)*arctan(1/2*
sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(
a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)]
```

3.203.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-(-a+bx^2)(a+bx^2)}(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(3/2)), x)
```

3.203.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

3.203.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{a^2-b^2\,x^4}\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(3/2)),x)
```

output
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(3/2)), x)
```

3.204 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

3.204.1 Optimal result

Integrand size = 28, antiderivative size = 168

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
1/8*x*(-b*x^2+a)/a^2/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)+9/32*x*(-b*x^2+a
)/a^3/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/64*arctan(x*2^(1/2)*b^(1/2)/
(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^3*2^(1/2)/b^(1/2)/(-b
^2*x^4+a^2)^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 2.69 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a-bx^2}(13a+9bx^2)+19\sqrt{2}(a+bx^2)^2\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)\right)}{64a^3\sqrt{b}\sqrt{a-bx^2}\,(a+bx^2)^{5/2}}$$

input
```
Integrate[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2]*(13*a + 9*b*x^2) + 19*Sq
rt[2]*(a + b*x^2)^2*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(64*a^3*
Sqrt[b]*Sqrt[a - b*x^2]*(a + b*x^2)^(5/2))
```

3.204.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1396, 316, 25, 27, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{\sqrt{a-bx^2}(bx^2+a)^3}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{x\sqrt{a-bx^2}}{8a^2(a+bx^2)^2}-\frac{\int -\frac{b(7a-2bx^2)}{\sqrt{a-bx^2}(bx^2+a)^2}dx}{8a^2b}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\int \frac{b(7a-2bx^2)}{\sqrt{a-bx^2}(bx^2+a)^2}dx}{8a^2b}+\frac{x\sqrt{a-bx^2}}{8a^2(a+bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\int \frac{7a-2bx^2}{\sqrt{a-bx^2}(bx^2+a)^2}dx}{8a^2}+\frac{x\sqrt{a-bx^2}}{8a^2(a+bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\frac{9x\sqrt{a-bx^2}}{4a(a+bx^2)}-\frac{\int -\frac{19a^2b}{\sqrt{a-bx^2}(bx^2+a)}dx}{4a^2b}}{8a^2}+\frac{x\sqrt{a-bx^2}}{8a^2(a+bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\frac{19}{4}\int \frac{1}{\sqrt{a-bx^2}(bx^2+a)}dx+\frac{9x\sqrt{a-bx^2}}{4a(a+bx^2)}}{8a^2}+\frac{x\sqrt{a-bx^2}}{8a^2(a+bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \; 291$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\frac{19}{4}\int \frac{1}{\frac{2abx^2}{a-bx^2}+a}d\frac{x}{\sqrt{a-bx^2}}+\frac{9x\sqrt{a-bx^2}}{4a(a+bx^2)}}{8a^2}+\frac{x\sqrt{a-bx^2}}{8a^2(a+bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \; 218$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\frac{19\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}a\sqrt{b}}+\frac{9x\sqrt{a-bx^2}}{4a(a+bx^2)}}{8a^2}+\frac{x\sqrt{a-bx^2}}{8a^2(a+bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

input
```
Int[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*((x*Sqrt[a - b*x^2])/(8*a^2*(a + b*x^2)^2
) + ((9*x*Sqrt[a - b*x^2])/(4*a*(a + b*x^2)) + (19*ArcTan[(Sqrt[2]*Sqrt[b]
*x)/Sqrt[a - b*x^2]])/(4*Sqrt[2]*a*Sqrt[b]))/(8*a^2)))/Sqrt[a^2 - b^2*x^4]
```

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.204. $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

3.204.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(138) = 276$.

Time = 0.23 (sec) , antiderivative size = 711, normalized size of antiderivative = 4.23

method	result
default	$-\frac{\sqrt{-b^2x^4+a^2}\, b^{\frac{9}{2}}\left(19\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^2+a}-\sqrt{-ab}\,x+a\right)}{bx-\sqrt{-ab}}\right)b^{\frac{5}{2}}x^4\sqrt{a}-19\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^2+a}+\sqrt{-ab}\,x+a\right)}{bx+\sqrt{-ab}}\right)b^{\frac{5}{2}}x^4\sqrt{a}+38}{}$

input
```
int(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-b^2*x^4+a^2)^(1/2)*b^(9/2)*(19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b
*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*b^(5/2)*x^4*a^(1/2)-19
*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(
-a*b)^(1/2)))*b^(5/2)*x^4*a^(1/2)+38*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x
^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*a^(3/2)*b^(3/2)*x^2-38*2
^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a
*b)^(1/2)))*a^(3/2)*b^(3/2)*x^2+16*arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*b^2*
x^4*(-a*b)^(1/2)-16*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1
/2)))^(1/2))*b^2*x^4*(-a*b)^(1/2)-36*b^(3/2)*(-a*b)^(1/2)*(-b*x^2+a)^(1/2)
*x^3+19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)
/(b*x-(-a*b)^(1/2)))*a^(5/2)*b^(1/2)-19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-
b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*a^(5/2)*b^(1/2)+32*ar
ctan(b^(1/2)*x/(-b*x^2+a)^(1/2))*a*b*x^2*(-a*b)^(1/2)-32*arctan(b^(1/2)*x/
(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a*b)^(1/2)))^(1/2))*a*b*x^2*(-a*b)^(1/2)-52*
b^(1/2)*(-a*b)^(1/2)*a*(-b*x^2+a)^(1/2)*x+16*arctan(b^(1/2)*x/(-b*x^2+a)^(
1/2))*a^2*(-a*b)^(1/2)-16*arctan(b^(1/2)*x/(1/b*(-b*x+(a*b)^(1/2))*(b*x+(a
*b)^(1/2)))^(1/2))*a^2*(-a*b)^(1/2))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/(-(-
a*b)^(1/2)+(a*b)^(1/2))^3/((-a*b)^(1/2)+(a*b)^(1/2))^3/(b*x+(-a*b)^(1/2))^
2/(b*x-(-a*b)^(1/2))^2/(-a*b)^(1/2)
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.17

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \left[-\frac{19\sqrt{2}(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-b}\log\left(-\frac{3\,b^2x^4+2\,abx^2-2\,\sqrt{2}\sqrt{-b^2x^4}}{b^2x^4+2\,abx^2}\right.}{128\,(a^3b^4x^6+3\,a^4b^3x^4+}\right.$$

$$-\frac{19\sqrt{2}(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{bx^2+a}\sqrt{b}}{2\,(b^2x^3+abx)}\right)-2\,\sqrt{-b^2x^4+a^2}(9\,b^2x^3+13\,abx)}{64\,(a^3b^4x^6+3\,a^4b^3x^4+3\,a^5b^2x^2+a^6b)}$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/128*(19*sqrt(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-b)*l
og(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a
)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(-b^2*x^4 + a^2)*
(9*b^2*x^3 + 13*a*b*x)*sqrt(b*x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a
^5*b^2*x^2 + a^6*b), -1/64*(19*sqrt(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^
2 + a^3)*sqrt(b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*s
qrt(b)/(b^2*x^3 + a*b*x)) - 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 + 13*a*b*x)*
sqrt(b*x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 + a^6*b)]
```

3.204.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-(-a+bx^2)\,(a+bx^2)}\,(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(5/2)), x)
```

3.204.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

3.204.8 Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{a^2-b^2\,x^4}\,(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(5/2)),x)
```

output
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(5/2)), x)
```

3.205 $\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

3.205.1 Optimal result

Integrand size = 29, antiderivative size = 152

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = -\frac{9ax\sqrt{a-bx^2}(a+bx^2)}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
-1/4*x*(-b*x^2+a)^(3/2)*(b*x^2+a)/(-b^2*x^4+a^2)^(1/2)-9/8*a*x*(b*x^2+a)*(
-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^
(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{1}{8}\left(\frac{x(-11a+2bx^2)\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}} - \frac{19a^2\log(-a+bx^2)}{\sqrt{b}} + \frac{19a^2\log\left(abx-b^2x^3+\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}\right)}{\sqrt{b}}\right)$$

input
```
Integrate[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

output
```
((x*(-11*a + 2*b*x^2)*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2] - (19*a^2*Log[-
a + b*x^2])/Sqrt[b] + (19*a^2*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2
]*Sqrt[a^2 - b^2*x^4]])/Sqrt[b])/8
```

3.205.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1396, 318, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}dx$$

$$\Big\downarrow \ 1396$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{(a-bx^2)^2}{\sqrt{bx^2+a}}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \ 318$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\int \frac{ab(5a-9bx^2)}{\sqrt{bx^2+a}}dx}{4b}-\frac{1}{4}x(a-bx^2)\sqrt{a+bx^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{1}{4}a\int \frac{5a-9bx^2}{\sqrt{bx^2+a}}dx-\frac{1}{4}x(a-bx^2)\sqrt{a+bx^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \ 299$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{1}{4}a\left(\frac{19}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{9}{2}x\sqrt{a+bx^2}\right)-\frac{1}{4}x(a-bx^2)\sqrt{a+bx^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \ 224$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{1}{4}a\left(\frac{19}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{9}{2}x\sqrt{a+bx^2}\right)-\frac{1}{4}x(a-bx^2)\sqrt{a+bx^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{1}{4}a\left(\frac{19a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\frac{9}{2}x\sqrt{a+bx^2}\right)-\frac{1}{4}x\left(a-bx^2\right)\sqrt{a+bx^2}\right)}{\sqrt{a^2-b^2x^4}}$$

input
```
Int[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*(-1/4*(x*(a - b*x^2)*Sqrt[a + b*x^2]) + (
a*((-9*x*Sqrt[a + b*x^2])/2 + (19*a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
(2*Sqrt[b])))/4))/Sqrt[a^2 - b^2*x^4]
```

3.205.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.205. $\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.205.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.62

method	result	siz
default	$\frac{\sqrt{-b^{2} x^{4}+a^{2}}\, \left(2 b^{\frac{3}{2}} x^{3} \sqrt{b \,x^{2}+a}-11 a x \sqrt{b}\, \sqrt{b \,x^{2}+a}+19 \ln \left(\sqrt{b}\, x +\sqrt{b \,x^{2}+a}\right) a^{2}\right)}{8 \sqrt{-b \,x^{2}+a}\, \sqrt{b \,x^{2}+a}\, \sqrt{b}}$	94
risch	$\frac{x \left(-2 b \,x^{2}+11 a \right) \sqrt{b \,x^{2}+a}\, \sqrt{\frac{\left(-b \,x^{2}+a \right) \left(-b^{2} x^{4}+a^{2}\right)}{\left(b \,x^{2}-a \right)^{2}}}\, \left(b \,x^{2}-a \right)}{8 \sqrt{-b \,x^{2}+a}\, \sqrt{-b^{2} x^{4}+a^{2}}}-\frac{19 a^{2} \ln \left(\sqrt{b}\, x +\sqrt{b \,x^{2}+a}\right) \sqrt{\frac{\left(-b \,x^{2}+a \right) \left(-b^{2} x^{4}+a^{2}\right)}{\left(b \,x^{2}-a \right)^{2}}}\, \left(b \,x^{2}-a \right)}{8 \sqrt{b}\, \sqrt{-b \,x^{2}+a}\, \sqrt{-b^{2} x^{4}+a^{2}}}$	18

input
```
int((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-b^2*x^4+a^2)^(1/2)*(2*b^(3/2)*x^3*(b*x^2+a)^(1/2)-11*a*x*b^(1/2)*(b*
x^2+a)^(1/2)+19*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*a^2)/(-b*x^2+a)^(1/2)/(b*x^2
+a)^(1/2)/b^(1/2)
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.74

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \left[\frac{19\,(a^2bx^2-a^3)\sqrt{b}\log\left(\frac{2\,b^2x^4-abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{bx^2-a}\right)-2\,\sqrt{-b^2x^4+a^2}(2\,b^2x^3}{16\,(b^2x^2-ab)}\right.$$

input
```
integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(19*(a^2*b*x^2 - a^3)*sqrt(b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2
*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b*x^2 - a)) - 2*sqrt(-b^2*x
^4 + a^2)*(2*b^2*x^3 - 11*a*b*x)*sqrt(-b*x^2 + a))/(b^2*x^2 - a*b), 1/8*(1
9*(a^2*b*x^2 - a^3)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*
sqrt(-b)/(b^2*x^3 - a*b*x)) - sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 - 11*a*b*x)*
sqrt(-b*x^2 + a))/(b^2*x^2 - a*b)]
```

3.205.6 Sympy [F]

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(a-bx^2)^{\frac{5}{2}}}{\sqrt{-(-a+bx^2)(a+bx^2)}}\,dx$$

input
```
integrate((-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.205.7 Maxima [F]

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.205.8 Giac [F]

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx=\int \frac{(a-b\,x^2)^{5/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

input
```
int((a - b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2),x)
```

output
```
int((a - b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.206 $\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx$

3.206.1 Optimal result

Integrand size = 29, antiderivative size = 109

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = -\frac{x\sqrt{a-bx^2}(a+bx^2)}{2\sqrt{a^2-b^2x^4}} + \frac{3a\sqrt{a-bx^2}\sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
-1/2*x*(b*x^2+a)*(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/2*a*arctanh(x*b^(
1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a
^2)^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{1}{2}\left(-\frac{x\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}} - \frac{3a\log(-a+bx^2)}{\sqrt{b}} + \frac{3a\log\left(abx-b^2x^3+\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}\right)}{\sqrt{b}}\right)$$

input
```
Integrate[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

output
```
(-((x*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2]) - (3*a*Log[-a + b*x^2])/Sqrt[b
] + (3*a*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]
])/Sqrt[b])/2
```

3.206.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1396, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{a-bx^2}{\sqrt{bx^2+a}}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{1}{2}x\sqrt{a+bx^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{1}{2}x\sqrt{a+bx^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\frac{1}{2}x\sqrt{a+bx^2}\right)}{\sqrt{a^2-b^2x^4}}$$

input
```
Int[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*(-1/2*(x*Sqrt[a + b*x^2]) + (3*a*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/Sqrt[a^2 - b^2*x^4]
```

3.206.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.206.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\sqrt{-b^2x^4+a^2}\left(-x\sqrt{b}\,\sqrt{b\,x^2+a}+3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)a\right)}{2\sqrt{-b\,x^2+a}\,\sqrt{b\,x^2+a}\,\sqrt{b}}$	74
risch	$\frac{x\sqrt{b\,x^2+a}\,\sqrt{\frac{\left(-b\,x^2+a\right)\left(-b^2x^4+a^2\right)}{\left(b\,x^2-a\right)^2}}\,\left(b\,x^2-a\right)}{2\sqrt{-b\,x^2+a}\,\sqrt{-b^2x^4+a^2}}-\frac{3a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)\sqrt{\frac{\left(-b\,x^2+a\right)\left(-b^2x^4+a^2\right)}{\left(b\,x^2-a\right)^2}}\,\left(b\,x^2-a\right)}{2\sqrt{b}\,\sqrt{-b\,x^2+a}\,\sqrt{-b^2x^4+a^2}}$	170

input
```
int((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b^2*x^4+a^2)^(1/2)*(-x*b^(1/2)*(b*x^2+a)^(1/2)+3*ln(b^(1/2)*x+(b*x^2
+a)^(1/2))*a)/(-b*x^2+a)^(1/2)/(b*x^2+a)^(1/2)/b^(1/2)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.17

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \left[\frac{2\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}bx+3\,(abx^2-a^2)\sqrt{b}\log\left(\frac{2\,b^2x^4-abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x}{bx^2-a}\right.}{4\,(b^2x^2-ab)}\right.$$

input
```
integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt
(b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqr
t(b)*x - a^2)/(b*x^2 - a)))/(b^2*x^2 - a*b), 1/2*(sqrt(-b^2*x^4 + a^2)*sqr
t(-b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)
*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/(b^2*x^2 - a*b)]
```

3.206.6 Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}}{\sqrt{-(-a+bx^2)(a+bx^2)}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.206.7 Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.206.8 Giac [F]

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{2}}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{(a-b\,x^2)^{3/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

input
```
int((a - b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2),x)
```

output
```
int((a - b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.207 $\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx$

3.207.1 Optimal result

Integrand size = 29, antiderivative size = 64

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2
)/(-b^2*x^4+a^2)^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{-\log\left(-a+bx^2\right)+\log\left(abx-b^2x^3+\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}\right)}{\sqrt{b}}$$

input
```
Integrate[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

output
```
(-Log[-a + b*x^2] + Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2
 - b^2*x^4]])/Sqrt[b]
```

3.207.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1396, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{\sqrt{bx^2+a}}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

input
```
Int[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sq
rt[b]*Sqrt[a^2 - b^2*x^4])
```

3.207.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.207.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\sqrt{-b^{2} x^{4}+a^{2}}\, \ln\left(\sqrt{b}\, x+\sqrt{b\, x^{2}+a}\right)}{\sqrt{-b\, x^{2}+a}\, \sqrt{b\, x^{2}+a}\, \sqrt{b}}$	54

input
```
int((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)/(b*x^2+a)^(1/2)*ln(b^(1/2)*x+(b*x^
2+a)^(1/2))/b^(1/2)
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \left[\frac{\log\left(\frac{2\,b^2x^4-abx^2-2\,\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{bx^2-a}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{-b}}{b^2x^3-abx}\right)}{b}\right]$$

input
```
integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sq
rt(b)*x - a^2)/(b*x^2 - a))/sqrt(b), sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*
sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x))/b]
```

3.207.6 Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{a-bx^2}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

3.207.7 Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

3.207.8 Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-b^2x^4+a^2}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \int \frac{\sqrt{a-b\,x^2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

input
```
int((a - b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2),x)
```

output
```
int((a - b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.208 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx$

3.208.1 Optimal result

Integrand size = 29, antiderivative size = 77

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
1/2*arctanh(x*2^(1/2)*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^
(1/2)/a*2^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx=\frac{\sqrt{a^2-b^2x^4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a^2 - b^2*x^4]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[2
]*a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

3.208.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1396, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{(a-bx^2)\sqrt{bx^2+a}}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{a-\frac{2abx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

input
```
Int[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

3.208.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.208.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.32

method	result
default	$\frac{\sqrt{-b^2x^4+a^2}\,\sqrt{b}\,\left(\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b\,x^2+a}-x\sqrt{ab}+a\right)}{bx+\sqrt{ab}}\right)\sqrt{b}-\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b\,x^2+a}+x\sqrt{ab}+a\right)}{bx-\sqrt{ab}}\right)\sqrt{b}+2\sqrt{ab}\,\ln\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}}{\sqrt{b}}\right.\right.}{2\sqrt{-b\,x^2+a}\,\sqrt{b\,x^2+a}\,\sqrt{ab}\left(\sqrt{-ab}-\sqrt{ab}\right)\left(\sqrt{-ab}+\sqrt{ab}\right)}$

input
```
int(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*
(b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)))*b^(1/2)-a^(1/2)*2^(1/2
)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2
)))*b^(1/2)+2*(a*b)^(1/2)*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))-2*ln((
b^(1/2)*(-1/b*(-b*x+(-a*b)^(1/2))*(b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*
(a*b)^(1/2))/(-b*x^2+a)^(1/2)/(b*x^2+a)^(1/2)/(a*b)^(1/2)/((-a*b)^(1/2)-(a
*b)^(1/2))/((-a*b)^(1/2)+(a*b)^(1/2))
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.01

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{3\,b^2x^4-2\,abx^2-2\,\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{b^2x^4-2\,abx^2+a^2}\right)}{4\,a\sqrt{b}}, \frac{\sqrt{2}\sqrt{-b}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{-b}}{2\,(b^2x^3-abx)}\right)}{2\,ab}\right]$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(2)*log(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*
sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2))/(a*sqrt(b))
, 1/2*sqrt(2)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2
 + a)*sqrt(-b)/(b^2*x^3 - a*b*x))/(a*b)]
```

3.208.6 Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-(-a+bx^2)(a+bx^2)}\sqrt{a-bx^2}}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a - b*x**2)), x)
```

3.208.7 Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)
```

3.208.8 Giac [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,\sqrt{a-b\,x^2}}\,dx$$

input
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(1/2)),x)
```

output
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(1/2)), x)
```

3.209 $\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

3.209.1 Optimal result

Integrand size = 29, antiderivative size = 124

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
1/4*x*(b*x^2+a)/a^2/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/8*arctanh(x*2^
(1/2)*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^2*2^(1/2
)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x\sqrt{a+bx^2}+3\sqrt{2}(a-bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)}{8a^2\sqrt{b}\,(a-bx^2)^{3/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a + b*x^2] + 3*Sqrt[2]*(a - b*x^2)*
ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(8*a^2*Sqrt[b]*(a - b*x^2)^
(3/2)*Sqrt[a + b*x^2])
```

3.209.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1396, 296, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{(a-bx^2)^2\sqrt{bx^2+a}}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{296}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3\int \frac{1}{(a-bx^2)\sqrt{bx^2+a}}dx}{4a}+\frac{x\sqrt{a+bx^2}}{4a^2(a-bx^2)}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3\int \frac{1}{a-\frac{2abx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{4a}+\frac{x\sqrt{a+bx^2}}{4a^2(a-bx^2)}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a^2\sqrt{b}}+\frac{x\sqrt{a+bx^2}}{4a^2(a-bx^2)}\right)}{\sqrt{a^2-b^2x^4}}$$

input
```
Int[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*((x*Sqrt[a + b*x^2])/(4*a^2*(a - b*x^2))
+ (3*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(4*Sqrt[2]*a^2*Sqrt[b])
))/Sqrt[a^2 - b^2*x^4]
```

3.209.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.209.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(100) = 200$.

Time = 0.23 (sec) , antiderivative size = 499, normalized size of antiderivative = 4.02

method	result
default	$-\frac{\sqrt{-b^{2} x^{4}+a^{2}}\, b^{\frac{5}{2}} \left(-3 \sqrt{2}\, \ln \left(\frac{2 b \left(\sqrt{2}\, \sqrt{a}\, \sqrt{b\, x^{2}+a}+x \sqrt{a b}+a\right)}{b x -\sqrt{a b}}\right) b^{\frac{3}{2}} x^{2} \sqrt{a}+3 \sqrt{2}\, \ln \left(\frac{2 b \left(\sqrt{2}\, \sqrt{a}\, \sqrt{b\, x^{2}+a}-x \sqrt{a b}+a\right)}{b x +\sqrt{a b}}\right) b^{\frac{3}{2}} x^{2} \sqrt{a}+3 \sqrt{2}\, \ln \left(\right.}{}$

input
```
int(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(-3*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x
^2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))*b^(3/2)*x^2*a^(1/2)+3*2^(1
/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1
/2)))*b^(3/2)*x^2*a^(1/2)+3*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2
)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))*a^(3/2)*b^(1/2)-3*2^(1/2)*ln(2*b*(2^
(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)))*a^(3/2)*
b^(1/2)+4*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*b*x^2*(a*b)^(1/2)-4*ln
((b^(1/2)*(-1/b*(-b*x+(-a*b)^(1/2))*(b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2)
)*b*x^2*(a*b)^(1/2)+4*(b*x^2+a)^(1/2)*b^(1/2)*(a*b)^(1/2)*x-4*ln((b^(1/2)*
(b*x^2+a)^(1/2)+b*x)/b^(1/2))*a*(a*b)^(1/2)+4*ln((b^(1/2)*(-1/b*(-b*x+(-a*
b)^(1/2))*(b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a*(a*b)^(1/2))/(-b*x^2+a
)^(1/2)/(b*x^2+a)^(1/2)/(-(-a*b)^(1/2)+(a*b)^(1/2))^2/((-a*b)^(1/2)+(a*b)^
(1/2))^2/(b*x-(a*b)^(1/2))/(b*x+(a*b)^(1/2))/(a*b)^(1/2)
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.44

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \left[\frac{4\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}bx+3\sqrt{2}(b^2x^4-2abx^2+a^2)\sqrt{b}\log\left(-\frac{3b^2x^4-2}{}\right.}{16\left(a^2b^3x^4-2a^3b^2x^2+a^4b\right)}\right.$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 -
2*a*b*x^2 + a^2)*sqrt(b)*log(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2
*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2))
)/(a^2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(
-b*x^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(-b)*arctan(1/
2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)
))/(a^2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b)]
```

3.209.6 Sympy [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-(-a+bx^2)(a+bx^2)}\,(a-bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(3/2)), x)
```

3.209.7 Maxima [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

3.209.8 Giac [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}(-bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{a^2-b^2\,x^4}\,(a-b\,x^2)^{3/2}}\,dx$$

input
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(3/2)),x)
```

output
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(3/2)), x)
```

3.210 $\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

3.210.1 Optimal result

Integrand size = 29, antiderivative size = 167

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

output
```
1/8*x*(b*x^2+a)/a^2/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)+9/32*x*(b*x^2+a)
/a^3/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/64*arctanh(x*2^(1/2)*b^(1/2)
/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^3*2^(1/2)/b^(1/2)/(-b
^2*x^4+a^2)^(1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}x(13a-9bx^2)\sqrt{a+bx^2}+19\sqrt{2}(a-bx^2)^2\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)}{64a^3\sqrt{b}(a-bx^2)^{5/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*(13*a - 9*b*x^2)*Sqrt[a + b*x^2] + 19*Sq
rt[2]*(a - b*x^2)^2*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(64*a^3
*Sqrt[b]*(a - b*x^2)^(5/2)*Sqrt[a + b*x^2])
```

3.210.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1396, 316, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\int \frac{1}{(a-bx^2)^3\sqrt{bx^2+a}}dx}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\int \frac{b(2bx^2+7a)}{(a-bx^2)^2\sqrt{bx^2+a}}dx}{8a^2b}+\frac{x\sqrt{a+bx^2}}{8a^2(a-bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\int \frac{2bx^2+7a}{(a-bx^2)^2\sqrt{bx^2+a}}dx}{8a^2}+\frac{x\sqrt{a+bx^2}}{8a^2(a-bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\frac{\int \frac{19a^2b}{(a-bx^2)\sqrt{bx^2+a}}dx}{4a^2b}+\frac{9x\sqrt{a+bx^2}}{4a(a-bx^2)}}{8a^2}+\frac{x\sqrt{a+bx^2}}{8a^2(a-bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\frac{19}{4}\int \frac{1}{(a-bx^2)\sqrt{bx^2+a}}dx+\frac{9x\sqrt{a+bx^2}}{4a(a-bx^2)}}{8a^2}+\frac{x\sqrt{a+bx^2}}{8a^2(a-bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \ 291$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\frac{19}{4}\int \frac{1}{a-\frac{2abx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{9x\sqrt{a+bx^2}}{4a(a-bx^2)}}{8a^2}+\frac{x\sqrt{a+bx^2}}{8a^2(a-bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

$$\Big\downarrow \ 221$$

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\left(\frac{\frac{19\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}a\sqrt{b}}+\frac{9x\sqrt{a+bx^2}}{4a(a-bx^2)}}{8a^2}+\frac{x\sqrt{a+bx^2}}{8a^2(a-bx^2)^2}\right)}{\sqrt{a^2-b^2x^4}}$$

input
```
Int[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

output
```
(Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*((x*Sqrt[a + b*x^2])/(8*a^2*(a - b*x^2)^2
) + ((9*x*Sqrt[a + b*x^2])/(4*a*(a - b*x^2)) + (19*ArcTanh[(Sqrt[2]*Sqrt[b
]*x)/Sqrt[a + b*x^2]])/(4*Sqrt[2]*a*Sqrt[b]))/(8*a^2)))/Sqrt[a^2 - b^2*x^4
]
```

3.210.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.210.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(137) = 274$.

Time = 0.24 (sec) , antiderivative size = 728, normalized size of antiderivative = 4.36

method	result
default	$-\frac{\sqrt{-b^2x^4+a^2}\, b^{\frac{9}{2}} \left(19\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\sqrt{a}\sqrt{b\,x^2+a}+x\sqrt{ab}+a\right)}{bx-\sqrt{ab}}\right) b^{\frac{5}{2}} x^4\sqrt{a}-19\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\sqrt{a}\sqrt{b\,x^2+a}-x\sqrt{ab}+a\right)}{bx+\sqrt{ab}}\right) b^{\frac{5}{2}} x^4\sqrt{a}-38\sqrt{2}\,\ln\left(\right.\right.}{}$

input
```
int(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-b^2*x^4+a^2)^(1/2)*b^(9/2)*(19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*
x^2+a)^(1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))*b^(5/2)*x^4*a^(1/2)-19*2^
(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^
(1/2)))*b^(5/2)*x^4*a^(1/2)-38*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(
1/2)+x*(a*b)^(1/2)+a)/(b*x-(a*b)^(1/2)))*a^(3/2)*b^(3/2)*x^2+38*2^(1/2)*ln
(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)))*
a^(3/2)*b^(3/2)*x^2+16*ln((b^(1/2)*(-1/b*(-b*x+(-a*b)^(1/2))*(b*x+(-a*b)^(
1/2)))^(1/2)+b*x)/b^(1/2))*b^2*x^4*(a*b)^(1/2)-16*ln((b^(1/2)*(b*x^2+a)^(1
/2)+b*x)/b^(1/2))*b^2*x^4*(a*b)^(1/2)-36*b^(3/2)*(a*b)^(1/2)*(b*x^2+a)^(1/
2)*x^3+19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+x*(a*b)^(1/2)+a)
/(b*x-(a*b)^(1/2)))*a^(5/2)*b^(1/2)-19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*
x^2+a)^(1/2)-x*(a*b)^(1/2)+a)/(b*x+(a*b)^(1/2)))*a^(5/2)*b^(1/2)-32*ln((b^
(1/2)*(-1/b*(-b*x+(-a*b)^(1/2))*(b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a*
b*x^2*(a*b)^(1/2)+32*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*a*b*x^2*(a*
b)^(1/2)+52*b^(1/2)*(a*b)^(1/2)*a*(b*x^2+a)^(1/2)*x+16*ln((b^(1/2)*(-1/b*(
-b*x+(-a*b)^(1/2))*(b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a^2*(a*b)^(1/2)
-16*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*a^2*(a*b)^(1/2))/(-b*x^2+a)^
(1/2)/(b*x^2+a)^(1/2)/((-a*b)^(1/2)-(a*b)^(1/2))^3/((-a*b)^(1/2)+(a*b)^(1/
2))^3/(b*x+(a*b)^(1/2))^2/(b*x-(a*b)^(1/2))^2/(a*b)^(1/2)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.25

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \left[\frac{19\sqrt{2}(b^3x^6-3ab^2x^4+3a^2bx^2-a^3)\sqrt{b}\log\left(-\frac{3b^2x^4-2abx^2-2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{\ }}{b^2x^4-2abx^2+a^2}\right.}{128\left(a^3b^4x^6-3a^4b^3x^4+3\right.}\right.$$

input
```
integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/128*(19*sqrt(2)*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3)*sqrt(b)*log
(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)
*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2)) + 4*sqrt(-b^2*x^4 + a^2)*(9
*b^2*x^3 - 13*a*b*x)*sqrt(-b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^
5*b^2*x^2 - a^6*b), 1/64*(19*sqrt(2)*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2
- a^3)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*s
qrt(-b)/(b^2*x^3 - a*b*x)) + 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 - 13*a*b*x)
*sqrt(-b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 - a^6*b)]
```

3.210.6 Sympy [F]

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-(-a+bx^2)(a+bx^2)}\,(a-bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(5/2)), x)
```

3.210.7 Maxima [F]

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

3.210.8 Giac [F]

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \int \frac{1}{\sqrt{-b^2x^4+a^2}(-bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx=\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,(a-b\,x^2)^{5/2}}\,dx$$

input
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(5/2)),x)
```

output
```
int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(5/2)), x)
```

3.211 $\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx$

3.211.1 Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx=\frac{\sqrt{-1+x^2}\sqrt{1+x^2}\text{arcsinh}(x)}{\sqrt{-1+x^4}}$$

output
```
arcsinh(x)*(x^2-1)^(1/2)*(x^2+1)^(1/2)/(x^4-1)^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx=-\log\left(1-x^2\right)+\log\left(-x+x^3+\sqrt{-1+x^2}\sqrt{-1+x^4}\right)$$

input
```
Integrate[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]
```

output
```
-Log[1 - x^2] + Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]]
```

3.211.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1396, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\int \frac{1}{\sqrt{x^2+1}}dx}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\text{arcsinh}(x)}{\sqrt{x^4-1}}$$

input
```
Int[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]
```

output
```
(Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]
```

3.211.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.211.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{x^4-1}\,\operatorname{arcsinh}(x)}{\sqrt{x^2-1}\,\sqrt{x^2+1}}$	25

input
```
int((x^2-1)^(1/2)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)
```

3.211.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(24) = 48.

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.43

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right) - \frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)$$

input
```
integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x)) - 1/2*log(-(x^3
 - sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x))
```

3.211.6 Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{(x-1)(x+1)}}{\sqrt{(x-1)(x+1)(x^2+1)}}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(x**4-1)**(1/2),x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.211.7 Maxima [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

3.211.8 Giac [F]

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2),x)
```

output
```
int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2), x)
```

3.212 $\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}} dx$

3.212.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = -\frac{\sqrt{-1+x^4}\arcsin(x)}{\sqrt{1-x^4}}$$

output
```
-arcsin(x)*(x^4-1)^(1/2)/(-x^4+1)^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = -\log\left(1+x^2\right)+\log\left(x+x^3+\sqrt{1+x^2}\sqrt{-1+x^4}\right)$$

input
```
Integrate[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
```

output
```
-Log[1 + x^2] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

3.212.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1396, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\int \frac{1}{\sqrt{x^2-1}}dx}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}$$

input
```
Int[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
```

output
```
(Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

3.212.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

3.212.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{\sqrt{x^{4}-1}\,\ln\left(x+\sqrt{x^{2}-1}\right)}{\sqrt{x^{2}+1}\,\sqrt{x^{2}-1}}$	33

input
```
int((x^2+1)^(1/2)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(x^2+1)^(1/2)*(x^4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

3.212.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(20) = 40.

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right) - \frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)$$

input
```
integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
 - sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x))
```

3.212.6 Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{x^2+1}}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}}\,dx$$

input
```
integrate((x**2+1)**(1/2)/(x**4-1)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + 1)/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.212.7 Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

3.212.8 Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

input
```
int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2),x)
```

output
```
int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2), x)
```

3.213 $\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

3.213.1 Optimal result

Integrand size = 31, antiderivative size = 73

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=-\frac{\sqrt{-1+x^4}\arcsin(x)}{\sqrt{1-x^2}\sqrt{1+x^2}}+\frac{\sqrt{-1+x^2}\sqrt{-1+x^4}\text{arcsinh}(x)}{(1-x^2)\sqrt{1+x^2}}$$

output
```
-arcsin(x)*(x^4-1)^(1/2)/(-x^2+1)^(1/2)/(x^2+1)^(1/2)+arcsinh(x)*(x^2-1)^(
1/2)*(x^4-1)^(1/2)/(-x^2+1)/(x^2+1)^(1/2)
```

3.213.2 Mathematica [A] (verified)

Time = 3.61 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\log(1-x^2)-\log(1+x^2)-\log\left(-x+x^3+\sqrt{-1+x^2}\sqrt{-1+x^4}\right)+\log\left(x+x^3+\sqrt{1+x^2}\sqrt{-1+x^4}\right)$$

input
```
Integrate[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

output
```
Log[1 - x^2] - Log[1 + x^2] - Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]
] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

3.213.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\sqrt{x^2+1}\text{arcsinh}(x)}{\sqrt{x^4-1}}$$

input
```
Int[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

output
```
-((Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]) + (Sqrt[-1 + x
^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

3.213.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.213.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{x^4-1}\,\operatorname{arcsinh}(x)}{\sqrt{x^2-1}\,\sqrt{x^2+1}}+\frac{\sqrt{x^4-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}\,\sqrt{x^2-1}}$	59

input
```
int((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)+1/(x^2+1)^(1/2)*(x
^4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

3.213.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(53) = 106.

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.88

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right) - \frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right) - \frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right) + \frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2-1}-x}{x^3-x}\right)$$

input
```
integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="frica
s")
```

output
```
1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
 - sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log((x^3 + sqrt(x^4 -
 1)*sqrt(x^2 - 1) - x)/(x^3 - x)) + 1/2*log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2
 - 1) - x)/(x^3 - x))
```

3.213.6 Sympy [F]

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{-\sqrt{x^2-1}+\sqrt{x^2+1}}{\sqrt{(x-1)(x+1)(x^2+1)}}\,dx$$

input
```
integrate((-(x**2-1)**(1/2)+(x**2+1)**(1/2))/(x**4-1)**(1/2),x)
```

output
```
Integral((-sqrt(x**2 - 1) + sqrt(x**2 + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1
)), x)
```

3.213.7 Maxima [F]

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

3.213.8 Giac [F]

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx=\int -\frac{\sqrt{x^2-1}-\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

input
```
int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2),x)
```

output
```
int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2), x)
```

3.214 $\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

3.214.1 Optimal result

Integrand size = 39, antiderivative size = 121

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{(7c^2d^2-5bcde+b^2e^2)\,x}{c^3}+\frac{e(4cd-be)x^3}{3c^2}+\frac{e^2x^5}{5c}-\frac{(2cd-be)^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}$$

output
```
(b^2*e^2-5*b*c*d*e+7*c^2*d^2)*x/c^3+1/3*e*(-b*e+4*c*d)*x^3/c^2+1/5*e^2*x^5
/c-(-b*e+2*c*d)^3*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(7/2)/e^(1
/2)/(-b*e+c*d)^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = -\frac{(-7c^2d^2+5bcde-b^2e^2)\,x}{c^3}-\frac{e(-4cd+be)x^3}{3c^2}+\frac{e^2x^5}{5c}-\frac{(-2cd+be)^3\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{c^{7/2}\sqrt{e}\sqrt{-cd+be}}$$

input
```
Integrate[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
-(((-7*c^2*d^2 + 5*b*c*d*e - b^2*e^2)*x)/c^3) - (e*(-4*c*d + b*e)*x^3)/(3*
c^2) + (e^2*x^5)/(5*c) - ((-2*c*d + b*e)^3*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt
[-(c*d) + b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

3.214.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1387, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^4}{bde+be^2x^2-cd^2+ce^2x^4}dx$$

$$\Big\downarrow \text{1387}$$

$$\int \frac{(d+ex^2)^3}{\frac{bde-cd^2}{d}+cex^2}dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{b^2e^2-5bcde+7c^2d^2}{c^3}+\frac{-b^3e^3+6b^2cde^2-12bc^2d^2e+8c^3d^3}{c^3(be-cd+cex^2)}+\frac{ex^2(4cd-be)}{c^2}+\frac{e^2x^4}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(2cd-be)^3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\sqrt{cd-be}}+\frac{x(b^2e^2-5bcde+7c^2d^2)}{c^3}+\frac{ex^3(4cd-be)}{3c^2}+\frac{e^2x^5}{5c}$$

input
```
Int[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
((7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)/c^3 + (e*(4*c*d - b*e)*x^3)/(3*c^2)
+ (e^2*x^5)/(5*c) - ((2*c*d - b*e)^3*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d
- b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

3.214.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.214.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11

method	result
default	$\frac{\frac{1}{5}e^2x^5c^2-\frac{1}{3}bc\,e^2x^3+\frac{4}{3}c^2de\,x^3+b^2e^2x-5bcdex+7c^2d^2x}{c^3}+\frac{\left(-b^3e^3+6b^2cd\,e^2-12b\,c^2d^2e+8c^3d^3\right)\arctan\left(\frac{xce}{\sqrt{(be-cd)ec}}\right)}{c^3\sqrt{(be-cd)ec}}$
risch	$\frac{e^2x^5}{5c}-\frac{b\,e^2x^3}{3c^2}+\frac{4de\,x^3}{3c}+\frac{b^2e^2x}{c^3}-\frac{5bdex}{c^2}+\frac{7d^2x}{c}-\frac{\ln\left(xce-\sqrt{-(be-cd)ec}\right)b^3e^3}{2c^3\sqrt{-(be-cd)ec}}+\frac{3\ln\left(xce-\sqrt{-(be-cd)ec}\right)b^2d\,e^2}{c^2\sqrt{-(be-cd)ec}}-$

input
```
int((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/5*e^2*x^5*c^2-1/3*b*c*e^2*x^3+4/3*c^2*d*e*x^3+b^2*e^2*x-5*b*c*d*e
*x+7*c^2*d^2*x)+(-b^3*e^3+6*b^2*c*d*e^2-12*b*c^2*d^2*e+8*c^3*d^3)/c^3/((b*
e-c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2))
```

3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(105) = 210$.

Time = 0.26 (sec) , antiderivative size = 446, normalized size of antiderivative = 3.69

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$$
$$= \left[\frac{6\,(c^4de^3-bc^3e^4)x^5+10\,(4\,c^4d^2e^2-5\,bc^3de^3+b^2c^2e^4)x^3-15\,(8\,c^3d^3-12\,bc^2d^2e+6\,b^2cde^2-b^3e^3)\sqrt{c^2}}{30\,(c^5de-bc^4e^2)}\right.$$

input
```
integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fric
as")
```

output
```
[1/30*(6*(c^4*d*e^3 - b*c^3*e^4)*x^5 + 10*(4*c^4*d^2*e^2 - 5*b*c^3*d*e^3 +
 b^2*c^2*e^4)*x^3 - 15*(8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e
^3)*sqrt(c^2*d*e - b*c*e^2)*log((c*e*x^2 + c*d - b*e + 2*sqrt(c^2*d*e - b*
c*e^2)*x)/(c*e*x^2 - c*d + b*e)) + 30*(7*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*
b^2*c^2*d*e^3 - b^3*c*e^4)*x)/(c^5*d*e - b*c^4*e^2), 1/15*(3*(c^4*d*e^3 -
b*c^3*e^4)*x^5 + 5*(4*c^4*d^2*e^2 - 5*b*c^3*d*e^3 + b^2*c^2*e^4)*x^3 - 15*
(8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*sqrt(-c^2*d*e + b*c
*e^2)*arctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) + 15*(7*c^4*d^3*e -
12*b*c^3*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e^4)*x)/(c^5*d*e - b*c^4*e^2)]
```

3.214.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(110) = 220$.

Time = 0.45 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.85

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$$
$$= x^3\left(-\frac{be^2}{3c^2}+\frac{4de}{3c}\right)+x\left(\frac{b^2e^2}{c^3}-\frac{5bde}{c^2}+\frac{7d^2}{c}\right)$$
$$+\frac{\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3\log\left(x+\frac{-bc^3e\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3+c^4d\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3}{b^3e^3-6b^2cde^2+12bc^2d^2e-8c^3d^3}\right)}{2}$$
$$-\frac{\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3\log\left(x+\frac{bc^3e\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3-c^4d\sqrt{-\frac{1}{c^7e(be-cd)}}(be-2cd)^3}{b^3e^3-6b^2cde^2+12bc^2d^2e-8c^3d^3}\right)}{2}$$
$$+\frac{e^2x^5}{5c}$$

input
```
integrate((e*x**2+d)**4/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

output
```
x**3*(-b*e**2/(3*c**2) + 4*d*e/(3*c)) + x*(b**2*e**2/c**3 - 5*b*d*e/c**2 +
 7*d**2/c) + sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log(x + (-b*c*
*3*e*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 + c**4*d*sqrt(-1/(c**7
*e*(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 + 12*b*c**
2*d**2*e - 8*c**3*d**3))/2 - sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**
3*log(x + (b*c**3*e*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 - c**4*
d*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*
e**2 + 12*b*c**2*d**2*e - 8*c**3*d**3))/2 + e**2*x**5/(5*c)
```

3.214.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxi
ma")
```

output
```
Timed out
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{\left(d+ex^2\right)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx &= \frac{\left(8\,c^3d^3-12\,bc^2d^2e+6\,b^2cde^2-b^3e^3\right)\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{\sqrt{-c^2de+bce^2}\,c^3}\\ &\quad+\frac{3\,c^4e^7x^5+20\,c^4de^6x^3-5\,bc^3e^7x^3+105\,c^4d^2e^5x-75\,bc^3de^6x+15\,b^2c^2e^7x}{15\,c^5e^5}\end{aligned}$$

input
```
integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac
")
```

output
```
(8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*arctan(c*e*x/sqrt(-
c^2*d*e + b*c*e^2))/(sqrt(-c^2*d*e + b*c*e^2)*c^3) + 1/15*(3*c^4*e^7*x^5 +
 20*c^4*d*e^6*x^3 - 5*b*c^3*e^7*x^3 + 105*c^4*d^2*e^5*x - 75*b*c^3*d*e^6*x
 + 15*b^2*c^2*e^7*x)/(c^5*e^5)
```

3.214.9 Mupad [B] (verification not implemented)

Time = 7.96 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = x\left(\frac{3\,d^2}{c}+\frac{\left(\frac{e\,(b\,e-c\,d)}{c^2}-\frac{3\,d\,e}{c}\right)\,(b\,e-c\,d)}{c\,e}\right)-x^3\left(\frac{e\,(b\,e-c\,d)}{3\,c^2}-\frac{d\,e}{c}\right)+\frac{e^2\,x^5}{5\,c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,e\,x\,(b\,e-2\,c\,d)^3}{\sqrt{b\,e^2-c\,d\,e}\,(b^3\,e^3-6\,b^2\,c\,d\,e^2+12\,b\,c^2\,d^2\,e-8\,c^3\,d^3)}\right)\,(b\,e-2\,c\,d)^3}{c^{7/2}\,\sqrt{b\,e^2-c\,d\,e}}$$

input
```
int((d + e*x^2)^4/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

output
```
x*((3*d^2)/c + (((e*(b*e - c*d))/c^2 - (3*d*e)/c)*(b*e - c*d))/(c*e)) - x^
3*((e*(b*e - c*d))/(3*c^2) - (d*e)/c) + (e^2*x^5)/(5*c) - (atan((c^(1/2)*e
*x*(b*e - 2*c*d)^3)/((b*e^2 - c*d*e)^(1/2)*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2
*d^2*e - 6*b^2*c*d*e^2)))*(b*e - 2*c*d)^3)/(c^(7/2)*(b*e^2 - c*d*e)^(1/2))
```

3.215 $\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4} dx$

3.215.1 Optimal result

Integrand size = 39, antiderivative size = 86

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4} dx = \frac{(3cd-be)x}{c^2}+\frac{ex^3}{3c}-\frac{(2cd-be)^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}$$

output
```
(-b*e+3*c*d)*x/c^2+1/3*e*x^3/c-(-b*e+2*c*d)^2*arctanh(x*c^(1/2)*e^(1/2)/(-
b*e+c*d)^(1/2))/c^(5/2)/e^(1/2)/(-b*e+c*d)^(1/2)
```

3.215.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4} dx = -\frac{(-3cd+be)x}{c^2}+\frac{ex^3}{3c}+\frac{(-2cd+be)^2\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{c^{5/2}\sqrt{e}\sqrt{-cd+be}}$$

input
```
Integrate[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
-(((-3*c*d + b*e)*x)/c^2) + (e*x^3)/(3*c) + ((-2*c*d + b*e)^2*ArcTan[(Sqrt
[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

3.215.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1387, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{bde+be^2x^2-cd^2+ce^2x^4}dx$$

$$\Big\downarrow \text{1387}$$

$$\int \frac{(d+ex^2)^2}{\frac{bde-cd^2}{d}+cex^2}dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{b^2e^2-4bcde+4c^2d^2}{c^2(be-cd+cex^2)}+\frac{3cd-be}{c^2}+\frac{ex^2}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(2cd-be)^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\sqrt{cd-be}}+\frac{x(3cd-be)}{c^2}+\frac{ex^3}{3c}$$

input
```
Int[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
((3*c*d - b*e)*x)/c^2 + (e*x^3)/(3*c) - ((2*c*d - b*e)^2*ArcTanh[(Sqrt[c]*
Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

3.215.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.215.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

method	result
default	$-\frac{-\frac{1}{3}c\,x^3e+bex-3cdx}{c^2}+\frac{(b^2e^2-4bcde+4c^2d^2)\arctan\left(\frac{xce}{\sqrt{(be-cd)ec}}\right)}{c^2\sqrt{(be-cd)ec}}$
risch	$\frac{e\,x^3}{3c}-\frac{bex}{c^2}+\frac{3dx}{c}-\frac{\ln\left(xce+\sqrt{-(be-cd)ec}\right)b^2e^2}{2c^2\sqrt{-(be-cd)ec}}+\frac{2\ln\left(xce+\sqrt{-(be-cd)ec}\right)bde}{c\sqrt{-(be-cd)ec}}-\frac{2\ln\left(xce+\sqrt{-(be-cd)ec}\right)d^2}{\sqrt{-(be-cd)ec}}+\frac{\ln\left(-x\right.}{}$

input
```
int((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(-1/3*c*x^3*e+b*e*x-3*c*d*x)+(b^2*e^2-4*b*c*d*e+4*c^2*d^2)/c^2/((b*
e-c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2))
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.62

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$$
$$= \left[\frac{2\,(c^3de^2-bc^2e^3)x^3+3\,(4\,c^2d^2-4\,bcde+b^2e^2)\sqrt{c^2de-bce^2}\log\left(\frac{cex^2+cd-be-2\,\sqrt{c^2de-bce^2}\,x}{cex^2-cd+be}\right)+6\,(3\,c^3d^2e-}{6\,(c^4de-bc^3e^2)}\right.$$

input
```
integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fric
as")
```

output
```
[1/6*(2*(c^3*d*e^2 - b*c^2*e^3)*x^3 + 3*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*
sqrt(c^2*d*e - b*c*e^2)*log((c*e*x^2 + c*d - b*e - 2*sqrt(c^2*d*e - b*c*e^
2)*x)/(c*e*x^2 - c*d + b*e)) + 6*(3*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)
*x)/(c^4*d*e - b*c^3*e^2), 1/3*((c^3*d*e^2 - b*c^2*e^3)*x^3 - 3*(4*c^2*d^2
 - 4*b*c*d*e + b^2*e^2)*sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-c^2*d*e + b
*c*e^2)*x/(c*d - b*e)) + 3*(3*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*x)/(c
^4*d*e - b*c^3*e^2)]
```

3.215.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(75) = 150$.

Time = 0.35 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.20

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$$
$$= x\left(-\frac{be}{c^2}+\frac{3d}{c}\right) - \frac{\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2\log\left(x+\frac{-bc^2e\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2+c^3d\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2}{b^2e^2-4bcde+4c^2d^2}\right)}{2} + \frac{\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2\log\left(x+\frac{bc^2e\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2-c^3d\sqrt{-\frac{1}{c^5e(be-cd)}}(be-2cd)^2}{b^2e^2-4bcde+4c^2d^2}\right)}{2} + \frac{ex^3}{3c}$$

input
```
integrate((e*x**2+d)**3/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

output
```
x*(-b*e/c**2 + 3*d/c) - sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log
(x + (-b*c**2*e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 + c**3*d*sq
rt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c
**2*d**2))/2 + sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(x + (b*c
**2*e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 - c**3*d*sqrt(-1/(c**
5*e*(b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2*d**2))
/2 + e*x**3/(3*c)
```

3.215.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-c*d)>0)', see `assume?` f
or more de
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{(4\,c^2d^2-4\,bcde+b^2e^2)\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{\sqrt{-c^2de+bce^2}c^2} + \frac{c^2e^4x^3+9\,c^2de^3x-3\,bce^4x}{3\,c^3e^3}$$

input
```
integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac
")
```

output
```
(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*arctan(c*e*x/sqrt(-c^2*d*e + b*c*e^2))/(
sqrt(-c^2*d*e + b*c*e^2)*c^2) + 1/3*(c^2*e^4*x^3 + 9*c^2*d*e^3*x - 3*b*c*e
^4*x)/(c^3*e^3)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 8.01 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = x\left(\frac{2\,d}{c}-\frac{b\,e-c\,d}{c^2}\right)+\frac{e\,x^3}{3\,c} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,e\,x\,(b\,e-2\,c\,d)^2}{\sqrt{b\,e^2-c\,d\,e}\,(b^2\,e^2-4\,b\,c\,d\,e+4\,c^2\,d^2)}\right)(b\,e-2\,c\,d)^2}{c^{5/2}\,\sqrt{b\,e^2-c\,d\,e}}$$

input
```
int((d + e*x^2)^3/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

output
```
x*((2*d)/c - (b*e - c*d)/c^2) + (e*x^3)/(3*c) + (atan((c^(1/2)*e*x*(b*e -
2*c*d)^2)/((b*e^2 - c*d*e)^(1/2)*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)))*(b*e
- 2*c*d)^2)/(c^(5/2)*(b*e^2 - c*d*e)^(1/2))
```

3.216 $\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

3.216.1 Optimal result

Integrand size = 39, antiderivative size = 64

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\frac{x}{c}-\frac{(2cd-be)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

output
```
x/c-(-b*e+2*c*d)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(3/2)/e^(1/
2)/(-b*e+c*d)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\frac{x}{c}-\frac{(-2cd+be)\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{c^{3/2}\sqrt{e}\sqrt{-cd+be}}$$

input
```
Integrate[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
x/c - ((-2*c*d + b*e)*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(
3/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

3.216.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1387, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{bde+be^2x^2-cd^2+ce^2x^4}dx$$

$$\Big\downarrow \text{1387}$$

$$\int \frac{d+ex^2}{\frac{bde-cd^2}{d}+cex^2}dx$$

$$\Big\downarrow \text{299}$$

$$\frac{(2cd-be)\int \frac{1}{cex^2-cd+be}dx}{c}+\frac{x}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{x}{c}-\frac{(2cd-be)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\sqrt{cd-be}}$$

input
```
Int[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
x/c - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^(3/2
)*Sqrt[e]*Sqrt[c*d - b*e])
```

3.216.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

3.216.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

method	result
default	$\frac{x}{c}+\frac{(-be+2cd)\arctan\left(\frac{xce}{\sqrt{(be-cd)ec}}\right)}{c\sqrt{(be-cd)ec}}$
risch	$\frac{x}{c}-\frac{\ln\left(xce-\sqrt{-(be-cd)ec}\right)be}{2c\sqrt{-(be-cd)ec}}+\frac{\ln\left(xce-\sqrt{-(be-cd)ec}\right)d}{\sqrt{-(be-cd)ec}}+\frac{\ln\left(-xce-\sqrt{-(be-cd)ec}\right)be}{2c\sqrt{-(be-cd)ec}}-\frac{\ln\left(-xce-\sqrt{-(be-cd)ec}\right)d}{\sqrt{-(be-cd)ec}}$

input
```
int((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

output
```
x/c+(-b*e+2*c*d)/c/((b*e-c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2
))
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.28

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \left[-\frac{\sqrt{c^2de-bce^2}(2\,cd-be)\log\left(\frac{cex^2+cd-be+2\,\sqrt{c^2de-bce^2}x}{cex^2-cd+be}\right)-2\,(c^2de-bce^2)x}{2\,(c^3de-bc^2e^2)},\ -\frac{\sqrt{-c^2de+bce^2}(2\,cd-be)\arctan\left(-\frac{\sqrt{-c^2de+bce^2}x}{cd-be}\right)-(c^2de-bce^2)x}{c^3de-bc^2e^2}\right]$$

input
```
integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fric
as")
```

output
```
[-1/2*(sqrt(c^2*d*e - b*c*e^2)*(2*c*d - b*e)*log((c*e*x^2 + c*d - b*e + 2*
sqrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*d + b*e)) - 2*(c^2*d*e - b*c*e^2)*
x)/(c^3*d*e - b*c^2*e^2), -(sqrt(-c^2*d*e + b*c*e^2)*(2*c*d - b*e)*arctan(
-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) - (c^2*d*e - b*c*e^2)*x)/(c^3*d*e
 - b*c^2*e^2)]
```

3.216.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(54) = 108$.

Time = 0.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.31

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)\log\left(x+\frac{-bce\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)+c^2d\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)}{be-2cd}\right)}{2} - \frac{\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)\log\left(x+\frac{bce\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)-c^2d\sqrt{-\frac{1}{c^3e(be-cd)}}(be-2cd)}{be-2cd}\right)}{2} + \frac{x}{c}$$

input
```
integrate((e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

output
```
sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(x + (-b*c*e*sqrt(-1/(c**3*
e*(b*e - c*d)))*(b*e - 2*c*d) + c**2*d*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e
- 2*c*d))/(b*e - 2*c*d))/2 - sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*l
og(x + (b*c*e*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d) - c**2*d*sqrt(-1
/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d))/(b*e - 2*c*d))/2 + x/c
```

3.216.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-c*d)>0)', see `assume?` f
or more de
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{(2\,cd-be)\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{\sqrt{-c^2de+bce^2}c}+\frac{x}{c}$$

input
```
integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac
")
```

output
```
(2*c*d - b*e)*arctan(c*e*x/sqrt(-c^2*d*e + b*c*e^2))/(sqrt(-c^2*d*e + b*c*
e^2)*c) + x/c
```

3.216.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{x}{c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,e\,x}{\sqrt{b\,e^2-c\,d\,e}}\right)\,(b\,e-2\,c\,d)}{c^{3/2}\,\sqrt{b\,e^2-c\,d\,e}}$$

input
```
int((d + e*x^2)^2/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

output
```
x/c - (atan((c^(1/2)*e*x)/(b*e^2 - c*d*e)^(1/2))*(b*e - 2*c*d))/(c^(3/2)*(
b*e^2 - c*d*e)^(1/2))
```

3.217 $\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

3.217.1 Optimal result

Integrand size = 37, antiderivative size = 49

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

output
```
-arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(1/2)/e^(1/2)/(-b*e+c*d)^(1
/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\frac{\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{-cd+be}}$$

input
```
Integrate[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[-(c*d
) + b*e])
```

3.217.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {1387, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{bde+be^2x^2-cd^2+ce^2x^4}\,dx$$
$$\Big\downarrow \text{ 1387}$$
$$\int \frac{1}{\frac{bde-cd^2}{d}+cex^2}dx$$
$$\Big\downarrow \text{ 221}$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{cd-be}}$$

input
```
Int[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
-(ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[c*d -
 b*e]))
```

3.217.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

3.217.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{xce}{\sqrt{(be-cd)ec}}\right)}{\sqrt{(be-cd)ec}}$	33
risch	$-\frac{\ln\left(xce+\sqrt{-(be-cd)ec}\right)}{2\sqrt{-(be-cd)ec}}+\frac{\ln\left(-xce+\sqrt{-(be-cd)ec}\right)}{2\sqrt{-(be-cd)ec}}$	75

input
```
int((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

output
```
1/((b*e-c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2))
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.73

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \left[\frac{\log\left(\frac{cex^2+cd-be-2\sqrt{c^2de-bce^2}x}{cex^2-cd+be}\right)}{2\sqrt{c^2de-bce^2}}, -\frac{\sqrt{-c^2de+bce^2}\arctan\left(-\frac{\sqrt{-c^2de+bce^2}x}{cd-be}\right)}{c^2de-bce^2}\right]$$

input
```
integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas
")
```

output
```
[1/2*log((c*e*x^2 + c*d - b*e - 2*sqrt(c^2*d*e - b*c*e^2)*x)/(c*e*x^2 - c*
d + b*e))/sqrt(c^2*d*e - b*c*e^2), -sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(
-c^2*d*e + b*c*e^2)*x/(c*d - b*e))/(c^2*d*e - b*c*e^2)]
```

3.217.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(44) = 88$.

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.53

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = -\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(-be\sqrt{-\frac{1}{ce(be-cd)}}+cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(be\sqrt{-\frac{1}{ce(be-cd)}}-cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}$$

input
```
integrate((e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

output
```
-sqrt(-1/(c*e*(b*e - c*d)))*log(-b*e*sqrt(-1/(c*e*(b*e - c*d))) + c*d*sqrt
(-1/(c*e*(b*e - c*d))) + x)/2 + sqrt(-1/(c*e*(b*e - c*d)))*log(b*e*sqrt(-1
/(c*e*(b*e - c*d))) - c*d*sqrt(-1/(c*e*(b*e - c*d))) + x)/2
```

3.217.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-c*d)>0)', see `assume?` f
or more de
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\frac{\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{\sqrt{-c^2de+bce^2}}$$

input
```
integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")
```

output
```
arctan(c*e*x/sqrt(-c^2*d*e + b*c*e^2))/sqrt(-c^2*d*e + b*c*e^2)
```

3.217.9 Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\frac{\mathrm{atan}\left(\frac{c\,e\,x}{\sqrt{b\,c\,e^2-c^2\,d\,e}}\right)}{\sqrt{b\,c\,e^2-c^2\,d\,e}}$$

input
```
int((d + e*x^2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

output
```
atan((c*e*x)/(b*c*e^2 - c^2*d*e)^(1/2))/(b*c*e^2 - c^2*d*e)^(1/2)
```

3.218 $\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

3.218.1 Optimal result

Integrand size = 39, antiderivative size = 136

$$\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = -\frac{x}{2d(2cd-be)(d+ex^2)} - \frac{(4cd-be)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2} - \frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^2}$$

output
```
-1/2*x/d/(-b*e+2*c*d)/(e*x^2+d)-1/2*(-b*e+4*c*d)*arctan(x*e^(1/2)/d^(1/2))
/d^(3/2)/(-b*e+2*c*d)^2/e^(1/2)-c^(3/2)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*
d)^(1/2))/(-b*e+2*c*d)^2/e^(1/2)/(-b*e+c*d)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

$$\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = -\frac{x}{2d(2cd-be)(d+ex^2)} + \frac{(-4cd+be)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}(2cd-be)^2} + \frac{c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{\sqrt{e}(-2cd+be)^2\sqrt{-cd+be}}$$

input
```
Integrate[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

output
```
-1/2*x/(d*(2*c*d - b*e)*(d + e*x^2)) + ((-4*c*d + b*e)*ArcTan[(Sqrt[e]*x)/
Sqrt[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) + (c^(3/2)*ArcTan[(Sqrt[c]*S
qrt[e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*(-2*c*d + b*e)^2*Sqrt[-(c*d) + b*e
])
```

3.218.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1387, 316, 25, 27, 397, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(bde+be^2x^2-cd^2+ce^2x^4)}dx$$

$$\Big\downarrow \text{1387}$$

$$\int \frac{1}{(d+ex^2)^2\left(\frac{bde-cd^2}{d}+cex^2\right)}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int -\frac{e(-cex^2+3cd-be)}{(ex^2+d)(-cex^2+cd-be)}dx}{2de(2cd-be)}-\frac{x}{2d(d+ex^2)(2cd-be)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e(-cex^2+3cd-be)}{(ex^2+d)(-cex^2+cd-be)}dx}{2de(2cd-be)}-\frac{x}{2d(d+ex^2)(2cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-cex^2+3cd-be}{(ex^2+d)(-cex^2+cd-be)}dx}{2d(2cd-be)}-\frac{x}{2d(d+ex^2)(2cd-be)}$$

$$\Big\downarrow \text{397}$$

$$-\frac{\frac{2c^2d\int \frac{1}{-cex^2+cd-be}dx}{2cd-be}+\frac{(4cd-be)\int \frac{1}{ex^2+d}dx}{2cd-be}}{2d(2cd-be)}-\frac{x}{2d(d+ex^2)(2cd-be)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{2c^2d\int \frac{1}{-cex^2+cd-be}dx}{2cd-be}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(4cd-be)}{\sqrt{d}\sqrt{e}(2cd-be)}}{2d(2cd-be)}-\frac{x}{2d\left(d+ex^2\right)(2cd-be)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(4cd-be)}{\sqrt{d}\sqrt{e}(2cd-be)}+\frac{2c^{3/2}d\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)}}{2d(2cd-be)}-\frac{x}{2d\left(d+ex^2\right)(2cd-be)}$$

input
```
Int[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

output
```
-1/2*x/(d*(2*c*d - b*e)*(d + e*x^2)) - (((4*c*d - b*e)*ArcTan[(Sqrt[e]*x)/
Sqrt[d]])/(Sqrt[d]*Sqrt[e]*(2*c*d - b*e)) + (2*c^(3/2)*d*ArcTanh[(Sqrt[c]*
Sqrt[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)))/(2*d
*(2*c*d - b*e))
```

3.218.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

3.218.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

method	result
default	$\frac{c^2 \arctan\left(\frac{xce}{\sqrt{(be-cd)ec}}\right)}{(be-2cd)^2\sqrt{(be-cd)ec}} + \frac{\frac{(be-2cd)x}{2d\left(e\,x^2+d\right)} + \frac{(be-4cd)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{(be-2cd)^2}$
risch	$\frac{x}{2d(be-2cd)(e\,x^2+d)} + \frac{\sqrt{-(be-cd)ec}\,c\ln\left(\left(-\sqrt{-(be-cd)ec}\,b^4e^4+10\sqrt{-(be-cd)ec}\,b^3cd\,e^3-37\sqrt{-(be-cd)ec}\,b^2c^2d^2e^2+48\sqrt{-(be\right.\right.}{}$

input
```
int(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

output
```
c^2/(b*e-2*c*d)^2/((b*e-c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2)
)+1/(b*e-2*c*d)^2*(1/2*(b*e-2*c*d)/d*x/(e*x^2+d)+1/2*(b*e-4*c*d)/d/(e*d)^(
1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 895, normalized size of antiderivative = 6.58

$$\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$$
$$= \left[\frac{2\,(cd^2e^2x^2+cd^3e)\sqrt{\frac{c}{cde-be^2}}\log\left(\frac{cex^2-2\,(cde-be^2)x\sqrt{\frac{c}{cde-be^2}}+cd-be}{cex^2-cd+be}\right)+(4\,cd^2-bde+(4\,cde-be^2)x^2)\sqrt{-de}}{4\,(4\,c^2d^5e-4\,bcd^4e^2+b^2d^3e^3+(4\,c^2d^4e^2-4\,bcd^3e^3+b^2d^2e^4)x^2} \right.$$
$$-\frac{(4\,cd^2-bde+(4\,cde-be^2)x^2)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)-(cd^2e^2x^2+cd^3e)\sqrt{\frac{c}{cde-be^2}}\log\left(\frac{cex^2-2\,(cde-be^2)x\sqrt{\frac{c}{cd}}}{cex^2-cd+b}\right.}{2\,(4\,c^2d^5e-4\,bcd^4e^2+b^2d^3e^3+(4\,c^2d^4e^2-4\,bcd^3e^3+b^2d^2e^4)x^2)}$$

input
```
integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fric
as")
```

output
```
[1/4*(2*(c*d^2*e^2*x^2 + c*d^3*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e*x^2 - 2
*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 - c*d + b
*e)) + (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2
*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 2*(2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e
- 4*b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e
^4)*x^2), -1/2*((4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(d*e)*arctan
(sqrt(d*e)*x/d) - (c*d^2*e^2*x^2 + c*d^3*e)*sqrt(c/(c*d*e - b*e^2))*log((c
*e*x^2 - 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2
 - c*d + b*e)) + (2*c*d^2*e - b*d*e^2)*x)/(4*c^2*d^5*e - 4*b*c*d^4*e^2 + b
^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2), 1/4*(4*(c
*d^2*e^2*x^2 + c*d^3*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(e*x*sqrt(-c/(c*d*e
 - b*e^2))) + (4*c*d^2 - b*d*e + (4*c*d*e - b*e^2)*x^2)*sqrt(-d*e)*log((e*
x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 2*(2*c*d^2*e - b*d*e^2)*x)/(4*c^2
*d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^
2*d^2*e^4)*x^2), 1/2*(2*(c*d^2*e^2*x^2 + c*d^3*e)*sqrt(-c/(c*d*e - b*e^2))
*arctan(e*x*sqrt(-c/(c*d*e - b*e^2))) - (4*c*d^2 - b*d*e + (4*c*d*e - b*e^
2)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - (2*c*d^2*e - b*d*e^2)*x)/(4*c^2*
d^5*e - 4*b*c*d^4*e^2 + b^2*d^3*e^3 + (4*c^2*d^4*e^2 - 4*b*c*d^3*e^3 + b^2
*d^2*e^4)*x^2)]
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

output
```
Timed out
```

3.218.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-c*d)>0)', see `assume?` f
or more de
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \frac{c^2\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{(4\,c^2d^2-4\,bcde+b^2e^2)\sqrt{-c^2de+bce^2}} - \frac{(4\,cd-be)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,(4\,c^2d^3-4\,bcd^2e+b^2de^2)\sqrt{de}} - \frac{x}{2\,(2\,cd^2-bde)(ex^2+d)}$$

input
```
integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac
")
```

output
```
c^2*arctan(c*e*x/sqrt(-c^2*d*e + b*c*e^2))/((4*c^2*d^2 - 4*b*c*d*e + b^2*e
^2)*sqrt(-c^2*d*e + b*c*e^2)) - 1/2*(4*c*d - b*e)*arctan(e*x/sqrt(d*e))/((
4*c^2*d^3 - 4*b*c*d^2*e + b^2*d*e^2)*sqrt(d*e)) - 1/2*x/((2*c*d^2 - b*d*e)
*(e*x^2 + d))
```

3.218.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 3901, normalized size of antiderivative = 28.68

$$\int \frac{1}{(d+ex^2)(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x^2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

output
```
- x/(2*(d + e*x^2)*(2*c*d^2 - b*d*e)) - (atan((((((96*c^7*d^6*e^6 - 224*b
*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9
 + 22*b^4*c^3*d^2*e^10)/(2*(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 12
*b*c^2*d^4*e)) - (x*(-c^3*e*(b*e - c*d))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b^
2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d
^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*
e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-c^3*e*(b*e - c*d))^(1/2))/(2*(b^3
*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)) - (x*(b^2*c^3*e^8 +
 20*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(4*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*
e)))*(-c^3*e*(b*e - c*d))^(1/2)*1i)/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e
^2 - 5*b^2*c*d*e^3) - (((((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*
d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(
2*(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e)) + (x*(-c^3
*e*(b*e - c*d))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384*b^3*c
^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*d^4 +
 b^2*d^2*e^2 - 4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b
^2*c*d*e^3)))*(-c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e^4 - 4*c^3*d^3*e + 8*b*
c^2*d^2*e^2 - 5*b^2*c*d*e^3)) + (x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c^4
*d*e^7))/(4*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))*(-c^3*e*(b*e - c*d))
^(1/2)*1i)/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3))/(...
```

3.219 $\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

3.219.1 Optimal result

Integrand size = 39, antiderivative size = 187

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx &= -\frac{x}{4d(2cd-be)(d+ex^2)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2(d+ex^2)}\\ &\quad -\frac{(28c^2d^2-16bcde+3b^2e^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}(2cd-be)^3}-\frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^3}\end{aligned}$$

output
```
-1/4*x/d/(-b*e+2*c*d)/(e*x^2+d)^2-1/8*(-3*b*e+10*c*d)*x/d^2/(-b*e+2*c*d)^2
/(e*x^2+d)-1/8*(3*b^2*e^2-16*b*c*d*e+28*c^2*d^2)*arctan(x*e^(1/2)/d^(1/2))
/d^(5/2)/(-b*e+2*c*d)^3/e^(1/2)-c^(5/2)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*
d)^(1/2))/(-b*e+2*c*d)^3/e^(1/2)/(-b*e+c*d)^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \frac{1}{8}\left(-\frac{(28c^2d^2-16bcde+3b^2e^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}(2cd-be)^3} - \frac{\frac{(-2cd+be)x(-be(5d+3ex^2)+2cd(7d+5ex^2))}{d^2(d+ex^2)^2}+\frac{8c^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{-cd+be}}\right)}{\sqrt{e}\sqrt{-cd+be}}}{(-2cd+be)^3}\right)$$

input
```
Integrate[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

output
```
(-(((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^
(5/2)*Sqrt[e]*(2*c*d - b*e)^3)) - (((-2*c*d + b*e)*x*(-(b*e*(5*d + 3*e*x^2
)) + 2*c*d*(7*d + 5*e*x^2)))/(d^2*(d + e*x^2)^2) + (8*c^(5/2)*ArcTan[(Sqrt
[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*Sqrt[-(c*d) + b*e]))/(-2*c*d
+ b*e)^3)/8
```

3.219.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {1387, 316, 25, 27, 402, 25, 27, 397, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2(bde+be^2x^2-cd^2+ce^2x^4)}dx$$

$$\Big\downarrow \text{ 1387}$$

$$\int \frac{1}{(d+ex^2)^3\left(\frac{bde-cd^2}{d}+cex^2\right)}dx$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\int -\frac{e(-3cex^2+7cd-3be)}{(ex^2+d)^2(-cex^2+cd-be)}dx}{4de(2cd-be)}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e(-3cex^2+7cd-3be)}{(ex^2+d)^2(-cex^2+cd-be)}dx}{4de(2cd-be)}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-3cex^2+7cd-3be}{(ex^2+d)^2(-cex^2+cd-be)}dx}{4d(2cd-be)}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be)}$$

$$\Big\downarrow \text{402}$$

$$-\frac{\frac{x(10cd-3be)}{2d(d+ex^2)(2cd-be)}-\frac{\int -\frac{e\left(18c^2d^2-13bced+3b^2e^2-ce(10cd-3be)x^2\right)}{\left(ex^2+d\right)\left(-cex^2+cd-be\right)}dx}{2de(2cd-be)}}{4d(2cd-be)}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{e\left(18c^2d^2-13bced+3b^2e^2-ce(10cd-3be)x^2\right)}{\left(ex^2+d\right)\left(-cex^2+cd-be\right)}dx}{2de(2cd-be)}+\frac{x(10cd-3be)}{2d(d+ex^2)(2cd-be)}}{4d(2cd-be)}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{18c^2d^2-13bced+3b^2e^2-ce(10cd-3be)x^2}{\left(ex^2+d\right)\left(-cex^2+cd-be\right)}dx}{2d(2cd-be)}+\frac{x(10cd-3be)}{2d(d+ex^2)(2cd-be)}}{4d(2cd-be)}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be)}$$

$$\Big\downarrow \text{397}$$

$$-\frac{\frac{\frac{\left(3b^2e^2-16bcde+28c^2d^2\right)\int \frac{1}{ex^2+d}dx}{2cd-be}+\frac{8c^3d^2\int \frac{1}{-cex^2+cd-be}dx}{2cd-be}}{2d(2cd-be)}+\frac{x(10cd-3be)}{2d(d+ex^2)(2cd-be)}}{4d(2cd-be)}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\frac{8c^3d^2\int \frac{1}{-cex^2+cd-be}dx}{2cd-be}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3b^2e^2-16bcde+28c^2d^2\right)}{\sqrt{d}\sqrt{e}(2cd-be)}}{2d(2cd-be)}+\frac{x(10cd-3be)}{2d(d+ex^2)(2cd-be)}}{4d(2cd-be)}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be)}$$

$$\Big\downarrow \text{221}$$

3.219. $\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)}dx$

$$-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3b^2e^2-16bcde+28c^2d^2\right)}{\sqrt{d}\sqrt{e}(2cd-be)}+\frac{8c^{5/2}d^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)}}{2d(2cd-be)}+\frac{x(10cd-3be)}{2d(d+ex^2)(2cd-be)}}{4d(2cd-be)}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be)}$$

input
```
Int[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

output
```
-1/4*x/(d*(2*c*d - b*e)*(d + e*x^2)^2) - (((10*c*d - 3*b*e)*x)/(2*d*(2*c*d
 - b*e)*(d + e*x^2)) + (((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]*(2*c*d - b*e)) + (8*c^(5/2)*d^2*ArcTanh
[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b
*e)))/(2*d*(2*c*d - b*e)))/(4*d*(2*c*d - b*e))
```

3.219.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

3.219.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{c^{3}\arctan\left(\frac{xce}{\sqrt{(be-cd)ec}}\right)}{(be-2cd)^{3}\sqrt{(be-cd)ec}}+\frac{\frac{\frac{e\left(3b^{2}e^{2}-16bcde+20c^{2}d^{2}\right)x^{3}}{8d^{2}}+\frac{\left(5b^{2}e^{2}-24bcde+28c^{2}d^{2}\right)x}{8d}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(3b^{2}e^{2}-16bcde+28c^{2}d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8d^{2}\sqrt{ed}}}{(be-2cd)^{3}}$	174
risch	Expression too large to display	101

input
```
int(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOS
E)
```

output
```
-c^3/(b*e-2*c*d)^3/((b*e-c*d)*e*c)^(1/2)*arctan(x*c*e/((b*e-c*d)*e*c)^(1/2
))+1/(b*e-2*c*d)^3*((1/8*e*(3*b^2*e^2-16*b*c*d*e+20*c^2*d^2)/d^2*x^3+1/8*(
5*b^2*e^2-24*b*c*d*e+28*c^2*d^2)/d*x)/(e*x^2+d)^2+1/8*(3*b^2*e^2-16*b*c*d*
e+28*c^2*d^2)/d^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(161) = 322.

Time = 0.85 (sec) , antiderivative size = 1765, normalized size of antiderivative = 9.44

$$\int \frac{1}{(d+ex^2)^2\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fr
icas")
```

output
```
[-1/16*(2*(20*c^2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 + 8*(c^2*d^3
*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(c/(c*d*e - b*e^2))*log((c*e
*x^2 + 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2 -
 c*d + b*e)) - (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2 + (28*c^2*d^2*e^
2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b
^2*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) +
2*(28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*d^8*e - 12*b*c
^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3 - 12*b*c^2*d^5
*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2 - 12*b*c^2*d^
6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/8*((20*c^2*d^3*e^2 - 16*b*
c*d^2*e^3 + 3*b^2*d*e^4)*x^3 + (28*c^2*d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2
+ (28*c^2*d^2*e^2 - 16*b*c*d*e^3 + 3*b^2*e^4)*x^4 + 2*(28*c^2*d^3*e - 16*b
*c*d^2*e^2 + 3*b^2*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 4*(c^2*d^
3*e^3*x^4 + 2*c^2*d^4*e^2*x^2 + c^2*d^5*e)*sqrt(c/(c*d*e - b*e^2))*log((c*
e*x^2 + 2*(c*d*e - b*e^2)*x*sqrt(c/(c*d*e - b*e^2)) + c*d - b*e)/(c*e*x^2
- c*d + b*e)) + (28*c^2*d^4*e - 24*b*c*d^3*e^2 + 5*b^2*d^2*e^3)*x)/(8*c^3*
d^8*e - 12*b*c^2*d^7*e^2 + 6*b^2*c*d^6*e^3 - b^3*d^5*e^4 + (8*c^3*d^6*e^3
- 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^4 + 2*(8*c^3*d^7*e^2
 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2), -1/16*(2*(20*c^
2*d^3*e^2 - 16*b*c*d^2*e^3 + 3*b^2*d*e^4)*x^3 - 16*(c^2*d^3*e^3*x^4 + 2...
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

output
```
Timed out
```

3.219.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-c*d)>0)', see `assume?` f
or more de
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \frac{c^3\arctan\left(\frac{cex}{\sqrt{-c^2de+bce^2}}\right)}{(8\,c^3d^3-12\,bc^2d^2e+6\,b^2cde^2-b^3e^3)\sqrt{-c^2de+bce^2}} - \frac{(28\,c^2d^2-16\,bcde+3\,b^2e^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,(8\,c^3d^5-12\,bc^2d^4e+6\,b^2cd^3e^2-b^3d^2e^3)\sqrt{de}} - \frac{10\,cdex^3-3\,be^2x^3+14\,cd^2x-5\,bdex}{8\,(4\,c^2d^4-4\,bcd^3e+b^2d^2e^2)(ex^2+d)^2}$$

input
```
integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="gi
ac")
```

output
```
c^3*arctan(c*e*x/sqrt(-c^2*d*e + b*c*e^2))/((8*c^3*d^3 - 12*b*c^2*d^2*e +
6*b^2*c*d*e^2 - b^3*e^3)*sqrt(-c^2*d*e + b*c*e^2)) - 1/8*(28*c^2*d^2 - 16*
b*c*d*e + 3*b^2*e^2)*arctan(e*x/sqrt(d*e))/((8*c^3*d^5 - 12*b*c^2*d^4*e +
6*b^2*c*d^3*e^2 - b^3*d^2*e^3)*sqrt(d*e)) - 1/8*(10*c*d*e*x^3 - 3*b*e^2*x^
3 + 14*c*d^2*x - 5*b*d*e*x)/((4*c^2*d^4 - 4*b*c*d^3*e + b^2*d^2*e^2)*(e*x^
2 + d)^2)
```

3.219.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 6267, normalized size of antiderivative = 33.51

$$\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x^2)^2*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

output
```
((x*(5*b*e - 14*c*d))/(8*d*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)) + (e*x^3*(3*
b*e - 10*c*d))/(8*d^2*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)))/(d^2 + e^2*x^4 +
 2*d*e*x^2) - (atan(((((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^
3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8 + b^4*d^4
*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - (((576*c^
10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7
*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12
 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^6*d^10 + b^6
*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 +
60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) - (x*(-c^5*e*(b*e - c*d))^(1/2)*(16
384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 4096
0*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*
b^7*c^2*d^4*e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b
^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2
 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(2*(b
^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e
^4)))*(-c^5*e*(b*e - c*d))^(1/2)*1i)/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3
*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4) + (((x*(9*b^4*c^3*e^10 + 848*c^
7*d^4*e^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(
64*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 3...
```

3.220 $\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

3.220.1 Optimal result

Integrand size = 41, antiderivative size = 139

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{(2cd-be)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{2cd-bex}}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}\sqrt{cd-be}}$$

output
```
1/2*(-2*b*e+5*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^2/e^(1/2)-(-b*e+2*
c*d)^(3/2)*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)
^(1/2))/c^2/e^(1/2)/(-b*e+c*d)^(1/2)+1/2*x*(e*x^2+d)^(1/2)/c
```

3.220.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = -\frac{-cx\sqrt{d+ex^2} + \frac{2(2cd-be)\sqrt{2c^2d^2-3bcde+b^2e^2}\text{arctanh}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{\sqrt{e}(cd-be)} + \frac{(5cd-2be)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{\sqrt{e}}}{2c^2}$$

input
```
Integrate[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
-1/2*(-(c*x*Sqrt[d + e*x^2]) + (2*(2*c*d - b*e)*Sqrt[2*c^2*d^2 - 3*b*c*d*e
 + b^2*e^2]*ArcTanh[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2]))/S
qrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]])/(Sqrt[e]*(c*d - b*e)) + ((5*c*d - 2
*b*e)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/Sqrt[e])/c^2
```

3.220.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.220$, Rules used = {1387, 318, 25, 27, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{5/2}}{bde+be^2x^2-cd^2+ce^2x^4}dx$$

$$\Big\downarrow \text{ 1387}$$

$$\int \frac{\left(d+ex^2\right)^{3/2}}{\frac{bde-cd^2}{d}+cex^2}dx$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\int -\frac{e\left(e(5cd-2be)x^2+d(3cd-be)\right)}{\sqrt{ex^2+d}\left(-cex^2+cd-be\right)}dx}{2ce}+\frac{x\sqrt{d+ex^2}}{2c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x\sqrt{d+ex^2}}{2c}-\frac{\int \frac{e\left(e(5cd-2be)x^2+d(3cd-be)\right)}{\sqrt{ex^2+d}\left(-cex^2+cd-be\right)}dx}{2ce}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x\sqrt{d+ex^2}}{2c}-\frac{\int \frac{e(5cd-2be)x^2+d(3cd-be)}{\sqrt{ex^2+d}\left(-cex^2+cd-be\right)}dx}{2c}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{x\sqrt{d+ex^2}}{2c}-\frac{\frac{2(2cd-be)^2\int \frac{1}{\sqrt{ex^2+d}\left(-cex^2+cd-be\right)}dx}{c}-\frac{(5cd-2be)\int \frac{1}{\sqrt{ex^2+d}}dx}{c}}{2c}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{x\sqrt{d+ex^2}}{2c}-\frac{\frac{2(2cd-be)^2\int\frac{1}{\sqrt{ex^2+d}\left(-cex^2+cd-be\right)}dx}{c}-\frac{(5cd-2be)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c}}{2c}$$

$$\Big\downarrow \; 219$$

$$\frac{x\sqrt{d+ex^2}}{2c}-\frac{\frac{2(2cd-be)^2\int\frac{1}{\sqrt{ex^2+d}\left(-cex^2+cd-be\right)}dx}{c}-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(5cd-2be)}{c\sqrt{e}}}{2c}$$

$$\Big\downarrow \; 291$$

$$\frac{x\sqrt{d+ex^2}}{2c}-\frac{\frac{2(2cd-be)^2\int\frac{1}{-\frac{(cde+(cd-be)e)x^2}{ex^2+d}+cd-be}d\frac{x}{\sqrt{ex^2+d}}}{c}-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(5cd-2be)}{c\sqrt{e}}}{2c}$$

$$\Big\downarrow \; 221$$

$$\frac{x\sqrt{d+ex^2}}{2c}-\frac{\frac{2(2cd-be)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c\sqrt{e}\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(5cd-2be)}{c\sqrt{e}}}{2c}$$

input
```
Int[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
(x*Sqrt[d + e*x^2])/(2*c) - (-(((5*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/(c*Sqrt[e])) + (2*(2*c*d - b*e)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[2*
c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c*Sqrt[e]*Sqrt[c*d - b*
e]))/(2*c)
```

3.220.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.220. $\int\frac{\left(d+ex^2\right)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

3.220. $\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

3.220.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{-\frac{2(be-2cd)^2 \operatorname{arctanh}\left(\frac{(be-cd)\sqrt{e\,x^2+d}}{x\sqrt{e(be-2cd)(be-cd)}}\right)}{\sqrt{e(be-2cd)(be-cd)}}-\frac{\sqrt{e\,x^2+d}\,cx\sqrt{e}-2\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)be+5\operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)cd}{\sqrt{e}}}{2c^2}$
risch	$\frac{x\sqrt{e\,x^2+d}}{2c}-\frac{\frac{(2be-5cd)\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{c\sqrt{e}}-\frac{\left(b^2e^2-4bcde+4c^2d^2\right)\ln\left(\frac{-\frac{2(be-2cd)}{c}-\frac{2\sqrt{-(be-cd)ec}\left(x+\frac{\sqrt{-(be-cd)ec}}{ec}\right)}{c}+2\sqrt{-\frac{be}{}}}{}\right)}{\sqrt{-(be-cd)ec}\,c}}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERB
OSE)
```

output
```
-1/2/c^2*(-2*(b*e-2*c*d)^2/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2)*arctanh((b*e-c*
d)*(e*x^2+d)^(1/2)/x/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2))-((e*x^2+d)^(1/2)*c*x
*e^(1/2)-2*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))*b*e+5*arctanh((e*x^2+d)^(1/2
)/x/e^(1/2))*c*d)/e^(1/2))
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 1079, normalized size of antiderivative = 7.76

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \left[\frac{2\sqrt{ex^2+d}cex-(5cd-2be)\sqrt{e}\log\left(-2ex^2+2\sqrt{ex^2+d}\sqrt{e}x-d\right)}{}\right.$$

input
```
integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="
fricas")
```

output
```
[1/4*(2*sqrt(e*x^2 + d)*c*e*x - (5*c*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 + 2*s
qrt(e*x^2 + d)*sqrt(e)*x - d) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*
e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 2
4*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e
^3)*x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e -
2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e -
b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e
^2)*x^2)))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x - 2*(5*c*d - 2*b*e)*sqrt(
-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b
*e)/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^
2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 +
4*b^2*d*e^3)*x^2 + 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2
*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/
(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*
e - b*c*e^2)*x^2)))/(c^2*e), 1/4*(2*sqrt(e*x^2 + d)*c*e*x + 2*(2*c*d*e - b
*e^2)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*
c*d*e - 2*b*e^2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))
/((2*c*d*e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)) - (5*c*d - 2*b*e)*sqrt(e)*
log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/(c^2*e), 1/2*(sqrt(e*x^2
+ d)*c*e*x - (5*c*d - 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)...
```

3.220.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(5/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

output
```
Timed out
```

3.220.7 Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="
maxima")
```

output
```
integrate((e*x^2 + d)^(5/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

3.220.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(e\,x^2+d)^{5/2}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

input
```
int((d + e*x^2)^(5/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

output
```
int((d + e*x^2)^(5/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

3.221 $\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

3.221.1 Optimal result

Integrand size = 41, antiderivative size = 108

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

output
```
arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(1/2)-arctanh(x*e^(1/2)*(-b*e+2*c*d
)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))*(-b*e+2*c*d)^(1/2)/c/e^(1/2)/(-b
*e+c*d)^(1/2)
```

3.221.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = -\frac{\sqrt{2c^2d^2-3bcde+b^2e^2}\operatorname{arctanh}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)+(cd-be)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{c\sqrt{e}(cd-be)}$$

input
```
Integrate[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
-((Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]*ArcTanh[(-(b*e) + c*(d - e*x^2 +
Sqrt[e]*x*Sqrt[d + e*x^2]))/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]] + (c*d
- b*e)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(c*Sqrt[e]*(c*d - b*e)))
```

3.221.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1387, 301, 25, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}}{bde+be^2x^2-cd^2+ce^2x^4}dx$$

$$\Big\downarrow \text{1387}$$

$$\int \frac{\sqrt{d+ex^2}}{\frac{bde-cd^2}{d}+cex^2}dx$$

$$\Big\downarrow \text{301}$$

$$\frac{(2cd-be)\int -\frac{1}{\sqrt{ex^2+d}(-cex^2+cd-be)}dx}{c}+\frac{\int \frac{1}{\sqrt{ex^2+d}}dx}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{1}{\sqrt{ex^2+d}}dx}{c}-\frac{(2cd-be)\int \frac{1}{\sqrt{ex^2+d}(-cex^2+cd-be)}dx}{c}$$

$$\Big\downarrow \text{224}$$

$$\frac{\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c}-\frac{(2cd-be)\int \frac{1}{\sqrt{ex^2+d}(-cex^2+cd-be)}dx}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{(2cd-be)\int \frac{1}{\sqrt{ex^2+d}(-cex^2+cd-be)}dx}{c}$$

$$\Big\downarrow \text{291}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{(2cd-be)\int \frac{1}{-\frac{(cde+(cd-be)e)x^2}{ex^2+d}+cd-be}d\frac{x}{\sqrt{ex^2+d}}}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\,\text{arctanh}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{c\sqrt{e}\sqrt{cd-be}}$$

input
```
Int[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(c*Sqrt[e]) - (Sqrt[2*c*d - b*e]*ArcT
anh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c*S
qrt[e]*Sqrt[c*d - b*e])
```

3.221.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

3.221. $\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

3.221.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$-\frac{\frac{(be-2cd)\,\mathrm{arctanh}\left(\frac{(be-cd)\sqrt{e\,x^{2}+d}}{x\sqrt{e(be-2cd)(be-cd)}}\right)}{\sqrt{e(be-2cd)(be-cd)}}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)}{\sqrt{e}}}{c}$	100
default	Expression too large to display	2436

input
```
int((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERB
OSE)
```

output
```
-1/c*((b*e-2*c*d)*arctanh((b*e-c*d)*(e*x^2+d)^(1/2)/x/(e*(b*e-2*c*d)*(b*e-
c*d))^(1/2))/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2)-1/e^(1/2)*arctanh((e*x^2+d)^(
1/2)/x/e^(1/2)))
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 940, normalized size of antiderivative = 8.70

$$\int \frac{\left(d+ex^2\right)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \left[\frac{e\sqrt{\frac{2\,cd-be}{cde-be^2}}\log\left(\frac{c^2d^4-2\,bcd^3e+b^2d^2e^2+\left(17\,c^2d^2e^2-24\,bcde^3+8\,b^2e^4\right)x^4+2\,\left(7\,c^2d^3e-11\,b\right.}{c^2e^2x^4+}\right.}{}\right.$$

input
```
integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="
fricas")
```

output
```
[1/4*(e*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*log((c^2*d^4 - 2*b*c*d^3*e + b
^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^
3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 - 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3
+ 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 +
 d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*
e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) + 2*sqrt(e)*log(-2*e*x^2 - 2*sqr
t(e*x^2 + d)*sqrt(e)*x - d))/(c*e), 1/4*(e*sqrt((2*c*d - b*e)/(c*d*e - b*e
^2))*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d
*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2
 - 4*((3*c^2*d^2*e^2 - 5*b*c*d*e^3 + 2*b^2*e^4)*x^3 + (c^2*d^3*e - 2*b*c*d
^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x^2 + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))
)/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2
)) - 4*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*e), 1/2*(e*sqrt(-(2
*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^
2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))/((2*c*d*e - b
*e^2)*x^3 + (2*c*d^2 - b*d*e)*x)) + sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 +
d)*sqrt(e)*x - d))/(c*e), 1/2*(e*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))*arct
an(1/2*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^2)*sqrt(e*x^2 + d)*sqrt(-(2*
c*d - b*e)/(c*d*e - b*e^2))/((2*c*d*e - b*e^2)*x^3 + (2*c*d^2 - b*d*e)*x))
 - 2*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*e)]
```

3.221.6 Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{\sqrt{d+ex^2}}{be-cd+cex^2}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

output
```
Integral(sqrt(d + e*x**2)/(b*e - c*d + c*e*x**2), x)
```

3.221.7 Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="
maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

3.221.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

input
```
int((d + e*x^2)^(3/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

output
```
int((d + e*x^2)^(3/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

3.222 $\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

3.222.1 Optimal result

Integrand size = 41, antiderivative size = 76

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

output
```
-arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/e^
(1/2)/(-b*e+c*d)^(1/2)/(-b*e+2*c*d)^(1/2)
```

3.222.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=-\frac{\operatorname{arctanh}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{\sqrt{e}\sqrt{2c^2d^2-3bcde+b^2e^2}}$$

input
```
Integrate[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
-(ArcTanh[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2]))/Sqrt[2*c^2*
d^2 - 3*b*c*d*e + b^2*e^2]]/(Sqrt[e]*Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]
))
```

3.222.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {1387, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{bde+be^2x^2-cd^2+ce^2x^4}dx$$
$$\Big\downarrow \ 1387$$
$$\int \frac{1}{\sqrt{d+ex^2}\left(\frac{bde-cd^2}{d}+cex^2\right)}dx$$
$$\Big\downarrow \ 291$$
$$\int \frac{1}{\frac{bde-cd^2}{d}-\frac{x^2\left(\frac{e\left(bde-cd^2\right)}{d}-cde\right)}{d+ex^2}}d\frac{x}{\sqrt{d+ex^2}}$$
$$\Big\downarrow \ 221$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

input
```
Int[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

output
```
-(ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])]
/(Sqrt[e]*Sqrt[c*d - b*e]*Sqrt[2*c*d - b*e]))
```

3.222.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

3.222.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{(be-cd)\sqrt{e\,x^2+d}}{x\sqrt{e(be-2cd)(be-cd)}}\right)}{\sqrt{e(be-2cd)(be-cd)}}$	64
default	Expression too large to display	1425

input
```
int((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERB
OSE)
```

output
```
1/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2)*arctanh((b*e-c*d)*(e*x^2+d)^(1/2)/x/(e*(
b*e-2*c*d)*(b*e-c*d))^(1/2))
```

3.222.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(62) = 124.

Time = 0.30 (sec) , antiderivative size = 432, normalized size of antiderivative = 5.68

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$$
$$= \left[\frac{\log\left(\frac{c^2d^4-2\,bcd^3e+b^2d^2e^2+\left(17\,c^2d^2e^2-24\,bcde^3+8\,b^2e^4\right)x^4+2\left(7\,c^2d^3e-11\,bcd^2e^2+4\,b^2de^3\right)x^2-4\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}\left(\left(3\,cde-2\,be^2\right)x^3\right.}{c^2e^2x^4+c^2d^2-2\,bcde+b^2e^2-2\left(c^2de-bce^2\right)x^2}}{4\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}}\right.$$
$$\left.-\frac{\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\arctan\left(-\frac{\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\left(cd^2-bde+\left(3\,cde-2\,be^2\right)x^2\right)\sqrt{ex^2+d}}{2\left(\left(2\,c^2d^2e^2-3\,bcde^3+b^2e^4\right)x^3+\left(2\,c^2d^3e-3\,bcd^2e^2+b^2de^3\right)x\right)}\right)}{2\left(2\,c^2d^2e-3\,bcde^2+b^2e^3\right)}\right]$$

input
```
integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="
fricas")
```

output
```
[1/4*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d
*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2
 - 4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*((3*c*d*e - 2*b*e^2)*x^3 +
(c*d^2 - b*d*e)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b
^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2))/sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*
e^3), -1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*arctan(-1/2*sqrt(-2*
c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*x^
2)*sqrt(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^2*d
^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x))/(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3
)]
```

3.222.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{1}{\sqrt{d+ex^2}\,(be-cd+cex^2)}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

output
```
Integral(1/(sqrt(d + e*x**2)*(b*e - c*d + c*e*x**2)), x)
```

3.222.7 Maxima [F]

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{\sqrt{ex^2+d}}{ce^2x^4+be^2x^2-cd^2+bde}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e), x)
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=-\frac{\arctan\left(\frac{\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2c-3\,cd+2\,be}{2\,\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}\right)}{\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}\sqrt{e}}$$

input
```
integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="
giac")
```

output
```
-arctan(1/2*((sqrt(e)*x - sqrt(e*x^2 + d))^2*c - 3*c*d + 2*b*e)/sqrt(-2*c^
2*d^2 + 3*b*c*d*e - b^2*e^2))/(sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2)*sqrt
(e))
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx=\int \frac{\sqrt{e\,x^2+d}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

output
```
int((d + e*x^2)^(1/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

3.223 $\int \frac{1}{\sqrt{d+ex^2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

3.223.1 Optimal result

Integrand size = 41, antiderivative size = 106

$$\int \frac{1}{\sqrt{d+ex^2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = -\frac{x}{d(2cd-be)\sqrt{d+ex^2}}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}$$

output
```
-c*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/
(-b*e+2*c*d)^(3/2)/e^(1/2)/(-b*e+c*d)^(1/2)-x/d/(-b*e+2*c*d)/(e*x^2+d)^(1/
2)
```

3.223.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{d+ex^2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = -\frac{\sqrt{e}\left(2c^2d^2-3bcde+b^2e^2\right)x+cd\sqrt{2c^2d^2-3bcde+b^2e^2}\sqrt{d+ex^2}\,\text{arctanh}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{d\sqrt{e}(cd-be)(-2cd+be)^2\sqrt{d+ex^2}}$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x
]
```

output
```
-((Sqrt[e]*(2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*x + c*d*Sqrt[2*c^2*d^2 - 3*b*
c*d*e + b^2*e^2]*Sqrt[d + e*x^2]*ArcTanh[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*
x*Sqrt[d + e*x^2]))/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]])/(d*Sqrt[e]*(c*
d - b*e)*(-2*c*d + b*e)^2*Sqrt[d + e*x^2]))
```

3.223.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {1387, 296, 25, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}\left(bde+be^2x^2-cd^2+ce^2x^4\right)}dx$$

$$\Big\downarrow \text{1387}$$

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(\frac{bde-cd^2}{d}+cex^2\right)}dx$$

$$\Big\downarrow \text{296}$$

$$\frac{c\int -\frac{1}{\sqrt{ex^2+d}\left(-cex^2+cd-be\right)}dx}{2cd-be}-\frac{x}{d\sqrt{d+ex^2}(2cd-be)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c\int \frac{1}{\sqrt{ex^2+d}\left(-cex^2+cd-be\right)}dx}{2cd-be}-\frac{x}{d\sqrt{d+ex^2}(2cd-be)}$$

$$\Big\downarrow \text{291}$$

$$-\frac{c\int \frac{1}{-\frac{(cde+(cd-be)e)x^2}{ex^2+d}+cd-be}d\frac{x}{\sqrt{ex^2+d}}}{2cd-be}-\frac{x}{d\sqrt{d+ex^2}(2cd-be)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{c\,\text{arctanh}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}-\frac{x}{d\sqrt{d+ex^2}(2cd-be)}$$

input
```
Int[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

output
```
-(x/(d*(2*c*d - b*e)*Sqrt[d + e*x^2])) - (c*ArcTanh[(Sqrt[e]*Sqrt[2*c*d -
b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*
d - b*e)^(3/2))
```

3.223.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

3.223. $\int \frac{1}{\sqrt{d+ex^2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

3.223.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$\frac{-cd\operatorname{arctanh}\left(\frac{(be-cd)\sqrt{e\,x^2+d}}{x\sqrt{e(be-2cd)(be-cd)}}\right)\sqrt{e\,x^2+d}+x\sqrt{e(be-2cd)(be-cd)}}{(be-2cd)\sqrt{e\,x^2+d}\,\sqrt{e(be-2cd)(be-cd)}\,d}$
default	$-\frac{c\sqrt{\left(x+\frac{\sqrt{-ed}}{e}\right)^2e-2\sqrt{-ed}\left(x+\frac{\sqrt{-ed}}{e}\right)}}{2d\left(\sqrt{-ed}\,c+\sqrt{-(be-cd)ec}\right)\left(-\sqrt{-ed}\,c+\sqrt{-(be-cd)ec}\right)\left(x+\frac{\sqrt{-ed}}{e}\right)}-\frac{c\sqrt{\left(x-\frac{\sqrt{-ed}}{e}\right)^2e+2\sqrt{-ed}\left(x-\right.}}{2d\left(\sqrt{-ed}\,c+\sqrt{-(be-cd)ec}\right)\left(-\sqrt{-ed}\,c+\sqrt{-(\right.}}$

input
```
int(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVE
RBOSE)
```

output
```
(-c*d*arctanh((b*e-c*d)*(e*x^2+d)^(1/2)/x/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2))
*(e*x^2+d)^(1/2)+x*(e*(b*e-2*c*d)*(b*e-c*d))^(1/2))/(b*e-2*c*d)/(e*x^2+d)^
(1/2)/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2)/d
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(90) = 180.

Time = 0.34 (sec) , antiderivative size = 701, normalized size of antiderivative = 6.61

$$\int \frac{1}{\sqrt{d+ex^2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$$
$$=\left[-\frac{4\left(2\,c^2d^2e-3\,bcde^2+b^2e^3\right)\sqrt{ex^2+d}x+\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}\left(cdex^2+cd^2\right)\log\left(\frac{c^2d^4-2\,bcd^3e+b^2d^2e^2}{\right.}}{4\left(4\,c^3d^5e-8\,bc^2d^4e^2+5\,b^2cd^3e^3-b^3d^2e^4+\right.}\right.$$
$$-\frac{2\left(2\,c^2d^2e-3\,bcde^2+b^2e^3\right)\sqrt{ex^2+d}x+\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\left(cdex^2+cd^2\right)\arctan\left(-\frac{\sqrt{-2\,c^2d^2e+3}}{2\left((2\,c^2d^2e^2\right.}\right.}{2\left(4\,c^3d^5e-8\,bc^2d^4e^2+5\,b^2cd^3e^3-b^3d^2e^4+\left(4\,c^3d^4e^2-8\,bc^2d^3e^3+5\,b^2cd^2e^4\right.\right.}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm
="fricas")
```

output
```
[-1/4*(4*(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x + sqrt(2*
c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*(c*d*e*x^2 + c*d^2)*log((c^2*d^4 - 2*b*
c*d^3*e + b^2*d^2*e^2 + (17*c^2*d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 +
2*(7*c^2*d^3*e - 11*b*c*d^2*e^2 + 4*b^2*d*e^3)*x^2 + 4*sqrt(2*c^2*d^2*e -
3*b*c*d*e^2 + b^2*e^3)*((3*c*d*e - 2*b*e^2)*x^3 + (c*d^2 - b*d*e)*x)*sqrt(
e*x^2 + d))/(c^2*e^2*x^4 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*
c*e^2)*x^2)))/(4*c^3*d^5*e - 8*b*c^2*d^4*e^2 + 5*b^2*c*d^3*e^3 - b^3*d^2*e
^4 + (4*c^3*d^4*e^2 - 8*b*c^2*d^3*e^3 + 5*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2),
 -1/2*(2*(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(e*x^2 + d)*x + sqrt(-2
*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d*e*x^2 + c*d^2)*arctan(-1/2*sqrt(-
2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c*d*e - 2*b*e^2)*
x^2)*sqrt(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^3 + (2*c^2
*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x)))/(4*c^3*d^5*e - 8*b*c^2*d^4*e^2 +
5*b^2*c*d^3*e^3 - b^3*d^2*e^4 + (4*c^3*d^4*e^2 - 8*b*c^2*d^3*e^3 + 5*b^2*c
*d^2*e^4 - b^3*d*e^5)*x^2)]
```

3.223.6 Sympy [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \int \frac{1}{(d+ex^2)^{\frac{3}{2}}\,(be-cd+cex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

output
```
Integral(1/((d + e*x**2)**(3/2)*(b*e - c*d + c*e*x**2)), x)
```

3.223.7 Maxima [F]

$$\int \frac{1}{\sqrt{d+ex^2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \int \frac{1}{(ce^2x^4+be^2x^2-cd^2+bde)\sqrt{ex^2+d}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm
="maxima")
```

output
```
integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*sqrt(e*x^2 + d)), x)
```

3.223.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{d+ex^2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \frac{c\sqrt{e}\arctan\left(-\frac{\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2c-3\,cd+2\,be}{2\,\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}\right)}{\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}(2\,cde-be^2)} - \frac{x}{(2\,cd^2-bde)\sqrt{ex^2+d}}$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm
="giac")
```

output
```
c*sqrt(e)*arctan(-1/2*((sqrt(e)*x - sqrt(e*x^2 + d))^2*c - 3*c*d + 2*b*e)/
sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2))/(sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2
*e^2)*(2*c*d*e - b*e^2)) - x/((2*c*d^2 - b*d*e)*sqrt(e*x^2 + d))
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex^2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2\right)}\,dx$$

input
```
int(1/((d + e*x^2)^(1/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

output
```
int(1/((d + e*x^2)^(1/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)), x)
```

3.224 $\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

3.224.1 Optimal result

Integrand size = 41, antiderivative size = 149

$$\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = -\frac{x}{3d(2cd-be)(d+ex^2)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}} - \frac{c^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{2cd-be}x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{5/2}}$$

output
```
-1/3*x/d/(-b*e+2*c*d)/(e*x^2+d)^(3/2)-c^2*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(
1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/(-b*e+2*c*d)^(5/2)/e^(1/2)/(-b*e+c*
d)^(1/2)-1/3*(-2*b*e+7*c*d)*x/d^2/(-b*e+2*c*d)^2/(e*x^2+d)^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.17

$$\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = -\frac{\frac{(-2cd+be)x(-be(3d+2ex^2)+cd(9d+7ex^2))}{d^2(d+ex^2)^{3/2}} + \frac{3c^2\sqrt{2c^2d^2-3bcde+b^2e^2}\operatorname{arctanh}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{\sqrt{e}(-cd+be)}}{3(-2cd+be)^3}$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4))
,x]
```

output
```
-1/3*(((-2*c*d + b*e)*x*(-(b*e*(3*d + 2*e*x^2)) + c*d*(9*d + 7*e*x^2)))/(d
^2*(d + e*x^2)^(3/2)) + (3*c^2*Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]*ArcTa
nh[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2]))/Sqrt[2*c^2*d^2 - 3
*b*c*d*e + b^2*e^2]])/(Sqrt[e]*(-(c*d) + b*e)))/(-2*c*d + b*e)^3
```

3.224.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {1387, 316, 25, 27, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2}(bde+be^2x^2-cd^2+ce^2x^4)}dx$$

$$\Big\downarrow \text{1387}$$

$$\int \frac{1}{(d+ex^2)^{5/2}\left(\frac{bde-cd^2}{d}+cex^2\right)}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int -\frac{e(-2cex^2+5cd-2be)}{(ex^2+d)^{3/2}(-cex^2+cd-be)}dx}{3de(2cd-be)}-\frac{x}{3d(d+ex^2)^{3/2}(2cd-be)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e(-2cex^2+5cd-2be)}{(ex^2+d)^{3/2}(-cex^2+cd-be)}dx}{3de(2cd-be)}-\frac{x}{3d(d+ex^2)^{3/2}(2cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-2cex^2+5cd-2be}{(ex^2+d)^{3/2}(-cex^2+cd-be)}dx}{3d(2cd-be)}-\frac{x}{3d(d+ex^2)^{3/2}(2cd-be)}$$

$$\Big\downarrow \text{402}$$

$$-\frac{\frac{x(7cd-2be)}{d\sqrt{d+ex^2}(2cd-be)}-\frac{\int -\frac{3c^2d^2e}{\sqrt{ex^2+d}(-cex^2+cd-be)}dx}{de(2cd-be)}}{3d(2cd-be)}-\frac{x}{3d(d+ex^2)^{3/2}(2cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3c^2d\int \frac{1}{\sqrt{ex^2+d}\left(-cex^2+cd-be\right)}dx}{2cd-be}+\frac{x(7cd-2be)}{d\sqrt{d+ex^2}(2cd-be)}}{3d(2cd-be)}-\frac{x}{3d\left(d+ex^2\right)^{3/2}(2cd-be)}$$

$$\Big\downarrow \text{ 291}$$

$$-\frac{\frac{3c^2d\int \frac{1}{-\frac{(cde+(cd-be)e)x^2}{ex^2+d}+cd-be}d\frac{x}{\sqrt{ex^2+d}}}{2cd-be}+\frac{x(7cd-2be)}{d\sqrt{d+ex^2}(2cd-be)}}{3d(2cd-be)}-\frac{x}{3d\left(d+ex^2\right)^{3/2}(2cd-be)}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{3c^2d\,\text{arctanh}\left(\frac{\sqrt{e}x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}(2cd-be)^{3/2}}+\frac{x(7cd-2be)}{d\sqrt{d+ex^2}(2cd-be)}}{3d(2cd-be)}-\frac{x}{3d\left(d+ex^2\right)^{3/2}(2cd-be)}$$

input
```
Int[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

output
```
-1/3*x/(d*(2*c*d - b*e)*(d + e*x^2)^(3/2)) - (((7*c*d - 2*b*e)*x)/(d*(2*c*
d - b*e)*Sqrt[d + e*x^2]) + (3*c^2*d*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)
/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e
)^(3/2)))/(3*d*(2*c*d - b*e))
```

3.224.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.224. $\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

3.224.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02

method	result	size
pseudoelliptic	$\frac{c^2 d^2 \operatorname{arctanh}\left(\frac{(be-cd)\sqrt{e x^2+d}}{x\sqrt{e(be-2cd)(be-cd)}}\right)(e x^2+d)^{\frac{3}{2}}+x\left(\frac{2b e^2 x^2}{3}+d\left(-\frac{7c x^2}{3}+b\right)e-3c d^2\right)\sqrt{e(be-2cd)(be-cd)}}{\sqrt{e(be-2cd)(be-cd)}\,(e x^2+d)^{\frac{3}{2}}(be-2cd)^2 d^2}$	152
default	Expression too large to display	1551

input
```
int(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVE
RBOSE)
```

output
```
(c^2*d^2*arctanh((b*e-c*d)*(e*x^2+d)^(1/2)/x/(e*(b*e-2*c*d)*(b*e-c*d))^(1/
2))*(e*x^2+d)^(3/2)+x*(2/3*b*e^2*x^2+d*(-7/3*c*x^2+b)*e-3*c*d^2)*(e*(b*e-2
*c*d)*(b*e-c*d))^(1/2))/(e*(b*e-2*c*d)*(b*e-c*d))^(1/2)/(e*x^2+d)^(3/2)/(b
*e-2*c*d)^2/d^2
```

3.224.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(127) = 254$.

Time = 0.51 (sec) , antiderivative size = 1063, normalized size of antiderivative = 7.13

$$\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \left[\frac{3\,(c^2d^2e^2x^4+2\,c^2d^3ex^2+c^2d^4)\sqrt{2\,c^2d^2e-3\,bcde^2+b^2}}{12}\right.$$

$$-\frac{3\,(c^2d^2e^2x^4+2\,c^2d^3ex^2+c^2d^4)\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\arctan\left(-\frac{\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\,(cd^2-bde+(3\,cde-2\,be^2}{2\,((2\,c^2d^2e^2-3\,bcde^3+b^2e^4)x^3+(2\,c^2d^3e-3\,bcd^2}\right.}{6\,(8\,c^4d^8e-20\,bc^3d^7e^2+18\,b^2c^2d^6e^3-7\,b^3cd^5e^4+b^4d^4e^5+(8\,c^4d^6e^3-20\,bc^3d^5e^4+}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm
="fricas")
```

output
```
[1/12*(3*(c^2*d^2*e^2*x^4 + 2*c^2*d^3*e*x^2 + c^2*d^4)*sqrt(2*c^2*d^2*e -
3*b*c*d*e^2 + b^2*e^3)*log((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (17*c^2*
d^2*e^2 - 24*b*c*d*e^3 + 8*b^2*e^4)*x^4 + 2*(7*c^2*d^3*e - 11*b*c*d^2*e^2
+ 4*b^2*d*e^3)*x^2 - 4*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*((3*c*d*e
 - 2*b*e^2)*x^3 + (c*d^2 - b*d*e)*x)*sqrt(e*x^2 + d))/(c^2*e^2*x^4 + c^2*d
^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*x^2)) - 4*((14*c^3*d^3*e^
2 - 25*b*c^2*d^2*e^3 + 13*b^2*c*d*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e -
11*b*c^2*d^3*e^2 + 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(8*c^4
*d^8*e - 20*b*c^3*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c*d^5*e^4 + b^4*d^4
*e^5 + (8*c^4*d^6*e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*c*d^
3*e^6 + b^4*d^2*e^7)*x^4 + 2*(8*c^4*d^7*e^2 - 20*b*c^3*d^6*e^3 + 18*b^2*c^
2*d^5*e^4 - 7*b^3*c*d^4*e^5 + b^4*d^3*e^6)*x^2), -1/6*(3*(c^2*d^2*e^2*x^4
+ 2*c^2*d^3*e*x^2 + c^2*d^4)*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*ar
ctan(-1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d^2 - b*d*e + (3*c
*d*e - 2*b*e^2)*x^2)*sqrt(e*x^2 + d)/((2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e
^4)*x^3 + (2*c^2*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x)) + 2*((14*c^3*d^3*e
^2 - 25*b*c^2*d^2*e^3 + 13*b^2*c*d*e^4 - 2*b^3*e^5)*x^3 + 3*(6*c^3*d^4*e -
 11*b*c^2*d^3*e^2 + 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(e*x^2 + d))/(8*c^
4*d^8*e - 20*b*c^3*d^7*e^2 + 18*b^2*c^2*d^6*e^3 - 7*b^3*c*d^5*e^4 + b^4*d^
4*e^5 + (8*c^4*d^6*e^3 - 20*b*c^3*d^5*e^4 + 18*b^2*c^2*d^4*e^5 - 7*b^3*...
```

3.224.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \int \frac{1}{(d+ex^2)^{\frac{5}{2}}(be-cd+cex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

output
```
Integral(1/((d + e*x**2)**(5/2)*(b*e - c*d + c*e*x**2)), x)
```

3.224.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \int \frac{1}{(ce^2x^4+be^2x^2-cd^2+bde)(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm
="maxima")
```

output
```
integrate(1/((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)*(e*x^2 + d)^(3/2)), x
)
```

3.224.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(127) = 254$.

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.30

$$\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = -\frac{c^2\sqrt{e}\arctan\left(\frac{\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2c-3\,cd+2\,be}{2\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}\right)}{(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}} - \frac{\left(\frac{(28\,c^3d^3e^2-36\,bc^2d^2e^3+15\,b^2cde^4-2\,b^3e^5)x^2}{16\,c^4d^6e-32\,bc^3d^5e^2+24\,b^2c^2d^4e^3-8\,b^3cd^3e^4+b^4d^2e^5}+\frac{3\,(12\,c^3d^4e-16\,bc^2d^3e^2+7\,b^2cd^2e^3-b^3de^4)}{16\,c^4d^6e-32\,bc^3d^5e^2+24\,b^2c^2d^4e^3-8\,b^3cd^3e^4+b^4d^2e^5}\right)x}{3\,(ex^2+d)^{\frac{3}{2}}}$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm
="giac")
```

output
```
-c^2*sqrt(e)*arctan(1/2*((sqrt(e)*x - sqrt(e*x^2 + d))^2*c - 3*c*d + 2*b*e
)/sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2))/((4*c^2*d^2*e - 4*b*c*d*e^2 + b^
2*e^3)*sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2)) - 1/3*((28*c^3*d^3*e^2 - 36
*b*c^2*d^2*e^3 + 15*b^2*c*d*e^4 - 2*b^3*e^5)*x^2/(16*c^4*d^6*e - 32*b*c^3*
d^5*e^2 + 24*b^2*c^2*d^4*e^3 - 8*b^3*c*d^3*e^4 + b^4*d^2*e^5) + 3*(12*c^3*
d^4*e - 16*b*c^2*d^3*e^2 + 7*b^2*c*d^2*e^3 - b^3*d*e^4)/(16*c^4*d^6*e - 32
*b*c^3*d^5*e^2 + 24*b^2*c^2*d^4*e^3 - 8*b^3*c*d^3*e^4 + b^4*d^2*e^5))*x/(e
*x^2 + d)^(3/2)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \int \frac{1}{(e\,x^2+d)^{3/2}\,(-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2)}\,dx$$

input
```
int(1/((d + e*x^2)^(3/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

output
```
int(1/((d + e*x^2)^(3/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)), x)
```

3.225 $\int (1+x^2)^3 \sqrt{1+x^2+x^4}\,dx$

3.225.1 Optimal result

Integrand size = 20, antiderivative size = 183

$$\int (1+x^2)^3 \sqrt{1+x^2+x^4}\,dx = \frac{26x\sqrt{1+x^2+x^4}}{45\,(1+x^2)} + \frac{2}{45}x(7+6x^2)\sqrt{1+x^2+x^4} + \frac{1}{3}x(1+x^2+x^4)^{3/2} + \frac{1}{9}x^3(1+x^2+x^4)^{3/2} - \frac{26(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{45\sqrt{1+x^2+x^4}} + \frac{7(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{15\sqrt{1+x^2+x^4}}$$

output
```
1/3*x*(x^4+x^2+1)^(3/2)+1/9*x^3*(x^4+x^2+1)^(3/2)+26/45*x*(x^4+x^2+1)^(1/2
)/(x^2+1)+2/45*x*(6*x^2+7)*(x^4+x^2+1)^(1/2)-26/45*(x^2+1)*(cos(2*arctan(x
))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/
(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+7/15*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)
/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^
(1/2)/(x^4+x^2+1)^(1/2)
```

3.225.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

$$\int (1+x^2)^3 \sqrt{1+x^2+x^4}\, dx$$
$$= \frac{x(29+61x^2+81x^4+57x^6+25x^8+5x^{10})+26\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\text{arcsinh}\left((-1)^{5/6}x\right.\right.}{45\sqrt{1+}}$$

input
```
Integrate[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

output
```
(x*(29 + 61*x^2 + 81*x^4 + 57*x^6 + 25*x^8 + 5*x^10) + 26*(-1)^(1/3)*Sqrt[
1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6
)*x], (-1)^(2/3)] + 2*(-1)^(5/6)*(9*I + 4*Sqrt[3])*Sqrt[1 + (-1)^(1/3)*x^2
]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])
/(45*Sqrt[1 + x^2 + x^4])
```

3.225.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1518, 27, 2207, 27, 1490, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2+1\right)^3 \sqrt{x^4+x^2+1}\, dx$$
$$\Big\downarrow \text{1518}$$
$$\frac{1}{9}\int 3\sqrt{x^4+x^2+1}\left(7x^4+8x^2+3\right)dx+\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{3}\int \sqrt{x^4+x^2+1}\left(7x^4+8x^2+3\right)dx+\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3$$
$$\Big\downarrow \text{2207}$$
$$\frac{1}{3}\left(\frac{1}{7}\int 14\left(2x^2+1\right)\sqrt{x^4+x^2+1}dx+x\left(x^4+x^2+1\right)^{3/2}\right)+\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(2\int \left(2x^2+1\right)\sqrt{x^4+x^2+1}dx+x\left(x^4+x^2+1\right)^{3/2}\right)+\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3$$

$$\Big\downarrow \; 1490$$

$$\frac{1}{3}\left(2\left(\frac{1}{15}\int \frac{13x^2+8}{\sqrt{x^4+x^2+1}}dx+\frac{1}{15}x\sqrt{x^4+x^2+1}\left(6x^2+7\right)\right)+x\left(x^4+x^2+1\right)^{3/2}\right)+\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3$$

$$\Big\downarrow \; 1511$$

$$\frac{1}{3}\left(2\left(\frac{1}{15}\left(21\int \frac{1}{\sqrt{x^4+x^2+1}}dx-13\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+\frac{1}{15}x\sqrt{x^4+x^2+1}\left(6x^2+7\right)\right)+x\left(x^4+x^2+1\right)^{3/2}\right.$$

$$\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{3}\left(2\left(\frac{1}{15}\left(\frac{21\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-13\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+\frac{1}{15}x\sqrt{x^4+x^2+1}\left(6x^2\right.\right.\right.$$

$$\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{3}\left(2\left(\frac{1}{15}\left(\frac{21\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-13\left(\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4}}{x}\right.\right.\right.\right.$$

$$\frac{1}{9}\left(x^4+x^2+1\right)^{3/2}x^3$$

input
```
Int[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

output
```
(x^3*(1 + x^2 + x^4)^(3/2))/9 + (x*(1 + x^2 + x^4)^(3/2) + 2*((x*(7 + 6*x^
2)*Sqrt[1 + x^2 + x^4])/15 + (-13*(-((x*Sqrt[1 + x^2 + x^4])/(1 + x^2)) +
((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/
Sqrt[1 + x^2 + x^4]) + (21*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ell
ipticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]))/15))/3
```

3.225. $\int \left(1+x^2\right)^3\sqrt{1+x^2+x^4}\,dx$

3.225.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.225.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.28

method	result
risch	$\frac{x(5x^6+20x^4+32x^2+29)\sqrt{x^4+x^2+1}}{45}+\frac{32\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{45\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{104\sqrt{1-\left(-\frac{1}{2}\cdots\right.}}{\cdots}$
default	$\frac{29x\sqrt{x^4+x^2+1}}{45}+\frac{32\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{45\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{104\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}\cdots\right.}}{\cdots}$
elliptic	$\frac{29x\sqrt{x^4+x^2+1}}{45}+\frac{32\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{45\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{104\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}\cdots\right.}}{\cdots}$

input
```
int((x^2+1)^3*(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/45*x*(5*x^6+20*x^4+32*x^2+29)*(x^4+x^2+1)^(1/2)+32/45/(-2+2*I*3^(1/2))^(
1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)
/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1
/2))^(1/2))-104/45/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/
2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(Ell
ipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(
1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.72

$$\int (1+x^2)^3 \sqrt{1+x^2+x^4}\,dx = \frac{13\sqrt{2}\left(\sqrt{-3}x-x\right)\sqrt{\sqrt{-3}-1}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right) \mid \frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)-\sqrt{2}\left(5\sqrt{-3}x-21x\right)\sqrt{\sqrt{-3}-1}F(a}{90x}$$

input
```
integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/90*(13*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2
*sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) - sqrt(2)*(5*sqrt(-3)*
x - 21*x)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) -
 1)/x), 1/2*sqrt(-3) - 1/2) + 2*(5*x^8 + 20*x^6 + 32*x^4 + 29*x^2 + 26)*sq
rt(x^4 + x^2 + 1))/x
```

3.225.6 Sympy [F]

$$\int (1+x^2)^3 \sqrt{1+x^2+x^4}\,dx = \int \sqrt{(x^2-x+1)(x^2+x+1)}\left(x^2+1\right)^3\,dx$$

input
```
integrate((x**2+1)**3*(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3, x)
```

3.225.7 Maxima [F]

$$\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\, dx = \int \sqrt{x^4+x^2+1}\left(x^2+1\right)^3 dx$$

input
```
integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

3.225.8 Giac [F]

$$\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\, dx = \int \sqrt{x^4+x^2+1}\left(x^2+1\right)^3 dx$$

input
```
integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\, dx = \int \left(x^2+1\right)^3 \sqrt{x^4+x^2+1}\, dx$$

input
```
int((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2),x)
```

output
```
int((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2), x)
```

3.226 $\int (1+x^2)^2 \sqrt{1+x^2+x^4}\, dx$

3.226.1 Optimal result

Integrand size = 20, antiderivative size = 164

$$\int (1+x^2)^2 \sqrt{1+x^2+x^4}\, dx = \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{2}{21}x(4+3x^2)\sqrt{1+x^2+x^4} + \frac{1}{7}x(1+x^2+x^4)^{3/2} - \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{4(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{7\sqrt{1+x^2+x^4}}$$

output
```
1/7*x*(x^4+x^2+1)^(3/2)+2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+2/21*x*(3*x^2+4)*(
x^4+x^2+1)^(1/2)-2/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*E
llipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(
1/2)+4/7*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin
(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.226.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.61 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99

$$\int (1+x^2)^2 \sqrt{1+x^2+x^4}\,dx$$
$$= \frac{x(11+20x^2+23x^4+12x^6+3x^8)+14\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/}\right.}{21\sqrt{1+x^2 -}}$$

input
```
Integrate[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```

output
```
(x*(11 + 20*x^2 + 23*x^4 + 12*x^6 + 3*x^8) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(
1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)
^(2/3)] + 2*(-1)^(1/3)*(-7 + 5*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1
 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(21*Sqr
t[1 + x^2 + x^4])
```

3.226.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1518, 27, 1490, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^2+1)^2 \sqrt{x^4+x^2+1}\,dx$$

$$\Big\downarrow \text{1518}$$

$$\frac{1}{7}\int 2(5x^2+3)\sqrt{x^4+x^2+1}dx+\frac{1}{7}x(x^4+x^2+1)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{7}\int (5x^2+3)\sqrt{x^4+x^2+1}dx+\frac{1}{7}x(x^4+x^2+1)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{2}{7}\left(\frac{1}{15}\int \frac{5(7x^2+5)}{\sqrt{x^4+x^2+1}}dx+\frac{1}{3}x\sqrt{x^4+x^2+1}(3x^2+4)\right)+\frac{1}{7}x(x^4+x^2+1)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{7}\left(\frac{1}{3}\int \frac{7x^2+5}{\sqrt{x^4+x^2+1}}dx+\frac{1}{3}x\sqrt{x^4+x^2+1}(3x^2+4)\right)+\frac{1}{7}x\left(x^4+x^2+1\right)^{3/2}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2}{7}\left(\frac{1}{3}\left(12\int \frac{1}{\sqrt{x^4+x^2+1}}dx-7\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+\frac{1}{3}x\sqrt{x^4+x^2+1}(3x^2+4)\right)+\frac{1}{7}x\left(x^4+x^2+1\right)^{3/2}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2}{7}\left(\frac{1}{3}\left(\frac{6(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-7\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+\frac{1}{3}x\sqrt{x^4+x^2+1}(3x^2+4)\right)+\frac{1}{7}x\left(x^4+x^2+1\right)^{3/2}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2}{7}\left(\frac{1}{3}\left(\frac{6(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-7\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+}}{x^2+1}\right.\right.\right.$$
$$\frac{1}{7}x\left(x^4+x^2+1\right)^{3/2}$$

input
```
Int[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```

output
```
(x*(1 + x^2 + x^4)^(3/2))/7 + (2*((x*(4 + 3*x^2)*Sqrt[1 + x^2 + x^4])/3 +
(-7*(-((x*Sqrt[1 + x^2 + x^4])/(1 + x^2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4
)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]) + (6*(1 +
 x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[
1 + x^2 + x^4])/3))/7
```

3.226. $\int (1+x^2)^2\sqrt{1+x^2+x^4}\,dx$

3.226.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.226.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.40

method	result
risch	$\frac{x(3x^4+9x^2+11)\sqrt{x^4+x^2+1}}{21}+\frac{20\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{21\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\cdots}{\cdots}$
default	$\frac{11x\sqrt{x^4+x^2+1}}{21}+\frac{20\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{21\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\cdots\right.}}{\cdots}$
elliptic	$\frac{11x\sqrt{x^4+x^2+1}}{21}+\frac{20\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{21\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\cdots\right.}}{\cdots}$

input
```
int((x^2+1)^2*(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*x*(3*x^4+9*x^2+11)*(x^4+x^2+1)^(1/2)+20/21/(-2+2*I*3^(1/2))^(1/2)*(1-
(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^
2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/
2))-8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2
-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*
x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2
*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.76

$$\int (1+x^2)^2 \sqrt{1+x^2+x^4}\,dx$$
$$= \frac{7\sqrt{2}(\sqrt{-3}x-x)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right) \mid \frac{1}{2}\sqrt{-3}-\frac{1}{2}) - 2\sqrt{2}(\sqrt{-3}x-6x)\sqrt{\sqrt{-3}-1}F(\text{arc}}{42x}$$

input
```
integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/42*(7*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*
sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) - 2*sqrt(2)*(sqrt(-3)*x
 - 6*x)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1
)/x), 1/2*sqrt(-3) - 1/2) + 2*(3*x^6 + 9*x^4 + 11*x^2 + 14)*sqrt(x^4 + x^2
 + 1))/x
```

3.226.6 Sympy [F]

$$\int (1+x^2)^2 \sqrt{1+x^2+x^4}\,dx = \int \sqrt{(x^2-x+1)(x^2+x+1)}(x^2+1)^2\,dx$$

input
```
integrate((x**2+1)**2*(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2, x)
```

3.226.7 Maxima [F]

$$\int (1+x^2)^2 \sqrt{1+x^2+x^4}\,dx = \int \sqrt{x^4+x^2+1}(x^2+1)^2\,dx$$

input
```
integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

3.226.8 Giac [F]

$$\int \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}\, dx = \int \sqrt{x^4+x^2+1}\left(x^2+1\right)^2 dx$$

input
```
integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}\, dx = \int \left(x^2+1\right)^2 \sqrt{x^4+x^2+1}\, dx$$

input
```
int((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2),x)
```

output
```
int((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2), x)
```

3.227 $\int (1+x^2)\sqrt{1+x^2+x^4}\,dx$

3.227.1 Optimal result

Integrand size = 18, antiderivative size = 145

$$\int (1+x^2)\sqrt{1+x^2+x^4}\,dx = \frac{3x\sqrt{1+x^2+x^4}}{5(1+x^2)}+\frac{1}{5}x(2+x^2)\sqrt{1+x^2+x^4} - \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{5\sqrt{1+x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{5\sqrt{1+x^2+x^4}}$$

output
```
3/5*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/5*x*(x^2+2)*(x^4+x^2+1)^(1/2)-3/5*(x^2+1
)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+3/5*(x^2+1)*(cos(2*arc
tan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^
2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.227.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.16

$$\int (1+x^2)\sqrt{1+x^2+x^4}\,dx$$
$$= \frac{2x+3x^3+3x^5+x^7+3\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\frac{3}{2}\sqrt{2+(1}{5\sqrt{1+x^2+x^4}}$$

input
```
Integrate[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

output
```
(2*x + 3*x^3 + 3*x^5 + x^7 + 3*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1
- (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (3*Sqrt
[2 + (1 - I*Sqrt[3])*x^2]*Sqrt[2 + (1 + I*Sqrt[3])*x^2]*EllipticF[ArcSin[(
x + I*Sqrt[3]*x)/2], (I/2)*(I + Sqrt[3])])/2)/(5*Sqrt[1 + x^2 + x^4])
```

3.227.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1490, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^2+1)\sqrt{x^4+x^2+1}\,dx$$
$$\Big\downarrow \text{1490}$$
$$\frac{1}{15}\int \frac{9(x^2+1)}{\sqrt{x^4+x^2+1}}dx+\frac{1}{5}x\sqrt{x^4+x^2+1}(x^2+2)$$
$$\Big\downarrow \text{27}$$
$$\frac{3}{5}\int \frac{x^2+1}{\sqrt{x^4+x^2+1}}dx+\frac{1}{5}x\sqrt{x^4+x^2+1}(x^2+2)$$
$$\Big\downarrow \text{1511}$$
$$\frac{3}{5}\left(2\int \frac{1}{\sqrt{x^4+x^2+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+\frac{1}{5}x\sqrt{x^4+x^2+1}(x^2+2)$$

$$\Big\downarrow \text{1416}$$

$$\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\int\frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+\frac{1}{5}x\sqrt{x^4+x^2+1}(x^2+2)$$

$$\Big\downarrow \text{1509}$$

$$\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}\right)+\frac{1}{5}x\sqrt{x^4+x^2+1}(x^2+2)$$

input
```
Int[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

output
```
(x*(2 + x^2)*Sqrt[1 + x^2 + x^4])/5 + (3*((x*Sqrt[1 + x^2 + x^4])/(1 + x^2
) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/
4])/Sqrt[1 + x^2 + x^4] + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ell
ipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]))/5
```

3.227.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.227.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.54

method	result
risch	$\frac{x(x^2+2)\sqrt{x^4+x^2+1}}{5}+\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{5\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{12\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-(\cdots}}{}$
default	$\frac{2x\sqrt{x^4+x^2+1}}{5}+\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{5\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{12\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\cdots\right.}}{5\cdots}$
elliptic	$\frac{2x\sqrt{x^4+x^2+1}}{5}+\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{5\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{12\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\cdots\right.}}{5\cdots}$

input
```
int((x^2+1)*(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x*(x^2+2)*(x^4+x^2+1)^(1/2)+6/5/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*
3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*E
llipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-12/5/(-2
+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1
/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3
^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))
^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int (1+x^2)\sqrt{1+x^2+x^4}\,dx$$
$$= \frac{3\sqrt{2}(\sqrt{-3}x-x)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})+6\sqrt{2}x\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-}}{2x}\right.}{20x}$$

input
```
integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/20*(3*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*
sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) + 6*sqrt(2)*x*sqrt(sqrt
(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3
) - 1/2) + 4*(x^4 + 2*x^2 + 3)*sqrt(x^4 + x^2 + 1))/x
```

3.227.6 Sympy [F]

$$\int (1+x^2)\sqrt{1+x^2+x^4}\,dx = \int \sqrt{(x^2-x+1)(x^2+x+1)}(x^2+1)\,dx$$

input
```
integrate((x**2+1)*(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1), x)
```

3.227.7 Maxima [F]

$$\int (1+x^2)\sqrt{1+x^2+x^4}\,dx = \int \sqrt{x^4+x^2+1}(x^2+1)\,dx$$

input
```
integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

3.227.8 Giac [F]

$$\int (1+x^2)\sqrt{1+x^2+x^4}\,dx = \int \sqrt{x^4+x^2+1}(x^2+1)\,dx$$

input
```
integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int (1+x^2)\sqrt{1+x^2+x^4}\,dx = \int (x^2+1)\sqrt{x^4+x^2+1}\,dx$$

input
```
int((x^2 + 1)*(x^2 + x^4 + 1)^(1/2),x)
```

output
```
int((x^2 + 1)*(x^2 + x^4 + 1)^(1/2), x)
```

3.228 $\int \frac{\sqrt{1+x^2+x^4}}{1+x^2} \, dx$

3.228.1 Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2} \, dx = \frac{x\sqrt{1+x^2+x^4}}{1+x^2} + \frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

output
```
1/2*arctan(x/(x^4+x^2+1)^(1/2))+x*(x^4+x^2+1)^(1/2)/(x^2+1)-(x^2+1)*(cos(2
*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^
4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+3/4*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^
2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.228.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.90 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx$$
$$= \frac{\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-\text{EllipticF}\left(i\text{arcsinh}\left((-1)^{5/6}x\right),(-\right.\right.}{\sqrt{1+x^2+x^4}}$$

input
```
Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

output
```
((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I
*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (
-1)^(2/3)] + (-1)^(1/3)*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-
1)^(2/3)]))/Sqrt[1 + x^2 + x^4]
```

3.228.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1520, 1509, 2214, 1416, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+x^2+1}}{x^2+1}dx$$
$$\Big\downarrow \ 1520$$
$$\int \frac{x^2+2}{(x^2+1)\sqrt{x^4+x^2+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx$$
$$\Big\downarrow \ 1509$$
$$\int \frac{x^2+2}{(x^2+1)\sqrt{x^4+x^2+1}}dx-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}$$
$$\Big\downarrow \ 2214$$

$$\frac{3}{2}\int \frac{1}{\sqrt{x^4+x^2+1}}dx+\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}$$

$$\Big\downarrow \text{2212}$$

$$\frac{1}{2}\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}$$

input
```
Int[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

output
```
(x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - ((1
+ x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt
[1 + x^2 + x^4] + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF
[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

3.228. $\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx$

3.228.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1520
```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-c/e^2   Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] + Int[(
2*a + b*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c, d
, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d
^2 - a*e^2, 0]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2214
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[(B*d + A*e)/(2*d*e)   Int[1/Sqrt[a + b*x
^2 + c*x^4], x], x] - Simp[(B*d - A*e)/(2*d*e)   Int[(d - e*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && E
qQ[c*d^2 - a*e^2, 0] && NeQ[B*d + A*e, 0]
```

3.228. $\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx$

3.228.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.14

method	result
default	$-\frac{4\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}+\frac{4\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,E\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}$
elliptic	$-\frac{4\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}+\frac{4\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,E\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}$

input
```
int((x^4+x^2+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-4/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1
/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*x*(-
2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+4/(-2+2*I*3^(1/2))^(1/2)*
(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x
^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(
-2+2*I*3^(1/2))^(1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3
^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*Ellipt
icPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1
/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx$$
$$= \frac{2\sqrt{2}\left(\sqrt{-3}x-x\right)\sqrt{\sqrt{-3}-1}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right) \mid \frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)-\sqrt{2}\left(\sqrt{-3}x-3x\right)\sqrt{\sqrt{-3}-1}F(\text{arcsi}}{8x}$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/8*(2*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*s
qrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) - sqrt(2)*(sqrt(-3)*x -
3*x)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x
), 1/2*sqrt(-3) - 1/2) + 4*x*arctan(x/sqrt(x^4 + x^2 + 1)) + 8*sqrt(x^4 +
x^2 + 1))/x
```

3.228.6 Sympy [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx = \int \frac{\sqrt{(x^2-x+1)(x^2+x+1)}}{x^2+1}\,dx$$

input
```
integrate((x**4+x**2+1)**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1), x)
```

3.228.7 Maxima [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

3.228.8 Giac [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

input
```
int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1),x)
```

output
```
int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1), x)
```

3.229 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx$

3.229.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx=\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}$$

output
```
1/2*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*ar
ctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.229.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx$$
$$=\frac{\frac{x+x^3+x^5}{1+x^2}+(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+\sqrt[3]{-1}\sqrt{1+\sqrt[3]{}}}{2\sqrt{1+x^2}}$$

input
```
Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

output
```
((x + x^3 + x^5)/(1 + x^2) + (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
(-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3
)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[
(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)
]))/(2*Sqrt[1 + x^2 + x^4])
```

3.229.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1553}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+x^2+1}}{\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \text{1553}$$

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{2\sqrt{x^4+x^2+1}}$$

input
```
Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

output
```
((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/
(2*Sqrt[1 + x^2 + x^4])
```

3.229.3.1 Defintions of rubi rules used

rule 1553
```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> With[{q = Rt[e/d, 2]}, Simp[c*(d + e*x^2)*(Sqrt[(e^2*(a + b*x^2 + c
*x^4))/(c*(d + e*x^2)^2)]/(2*d*e^2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*
ArcTan[q*x], (2*c*d - b*e)/(4*c*d)], x]] /; FreeQ[{a, b, c, d, e}, x] && Ne
Q[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
 && PosQ[e/d]
```

3.229.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 224, normalized size of antiderivative = 4.57

method	result
default	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}+\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{\sqrt{-}}$
risch	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}+\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{\sqrt{-}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}+\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{\sqrt{-}}$

input
```
int((x^4+x^2+1)^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^
(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*Ell
ipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*
3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*
x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2
))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.31

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx =$$
$$-\frac{2\sqrt{2}\sqrt{-3}(x^2+1)\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})+\sqrt{2}(x^2-\sqrt{-3}(x^2+1)+}{8\,(x^2+1)}$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(2)*sqrt(-3)*(x^2 + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/
2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + sqrt(2)*(x^2 - sqrt
(-3)*(x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*x*sqr
t(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 4*sqrt(x^4 + x^2 + 1)*x)/(x^2 + 1)
```

3.229.6 Sympy [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{(x^2-x+1)(x^2+x+1)}}{(x^2+1)^2}\,dx$$

input
```
integrate((x**4+x**2+1)**(1/2)/(x**2+1)**2,x)
```

output
```
Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**2, x)
```

3.229.7 Maxima [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^2}\,dx$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

3.229.8 Giac [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^2}\,dx$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^2}\,dx$$

input
```
int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^2,x)
```

output
```
int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^2, x)
```

3.230 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3} dx$

3.230.1 Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3} dx = \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{1}{4}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

output
```
1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/4*(x^2
+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x))
,1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.230.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3} dx = \frac{\frac{x(2+x^2)(1+x^2+x^4)}{(1+x^2)^2} + \sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(-E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right) + \text{EllipticF}\left(i\text{ar}\right.\right.}{4}$$

input
```
Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

output
```
((x*(2 + x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 + (-1)^(1/3)*Sqrt[1 + (-1)^(1/3
)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^
(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) + 2*(-1)^(2/3)*Sq
rt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*A
rcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(4*Sqrt[1 + x^2 + x^4])
```

3.230.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1554, 25, 2210, 27, 2230, 1509, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3} \, dx$$

$$\Big\downarrow \text{1554}$$

$$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2}-\frac{1}{4}\int -\frac{x^4+2x^2+3}{(x^2+1)^2\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{x^4+2x^2+3}{(x^2+1)^2\sqrt{x^4+x^2+1}}dx+\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}$$

$$\Big\downarrow \text{2210}$$

$$\frac{1}{4}\left(\frac{x\sqrt{x^4+x^2+1}}{x^2+1}-\frac{1}{2}\int -\frac{2(-x^4-x^2+2)}{(x^2+1)\sqrt{x^4+x^2+1}}dx\right)+\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\int \frac{-x^4-x^2+2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}\right)+\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}$$

$$\Big\downarrow \text{2230}$$

$$\frac{1}{4}\left(\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx+\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}\right)+\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{4}\left(\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}\right)+\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}$$

$$\Big\downarrow \text{2212}$$

$$\frac{1}{4}\left(\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}\right)+\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}\right)+\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}$$

input
```
Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

output
```
(x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + (ArcTan[x/Sqrt[1 + x^2 + x^4]] +
 ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])
/Sqrt[1 + x^2 + x^4])/4
```

3.230.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1554
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(q +
 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*((a*(2*q + 3) +
2*b*(q + 2)*x^2 + c*(2*q + 5)*x^4)/Sqrt[a + b*x^2 + c*x^4]), x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && ILtQ[q, -1]
```

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2230
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-C/e^2   Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]
+ Simp[1/e^2   Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2
+ c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && Eq
Q[c*d^2 - a*e^2, 0]
```

3.230. $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3} \, dx$

3.230.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.70

method	result
risch	$\frac{\sqrt{x^4+x^2+1}\,x(x^2+2)}{4(x^2+1)^2}+\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\left(F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-E\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(1+i\sqrt{3}\right)}$
default	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2}+\frac{x\sqrt{x^4+x^2+1}}{4x^2+4}+\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(1+i\sqrt{3}\right)}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{i}{}}}{\sqrt{-2+2i\sqrt{3}}\sqrt{}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2}+\frac{x\sqrt{x^4+x^2+1}}{4x^2+4}+\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(1+i\sqrt{3}\right)}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{i}{}}}{\sqrt{-2+2i\sqrt{3}}\sqrt{}}$

input
```
int((x^4+x^2+1)^(1/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^4+x^2+1)^(1/2)*x*(x^2+2)/(x^2+1)^2+1/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/
2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)
^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3
^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))
^(1/2)))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2
)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1
/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(
-1/2+1/2*I*3^(1/2))^(1/2))
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx =$$
$$-\frac{2\sqrt{2}\sqrt{-3}(x^4+2x^2+1)\sqrt{\sqrt{-3}-1}F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)+\sqrt{2}(x^4+2x^2-\sqrt{-}}{}$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*sqrt(2)*sqrt(-3)*(x^4 + 2*x^2 + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(
arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + sqrt(2)*(x
^4 + 2*x^2 - sqrt(-3)*(x^4 + 2*x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e
(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 4*(x^4 +
2*x^2 + 1)*arctan(x/sqrt(x^4 + x^2 + 1)) - 4*sqrt(x^4 + x^2 + 1)*(x^3 + 2*
x))/(x^4 + 2*x^2 + 1)
```

3.230.6 Sympy [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx = \int \frac{\sqrt{(x^2-x+1)(x^2+x+1)}}{(x^2+1)^3}\,dx$$

input
```
integrate((x**4+x**2+1)**(1/2)/(x**2+1)**3,x)
```

output
```
Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**3, x)
```

3.230.7 Maxima [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3}\,dx$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

3.230.8 Giac [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3}\,dx$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^3}\,dx$$

input
```
int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^3,x)
```

output
```
int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^3, x)
```

3.231 $\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx$

3.231.1 Optimal result

Integrand size = 20, antiderivative size = 166

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx = \frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3}+\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2}+\frac{1}{4}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{8\sqrt{1+x^2+x^4}}$$

output

```
1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^3+1/6*x*(x
^4+x^2+1)^(1/2)/(x^2+1)^2+1/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arc
tan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4
+x^2+1)^(1/2)-1/8*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Elli
pticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2
)
```

3.231.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx = \frac{\frac{x(1+x^2+x^4)(4+5x^2+2x^4)}{(1+x^2)^3} - 2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right) - \text{EllipticF}\right.}{}$$

input
```
Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```

output
```
((x*(1 + x^2 + x^4)*(4 + 5*x^2 + 2*x^4))/(1 + x^2)^3 - 2*(-1)^(1/3)*Sqrt[1
 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6
)*x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - (-1)
^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSi
nh[(-1)^(5/6)*x], (-1)^(2/3)] + 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt
[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^
(2/3)])/(6*Sqrt[1 + x^2 + x^4])
```

3.231.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.30, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {1554, 25, 2210, 27, 2210, 27, 2230, 27, 1509, 1654, 1416, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^4}\,dx$$

$$\Big\downarrow \text{1554}$$

$$\frac{x\sqrt{x^4+x^2+1}}{6(x^2+1)^3} - \frac{1}{6}\int -\frac{3x^4+4x^2+5}{(x^2+1)^3\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\int \frac{3x^4+4x^2+5}{(x^2+1)^3\sqrt{x^4+x^2+1}}dx + \frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{2210}$$

$$\frac{1}{6}\left(\frac{x\sqrt{x^4+x^2+1}}{(x^2+1)^2}-\frac{1}{4}\int -\frac{4(x^4+x^2+4)}{(x^2+1)^2\sqrt{x^4+x^2+1}}dx\right)+\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\int \frac{x^4+x^2+4}{(x^2+1)^2\sqrt{x^4+x^2+1}}dx+\frac{\sqrt{x^4+x^2+1}x}{(x^2+1)^2}\right)+\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{2210}$$

$$\frac{1}{6}\left(-\frac{1}{2}\int -\frac{2(-2x^4-3x^2+2)}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{2\sqrt{x^4+x^2+1}x}{x^2+1}+\frac{\sqrt{x^4+x^2+1}x}{(x^2+1)^2}\right)+\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\int \frac{-2x^4-3x^2+2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{2\sqrt{x^4+x^2+1}x}{x^2+1}+\frac{\sqrt{x^4+x^2+1}x}{(x^2+1)^2}\right)+\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{2230}$$

$$\frac{1}{6}\left(2\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx+\int -\frac{3x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{2\sqrt{x^4+x^2+1}x}{x^2+1}+\frac{\sqrt{x^4+x^2+1}x}{(x^2+1)^2}\right)+\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(2\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx-3\int \frac{x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{2\sqrt{x^4+x^2+1}x}{x^2+1}+\frac{\sqrt{x^4+x^2+1}x}{(x^2+1)^2}\right)+\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{6}\left(-3\int \frac{x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)+\frac{2\sqrt{x^4+x^2}}{x^2+}\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{1654}$$

3.231. $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx$

$$\frac{1}{6}\left(-3\left(\frac{1}{2}\int\frac{1}{\sqrt{x^4+x^2+1}}dx-\frac{1}{2}\int\frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx\right)+2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x}{}\right.\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{6}\left(-3\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{1}{2}\int\frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx\right)+2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+}{(x^2+1)}}}{\sqrt{x^4}}\right.\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{2212}$$

$$\frac{1}{6}\left(-3\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{1}{2}\int\frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}\right)+2\left(\frac{(x^2+1)\sqrt{\frac{x^4+}{(x^2}}}{\sqrt{x}}\right.\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}\left(-3\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)\right)+2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E}{\sqrt{x^4+x^2}}\right.\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{6(x^2+1)^3}$$

input
```
Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```

output
```
(x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^3) + ((x*Sqrt[1 + x^2 + x^4])/(1 + x^
2)^2 + (2*x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + 2*(-((x*Sqrt[1 + x^2 + x^4])/
(1 + x^2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcT
an[x], 1/4])/Sqrt[1 + x^2 + x^4]) - 3*(-1/2*ArcTan[x/Sqrt[1 + x^2 + x^4]]
+ ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4]
)/(4*Sqrt[1 + x^2 + x^4])))/6
```

3.231. $\int\frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}dx$

3.231.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1554
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(q +
 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*((a*(2*q + 3) +
2*b*(q + 2)*x^2 + c*(2*q + 5)*x^4)/Sqrt[a + b*x^2 + c*x^4]), x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && ILtQ[q, -1]
```

rule 1654
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Simp[1/(2*e)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[
1/(2*e)   Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && EqQ[c*d^2
- a*e^2, 0]
```

3.231. $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx$

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2230
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-C/e^2   Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]
+ Simp[1/e^2   Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2
+ c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && Eq
Q[c*d^2 - a*e^2, 0]
```

3.231.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.05

method	result
risch	$\frac{\sqrt{x^4+x^2+1}\,x(2x^4+5x^2+4)}{6(x^2+1)^3}-\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{}}{}$
default	$\frac{x\sqrt{x^4+x^2+1}}{6(x^2+1)^3}+\frac{x\sqrt{x^4+x^2+1}}{6(x^2+1)^2}+\frac{x\sqrt{x^4+x^2+1}}{3x^2+3}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1+\frac{x^2}{2}-}}{}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{6(x^2+1)^3}+\frac{x\sqrt{x^4+x^2+1}}{6(x^2+1)^2}+\frac{x\sqrt{x^4+x^2+1}}{3x^2+3}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1+\frac{x^2}{2}-}}{}$

input
```
int((x^4+x^2+1)^(1/2)/(x^2+1)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(x^4+x^2+1)^(1/2)*x*(2*x^4+5*x^2+4)/(x^2+1)^3-1/3/(-2+2*I*3^(1/2))^(1/
2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(
x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2
))^(1/2))+4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1
-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(Elliptic
F(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x
*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))+1/2/(-1/2+1/2*I*3^(1/
2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2)
)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2
+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx =$$
$$-\frac{4\sqrt{2}\left(x^6+3x^4+3x^2-\sqrt{-3}(x^6+3x^4+3x^2+1)+1\right)\sqrt{\sqrt{-3}-1}\,E\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\right.}{}$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="fricas")
```

output
```
-1/48*(4*sqrt(2)*(x^6 + 3*x^4 + 3*x^2 - sqrt(-3)*(x^6 + 3*x^4 + 3*x^2 + 1)
 + 1)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1
)), 1/2*sqrt(-3) - 1/2) - sqrt(2)*(3*x^6 + 9*x^4 + 9*x^2 - 5*sqrt(-3)*(x^6
 + 3*x^4 + 3*x^2 + 1) + 3)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2
)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 12*(x^6 + 3*x^4 + 3*x^2 + 1
)*arctan(x/sqrt(x^4 + x^2 + 1)) - 8*(2*x^5 + 5*x^3 + 4*x)*sqrt(x^4 + x^2 +
 1))/(x^6 + 3*x^4 + 3*x^2 + 1)
```

3.231.6 Sympy [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx = \int \frac{\sqrt{(x^2-x+1)(x^2+x+1)}}{(x^2+1)^4}\,dx$$

input
```
integrate((x**4+x**2+1)**(1/2)/(x**2+1)**4,x)
```

output
```
Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**4, x)
```

3.231.7 Maxima [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^4}\,dx$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

3.231.8 Giac [F]

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^4}\,dx$$

input
```
integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx = \int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^4}\,dx$$

input
```
int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^4,x)
```

output
```
int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^4, x)
```

3.232 $\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx$

3.232.1 Optimal result

Integrand size = 20, antiderivative size = 159

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{14x\sqrt{1+x^2+x^4}}{15(1+x^2)} - \frac{14(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{15\sqrt{1+x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{5\sqrt{1+x^2+x^4}}$$

output
```
11/15*x*(x^4+x^2+1)^(1/2)+1/5*x^3*(x^4+x^2+1)^(1/2)+14/15*x*(x^4+x^2+1)^(1
/2)/(x^2+1)-14/15*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Elli
pticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2
)+3/5*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*
arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.232.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{x(11+14x^2+14x^4+3x^6)+14\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+2\sqrt[3]{}}{15\sqrt{1+x^2+x^4}}$$

input
```
Integrate[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

output
```
(x*(11 + 14*x^2 + 14*x^4 + 3*x^6) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]
*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] +
 2*(-1)^(1/3)*(-7 + 2*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(
2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(15*Sqrt[1 + x^2
 + x^4])
```

3.232.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1518, 2207, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^3}{\sqrt{x^4+x^2+1}}dx$$
$$\Big\downarrow \text{ 1518}$$
$$\frac{1}{5}\int \frac{11x^4+12x^2+5}{\sqrt{x^4+x^2+1}}dx+\frac{1}{5}\sqrt{x^4+x^2+1}x^3$$
$$\Big\downarrow \text{ 2207}$$
$$\frac{1}{5}\left(\frac{1}{3}\int \frac{2(7x^2+2)}{\sqrt{x^4+x^2+1}}dx+\frac{11}{3}\sqrt{x^4+x^2+1}x\right)+\frac{1}{5}\sqrt{x^4+x^2+1}x^3$$
$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{7x^2+2}{\sqrt{x^4+x^2+1}}dx+\frac{11}{3}\sqrt{x^4+x^2+1}x\right)+\frac{1}{5}\sqrt{x^4+x^2+1}x^3$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(9\int \frac{1}{\sqrt{x^4+x^2+1}}dx-7\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+\frac{11}{3}\sqrt{x^4+x^2+1}x\right)+\frac{1}{5}\sqrt{x^4+x^2+1}x^3$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(\frac{9(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-7\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+\frac{11}{3}\sqrt{x^4+x^2+1}x\right)+ \frac{1}{5}\sqrt{x^4+x^2+1}x^3$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{5}\left(\frac{2}{3}\left(\frac{9(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-7\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+}}{x^2+1}\right.\right.\right.$$
$$\frac{1}{5}\sqrt{x^4+x^2+1}x^3$$

input
```
Int[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

output
```
(x^3*Sqrt[1 + x^2 + x^4])/5 + ((11*x*Sqrt[1 + x^2 + x^4])/3 + (2*(-7*(-((x
*Sqrt[1 + x^2 + x^4])/(1 + x^2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^
2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]) + (9*(1 + x^2)*Sqr
t[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^
2 + x^4])))/3)/5
```

3.232.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.232. $\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx$

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.232. $\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx$

3.232.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.42

method	result
risch	$\frac{x(3x^2+11)\sqrt{x^4+x^2+1}}{15}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{15\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{56\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-}$
default	$\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{15\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{x^3\sqrt{x^4+x^2+1}}{5}+\frac{11x\sqrt{x^4+x^2+1}}{15}-\frac{56\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{}}{2}\right.}}{}$
elliptic	$\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{15\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{x^3\sqrt{x^4+x^2+1}}{5}+\frac{11x\sqrt{x^4+x^2+1}}{15}-\frac{56\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{}}{2}\right.}}{}$

input
```
int((x^2+1)^3/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*x^2+11)*(x^4+x^2+1)^(1/2)+8/15/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1
/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1
/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-56/
15/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*
I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2
+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^
(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.76

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \frac{7\sqrt{2}\left(\sqrt{-3}x-x\right)\sqrt{\sqrt{-3}-1}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)-\sqrt{2}\left(5\sqrt{-3}x-9x\right)\sqrt{\sqrt{-3}-1}F(\mathrm{arc}}{30\,x}$$

input
```
integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(7*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*
sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) - sqrt(2)*(5*sqrt(-3)*x
 - 9*x)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1
)/x), 1/2*sqrt(-3) - 1/2) + 2*(3*x^4 + 11*x^2 + 14)*sqrt(x^4 + x^2 + 1))/x
```

3.232.6 Sympy [F]

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^3}{\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

input
```
integrate((x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral((x**2 + 1)**3/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.232.7 Maxima [F]

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^3}{\sqrt{x^4+x^2+1}}\,dx$$

input
```
integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

3.232.8 Giac [F]

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^3}{\sqrt{x^4+x^2+1}}\,dx$$

input
```
integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^3}{\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((x^2 + 1)^3/(x^2 + x^4 + 1)^(1/2),x)
```

output
```
int((x^2 + 1)^3/(x^2 + x^4 + 1)^(1/2), x)
```

3.233 $\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx$

3.233.1 Optimal result

Integrand size = 20, antiderivative size = 137

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \frac{1}{3}x\sqrt{1+x^2+x^4}+\frac{4x\sqrt{1+x^2+x^4}}{3(1+x^2)} - \frac{4(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}$$

output
```
1/3*x*(x^4+x^2+1)^(1/2)+4/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)-4/3*(x^2+1)*(cos(2
*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^
4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+(x^2+1)*(cos(2*arctan(x))^2)^(
1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)
^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.233.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \frac{x+x^3+x^5+4\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)\left|(-1)^{2/3}\right.\right)+2\sqrt[3]{-1}\left(-2+\sqrt[3]{-1}\right.}{3\sqrt{1+x^2+x^4}}$$

input
```
Integrate[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

output
```
(x + x^3 + x^5 + 4*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)
*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(1/3)*(-2 +
(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(3*Sqrt[1 + x^2 + x^4])
```

3.233.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1518, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^2}{\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{1518}$$

$$\frac{1}{3}\int \frac{2(2x^2+1)}{\sqrt{x^4+x^2+1}}dx+\frac{1}{3}\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{2x^2+1}{\sqrt{x^4+x^2+1}}dx+\frac{1}{3}\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \text{1511}$$

$$\frac{2}{3}\left(3\int \frac{1}{\sqrt{x^4+x^2+1}}dx-2\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+\frac{1}{3}\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \text{1416}$$

$$\frac{2}{3}\left(\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-2\int\frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+\frac{1}{3}\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \text{1509}$$

$$\frac{2}{3}\left(\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-2\left(\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right.\right.$$

$$\frac{1}{3}\sqrt{x^4+x^2+1}x$$

input
```
Int[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

output
```
(x*Sqrt[1 + x^2 + x^4])/3 + (2*(-2*(-((x*Sqrt[1 + x^2 + x^4])/(1 + x^2)) +
 ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])
/Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ell
ipticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4])))/3
```

3.233.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.233.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.59

method	result
default	$\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{x\sqrt{x^4+x^2+1}}{3}-\frac{16\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{3\sqrt{}}$
risch	$\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{x\sqrt{x^4+x^2+1}}{3}-\frac{16\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{3\sqrt{}}$
elliptic	$\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{x\sqrt{x^4+x^2+1}}{3}-\frac{16\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{3\sqrt{}}$

input
```
int((x^2+1)^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2
*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^
(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/3*x*(x^4+x^2+1)^(1/2)-16/3/(-2+2*I*3^(
1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2
)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^
(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1
/2*(-2+2*I*3^(1/2))^(1/2)))
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \frac{2\sqrt{2}(\sqrt{-3}x-x)\sqrt{\sqrt{-3}-1}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)-\sqrt{2}(\sqrt{-3}x-3x)\sqrt{\sqrt{-3}-1}F(\text{arcsi}}{6x}$$

input
```
integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*s
qrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) - sqrt(2)*(sqrt(-3)*x -
3*x)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x
), 1/2*sqrt(-3) - 1/2) + 2*sqrt(x^4 + x^2 + 1)*(x^2 + 4))/x
```

3.233.6 Sympy [F]

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^2}{\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

input
```
integrate((x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral((x**2 + 1)**2/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.233.7 Maxima [F]

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^2}{\sqrt{x^4+x^2+1}}\,dx$$

input
```
integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

3.233.8 Giac [F]

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^2}{\sqrt{x^4+x^2+1}}\,dx$$

input
```
integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{(x^2+1)^2}{\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((x^2 + 1)^2/(x^2 + x^4 + 1)^(1/2),x)
```

output
```
int((x^2 + 1)^2/(x^2 + x^4 + 1)^(1/2), x)
```

3.234 $\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx$

3.234.1 Optimal result

Integrand size = 18, antiderivative size = 115

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \frac{x\sqrt{1+x^2+x^4}}{1+x^2} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}$$

output
```
x*(x^4+x^2+1)^(1/2)/(x^2+1)-(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arcta
n(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x
^2+1)^(1/2)+(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(
sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.234.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\,\text{arcsinh}\left((-1)^{5/6}x\right)\,\middle|\,(-1)^{2/3}\right)+\left(-1+\sqrt[3]{-1}\right)\text{EllipticF}\left(i\,\text{arcsinh}(\right.\right.}{\sqrt{1+x^2+x^4}}$$

input
```
Integrate[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```

output
```
((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I
*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1 + (-1)^(1/3))*EllipticF[I*ArcSin
h[(-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4]
```

3.234.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{\sqrt{x^4+x^2+1}}dx$$
$$\Big\downarrow \text{ 1511}$$
$$2\int \frac{1}{\sqrt{x^4+x^2+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx$$
$$\Big\downarrow \text{ 1416}$$
$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx$$
$$\Big\downarrow \text{ 1509}$$
$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}$$

input
```
Int[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```

output
```
(x*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x
^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] + ((1 + x^2)*Sqrt[
(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x
^4]
```

3.234.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.234.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.78

method	result
default	$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(F\left(\frac{x\sqrt{-2+2}}{2}\right.\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x}}$
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(F\left(\frac{x\sqrt{-2+2}}{2}\right.\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x}}$

input
```
int((x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I
*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2))-4/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^
(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+
I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1
/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{2}(\sqrt{-3}x-x)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right) \mid \frac{1}{2}\sqrt{-3}-\frac{1}{2}) + 2\sqrt{2}x\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right.}{4x}$$

input
```
integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*(sqrt(-3)*x - x)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*sqr
t(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2) + 2*sqrt(2)*x*sqrt(sqrt(-3
) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) -
 1/2) + 4*sqrt(x^4 + x^2 + 1))/x
```

3.234.6 Sympy [F]

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

input
```
integrate((x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral((x**2 + 1)/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

3.234.7 Maxima [F]

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

input
```
integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

3.234.8 Giac [F]

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

input
```
integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((x^2 + 1)/(x^2 + x^4 + 1)^(1/2),x)
```

output
```
int((x^2 + 1)/(x^2 + x^4 + 1)^(1/2), x)
```

3.235 $\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

3.235.1 Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

output
```
1/2*arctan(x/(x^4+x^2+1)^(1/2))+1/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos
(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2
)/(x^4+x^2+1)^(1/2)
```

3.235.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticPi}\left(\sqrt[3]{-1},i\text{arcsinh}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)}{\sqrt{1+x^2+x^4}}$$

input
```
Integrate[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(
-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/Sqrt[1 + x^2 + x^4]
```

3.235.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1534, 1416, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{1534}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+x^2+1}}dx+\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{2212}$$

$$\frac{1}{2}\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

input
```
Int[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2
)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

3.235.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1534
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Simp[1/(2*d)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[1/(2*
d)   Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0
] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

3.235.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.51

method	result	size
default	$\frac{\sqrt{1+\frac{x^{2}}{2}-\frac{i x^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{i x^{2}\sqrt{3}}{2}}\,\Pi\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^{4}+x^{2}+1}}$	104
elliptic	$\frac{\sqrt{1+\frac{x^{2}}{2}-\frac{i x^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{i x^{2}\sqrt{3}}{2}}\,\Pi\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^{4}+x^{2}+1}}$	104

input
```
int(1/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^
2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2
))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*
3^(1/2))^(1/2))
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\frac{1}{8}\sqrt{2}(\sqrt{-3}+1)\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)|\frac{1}{2}\sqrt{-3}-\frac{1}{2}) + \frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

input
```
integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*sqrt(2)*(sqrt(-3) + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(
2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + 1/2*arctan(x/sqrt(x^4 + x^
2 + 1))
```

3.235.6 Sympy [F]

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)}\,dx$$

input
```
integrate(1/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)), x)
```

3.235.7 Maxima [F]

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

3.235.8 Giac [F]

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}\,dx$$

input
```
int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

3.236 $\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx$

3.236.1 Optimal result

Integrand size = 20, antiderivative size = 118

$$\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx = \frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

output
```
1/2*arctan(x/(x^4+x^2+1)^(1/2))+1/2*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos
(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2
)/(x^4+x^2+1)^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x)
)*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1
)^(1/2)
```

3.236.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.92

$$\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{\frac{x+x^3+x^5}{1+x^2} - (-1)^{2/3}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+\sqrt[3]{-1}\sqrt{1+\sqrt[3]{}}}{}$$

input
```
Integrate[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

output
```
((x + x^3 + x^5)/(1 + x^2) - (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
(-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3
)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[
(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)
]) + 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*Ellipt
icPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(2*Sqrt[1 + x^2 + x
^4])
```

3.236.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.43, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1551, 25, 2230, 27, 1509, 1654, 1416, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)^2\sqrt{x^4+x^2+1}}dx$$
$$\Big\downarrow \text{1551}$$
$$\frac{x\sqrt{x^4+x^2+1}}{2(x^2+1)} - \frac{1}{2}\int -\frac{-x^4-2x^2+1}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{2}\int \frac{-x^4-2x^2+1}{(x^2+1)\sqrt{x^4+x^2+1}}dx + \frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{2230}$$

$$\frac{1}{2}\left(\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx+\int -\frac{2x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx\right)+\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx-2\int \frac{x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx\right)+\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{2}\left(-2\int \frac{x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{\sqrt{x^4+x^2+1}x}{x^2+1}\right)+\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{1654}$$

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\int \frac{1}{\sqrt{x^4+x^2+1}}dx-\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\middle|\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{\sqrt{x^4}}{}\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{2}\left(-2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E}{\sqrt{x^4+x^2}}\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{2212}$$

$$\frac{1}{2}\left(-2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{1}{2}\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+}{(x^2+1)^2}}}{\sqrt{x^4+}}\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{216}$$

3.236. $\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx$

$$\frac{1}{2}\left(-2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E(2\,\text{a}}{\sqrt{x^4+x^2+}}\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

input
```
Int[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

output
```
(x*Sqrt[1 + x^2 + x^4])/(2*(1 + x^2)) + (-((x*Sqrt[1 + x^2 + x^4])/(1 + x^
2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
1/4])/Sqrt[1 + x^2 + x^4] - 2*(-1/2*ArcTan[x/Sqrt[1 + x^2 + x^4]] + ((1 +
x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqr
t[1 + x^2 + x^4])))/2
```

3.236.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 1654
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Simp[1/(2*e)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[
1/(2*e)   Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && EqQ[c*d^2
- a*e^2, 0]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2230
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-C/e^2   Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]
+ Simp[1/e^2   Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2
+ c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && Eq
Q[c*d^2 - a*e^2, 0]
```

3.236. $\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx$

3.236.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.78

method	result
risch	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{\sqrt{-}}$
default	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(1+i\sqrt{}\right.}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(1+i\sqrt{}\right.}$

input
```
int(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^
(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*Ell
ipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*
3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*
x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2
))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2
),1/2*(-2+2*I*3^(1/2))^(1/2)))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2
*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2
)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/
2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.19

$$\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx = -\frac{\sqrt{2}(x^2-\sqrt{-3}(x^2+1)+1)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})-\sqrt{2}(x^2-\sqrt{-3}(x}{}$$

input
```
integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(2)*(x^2 - sqrt(-3)*(x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e
(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - sqrt(2)*(
x^2 - sqrt(-3)*(x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqr
t(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 4*(x^2 + 1)*arctan(x/sqr
t(x^4 + x^2 + 1)) - 4*sqrt(x^4 + x^2 + 1)*x)/(x^2 + 1)
```

3.236.6 Sympy [F]

$$\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)^2}\,dx$$

input
```
integrate(1/(x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2), x)
```

3.236.7 Maxima [F]

$$\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)^2}\,dx$$

input
```
integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

3.236.8 Giac [F]

$$\int \frac{1}{(1+x^2)^2 \sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)^2}\,dx$$

input
```
integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)^2 \sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{(x^2+1)^2 \sqrt{x^4+x^2+1}}\,dx$$

input
```
int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)), x)
```

3.237 $\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx$

3.237.1 Optimal result

Integrand size = 20, antiderivative size = 142

$$\begin{aligned}\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx &= \frac{x\sqrt{1+x^2+x^4}}{4\,(1+x^2)^2}+\frac{1}{4}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\\ &\quad+\frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}\\ &\quad-\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}\end{aligned}$$

output
```
1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+3/4*(x^2
+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x))
,1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)-1/2*(x^2+1)*(cos(2*a
rctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+
x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.237.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.65

$$\int \frac{1}{(1+x^2)^3 \sqrt{1+x^2+x^4}} dx = \frac{\frac{x(4+3x^2)(1+x^2+x^4)}{(1+x^2)^2} - 3\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right) - \text{EllipticF}\left(ia\right.\right.}{}$$

input
```
Integrate[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

output
```
((x*(4 + 3*x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 - 3*(-1)^(1/3)*Sqrt[1 + (-1)^
(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-
1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - 2*(-1)^(2/3)
*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1
)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]
)/(4*Sqrt[1 + x^2 + x^4])
```

3.237.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {1551, 25, 2210, 27, 2027, 2230, 1509, 2214, 1416, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)^3 \sqrt{x^4+x^2+1}} dx$$

$$\Big\downarrow \text{1551}$$

$$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2} - \frac{1}{4}\int -\frac{x^4-2x^2+3}{(x^2+1)^2\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{x^4-2x^2+3}{(x^2+1)^2\sqrt{x^4+x^2+1}}dx + \frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}$$

$$\Big\downarrow \; 2210$$

$$\frac{1}{4}\left(\frac{3x\sqrt{x^4+x^2+1}}{x^2+1}-\frac{1}{2}\int \frac{2\left(3x^4+5x^2\right)}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx\right)+\frac{\sqrt{x^4+x^2+1}x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(\frac{3x\sqrt{x^4+x^2+1}}{x^2+1}-\int \frac{3x^4+5x^2}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx\right)+\frac{\sqrt{x^4+x^2+1}x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 2027$$

$$\frac{1}{4}\left(\frac{3x\sqrt{x^4+x^2+1}}{x^2+1}-\int \frac{x^2\left(3x^2+5\right)}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx\right)+\frac{\sqrt{x^4+x^2+1}x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 2230$$

$$\frac{1}{4}\left(3\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx-\int \frac{5x^2+3}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx+\frac{3\sqrt{x^4+x^2+1}x}{x^2+1}\right)+\frac{\sqrt{x^4+x^2+1}x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{4}\left(-\int \frac{5x^2+3}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx+3\left(\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)+\frac{3\sqrt{x^4+x^2\cdots}}{x^2+1}\cdots\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 2214$$

$$\frac{1}{4}\left(-4\int \frac{1}{\sqrt{x^4+x^2+1}}dx+\int \frac{1-x^2}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx+3\left(\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+\cdots}}{x^2+\cdots}\cdots\right.\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{4}\left(\int \frac{1-x^2}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx-\frac{2\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}+3\left(\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\,\text{ar}\cdots\right.}{\sqrt{x^4+x^2+1}\cdots}\right.\right.$$
$$\frac{\sqrt{x^4+x^2+1}x}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 2212$$

3.237. $\int \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx$

$$\frac{1}{4}\left(\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}+3\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E(2\right.\right.$$

$$\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}$$

$\downarrow$ 216

$$\frac{1}{4}\left(\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}+3\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E(2\arctan\right.\right.$$

$$\frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2}$$

input
```
Int[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

output
```
(x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ((3*x*Sqrt[1 + x^2 + x^4])/(1 +
x^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]] + 3*(-((x*Sqrt[1 + x^2 + x^4])/(1 + x
^2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
 1/4])/Sqrt[1 + x^2 + x^4]) - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^
2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4])/4
```

3.237.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

3.237. $\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx$

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2214
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[(B*d + A*e)/(2*d*e)   Int[1/Sqrt[a + b*x
^2 + c*x^4], x], x] - Simp[(B*d - A*e)/(2*d*e)   Int[(d - e*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && E
qQ[c*d^2 - a*e^2, 0] && NeQ[B*d + A*e, 0]
```

rule 2230
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-C/e^2   Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]
+ Simp[1/e^2   Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2
+ c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && Eq
Q[c*d^2 - a*e^2, 0]
```

3.237.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.37

method	result
risch	$\frac{\sqrt{x^4+x^2+1}\,x(3x^2+4)}{4(x^2+1)^2}-\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{3\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\right.}}{}$
default	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2}+\frac{3x\sqrt{x^4+x^2+1}}{4(x^2+1)}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{3\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}-}}{\sqrt{-2+2i\sqrt{3}}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2}+\frac{3x\sqrt{x^4+x^2+1}}{4(x^2+1)}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{3\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}-}}{\sqrt{-2+2i\sqrt{3}}}$

input
```
int(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^4+x^2+1)^(1/2)*x*(3*x^2+4)/(x^2+1)^2-1/(-2+2*I*3^(1/2))^(1/2)*(1-(-
1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+
1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)
)+3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2
*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-
2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3
^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*
(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x
^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(
1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.27

$$\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx = -\frac{3\sqrt{2}\left(x^4+2x^2-\sqrt{-3}(x^4+2x^2+1)+1\right)\sqrt{\sqrt{-3}-1}E\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)-2}{}$$

input
```
integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*(3*sqrt(2)*(x^4 + 2*x^2 - sqrt(-3)*(x^4 + 2*x^2 + 1) + 1)*sqrt(sqrt(
-3) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3)
 - 1/2) - 2*sqrt(2)*(2*x^4 + 4*x^2 - sqrt(-3)*(x^4 + 2*x^2 + 1) + 2)*sqrt(
sqrt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqr
t(-3) - 1/2) - 4*(x^4 + 2*x^2 + 1)*arctan(x/sqrt(x^4 + x^2 + 1)) - 4*sqrt(
x^4 + x^2 + 1)*(3*x^3 + 4*x))/(x^4 + 2*x^2 + 1)
```

3.237.6 Sympy [F]

$$\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)^3}\,dx$$

input
```
integrate(1/(x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3), x)
```

3.237.7 Maxima [F]

$$\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)^3}\,dx$$

input
```
integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

3.237.8 Giac [F]

$$\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+x^2+1}(x^2+1)^3}\,dx$$

input
```
integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx = \int \frac{1}{(x^2+1)^3\sqrt{x^4+x^2+1}}\,dx$$

input
```
int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2)), x)
```

3.238 $\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} dx$

3.238.1 Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} dx = -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} - \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}$$

output
```
-1/3*x*(-x^2+1)/(x^4+x^2+1)^(1/2)+2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)-2/3*(x^2
+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x))
,1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+(x^2+1)*(cos(2*arcta
n(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+
1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.238.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} dx = \frac{-x+x^3+2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+}{3\sqrt{1+}}$$

input
```
Integrate[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```

output
```
(-x + x^3 + 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]
*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(5/6)*Sqrt[3 + 3*
(-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x]
, (-1)^(2/3)])/(3*Sqrt[1 + x^2 + x^4])
```

3.238.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1517, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^3}{(x^4+x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{1517}$$

$$\frac{1}{3}\int \frac{2(x^2+2)}{\sqrt{x^4+x^2+1}}dx-\frac{x(1-x^2)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{x^2+2}{\sqrt{x^4+x^2+1}}dx-\frac{x(1-x^2)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2}{3}\left(3\int \frac{1}{\sqrt{x^4+x^2+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)-\frac{x(1-x^2)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2}{3}\left(\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)-\frac{x(1-x^2)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2}{3}\left(\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{x^2+1}\right)-\frac{x(1-x^2)}{3\sqrt{x^4+x^2+1}}$$

input
```
Int[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```

output
```
-1/3*(x*(1 - x^2))/Sqrt[1 + x^2 + x^4] + (2*((x*Sqrt[1 + x^2 + x^4])/(1 +
x^2) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
 1/4])/Sqrt[1 + x^2 + x^4] + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2
]*EllipticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4])))/3
```

3.238.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.238.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.55

method	result
risch	$\frac{x(x^2-1)}{3\sqrt{x^4+x^2+1}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\cdots$
elliptic	$-\frac{2\left(\frac{1}{6}x-\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\cdots$
default	$-\frac{4\left(-\frac{1}{6}x+\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\cdots$

input
```
int((x^2+1)^3/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(x^2-1)/(x^4+x^2+1)^(1/2)+8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*
3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*E
llipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/(-2+
2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/
2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^
(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^
(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}(x^5+x^3-\sqrt{-3}(x^5+x^3+x)+x)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-3}-1}}{2x}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})-\sqrt{2}(3x^5+3x^3}{6(x^5}$$

input
```
integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(2)*(x^5 + x^3 - sqrt(-3)*(x^5 + x^3 + x) + x)*sqrt(sqrt(-3) - 1
)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1/2)
 - sqrt(2)*(3*x^5 + 3*x^3 + sqrt(-3)*(x^5 + x^3 + x) + 3*x)*sqrt(sqrt(-3)
- 1)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-3) - 1)/x), 1/2*sqrt(-3) - 1
/2) - 2*(3*x^4 + x^2 + 2)*sqrt(x^4 + x^2 + 1))/(x^5 + x^3 + x)
```

3.238.6 Sympy [F]

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^3}{((x^2-x+1)(x^2+x+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

output
```
Integral((x**2 + 1)**3/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

3.238.7 Maxima [F]

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^3}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)
```

3.238.8 Giac [F]

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^3}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^3}{(x^4+x^2+1)^{3/2}}\,dx$$

input
```
int((x^2 + 1)^3/(x^2 + x^4 + 1)^(3/2),x)
```

output
```
int((x^2 + 1)^3/(x^2 + x^4 + 1)^(3/2), x)
```

3.239 $\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}} \, dx$

3.239.1 Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}} \, dx = \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} - \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

output
```
1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)-2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+2/3*(x^2
+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x))
,1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.239.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.61

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}} \, dx = \frac{x + 2x^3 - 2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right) -}{3\sqrt{1}}$$

input
```
Integrate[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
```

output
```
(x + 2*x^3 - 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2
]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - I*Sqrt[2 + (1 + I*Sqrt[
3])*x^2]*Sqrt[6 + (3 - (3*I)*Sqrt[3])*x^2]*EllipticF[ArcSin[(x + I*Sqrt[3]
*x)/2], (I/2)*(I + Sqrt[3])])/(3*Sqrt[1 + x^2 + x^4])
```

3.239.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1517, 27, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^2}{(x^4+x^2+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{1517}$$

$$\frac{1}{3}\int \frac{2(1-x^2)}{\sqrt{x^4+x^2+1}}dx+\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx+\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2}{3}\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)+\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}$$

input
```
Int[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
```

output
```
(x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) + (2*(-((x*Sqrt[1 + x^2 + x^4])/(1
 + x^2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan
[x], 1/4])/Sqrt[1 + x^2 + x^4]))/3
```

3.239.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.239.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.30

method	result
risch	$\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{3\sqrt{}}$
elliptic	$-\frac{2\left(-\frac{1}{3}x^3-\frac{1}{6}x\right)}{\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\right.}}{}$
default	$-\frac{2\left(-\frac{1}{6}x+\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\right.}}{}$

input
```
int((x^2+1)^2/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)+4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*
I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)
*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+8/3/(-
2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(
1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*
3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2)
)^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.34

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{2}\sqrt{-3}(x^4+x^2+1)\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right) \mid \frac{1}{2}\sqrt{-3}-\frac{1}{2}) + \sqrt{2}(x^4+x^2-\sqrt{-3}(x^4}{6\,(x^4+x^2}$$

input
```
integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(2)*sqrt(-3)*(x^4 + x^2 + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(arc
sin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + sqrt(2)*(x^4
+ x^2 - sqrt(-3)*(x^4 + x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin
(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 2*sqrt(x^4 + x^2
 + 1)*(2*x^3 + x))/(x^4 + x^2 + 1)
```

3.239.6 Sympy [F]

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^2}{((x^2-x+1)(x^2+x+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((x**2+1)**2/(x**4+x**2+1)**(3/2),x)
```

output
```
Integral((x**2 + 1)**2/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

3.239.7 Maxima [F]

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^2}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

3.239.8 Giac [F]

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^2}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+1)^2}{(x^4+x^2+1)^{3/2}}\,dx$$

input
```
int((x^2 + 1)^2/(x^2 + x^4 + 1)^(3/2),x)
```

output
```
int((x^2 + 1)^2/(x^2 + x^4 + 1)^(3/2), x)
```

3.240 $\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}}\,dx$

3.240.1 Optimal result

Integrand size = 18, antiderivative size = 96

$$\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}}\,dx = \frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} - \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

output
```
1/3*x*(x^2+2)/(x^4+x^2+1)^(1/2)-1/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/3*(x^2+1
)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.240.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.86 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.67

$$\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}}\,dx = \frac{2x+x^3-\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)-\frac{1}{2}}{3\sqrt{1}}$$

input
```
Integrate[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

output
```
(2*x + x^3 - (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*
EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - (I/2)*Sqrt[2 + (1 + I*Sqr
t[3])*x^2]*Sqrt[6 + (3 - (3*I)*Sqrt[3])*x^2]*EllipticF[ArcSin[(x + I*Sqrt[
3]*x)/2], (I/2)*(I + Sqrt[3])])/(3*Sqrt[1 + x^2 + x^4])
```

3.240.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1492, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{(x^4+x^2+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1492}$$

$$\frac{1}{3}\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx+\frac{x(x^2+2)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{1}{3}\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)+\frac{x(x^2+2)}{3\sqrt{x^4+x^2+1}}$$

input
```
Int[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

output
```
(x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) + (-((x*Sqrt[1 + x^2 + x^4])/(1 + x^
2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
1/4])/Sqrt[1 + x^2 + x^4])/3
```

3.240.3.1 Defintions of rubi rules used

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

3.240.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.32

method	result
risch	$\frac{x(x^2+2)}{3\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{3\sqrt{}}$
elliptic	$-\frac{2\left(-\frac{1}{6}x^3-\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\right.}}{}$
default	$-\frac{2\left(-\frac{1}{6}x+\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\right.}}{}$

input
```
int((x^2+1)/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(x^2+2)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*
3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*E
llipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+4/3/(-2+
2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/
2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^
(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^
(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.36

$$\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{2}\sqrt{-3}(x^4+x^2+1)\sqrt{\sqrt{-3}-1}F(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2})+\sqrt{2}(x^4+x^2-\sqrt{-3}(x^4}{12\,(x^4+x^2}$$

input
```
integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(2)*sqrt(-3)*(x^4 + x^2 + 1)*sqrt(sqrt(-3) - 1)*elliptic_f(ar
csin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + sqrt(2)*(x^4
 + x^2 - sqrt(-3)*(x^4 + x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsi
n(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 4*sqrt(x^4 + x^
2 + 1)*(x^3 + 2*x))/(x^4 + x^2 + 1)
```

3.240.6 Sympy [F]

$$\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{x^2+1}{((x^2-x+1)(x^2+x+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((x**2+1)/(x**4+x**2+1)**(3/2),x)
```

output
```
Integral((x**2 + 1)/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

3.240.7 Maxima [F]

$$\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{x^2+1}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

3.240.8 Giac [F]

$$\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{x^2+1}{(x^4+x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}}\,dx = \int \frac{x^2+1}{(x^4+x^2+1)^{3/2}}\,dx$$

input
```
int((x^2 + 1)/(x^2 + x^4 + 1)^(3/2),x)
```

output
```
int((x^2 + 1)/(x^2 + x^4 + 1)^(3/2), x)
```

3.241 $\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx$

3.241.1 Optimal result

Integrand size = 20, antiderivative size = 166

$$\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx = -\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)-\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

output
```
1/2*arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)+2/3*x*(x
^4+x^2+1)^(1/2)/(x^2+1)-2/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arcta
n(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x
^2+1)^(1/2)+3/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Ellipt
icF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.241.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.23

$$\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx = \frac{-x-2x^3+2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)|\left(\right.\right.}{}$$

input
```
Integrate[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

output
```
(-x - 2*x^3 + 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^
2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*(-2 + (-1)^
(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSi
nh[(-1)^(5/6)*x], (-1)^(2/3)] + 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt
[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^
(2/3)])/(3*Sqrt[1 + x^2 + x^4])
```

3.241.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1545, 25, 1439, 1511, 1416, 1509, 1534, 1416, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)(x^4+x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 1545}$$

$$\int -\frac{x^2}{(x^4+x^2+1)^{3/2}}dx+\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx-\int \frac{x^2}{(x^4+x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 1439}$$

$$\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{1}{3}\int \frac{2x^2+1}{\sqrt{x^4+x^2+1}}dx-\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{3}\left(3\int \frac{1}{\sqrt{x^4+x^2+1}}dx-2\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx-\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{3}\left(\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-2\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx\right)+ \int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx-\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{1509}$$

$$\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx+ \frac{1}{3}\left(\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)\right) \frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{1534}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+x^2+1}}dx+\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+ \frac{1}{3}\left(\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)\right) \frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+ \frac{1}{3}\left(\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)\right) \frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{2212}$$

3.241. $\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx$

$$\frac{1}{2}\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{1}{3}\left(\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)\right)-\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{1}{3}\left(\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}-2\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)\right)-\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}$$

input
```
Int[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

output
```
-1/3*(x*(1 + 2*x^2))/Sqrt[1 + x^2 + x^4] + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2
 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4
])/(4*Sqrt[1 + x^2 + x^4]) + (-2*(-((x*Sqrt[1 + x^2 + x^4])/(1 + x^2)) + (
(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/S
qrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ellip
ticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]))/3
```

3.241.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.241. $\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx$

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1534
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Simp[1/(2*d)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[1/(2*
d)   Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0
] && EqQ[c*d^2 - a*e^2, 0]
```

rule 1545
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x^2)*(a + b*x^
2 + c*x^4)^p, x], x] + Simp[e^2/(c*d^2 - b*d*e + a*e^2)   Int[(a + b*x^2 +
c*x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0] && (EqQ[c*
d^2 - a*e^2, 0] || NiceSqrtQ[b^2 - 4*a*c])
```

3.241. $\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx$

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

3.241.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.98

method	result
risch	$-\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i}{}\right.}}{3}$
default	$-\frac{2\left(\frac{1}{3}x^3+\frac{1}{6}x\right)}{\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{8\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-}}{}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\left(1\right.}$
elliptic	$-\frac{2\left(\frac{1}{3}x^3+\frac{1}{6}x\right)}{\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{8\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-}}{}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\left(1\right.}$

input
```
int(1/(x^2+1)/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2
*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2
)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/(
-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^
(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I
*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2
))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*
x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+
1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(
-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

$$\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx = \frac{4\sqrt{2}(x^4+x^2-\sqrt{-3}(x^4+x^2+1)+1)\sqrt{\sqrt{-3}-1}E(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-}\right.}{}$$

input
```
integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(4*sqrt(2)*(x^4 + x^2 - sqrt(-3)*(x^4 + x^2 + 1) + 1)*sqrt(sqrt(-3) -
 1)*elliptic_e(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/
2) - sqrt(2)*(9*x^4 + 9*x^2 + sqrt(-3)*(x^4 + x^2 + 1) + 9)*sqrt(sqrt(-3)
- 1)*elliptic_f(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1
/2) + 12*(x^4 + x^2 + 1)*arctan(x/sqrt(x^4 + x^2 + 1)) - 8*sqrt(x^4 + x^2
+ 1)*(2*x^3 + x))/(x^4 + x^2 + 1)
```

3.241.6 Sympy [F]

$$\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx = \int \frac{1}{((x^2-x+1)(x^2+x+1))^{\frac{3}{2}}(x^2+1)}\,dx$$

input
```
integrate(1/(x**2+1)/(x**4+x**2+1)**(3/2),x)
```

output
```
Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)), x)
```

3.241.7 Maxima [F]

$$\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+x^2+1)^{\frac{3}{2}}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

3.241.8 Giac [F]

$$\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+x^2+1)^{\frac{3}{2}}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^2+1)\,(x^4+x^2+1)^{3/2}}\,dx$$

input
```
int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(3/2)),x)
```

output
```
int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(3/2)), x)
```

3.242 $\int \frac{1}{(1+x^2)^2(1+x^2+x^4)^{3/2}}\,dx$

3.242.1 Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{1}{(1+x^2)^2(1+x^2+x^4)^{3/2}}\,dx = -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{6\sqrt{1+x^2+x^4}}$$

output
```
arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(x^2+2)/(x^4+x^2+1)^(1/2)+1/3*x*(x^4+x^2
+1)^(1/2)/(x^2+1)+1/6*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*
EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^
(1/2)
```

3.242.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.51

$$\int \frac{1}{(1+x^2)^2(1+x^2+x^4)^{3/2}}\,dx = \frac{-2x(1+x^2)(2+x^2)+3x(1+x^2+x^4)-\sqrt[3]{-1}(1+x^2)\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{\ }}{}$$

input
```
Integrate[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

output
```
(-2*x*(1 + x^2)*(2 + x^2) + 3*x*(1 + x^2 + x^4) - (-1)^(1/3)*(1 + x^2)*Sqr
t[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(
5/6)*x], (-1)^(2/3)] + (-1 + 5*(-1)^(1/3))*EllipticF[I*ArcSinh[(-1)^(5/6)*
x], (-1)^(2/3)] - 12*(-1)^(1/3)*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6
)*x], (-1)^(2/3)]))/(6*(1 + x^2)*Sqrt[1 + x^2 + x^4])
```

3.242.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)^2 (x^4+x^2+1)^{3/2}} \, dx$$

$$\Big\downarrow \ 1556$$

$$\int \left(\frac{-x^2-1}{(x^4+x^2+1)^{3/2}}+\frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}+\frac{1}{(x^2+1)^2\sqrt{x^4+x^2+1}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{6\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)}-\frac{(x^2+2)x}{3\sqrt{x^4+x^2+1}}$$

input
```
Int[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

output
```
-1/3*(x*(2 + x^2))/Sqrt[1 + x^2 + x^4] + (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x
^2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]] + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1
+ x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(6*Sqrt[1 + x^2 + x^4])
```

3.242.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.242.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.05

method	result
risch	$\frac{x(x^4-3x^2-1)}{6(x^2+1)\sqrt{x^4+x^2+1}}-\frac{5\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\right.}}{}$
default	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{2\left(\frac{1}{6}x^3+\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}}-\frac{5\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+}}{3\sqrt{-2+2i\sqrt{3}}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{2\left(\frac{1}{6}x^3+\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}}-\frac{5\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+}}{3\sqrt{-2+2i\sqrt{3}}}$

input
```
int(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(x^4-3*x^2-1)/(x^2+1)/(x^4+x^2+1)^(1/2)-5/3/(-2+2*I*3^(1/2))^(1/2)*(
1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+
x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(
1/2))+2/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1
/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/
2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2
+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))+2/(-1/2+1/2*I*3^(1/2))^(1
/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2
)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I
*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.75

$$\int \frac{1}{(1+x^2)^2 (1+x^2+x^4)^{3/2}}\, dx =$$
$$-\frac{2\sqrt{2}\sqrt{-3}(x^6+2x^4+2x^2+1)\sqrt{\sqrt{-3}-1}F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\sqrt{\sqrt{-3}-1}\right) \mid \frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)+\sqrt{2}(x^6+2x^4+}{}$$

input
```
integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/24*(2*sqrt(2)*sqrt(-3)*(x^6 + 2*x^4 + 2*x^2 + 1)*sqrt(sqrt(-3) - 1)*ell
iptic_f(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) + sq
rt(2)*(x^6 + 2*x^4 + 2*x^2 - sqrt(-3)*(x^6 + 2*x^4 + 2*x^2 + 1) + 1)*sqrt(
sqrt(-3) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqr
t(-3) - 1/2) - 24*(x^6 + 2*x^4 + 2*x^2 + 1)*arctan(x/sqrt(x^4 + x^2 + 1))
- 4*(x^5 - 3*x^3 - x)*sqrt(x^4 + x^2 + 1))/(x^6 + 2*x^4 + 2*x^2 + 1)
```

3.242.6 Sympy [F]

$$\int \frac{1}{(1+x^2)^2 (1+x^2+x^4)^{3/2}}\, dx = \int \frac{1}{((x^2-x+1)(x^2+x+1))^{\frac{3}{2}} (x^2+1)^2}\, dx$$

input
```
integrate(1/(x**2+1)**2/(x**4+x**2+1)**(3/2),x)
```

output
```
Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**2), x)
```

3.242.7 Maxima [F]

$$\int \frac{1}{(1+x^2)^2 (1+x^2+x^4)^{3/2}} \, dx = \int \frac{1}{(x^4+x^2+1)^{\frac{3}{2}} (x^2+1)^2} \, dx$$

input
```
integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

3.242.8 Giac [F]

$$\int \frac{1}{(1+x^2)^2 (1+x^2+x^4)^{3/2}} \, dx = \int \frac{1}{(x^4+x^2+1)^{\frac{3}{2}} (x^2+1)^2} \, dx$$

input
```
integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)^2 (1+x^2+x^4)^{3/2}} \, dx = \int \frac{1}{(x^2+1)^2 (x^4+x^2+1)^{3/2}} \, dx$$

input
```
int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(3/2)),x)
```

output
```
int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(3/2)), x)
```

3.243 $\int \frac{1}{(1+x^2)^3(1+x^2+x^4)^{3/2}}\,dx$

3.243.1 Optimal result

Integrand size = 20, antiderivative size = 190

$$\int \frac{1}{(1+x^2)^3(1+x^2+x^4)^{3/2}}\,dx = -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{3}{4}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{19(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{12\sqrt{1+x^2+x^4}} - \frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

output

```
3/4*arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(-x^2+1)/(x^4+x^2+1)^(1/2)+1/4*x*(x^
4+x^2+1)^(1/2)/(x^2+1)^2-1/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+19/12*(x^2+1)*(co
s(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*(
(x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)-5/4*(x^2+1)*(cos(2*arctan(x
))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/
(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

3.243.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01

$$\int \frac{1}{(1+x^2)^3 (1+x^2+x^4)^{3/2}} \, dx = \frac{4x(-1+x^2)(1+x^2)^2 + 3x(1+x^2+x^4) + 15x(1+x^2)(1+x^2+x^4) -}{}$$

input
```
Integrate[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```

output
```
(4*x*(-1 + x^2)*(1 + x^2)^2 + 3*x*(1 + x^2 + x^4) + 15*x*(1 + x^2)*(1 + x^
2 + x^4) - (-1)^(1/3)*(1 + x^2)^2*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(
2/3)*x^2]*(19*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-9 + (10*I
)*Sqrt[3])*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 18*(-1)^(1/3)*
EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/(12*(1 + x^2
)^2*Sqrt[1 + x^2 + x^4])
```

3.243.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)^3 (x^4+x^2+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{1556}$$

$$\int \left(\frac{1}{(x^2+1)^2 \sqrt{x^4+x^2+1}} + \frac{1}{(x^2+1)^3 \sqrt{x^4+x^2+1}} - \frac{1}{(x^4+x^2+1)^{3/2}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4} \arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - \frac{5(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} \text{EllipticF}\left(2\arctan(x), \frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}} + \frac{19(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}} E\left(2\arctan(x) \middle| \frac{1}{4}\right)}{12\sqrt{x^4+x^2+1}} - \frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)} + \frac{\sqrt{x^4+x^2+1}x}{4(x^2+1)^2} - \frac{(1-x^2)x}{3\sqrt{x^4+x^2+1}}$$

input
```
Int[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```

output
```
-1/3*(x*(1 - x^2))/Sqrt[1 + x^2 + x^4] + (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x
^2)^2) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (3*ArcTan[x/Sqrt[1 + x^2
+ x^4]])/4 + (19*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*A
rcTan[x], 1/4])/(12*Sqrt[1 + x^2 + x^4]) - (5*(1 + x^2)*Sqrt[(1 + x^2 + x^
4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

3.243.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.243.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.82

method	result
risch	$\frac{x(19x^6+37x^4+29x^2+14)}{12(x^2+1)^2\sqrt{x^4+x^2+1}} - \frac{10\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}, \frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}} + 19\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}$
default	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2} + \frac{5x\sqrt{x^4+x^2+1}}{4(x^2+1)} - \frac{2\left(\frac{1}{6}x-\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}} - \frac{10\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}, \frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}} + 19\sqrt{1+}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2} + \frac{5x\sqrt{x^4+x^2+1}}{4(x^2+1)} - \frac{2\left(\frac{1}{6}x-\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}} - \frac{10\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,F\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}, \frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}} + 19\sqrt{1+}$

input
```
int(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x*(19*x^6+37*x^4+29*x^2+14)/(x^2+1)^2/(x^4+x^2+1)^(1/2)-10/3/(-2+2*I*
3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*
x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-
2+2*I*3^(1/2))^(1/2))+19/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*
x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/
2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-El
lipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))+3/2/(-1/
2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*
I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/
2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2
))^(1/2))
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.32

$$\int \frac{1}{(1+x^2)^3\,(1+x^2+x^4)^{3/2}}\,dx =$$

$$-\frac{19\sqrt{2}\big(x^8+3\,x^6+4\,x^4+3\,x^2-\sqrt{-3}(x^8+3\,x^6+4\,x^4+3\,x^2+1)+1\big)\sqrt{\sqrt{-3}-1}E\Big(\arcsin\Big(\tfrac{1}{2}\sqrt{2}x\sqrt{\sqrt{-}}$$

input
```
integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/48*(19*sqrt(2)*(x^8 + 3*x^6 + 4*x^4 + 3*x^2 - sqrt(-3)*(x^8 + 3*x^6 + 4
*x^4 + 3*x^2 + 1) + 1)*sqrt(sqrt(-3) - 1)*elliptic_e(arcsin(1/2*sqrt(2)*x*
sqrt(sqrt(-3) - 1)), 1/2*sqrt(-3) - 1/2) - 2*sqrt(2)*(15*x^8 + 45*x^6 + 60
*x^4 + 45*x^2 - 4*sqrt(-3)*(x^8 + 3*x^6 + 4*x^4 + 3*x^2 + 1) + 15)*sqrt(sq
rt(-3) - 1)*elliptic_f(arcsin(1/2*sqrt(2)*x*sqrt(sqrt(-3) - 1)), 1/2*sqrt(
-3) - 1/2) - 36*(x^8 + 3*x^6 + 4*x^4 + 3*x^2 + 1)*arctan(x/sqrt(x^4 + x^2
+ 1)) - 4*(19*x^7 + 37*x^5 + 29*x^3 + 14*x)*sqrt(x^4 + x^2 + 1))/(x^8 + 3*
x^6 + 4*x^4 + 3*x^2 + 1)
```

3.243.6 Sympy [F]

$$\int \frac{1}{(1+x^2)^3 (1+x^2+x^4)^{3/2}} \, dx = \int \frac{1}{((x^2-x+1)(x^2+x+1))^{\frac{3}{2}} (x^2+1)^3} \, dx$$

input
```
integrate(1/(x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

output
```
Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**3), x)
```

3.243.7 Maxima [F]

$$\int \frac{1}{(1+x^2)^3 (1+x^2+x^4)^{3/2}} \, dx = \int \frac{1}{(x^4+x^2+1)^{\frac{3}{2}} (x^2+1)^3} \, dx$$

input
```
integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

3.243.8 Giac [F]

$$\int \frac{1}{(1+x^2)^3 (1+x^2+x^4)^{3/2}} \, dx = \int \frac{1}{(x^4+x^2+1)^{\frac{3}{2}} (x^2+1)^3} \, dx$$

input
```
integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)^3 (1+x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^2+1)^3 (x^4+x^2+1)^{3/2}}\,dx$$

input
```
int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(3/2)),x)
```

output
```
int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(3/2)), x)
```

3.244 $\int (d+ex^2)^4 (a+bx^2+cx^4)\, dx$

3.244.1 Optimal result

Integrand size = 22, antiderivative size = 135

$$\int (d+ex^2)^4 (a+bx^2+cx^4)\, dx = ad^4x + \frac{1}{3}d^3(bd+4ae)x^3 + \frac{1}{5}d^2(cd^2+4bde+6ae^2)\, x^5 + \frac{2}{7}de(2cd^2+e(3bd+2ae))\, x^7 + \frac{1}{9}e^2(6cd^2+e(4bd+ae))\, x^9 + \frac{1}{11}e^3(4cd+be)x^{11} + \frac{1}{13}ce^4x^{13}$$

output
```
a*d^4*x+1/3*d^3*(4*a*e+b*d)*x^3+1/5*d^2*(6*a*e^2+4*b*d*e+c*d^2)*x^5+2/7*d*
e*(2*c*d^2+e*(2*a*e+3*b*d))*x^7+1/9*e^2*(6*c*d^2+e*(a*e+4*b*d))*x^9+1/11*e
^3*(b*e+4*c*d)*x^11+1/13*c*e^4*x^13
```

3.244.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^4 (a+bx^2+cx^4)\, dx = ad^4x + \frac{1}{3}d^3(bd+4ae)x^3 + \frac{1}{5}d^2(cd^2+4bde+6ae^2)\, x^5 + \frac{2}{7}de(2cd^2+3bde+2ae^2)\, x^7 + \frac{1}{9}e^2(6cd^2+4bde+ae^2)\, x^9 + \frac{1}{11}e^3(4cd+be)x^{11} + \frac{1}{13}ce^4x^{13}$$

input
```
Integrate[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

output
```
a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5
)/5 + (2*d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + 4*b*d*
e + a*e^2)*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13
```

3.244.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^4 \left(a+bx^2+cx^4\right) \, dx$$

$$\Big\downarrow \; 1467$$

$$\int \left(e^2x^8\left(e(ae+4bd)+6cd^2\right)+d^2x^4\left(6ae^2+4bde+cd^2\right)+2dex^6\left(e(2ae+3bd)+2cd^2\right)+d^3x^2(4ae+bd)+ad^4 -$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{9}e^2x^9\left(e(ae+4bd)+6cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right)+\frac{2}{7}dex^7\left(e(2ae+3bd)+2cd^2\right)+ \frac{1}{3}d^3x^3(4ae+bd)+ad^4x+\frac{1}{11}e^3x^{11}(be+4cd)+\frac{1}{13}ce^4x^{13}$$

input
```
Int[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

output
```
a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5
)/5 + (2*d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^7)/7 + (e^2*(6*c*d^2 + e*(4*b
*d + a*e))*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13
```

3.244.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.244.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

method	result
norman	$\frac{c\,e^4x^{13}}{13}+\left(\frac{1}{11}e^4b+\frac{4}{11}d\,e^3c\right)x^{11}+\left(\frac{1}{9}e^4a+\frac{4}{9}d\,e^3b+\frac{2}{3}e^2d^2c\right)x^9+\left(\frac{4}{7}d\,e^3a+\frac{6}{7}e^2d^2b+\frac{4}{7}d^3ec\right)x$
default	$\frac{c\,e^4x^{13}}{13}+\frac{(e^4b+4d\,e^3c)x^{11}}{11}+\frac{(e^4a+4d\,e^3b+6e^2d^2c)x^9}{9}+\frac{(4d\,e^3a+6e^2d^2b+4d^3ec)x^7}{7}+\frac{(6e^2d^2a+4d^3eb+d^4c)x^5}{5}+\frac{(4}{}$
gosper	$\frac{1}{13}c\,e^4x^{13}+\frac{1}{11}x^{11}e^4b+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{4}{9}x^9d\,e^3b+\frac{2}{3}x^9e^2d^2c+\frac{4}{7}x^7d\,e^3a+\frac{6}{7}x^7e^2d^2b+$
risch	$\frac{1}{13}c\,e^4x^{13}+\frac{1}{11}x^{11}e^4b+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{4}{9}x^9d\,e^3b+\frac{2}{3}x^9e^2d^2c+\frac{4}{7}x^7d\,e^3a+\frac{6}{7}x^7e^2d^2b+$
parallelrisch	$\frac{1}{13}c\,e^4x^{13}+\frac{1}{11}x^{11}e^4b+\frac{4}{11}cd\,e^3x^{11}+\frac{1}{9}x^9e^4a+\frac{4}{9}x^9d\,e^3b+\frac{2}{3}x^9e^2d^2c+\frac{4}{7}x^7d\,e^3a+\frac{6}{7}x^7e^2d^2b+$

input
```
int((e*x^2+d)^4*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/13*c*e^4*x^13+(1/11*e^4*b+4/11*d*e^3*c)*x^11+(1/9*e^4*a+4/9*d*e^3*b+2/3*
e^2*d^2*c)*x^9+(4/7*d*e^3*a+6/7*e^2*d^2*b+4/7*d^3*e*c)*x^7+(6/5*e^2*d^2*a+
4/5*d^3*e*b+1/5*d^4*c)*x^5+(4/3*d^3*e*a+1/3*d^4*b)*x^3+a*d^4*x
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (d+ex^2)^4\left(a+bx^2+cx^4\right)\,dx &= \frac{1}{13}\,ce^4x^{13}+\frac{1}{11}\left(4\,cde^3+be^4\right)x^{11} \\ &+\frac{1}{9}\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^9 \\ &+\frac{2}{7}\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^7+ad^4x \\ &+\frac{1}{5}\left(cd^4+4\,bd^3e+6\,ad^2e^2\right)x^5+\frac{1}{3}\left(bd^4+4\,ad^3e\right)x^3\end{aligned}$$

input
```
integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/13*c*e^4*x^13 + 1/11*(4*c*d*e^3 + b*e^4)*x^11 + 1/9*(6*c*d^2*e^2 + 4*b*d
*e^3 + a*e^4)*x^9 + 2/7*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^7 + a*d^4*
x + 1/5*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^5 + 1/3*(b*d^4 + 4*a*d^3*e)*x^
3
```

3.244.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16

$$\int (d+ex^2)^4 (a+bx^2+cx^4)\,dx = ad^4x + \frac{ce^4x^{13}}{13} + x^{11}\left(\frac{be^4}{11} + \frac{4cde^3}{11}\right) + x^9\left(\frac{ae^4}{9} + \frac{4bde^3}{9} + \frac{2cd^2e^2}{3}\right) + x^7 \cdot \left(\frac{4ade^3}{7} + \frac{6bd^2e^2}{7} + \frac{4cd^3e}{7}\right) + x^5 \cdot \left(\frac{6ad^2e^2}{5} + \frac{4bd^3e}{5} + \frac{cd^4}{5}\right) + x^3 \cdot \left(\frac{4ad^3e}{3} + \frac{bd^4}{3}\right)$$

input
```
integrate((e*x**2+d)**4*(c*x**4+b*x**2+a),x)
```

output
```
a*d**4*x + c*e**4*x**13/13 + x**11*(b*e**4/11 + 4*c*d*e**3/11) + x**9*(a*e
**4/9 + 4*b*d*e**3/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 6*b*d**2*e*
*2/7 + 4*c*d**3*e/7) + x**5*(6*a*d**2*e**2/5 + 4*b*d**3*e/5 + c*d**4/5) +
x**3*(4*a*d**3*e/3 + b*d**4/3)
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^4 (a+bx^2+cx^4)\,dx = \frac{1}{13}\,ce^4x^{13} + \frac{1}{11}\,(4\,cde^3 + be^4)x^{11} + \frac{1}{9}\,(6\,cd^2e^2 + 4\,bde^3 + ae^4)x^9 + \frac{2}{7}\,(2\,cd^3e + 3\,bd^2e^2 + 2\,ade^3)x^7 + ad^4x + \frac{1}{5}\,(cd^4 + 4\,bd^3e + 6\,ad^2e^2)x^5 + \frac{1}{3}\,(bd^4 + 4\,ad^3e)x^3$$

input
```
integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/13*c*e^4*x^13 + 1/11*(4*c*d*e^3 + b*e^4)*x^11 + 1/9*(6*c*d^2*e^2 + 4*b*d
*e^3 + a*e^4)*x^9 + 2/7*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^7 + a*d^4*
x + 1/5*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^5 + 1/3*(b*d^4 + 4*a*d^3*e)*x^
3
```

3.244.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10

$$\int (d+ex^2)^4 (a+bx^2+cx^4)\,dx = \frac{1}{13}ce^4x^{13} + \frac{4}{11}cde^3x^{11} + \frac{1}{11}be^4x^{11} + \frac{2}{3}cd^2e^2x^9 + \frac{4}{9}bde^3x^9 + \frac{1}{9}ae^4x^9 + \frac{4}{7}cd^3ex^7 + \frac{6}{7}bd^2e^2x^7 + \frac{4}{7}ade^3x^7 + \frac{1}{5}cd^4x^5 + \frac{4}{5}bd^3ex^5 + \frac{6}{5}ad^2e^2x^5 + \frac{1}{3}bd^4x^3 + \frac{4}{3}ad^3ex^3 + ad^4x$$

input
```
integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/13*c*e^4*x^13 + 4/11*c*d*e^3*x^11 + 1/11*b*e^4*x^11 + 2/3*c*d^2*e^2*x^9
+ 4/9*b*d*e^3*x^9 + 1/9*a*e^4*x^9 + 4/7*c*d^3*e*x^7 + 6/7*b*d^2*e^2*x^7 +
4/7*a*d*e^3*x^7 + 1/5*c*d^4*x^5 + 4/5*b*d^3*e*x^5 + 6/5*a*d^2*e^2*x^5 + 1/
3*b*d^4*x^3 + 4/3*a*d^3*e*x^3 + a*d^4*x
```

3.244.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.97

$$\int (d+ex^2)^4 (a+bx^2+cx^4)\,dx = x^3\left(\frac{b\,d^4}{3}+\frac{4\,a\,e\,d^3}{3}\right)+x^{11}\left(\frac{b\,e^4}{11}+\frac{4\,c\,d\,e^3}{11}\right) + x^5\left(\frac{c\,d^4}{5}+\frac{4\,b\,d^3\,e}{5}+\frac{6\,a\,d^2\,e^2}{5}\right) + x^9\left(\frac{2\,c\,d^2\,e^2}{3}+\frac{4\,b\,d\,e^3}{9}+\frac{a\,e^4}{9}\right)+\frac{c\,e^4\,x^{13}}{13} + a\,d^4\,x+\frac{2\,d\,e\,x^7\,(2\,c\,d^2+3\,b\,d\,e+2\,a\,e^2)}{7}$$

input
```
int((d + e*x^2)^4*(a + b*x^2 + c*x^4),x)
```

output
```
x^3*((b*d^4)/3 + (4*a*d^3*e)/3) + x^11*((b*e^4)/11 + (4*c*d*e^3)/11) + x^5
*((c*d^4)/5 + (6*a*d^2*e^2)/5 + (4*b*d^3*e)/5) + x^9*((a*e^4)/9 + (2*c*d^2
*e^2)/3 + (4*b*d*e^3)/9) + (c*e^4*x^13)/13 + a*d^4*x + (2*d*e*x^7*(2*a*e^2
 + 2*c*d^2 + 3*b*d*e))/7
```

3.244. $\int (d+ex^2)^4 \left(a+bx^2+cx^4\right) \, dx$

3.245 $\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right) \, dx$

3.245.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right) \, dx = ad^3x+\frac{1}{3}d^2(bd+3ae)x^3+\frac{1}{5}d\left(cd^2+3e(bd+ae)\right)x^5 +\frac{1}{7}e\left(3cd^2+e(3bd+ae)\right)x^7+\frac{1}{9}e^2(3cd+be)x^9+\frac{1}{11}ce^3x^{11}$$

output
```
a*d^3*x+1/3*d^2*(3*a*e+b*d)*x^3+1/5*d*(c*d^2+3*e*(a*e+b*d))*x^5+1/7*e*(3*c
*d^2+e*(a*e+3*b*d))*x^7+1/9*e^2*(b*e+3*c*d)*x^9+1/11*c*e^3*x^11
```

3.245.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right) \, dx = ad^3x+\frac{1}{3}d^2(bd+3ae)x^3+\frac{1}{5}d\left(cd^2+3bde+3ae^2\right)x^5 +\frac{1}{7}e\left(3cd^2+3bde+ae^2\right)x^7+\frac{1}{9}e^2(3cd+be)x^9+\frac{1}{11}ce^3x^{11}$$

input
```
Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]
```

output
```
a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^5)/
5 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c
*e^3*x^11)/11
```

3.245.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^3 (a+bx^2+cx^4)\,dx$$

$$\Big\downarrow \text{1467}$$

$$\int \left(ex^6 (e(ae+3bd)+3cd^2)+dx^4 (3e(ae+bd)+cd^2)+d^2x^2 (3ae+bd)+ad^3+e^2x^8 (be+3cd)+ce^3x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}ex^7 (e(ae+3bd)+3cd^2)+\frac{1}{5}dx^5 (3e(ae+bd)+cd^2)+\frac{1}{3}d^2x^3 (3ae+bd)+ad^3x+\frac{1}{9}e^2x^9 (be+3cd)+\frac{1}{11}ce^3x^{11}$$

input
```
Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]
```

output
```
a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^5)/5
+ (e*(3*c*d^2 + e*(3*b*d + a*e))*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e
^3*x^11)/11
```

3.245.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.245.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

method	result
norman	$\frac{c\,e^3x^{11}}{11}+\left(\frac{1}{9}e^3b+\frac{1}{3}d\,e^2c\right)x^9+\left(\frac{1}{7}a\,e^3+\frac{3}{7}d\,e^2b+\frac{3}{7}c\,d^2e\right)x^7+\left(\frac{3}{5}d\,e^2a+\frac{3}{5}d^2eb+\frac{1}{5}d^3c\right)x^5+($
default	$\frac{c\,e^3x^{11}}{11}+\frac{(e^3b+3d\,e^2c)x^9}{9}+\frac{(a\,e^3+3d\,e^2b+3c\,d^2e)x^7}{7}+\frac{(3d\,e^2a+3d^2eb+d^3c)x^5}{5}+\frac{(3d^2ea+d^3b)x^3}{3}+a\,d^3x$
gosper	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{9}x^9e^3b+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7d\,e^2b+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{3}{5}x^5d^2eb+\frac{1}{5}x^5$
risch	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{9}x^9e^3b+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7d\,e^2b+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{3}{5}x^5d^2eb+\frac{1}{5}x^5$
parallelrisch	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{9}x^9e^3b+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7d\,e^2b+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{3}{5}x^5d^2eb+\frac{1}{5}x^5$

input
```
int((e*x^2+d)^3*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/11*c*e^3*x^11+(1/9*e^3*b+1/3*d*e^2*c)*x^9+(1/7*a*e^3+3/7*d*e^2*b+3/7*c*d
^2*e)*x^7+(3/5*d*e^2*a+3/5*d^2*e*b+1/5*d^3*c)*x^5+(d^2*e*a+1/3*d^3*b)*x^3+
a*d^3*x
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int (d+ex^2)^3\,(a+bx^2+cx^4)\ dx=\frac{1}{11}\,ce^3x^{11}+\frac{1}{9}\left(3\,cde^2+be^3\right)x^9+\frac{1}{7}\left(3\,cd^2e+3\,bde^2+ae^3\right)x^7+\frac{1}{5}\left(cd^3+3\,bd^2e+3\,ade^2\right)x^5+ad^3x+\frac{1}{3}\left(bd^3+3\,ad^2e\right)x^3$$

input
```
integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/11*c*e^3*x^11 + 1/9*(3*c*d*e^2 + b*e^3)*x^9 + 1/7*(3*c*d^2*e + 3*b*d*e^2
 + a*e^3)*x^7 + 1/5*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^5 + a*d^3*x + 1/3*(b
*d^3 + 3*a*d^2*e)*x^3
```

3.245.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

$$\int (d+ex^2)^3 (a+bx^2+cx^4)\ dx = ad^3x + \frac{ce^3x^{11}}{11} + x^9\left(\frac{be^3}{9}+\frac{cde^2}{3}\right) + x^7\left(\frac{ae^3}{7}+\frac{3bde^2}{7}+\frac{3cd^2e}{7}\right) + x^5\cdot\left(\frac{3ade^2}{5}+\frac{3bd^2e}{5}+\frac{cd^3}{5}\right) + x^3\left(ad^2e+\frac{bd^3}{3}\right)$$

input
```
integrate((e*x**2+d)**3*(c*x**4+b*x**2+a),x)
```

output
```
a*d**3*x + c*e**3*x**11/11 + x**9*(b*e**3/9 + c*d*e**2/3) + x**7*(a*e**3/7
 + 3*b*d*e**2/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + 3*b*d**2*e/5 + c*d*
*3/5) + x**3*(a*d**2*e + b*d**3/3)
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int (d+ex^2)^3 (a+bx^2+cx^4)\ dx = \frac{1}{11}\,ce^3x^{11} + \frac{1}{9}\left(3\,cde^2+be^3\right)x^9 + \frac{1}{7}\left(3\,cd^2e+3\,bde^2+ae^3\right)x^7 + \frac{1}{5}\left(cd^3+3\,bd^2e+3\,ade^2\right)x^5 + ad^3x + \frac{1}{3}\left(bd^3+3\,ad^2e\right)x^3$$

input
```
integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/11*c*e^3*x^11 + 1/9*(3*c*d*e^2 + b*e^3)*x^9 + 1/7*(3*c*d^2*e + 3*b*d*e^2
 + a*e^3)*x^7 + 1/5*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^5 + a*d^3*x + 1/3*(b
*d^3 + 3*a*d^2*e)*x^3
```

3.245.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08

$$\int (d+ex^2)^3 (a+bx^2+cx^4)\ dx = \frac{1}{11}ce^3x^{11} + \frac{1}{3}cde^2x^9 + \frac{1}{9}be^3x^9 + \frac{3}{7}cd^2ex^7 + \frac{3}{7}bde^2x^7 + \frac{1}{7}ae^3x^7 + \frac{1}{5}cd^3x^5 + \frac{3}{5}bd^2ex^5 + \frac{3}{5}ade^2x^5 + \frac{1}{3}bd^3x^3 + ad^2ex^3 + ad^3x$$

input
```
integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/11*c*e^3*x^11 + 1/3*c*d*e^2*x^9 + 1/9*b*e^3*x^9 + 3/7*c*d^2*e*x^7 + 3/7*
b*d*e^2*x^7 + 1/7*a*e^3*x^7 + 1/5*c*d^3*x^5 + 3/5*b*d^2*e*x^5 + 3/5*a*d*e^
2*x^5 + 1/3*b*d^3*x^3 + a*d^2*e*x^3 + a*d^3*x
```

3.245.9 Mupad [B] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int (d+ex^2)^3 (a+bx^2+cx^4)\ dx = x^3\left(\frac{b\,d^3}{3}+a\,e\,d^2\right) + x^9\left(\frac{b\,e^3}{9}+\frac{c\,d\,e^2}{3}\right) + x^5\left(\frac{c\,d^3}{5}+\frac{3\,b\,d^2\,e}{5}+\frac{3\,a\,d\,e^2}{5}\right) + x^7\left(\frac{3\,c\,d^2\,e}{7}+\frac{3\,b\,d\,e^2}{7}+\frac{a\,e^3}{7}\right) + \frac{c\,e^3\,x^{11}}{11} + a\,d^3\,x$$

input
```
int((d + e*x^2)^3*(a + b*x^2 + c*x^4),x)
```

output
```
x^3*((b*d^3)/3 + a*d^2*e) + x^9*((b*e^3)/9 + (c*d*e^2)/3) + x^5*((c*d^3)/5
 + (3*a*d*e^2)/5 + (3*b*d^2*e)/5) + x^7*((a*e^3)/7 + (3*b*d*e^2)/7 + (3*c*
d^2*e)/7) + (c*e^3*x^11)/11 + a*d^3*x
```

3.246 $\int (d+ex^2)^2 (a+bx^2+cx^4)\,dx$

3.246.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int (d+ex^2)^2 (a+bx^2+cx^4)\,dx = ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5 + \frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9$$

output
```
a*d^2*x+1/3*d*(2*a*e+b*d)*x^3+1/5*(c*d^2+e*(a*e+2*b*d))*x^5+1/7*e*(b*e+2*c
*d)*x^7+1/9*c*e^2*x^9
```

3.246.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (d+ex^2)^2 (a+bx^2+cx^4)\,dx = ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+2bde+ae^2\right)x^5 + \frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9$$

input
```
Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

output
```
a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e
*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

3.246.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx$$

$$\Big\downarrow \text{ 1467}$$

$$\int \left(x^4\left(e(ae+2bd)+cd^2\right)+dx^2(2ae+bd)+ad^2+ex^6(be+2cd)+ce^2x^8\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

input
```
Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

output
```
a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e
*(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

3.246.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.246.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result	siz
default	$\frac{c\,e^{2} x^{9}}{9}+\frac{\left(b\,e^{2}+2 d c e \right) x^{7}}{7}+\frac{\left(a\,e^{2}+2 b d e +c\,d^{2}\right) x^{5}}{5}+\frac{\left(2 e d a +b\,d^{2}\right) x^{3}}{3}+a\,d^{2} x$	70
norman	$\frac{c\,e^{2} x^{9}}{9}+\left(\frac{1}{7} b\,e^{2}+\frac{2}{7} d c e \right) x^{7}+\left(\frac{1}{5} a\,e^{2}+\frac{2}{5} b d e +\frac{1}{5} c\,d^{2}\right) x^{5}+\left(\frac{2}{3} e d a +\frac{1}{3} b\,d^{2}\right) x^{3}+a\,d^{2} x$	71
gosper	$\frac{1}{9} c\,e^{2} x^{9}+\frac{1}{7} x^{7} b\,e^{2}+\frac{2}{7} c d e\,x^{7}+\frac{1}{5} x^{5} a\,e^{2}+\frac{2}{5} x^{5} b d e +\frac{1}{5} x^{5} c\,d^{2}+\frac{2}{3} a d e\,x^{3}+\frac{1}{3} x^{3} b\,d^{2}+a\,d^{2} x$	77
risch	$\frac{1}{9} c\,e^{2} x^{9}+\frac{1}{7} x^{7} b\,e^{2}+\frac{2}{7} c d e\,x^{7}+\frac{1}{5} x^{5} a\,e^{2}+\frac{2}{5} x^{5} b d e +\frac{1}{5} x^{5} c\,d^{2}+\frac{2}{3} a d e\,x^{3}+\frac{1}{3} x^{3} b\,d^{2}+a\,d^{2} x$	77
parallelrisch	$\frac{1}{9} c\,e^{2} x^{9}+\frac{1}{7} x^{7} b\,e^{2}+\frac{2}{7} c d e\,x^{7}+\frac{1}{5} x^{5} a\,e^{2}+\frac{2}{5} x^{5} b d e +\frac{1}{5} x^{5} c\,d^{2}+\frac{2}{3} a d e\,x^{3}+\frac{1}{3} x^{3} b\,d^{2}+a\,d^{2} x$	77

input
```
int((e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x^5+1/3*(2
*a*d*e+b*d^2)*x^3+a*d^2*x
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)\,dx = \frac{1}{9}\,ce^{2}x^{9} + \frac{1}{7}\,{\left(2\,cde + be^{2}\right)}x^{7} + \frac{1}{5}\,{\left(cd^{2} + 2\,bde + ae^{2}\right)}x^{5} + ad^{2}x + \frac{1}{3}\,{\left(bd^{2} + 2\,ade\right)}x^{3}$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*
x^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

3.246.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

$$\int (d+ex^2)^2 (a+bx^2+cx^4) \, dx = ad^2x + \frac{ce^2x^9}{9} + x^7\left(\frac{be^2}{7} + \frac{2cde}{7}\right) + x^5\left(\frac{ae^2}{5} + \frac{2bde}{5} + \frac{cd^2}{5}\right) + x^3 \cdot \left(\frac{2ade}{3} + \frac{bd^2}{3}\right)$$

input
```
integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

output
```
a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 +
2*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (d+ex^2)^2 (a+bx^2+cx^4) \, dx = \frac{1}{9}ce^2x^9 + \frac{1}{7}(2cde+be^2)x^7 + \frac{1}{5}(cd^2+2bde+ae^2)x^5 + ad^2x + \frac{1}{3}(bd^2+2ade)x^3$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*
x^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int (d+ex^2)^2 (a+bx^2+cx^4) \, dx = \frac{1}{9}ce^2x^9 + \frac{2}{7}cdex^7 + \frac{1}{7}be^2x^7 + \frac{1}{5}cd^2x^5 + \frac{2}{5}bdex^5 + \frac{1}{5}ae^2x^5 + \frac{1}{3}bd^2x^3 + \frac{2}{3}adex^3 + ad^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/9*c*e^2*x^9 + 2/7*c*d*e*x^7 + 1/7*b*e^2*x^7 + 1/5*c*d^2*x^5 + 2/5*b*d*e*
x^5 + 1/5*a*e^2*x^5 + 1/3*b*d^2*x^3 + 2/3*a*d*e*x^3 + a*d^2*x
```

3.246.9 Mupad [B] (verification not implemented)

Time = 7.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int (d+ex^2)^2 (a+bx^2+cx^4)\,dx = x^5\left(\frac{c\,d^2}{5}+\frac{2\,b\,d\,e}{5}+\frac{a\,e^2}{5}\right)+x^3\left(\frac{b\,d^2}{3}+\frac{2\,a\,e\,d}{3}\right)+x^7\left(\frac{b\,e^2}{7}+\frac{2\,c\,d\,e}{7}\right)+\frac{c\,e^2\,x^9}{9}+a\,d^2\,x$$

input
```
int((d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

output
```
x^5*((a*e^2)/5 + (c*d^2)/5 + (2*b*d*e)/5) + x^3*((b*d^2)/3 + (2*a*d*e)/3)
+ x^7*((b*e^2)/7 + (2*c*d*e)/7) + (c*e^2*x^9)/9 + a*d^2*x
```

3.247 $\int (d+ex^2)(a+bx^2+cx^4)\,dx$

3.247.1 Optimal result

Integrand size = 20, antiderivative size = 42

$$\int (d+ex^2)(a+bx^2+cx^4)\,dx = adx+\frac{1}{3}(bd+ae)x^3+\frac{1}{5}(cd+be)x^5+\frac{1}{7}cex^7$$

output
```
a*d*x+1/3*(a*e+b*d)*x^3+1/5*(b*e+c*d)*x^5+1/7*c*e*x^7
```

3.247.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (d+ex^2)(a+bx^2+cx^4)\,dx = adx+\frac{1}{3}(bd+ae)x^3+\frac{1}{5}(cd+be)x^5+\frac{1}{7}cex^7$$

input
```
Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

3.247.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)(a+bx^2+cx^4)\,dx$$

$$\Big\downarrow \text{1467}$$

$$\int \left(x^2(ae+bd)+ad+x^4(be+cd)+cex^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

input
```
Int[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

3.247.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.247.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
default	$adx+\frac{(ae+bd)x^3}{3}+\frac{(be+cd)x^5}{5}+\frac{ce\,x^7}{7}$	37
norman	$\frac{ce\,x^7}{7}+\left(\frac{be}{5}+\frac{cd}{5}\right)x^5+\left(\frac{ae}{3}+\frac{bd}{3}\right)x^3+adx$	39
gosper	$\frac{1}{7}ce\,x^7+\frac{1}{5}x^5be+\frac{1}{5}cd\,x^5+\frac{1}{3}ae\,x^3+\frac{1}{3}x^3bd+adx$	41
risch	$\frac{1}{7}ce\,x^7+\frac{1}{5}x^5be+\frac{1}{5}cd\,x^5+\frac{1}{3}ae\,x^3+\frac{1}{3}x^3bd+adx$	41
parallelrisch	$\frac{1}{7}ce\,x^7+\frac{1}{5}x^5be+\frac{1}{5}cd\,x^5+\frac{1}{3}ae\,x^3+\frac{1}{3}x^3bd+adx$	41

input
```
int((e*x^2+d)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
a*d*x+1/3*(a*e+b*d)*x^3+1/5*(b*e+c*d)*x^5+1/7*c*e*x^7
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (d+ex^2)(a+bx^2+cx^4)\,dx=\frac{1}{7}cex^7+\frac{1}{5}(cd+be)x^5+\frac{1}{3}(bd+ae)x^3+adx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/7*c*e*x^7 + 1/5*(c*d + b*e)*x^5 + 1/3*(b*d + a*e)*x^3 + a*d*x
```

3.247.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int (d+ex^2)(a+bx^2+cx^4)\,dx=adx+\frac{cex^7}{7}+x^5\left(\frac{be}{5}+\frac{cd}{5}\right)+x^3\left(\frac{ae}{3}+\frac{bd}{3}\right)$$

input
```
integrate((e*x**2+d)*(c*x**4+b*x**2+a),x)
```

output
```
a*d*x + c*e*x**7/7 + x**5*(b*e/5 + c*d/5) + x**3*(a*e/3 + b*d/3)
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (d+ex^2) (a+bx^2+cx^4) \, dx = \frac{1}{7}cex^7 + \frac{1}{5}(cd+be)x^5 + \frac{1}{3}(bd+ae)x^3 + adx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/7*c*e*x^7 + 1/5*(c*d + b*e)*x^5 + 1/3*(b*d + a*e)*x^3 + a*d*x
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int (d+ex^2) (a+bx^2+cx^4) \, dx = \frac{1}{7}cex^7 + \frac{1}{5}cdx^5 + \frac{1}{5}bex^5 + \frac{1}{3}bdx^3 + \frac{1}{3}aex^3 + adx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/7*c*e*x^7 + 1/5*c*d*x^5 + 1/5*b*e*x^5 + 1/3*b*d*x^3 + 1/3*a*e*x^3 + a*d*
x
```

3.247.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int (d+ex^2) (a+bx^2+cx^4) \, dx = \frac{c\,e\,x^7}{7} + \left(\frac{b\,e}{5} + \frac{c\,d}{5}\right) x^5 + \left(\frac{a\,e}{3} + \frac{b\,d}{3}\right) x^3 + a\,d\,x$$

input
```
int((d + e*x^2)*(a + b*x^2 + c*x^4),x)
```

output
```
x^3*((a*e)/3 + (b*d)/3) + x^5*((b*e)/5 + (c*d)/5) + a*d*x + (c*e*x^7)/7
```

3.248 $\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx$

3.248.1 Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx=-\frac{(cd-be)x}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

output
```
-(-b*e+c*d)*x/e^2+1/3*c*x^3/e+(a*e^2-b*d*e+c*d^2)*arctan(x*e^(1/2)/d^(1/2)
)/e^(5/2)/d^(1/2)
```

3.248.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx=\frac{(-cd+be)x}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
```

output
```
((-(c*d) + b*e)*x)/e^2 + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

3.248.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx$$

$$\Big\downarrow \text{ 1467}$$

$$\int \left(\frac{ae^2-bde+cd^2}{e^2\left(d+ex^2\right)}-\frac{cd-be}{e^2}+\frac{cx^2}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{d}e^{5/2}}-\frac{x(cd-be)}{e^2}+\frac{cx^3}{3e}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
```

output
```
-(((c*d - b*e)*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

3.248.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.248.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result
default	$\frac{\frac{1}{3}c\,x^3e+bex-cdx}{e^2}+\frac{(a\,e^2-bde+c\,d^2)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{e^2\sqrt{ed}}$
risch	$\frac{c\,x^3}{3e}+\frac{bx}{e}-\frac{cdx}{e^2}-\frac{\ln(ex+\sqrt{-ed})a}{2\sqrt{-ed}}+\frac{\ln(ex+\sqrt{-ed})bd}{2e\sqrt{-ed}}-\frac{\ln(ex+\sqrt{-ed})c\,d^2}{2e^2\sqrt{-ed}}+\frac{\ln(-ex+\sqrt{-ed})a}{2\sqrt{-ed}}-\frac{\ln(-ex+\sqrt{-ed})bd}{2e\sqrt{-ed}}+$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(1/3*c*x^3*e+b*e*x-c*d*x)+(a*e^2-b*d*e+c*d^2)/e^2/(e*d)^(1/2)*arctan
(e*x/(e*d)^(1/2))
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.41

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx$$
$$=\left[\frac{2\,cde^2x^3-3\,(cd^2-bde+ae^2)\sqrt{-de}\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right)-6\,(cd^2e-bde^2)x}{6\,de^3},\frac{cde^2x^3+3\,(cd^2-bde+ae\right.$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/6*(2*c*d*e^2*x^3 - 3*(c*d^2 - b*d*e + a*e^2)*sqrt(-d*e)*log((e*x^2 - 2*
sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(c*d^2*e - b*d*e^2)*x)/(d*e^3), 1/3*(c*
d*e^2*x^3 + 3*(c*d^2 - b*d*e + a*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*
(c*d^2*e - b*d*e^2)*x)/(d*e^3)]
```

3.248.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(58) = 116$.

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.77

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = \frac{cx^3}{3e} + x\left(\frac{b}{e} - \frac{cd}{e^2}\right) - \frac{\sqrt{-\frac{1}{de^5}}\,(ae^2 - bde + cd^2)\log\left(-de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^5}}\,(ae^2 - bde + cd^2)\log\left(de^2\sqrt{-\frac{1}{de^5}} + x\right)}{2}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d),x)
```

output
```
c*x**3/(3*e) + x*(b/e - c*d/e**2) - sqrt(-1/(d*e**5))*(a*e**2 - b*d*e + c*
d**2)*log(-d*e**2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 - b
*d*e + c*d**2)*log(d*e**2*sqrt(-1/(d*e**5)) + x)/2
```

3.248.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = \frac{(cd^2-bde+ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^2} + \frac{ce^2x^3-3\,cdex+3\,be^2x}{3\,e^3}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="giac")
```

output
```
(c*d^2 - b*d*e + a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) + 1/3*(c*e^2
*x^3 - 3*c*d*e*x + 3*b*e^2*x)/e^3
```

3.248.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = x\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)+\frac{c\,x^3}{3\,e}+\frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(c\,d^2-b\,d\,e+a\,e^2)}{\sqrt{d}\,e^{5/2}}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2),x)
```

output
```
x*(b/e - (c*d)/e^2) + (c*x^3)/(3*e) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 +
c*d^2 - b*d*e))/(d^(1/2)*e^(5/2))
```

3.249 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} \, dx$

3.249.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} \, dx = \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

output
```
c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*a
rctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

3.249.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} \, dx = \frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{2de^2(d+ex^2)} - \frac{(3cd^2-bde-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

output
```
(c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2
- b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

3.249.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1471, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2} \, dx$$

$$\Big\downarrow \ 1471$$

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\int \frac{cd^2-2cex^2d-e(bd+ae)}{e^2\left(ex^2+d\right)}dx}{2d}$$

$$\Big\downarrow \ 27$$

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\int \frac{cd^2-2cex^2d-e(bd+ae)}{ex^2+d}dx}{2de^2}$$

$$\Big\downarrow \ 299$$

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\left(3cd^2-e(ae+bd)\right)\int \frac{1}{ex^2+d}dx-2cdx}{2de^2}$$

$$\Big\downarrow \ 218$$

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)}-\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{\sqrt{d}\sqrt{e}}-2cdx}{2de^2}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - (-2*c*d*x + ((3*c*d^2
- e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]))/(2*d*e^2)
```

3.249.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.249.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
default	$\frac{cx}{e^2}+\frac{\frac{\left(a\,e^2-bde+c\,d^2\right)x}{2d\left(e\,x^2+d\right)}+\frac{\left(a\,e^2+bde-3c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^2}$
risch	$\frac{cx}{e^2}+\frac{\left(a\,e^2-bde+c\,d^2\right)x}{2d\,e^2\left(e\,x^2+d\right)}-\frac{\ln\left(ex+\sqrt{-ed}\right)a}{4\sqrt{-ed}\,d}-\frac{\ln\left(ex+\sqrt{-ed}\right)b}{4e\sqrt{-ed}}+\frac{3d\ln\left(ex+\sqrt{-ed}\right)c}{4e^2\sqrt{-ed}}+\frac{\ln\left(-ex+\sqrt{-ed}\right)a}{4\sqrt{-ed}\,d}+\frac{\ln\left(-ex+\sqrt{-ed}\right)}{4e\sqrt{-ed}}$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*
d^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx$$
$$= \left[\frac{4\,cd^2e^2x^3+(3\,cd^3-bd^2e-ade^2+(3\,cd^2e-bde^2-ae^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)+2\,(3\,cd^3e-bd^2}{4\,(d^2e^4x^2+d^3e^3)}\right.$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^
2 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) +
 2*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*
d^2*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)
*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*
x)/(d^2*e^4*x^2 + d^3*e^3)]
```

3.249.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(75) = 150.

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.84

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2}+\frac{x(ae^2-bde+cd^2)}{2d^2e^2+2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

output
```
c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqr
t(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3
*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2
*e**2*sqrt(-1/(d**3*e**5)) + x)/4
```

3.249.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} - \frac{(3\,cd^2-bde-ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^2} + \frac{cd^2x-bdex+ae^2x}{2\,(ex^2+d)de^2}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
c*x/e^2 - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d
*e^2) + 1/2*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*d*e^2)
```

3.249.9 Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2+b\,d\,e+a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

output
```
(c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/
2)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.250 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx$

3.250.1 Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d(d+ex^2)^2} - \frac{(5cd^2-e(bd+3ae))x}{8d^2e^2(d+ex^2)} + \frac{(3cd^2+e(bd+3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

output
```
1/4*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^2-1/8*(5*c*d^2-e*(3*a*e+b*d))*x/d^2
/e^2/(e*x^2+d)+1/8*(3*c*d^2+e*(3*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(5/
2)/e^(5/2)
```

3.250.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx = \frac{x(-cd^2(3d+5ex^2)+e(bd(-d+ex^2)+ae(5d+3ex^2)))}{8d^2e^2(d+ex^2)^2} + \frac{(3cd^2+e(bd+3ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

output
```
(x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2)))
)/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

3.250.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1471, 25, 27, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} \, dx$$

$$\Big\downarrow \; 1471$$

$$\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}-\frac{\int -\frac{4cdx^2+e\left(3a-\frac{d(cd-be)}{e^2}\right)}{e(ex^2+d)^2}dx}{4d}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{cd^2}{e}+4cx^2d+bd+3ae}{e(ex^2+d)^2}dx}{4d}+\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-\frac{cd^2}{e}+4cx^2d+bd+3ae}{(ex^2+d)^2}dx}{4de}+\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{1}{2}\left(\frac{3ae}{d}+b+\frac{3cd}{e}\right)\int \frac{1}{ex^2+d}dx+\frac{x\left(3ae+bd-\frac{5cd^2}{e}\right)}{2d(d+ex^2)}}{4de}+\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(\frac{3ae}{d}+b+\frac{3cd}{e}\right)}{2\sqrt{d}\sqrt{e}}+\frac{x\left(3ae+bd-\frac{5cd^2}{e}\right)}{2d(d+ex^2)}}{4de}+\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(4*d*e^2*(d + e*x^2)^2) + (((b*d - (5*c*d^2)/e
 + 3*a*e)*x)/(2*d*(d + e*x^2)) + ((b + (3*c*d)/e + (3*a*e)/d)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(2*Sqrt[d]*Sqrt[e]))/(4*d*e)
```

3.250.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.250.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{\left(3a\,e^{2}+bde-5c\,d^{2}\right)x^{3}}{8d^{2}e}+\frac{\left(5a\,e^{2}-bde-3c\,d^{2}\right)x}{8d\,e^{2}}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(3a\,e^{2}+bde+3c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{8e^{2}d^{2}\sqrt{ed}}$
risch	$\frac{\frac{\left(3a\,e^{2}+bde-5c\,d^{2}\right)x^{3}}{8d^{2}e}+\frac{\left(5a\,e^{2}-bde-3c\,d^{2}\right)x}{8d\,e^{2}}}{\left(e\,x^{2}+d\right)^{2}}-\frac{3\ln\left(ex+\sqrt{-ed}\right)a}{16\sqrt{-ed}\,d^{2}}-\frac{\ln\left(ex+\sqrt{-ed}\right)b}{16\sqrt{-ed}\,ed}-\frac{3\ln\left(ex+\sqrt{-ed}\right)c}{16\sqrt{-ed}\,e^{2}}+\frac{3\ln\left(-ex+\sqrt{-ed}\right)a}{16\sqrt{-ed}\,d^{2}}-$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/d/e^2*x
)/(e*x^2+d)^2+1/8*(3*a*e^2+b*d*e+3*c*d^2)/e^2/d^2/(e*d)^(1/2)*arctan(e*x/(
e*d)^(1/2))
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.40

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx$$
$$=\left[-\frac{2\,(5\,cd^3e^2-bd^2e^3-3\,ade^4)x^3+(3\,cd^4+bd^3e+3\,ad^2e^2+(3\,cd^2e^2+bde^3+3\,ae^4)x^4+2\,(3\,cd^3e+bd^2}{16\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}\right.$$
$$-\frac{(5\,cd^3e^2-bd^2e^3-3\,ade^4)x^3-(3\,cd^4+bd^3e+3\,ad^2e^2+(3\,cd^2e^2+bde^3+3\,ae^4)x^4+2\,(3\,cd^3e+bd^2e^2}{8\,(d^3e^5x^4+2\,d^4e^4x^2+d^5e^3)}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 + (3*c*d^4 + b*d^3*e +
 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^
2*e^2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2
 + d)) + 2*(3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e
^4*x^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 - (3*c*
d^4 + b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3
*c*d^3*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (
3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5
*e^3)]
```

3.250.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.70

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{d^5e^5}}\cdot(3ae^2+bde+3cd^2)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^5}}\cdot(3ae^2+bde+3cd^2)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16} + \frac{x^3\cdot(3ae^3+bde^2-5cd^2e)+x(5ade^2-bd^2e-3cd^3)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

output
```
-sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1
/(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)
*log(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 -
 5*c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**
3*e**3*x**2 + 8*d**2*e**4*x**4)
```

3.250.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.250.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \frac{(3\,cd^2+bde+3\,ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^2e^2} - \frac{5\,cd^2ex^3-bde^2x^3-3\,ae^3x^3+3\,cd^3x+bd^2ex-5\,ade^2x}{8\,(ex^2+d)^2d^2e^2}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^2)
- 1/8*(5*c*d^2*e*x^3 - b*d*e^2*x^3 - 3*a*e^3*x^3 + 3*c*d^3*x + b*d^2*e*x -
 5*a*d*e^2*x)/((e*x^2 + d)^2*d^2*e^2)
```

3.250.9 Mupad [B] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}} - \frac{\frac{x\,\left(3\,c\,d^2+b\,d\,e-5\,a\,e^2\right)}{8\,d\,e^2}-\frac{x^3\,\left(-5\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^2\,e}}{d^2+2\,d\,e\,x^2+e^2\,x^4}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^3,x)
```

output
```
(atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 3*c*d^2 + b*d*e))/(8*d^(5/2)*e^(5/2)
) - ((x*(3*c*d^2 - 5*a*e^2 + b*d*e))/(8*d*e^2) - (x^3*(3*a*e^2 - 5*c*d^2 +
 b*d*e))/(8*d^2*e))/(d^2 + e^2*x^4 + 2*d*e*x^2)
```

3.251 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx$

3.251.1 Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\left(d+ex^2\right)^3} - \frac{(7cd^2-e(bd+5ae))\,x}{24d^2e^2\left(d+ex^2\right)^2} + \frac{(cd^2+e(bd+5ae))\,x}{16d^3e^2\left(d+ex^2\right)} + \frac{(cd^2+e(bd+5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

output
```
1/6*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^3-1/24*(7*c*d^2-e*(5*a*e+b*d))*x/d^
2/e^2/(e*x^2+d)^2+1/16*(c*d^2+e*(5*a*e+b*d))*x/d^3/e^2/(e*x^2+d)+1/16*(c*d
^2+e*(5*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(5/2)
```

3.251.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = \frac{x(cd^2(-3d^2-8dex^2+3e^2x^4)+e(bd(-3d^2+8dex^2+3e^2x^4)+ae(33d^2+40dex^2+15e^2x^4)))}{48d^3e^2\left(d+ex^2\right)^3} + \frac{(cd^2+e(bd+5ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```

output
```
(x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + e*(b*d*(-3*d^2 + 8*d*e*x^2 +
3*e^2*x^4) + a*e*(33*d^2 + 40*d*e*x^2 + 15*e^2*x^4))))/(48*d^3*e^2*(d + e*
x^2)^3) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7
/2)*e^(5/2))
```

3.251.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1471, 25, 27, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4} \, dx$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^2)^3}-\frac{\int -\frac{6cdx^2+e\left(5a-\frac{d(cd-be)}{e^2}\right)}{e(ex^2+d)^3}dx}{6d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-\frac{cd^2}{e}+6cx^2d+bd+5ae}{e(ex^2+d)^3}dx}{6d}+\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^2)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-\frac{cd^2}{e}+6cx^2d+bd+5ae}{(ex^2+d)^3}dx}{6de}+\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^2)^3}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{3}{4}\left(\frac{5ae}{d}+b+\frac{cd}{e}\right)\int \frac{1}{(ex^2+d)^2}dx+\frac{x\left(5ae+bd-\frac{7cd^2}{e}\right)}{4d(d+ex^2)^2}}{6de}+\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^2)^3}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{3}{4}\left(\frac{5ae}{d}+b+\frac{cd}{e}\right)\left(\frac{\int \frac{1}{ex^2+d}dx}{2d}+\frac{x}{2d(d+ex^2)}\right)+\frac{x\left(5ae+bd-\frac{7cd^2}{e}\right)}{4d(d+ex^2)^2}}{6de}+\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^2)^3}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{3}{4}\left(\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}+\frac{x}{2d(d+ex^2)}\right)\left(\frac{5ae}{d}+b+\frac{cd}{e}\right)+\frac{x\left(5ae+bd-\frac{7cd^2}{e}\right)}{4d(d+ex^2)^2}}{6de}+\frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^2)^3}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^2)^3) + (((b*d - (7*c*d^2)/e
 + 5*a*e)*x)/(4*d*(d + e*x^2)^2) + (3*(b + (c*d)/e + (5*a*e)/d)*(x/(2*d*(d
 + e*x^2)) + ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(2*d^(3/2)*Sqrt[e])))/4)/(6*d*e)
```

3.251.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.251.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

method	result
default	$\frac{\frac{(5a e^2+bde+c d^2)x^5}{16d^3}+\frac{(5a e^2+bde-c d^2)x^3}{6d^2 e}+\frac{(11a e^2-bde-c d^2)x}{16d e^2}}{(e x^2+d)^3}+\frac{(5a e^2+bde+c d^2)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{16d^3 e^2\sqrt{ed}}$
risch	$\frac{\frac{(5a e^2+bde+c d^2)x^5}{16d^3}+\frac{(5a e^2+bde-c d^2)x^3}{6d^2 e}+\frac{(11a e^2-bde-c d^2)x}{16d e^2}}{(e x^2+d)^3}-\frac{5\ln(ex+\sqrt{-ed})a}{32\sqrt{-ed}\,d^3}-\frac{\ln(ex+\sqrt{-ed})b}{32\sqrt{-ed}\,e\,d^2}-\frac{\ln(ex+\sqrt{-ed})c}{32\sqrt{-ed}\,e^2 d}+\frac{5\ln}{}$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*a*e^2+b*d*e+c*d^2)/d^3*x^5+1/6*(5*a*e^2+b*d*e-c*d^2)/d^2/e*x^3+1/
16*(11*a*e^2-b*d*e-c*d^2)/d/e^2*x)/(e*x^2+d)^3+1/16*(5*a*e^2+b*d*e+c*d^2)/
d^3/e^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.53

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx$$
$$=\left[\frac{6\,(cd^3e^3+bd^2e^4+5\,ade^5)x^5-16\,(cd^4e^2-bd^3e^3-5\,ad^2e^4)x^3-3\,((cd^2e^3+bde^4+5\,ae^5)x^6+cd^5+bd^4}{96\,(d^4}\right.$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="fricas")
```

output
```
[1/96*(6*(c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^5 - 16*(c*d^4*e^2 - b*d^3*e
^3 - 5*a*d^2*e^4)*x^3 - 3*((c*d^2*e^3 + b*d*e^4 + 5*a*e^5)*x^6 + c*d^5 + b
*d^4*e + 5*a*d^3*e^2 + 3*(c*d^3*e^2 + b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^
4*e + b*d^3*e^2 + 5*a*d^2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x
 - d)/(e*x^2 + d)) - 6*(c*d^5*e + b*d^4*e^2 - 11*a*d^3*e^3)*x)/(d^4*e^6*x^
6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3), 1/48*(3*(c*d^3*e^3 + b*d^2*e
^4 + 5*a*d*e^5)*x^5 - 8*(c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^3 + 3*((c*
d^2*e^3 + b*d*e^4 + 5*a*e^5)*x^6 + c*d^5 + b*d^4*e + 5*a*d^3*e^2 + 3*(c*d^
3*e^2 + b*d^2*e^3 + 5*a*d*e^4)*x^4 + 3*(c*d^4*e + b*d^3*e^2 + 5*a*d^2*e^3)
*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(c*d^5*e + b*d^4*e^2 - 11*a*d^3*
e^3)*x)/(d^4*e^6*x^6 + 3*d^5*e^5*x^4 + 3*d^6*e^4*x^2 + d^7*e^3)]
```

3.251.6 Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.61

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = -\frac{\sqrt{-\frac{1}{d^7e^5}}\cdot(5ae^2+bde+cd^2)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7e^5}}\cdot(5ae^2+bde+cd^2)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32} + \frac{x^5\cdot(15ae^4+3bde^3+3cd^2e^2)+x^3\cdot(40ade^3+8bd^2e^2-8cd^3e)+x(33ad^2e^2-3bd^3e-3cd^4)}{48d^6e^2+144d^5e^3x^2+144d^4e^4x^4+48d^3e^5x^6}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**4,x)
```

output
```
-sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(-d**4*e**2*sqrt(-1/(
d**7*e**5)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log
(d**4*e**2*sqrt(-1/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*b*d*e**3 +
3*c*d**2*e**2) + x**3*(40*a*d*e**3 + 8*b*d**2*e**2 - 8*c*d**3*e) + x*(33*a
*d**2*e**2 - 3*b*d**3*e - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*e**3*x**2 +
144*d**4*e**4*x**4 + 48*d**3*e**5*x**6)
```

3.251.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = \frac{(cd^2+bde+5\,ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}d^3e^2} + \frac{3\,cd^2e^2x^5+3\,bde^3x^5+15\,ae^4x^5-8\,cd^3ex^3+8\,bd^2e^2x^3+40\,ade^3x^3-3\,cd^4x-3\,bd^3ex+33\,ad^2e^2x}{48\,(ex^2+d)^3d^3e^2}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="giac")
```

output
```
1/16*(c*d^2 + b*d*e + 5*a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e^2) +
 1/48*(3*c*d^2*e^2*x^5 + 3*b*d*e^3*x^5 + 15*a*e^4*x^5 - 8*c*d^3*e*x^3 + 8*
b*d^2*e^2*x^3 + 40*a*d*e^3*x^3 - 3*c*d^4*x - 3*b*d^3*e*x + 33*a*d^2*e^2*x)
/((e*x^2 + d)^3*d^3*e^2)
```

3.251.9 Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx = \frac{\frac{x^5\,(c\,d^2+b\,d\,e+5\,a\,e^2)}{16\,d^3}-\frac{x\,(c\,d^2+b\,d\,e-11\,a\,e^2)}{16\,d\,e^2}+\frac{x^3\,(-c\,d^2+b\,d\,e+5\,a\,e^2)}{6\,d^2\,e}}{d^3+3\,d^2\,e\,x^2+3\,d\,e^2\,x^4+e^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(c\,d^2+b\,d\,e+5\,a\,e^2)}{16\,d^{7/2}\,e^{5/2}}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^4,x)
```

output
```
((x^5*(5*a*e^2 + c*d^2 + b*d*e))/(16*d^3) - (x*(c*d^2 - 11*a*e^2 + b*d*e))
/(16*d*e^2) + (x^3*(5*a*e^2 - c*d^2 + b*d*e))/(6*d^2*e))/(d^3 + e^3*x^6 +
3*d^2*e*x^2 + 3*d*e^2*x^4) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2 +
 b*d*e))/(16*d^(7/2)*e^(5/2))
```

3.252 $\int (d+ex^2)^3 (a+bx^2+cx^4)^2\, dx$

3.252.1 Optimal result

Integrand size = 24, antiderivative size = 223

$$\begin{aligned}\int (d+ex^2)^3 (a+bx^2+cx^4)^2\, dx = {} & a^2d^3x+\frac{1}{3}ad^2(2bd+3ae)x^3 \\ & +\frac{1}{5}d\left(b^2d^2+6abde+a\left(2cd^2+3ae^2\right)\right)x^5 \\ & +\frac{1}{7}\left(2bcd^3+3b^2d^2e+6acd^2e+6abde^2+a^2e^3\right)x^7 \\ & +\frac{1}{9}\left(c^2d^3+6cde(bd+ae)+be^2(3bd+2ae)\right)x^9 \\ & +\frac{1}{11}e\left(3c^2d^2+b^2e^2+2ce(3bd+ae)\right)x^{11} \\ & +\frac{1}{13}ce^2(3cd+2be)x^{13}+\frac{1}{15}c^2e^3x^{15}\end{aligned}$$

output
```
a^2*d^3*x+1/3*a*d^2*(3*a*e+2*b*d)*x^3+1/5*d*(b^2*d^2+6*a*b*d*e+a*(3*a*e^2+
2*c*d^2))*x^5+1/7*(a^2*e^3+6*a*b*d*e^2+6*a*c*d^2*e+3*b^2*d^2*e+2*b*c*d^3)*
x^7+1/9*(c^2*d^3+6*c*d*e*(a*e+b*d)+b*e^2*(2*a*e+3*b*d))*x^9+1/11*e*(3*c^2*
d^2+b^2*e^2+2*c*e*(a*e+3*b*d))*x^11+1/13*c*e^2*(2*b*e+3*c*d)*x^13+1/15*c^2
*e^3*x^15
```

3.252.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx = a^2d^3x+\frac{1}{3}ad^2(2bd+3ae)x^3 +\frac{1}{5}d\left(b^2d^2+6abde+a\left(2cd^2+3ae^2\right)\right)x^5 +\frac{1}{7}\left(2bcd^3+3b^2d^2e+6acd^2e+6abde^2+a^2e^3\right)x^7 +\frac{1}{9}\left(c^2d^3+6cde(bd+ae)+be^2(3bd+2ae)\right)x^9 +\frac{1}{11}e\left(3c^2d^2+b^2e^2+2ce(3bd+ae)\right)x^{11} +\frac{1}{13}ce^2(3cd+2be)x^{13}+\frac{1}{15}c^2e^3x^{15}$$

input
```
Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2
*c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*
b*d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d
 + 2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/1
1 + (c*e^2*(3*c*d + 2*b*e)*x^13)/13 + (c^2*e^3*x^15)/15
```

3.252.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx$$

$$\Big\downarrow \text{1467}$$

$$\int \left(x^6\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3+ex^{10}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+dx^4\left(6abde+a\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7\left(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\right)+a^2d^3x+\frac{1}{11}ex^{11}\left(2ce(ae+3bd)+b^2e^2+3c^2d^2\right)+\frac{1}{5}dx^5\left(6abde+a\left(3ae^2+2cd^2\right)+b^2d^2\right)+\frac{1}{9}x^9\left(6cde(ae+bd)+be^2(2ae+3bd)+c^2d^3\right)+\frac{1}{3}ad^2x^3(3ae+2bd)+\frac{1}{13}ce^2x^{13}(2be+3cd)+\frac{1}{15}c^2e^3x^{15}$$

input
```
Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2
*c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*
b*d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d
 + 2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/1
1 + (c*e^2*(3*c*d + 2*b*e)*x^13)/13 + (c^2*e^3*x^15)/15
```

3.252.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.252.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.98

method	result
default	$\frac{c^2e^3x^{15}}{15}+\frac{(2e^3bc+3d\,e^2c^2)x^{13}}{13}+\frac{(3d^2e\,c^2+6d\,e^2bc+e^3(2ac+b^2))x^{11}}{11}+\frac{(c^2d^3+6d^2ebc+3d\,e^2(2ac+b^2)+2e^3ab)x^9}{9}+$
norman	$a^2d^3x+\left(d^2e\,a^2+\frac{2}{3}a\,d^3b\right)x^3+\left(\frac{3}{5}d\,e^2a^2+\frac{6}{5}d^2eab+\frac{2}{5}d^3ac+\frac{1}{5}b^2d^3\right)x^5+\left(\frac{1}{7}e^3a^2+\frac{6}{7}abd\,e^2\right.$
gosper	$a^2d^3x+a^2d^2e\,x^3+\frac{2}{3}x^3a\,d^3b+\frac{3}{5}x^5d\,e^2a^2+\frac{6}{5}x^5d^2eab+\frac{2}{5}x^5d^3ac+\frac{1}{5}x^5b^2d^3+\frac{1}{7}x^7e^3a^2+\frac{6}{7}x^7$
risch	$a^2d^3x+a^2d^2e\,x^3+\frac{2}{3}x^3a\,d^3b+\frac{3}{5}x^5d\,e^2a^2+\frac{6}{5}x^5d^2eab+\frac{2}{5}x^5d^3ac+\frac{1}{5}x^5b^2d^3+\frac{1}{7}x^7e^3a^2+\frac{6}{7}x^7$
parallelrisch	$a^2d^3x+a^2d^2e\,x^3+\frac{2}{3}x^3a\,d^3b+\frac{3}{5}x^5d\,e^2a^2+\frac{6}{5}x^5d^2eab+\frac{2}{5}x^5d^3ac+\frac{1}{5}x^5b^2d^3+\frac{1}{7}x^7e^3a^2+\frac{6}{7}x^7$

input
```
int((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*c^2*e^3*x^15+1/13*(2*b*c*e^3+3*c^2*d*e^2)*x^13+1/11*(3*d^2*e*c^2+6*d*
e^2*b*c+e^3*(2*a*c+b^2))*x^11+1/9*(c^2*d^3+6*d^2*e*b*c+3*d*e^2*(2*a*c+b^2)
+2*e^3*a*b)*x^9+1/7*(2*b*c*d^3+3*d^2*e*(2*a*c+b^2)+6*a*b*d*e^2+e^3*a^2)*x^
7+1/5*(d^3*(2*a*c+b^2)+6*d^2*e*a*b+3*d*e^2*a^2)*x^5+1/3*(3*a^2*d^2*e+2*a*b
*d^3)*x^3+a^2*d^3*x
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.98

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \,dx = \frac{1}{15}c^2e^3x^{15} + \frac{1}{13}\left(3\,c^2de^2 + 2\,bce^3\right)x^{13} + \frac{1}{11}\left(3\,c^2d^2e + 6\,bcde^2 + \left(b^2+2\,ac\right)e^3\right)x^{11} + \frac{1}{9}\left(c^2d^3 + 6\,bcd^2e + 2\,abe^3 + 3\,\left(b^2+2\,ac\right)de^2\right)x^9 + \frac{1}{7}\left(2\,bcd^3 + 6\,abde^2 + a^2e^3 + 3\,\left(b^2+2\,ac\right)d^2e\right)x^7 + a^2d^3x + \frac{1}{5}\left(6\,abd^2e + 3\,a^2de^2 + \left(b^2+2\,ac\right)d^3\right)x^5 + \frac{1}{3}\left(2\,abd^3 + 3\,a^2d^2e\right)x^3$$

input
```
integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/15*c^2*e^3*x^15 + 1/13*(3*c^2*d*e^2 + 2*b*c*e^3)*x^13 + 1/11*(3*c^2*d^2*
e + 6*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^11 + 1/9*(c^2*d^3 + 6*b*c*d^2*e + 2
*a*b*e^3 + 3*(b^2 + 2*a*c)*d*e^2)*x^9 + 1/7*(2*b*c*d^3 + 6*a*b*d*e^2 + a^2
*e^3 + 3*(b^2 + 2*a*c)*d^2*e)*x^7 + a^2*d^3*x + 1/5*(6*a*b*d^2*e + 3*a^2*d
*e^2 + (b^2 + 2*a*c)*d^3)*x^5 + 1/3*(2*a*b*d^3 + 3*a^2*d^2*e)*x^3
```

3.252.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.22

$$\int (d+ex^2)^3 (a+bx^2+cx^4)^2 \, dx = a^2d^3x + \frac{c^2e^3x^{15}}{15} + x^{13} \cdot \left(\frac{2bce^3}{13} + \frac{3c^2de^2}{13}\right) + x^{11} \cdot \left(\frac{2ace^3}{11} + \frac{b^2e^3}{11} + \frac{6bcde^2}{11} + \frac{3c^2d^2e}{11}\right) + x^9 \cdot \left(\frac{2abe^3}{9} + \frac{2acde^2}{3} + \frac{b^2de^2}{3} + \frac{2bcd^2e}{3} + \frac{c^2d^3}{9}\right) + x^7\left(\frac{a^2e^3}{7} + \frac{6abde^2}{7} + \frac{6acd^2e}{7} + \frac{3b^2d^2e}{7} + \frac{2bcd^3}{7}\right) + x^5 \cdot \left(\frac{3a^2de^2}{5} + \frac{6abd^2e}{5} + \frac{2acd^3}{5} + \frac{b^2d^3}{5}\right) + x^3\left(a^2d^2e + \frac{2abd^3}{3}\right)$$

input
```
integrate((e*x**2+d)**3*(c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*d**3*x + c**2*e**3*x**15/15 + x**13*(2*b*c*e**3/13 + 3*c**2*d*e**2/13
) + x**11*(2*a*c*e**3/11 + b**2*e**3/11 + 6*b*c*d*e**2/11 + 3*c**2*d**2*e/
11) + x**9*(2*a*b*e**3/9 + 2*a*c*d*e**2/3 + b**2*d*e**2/3 + 2*b*c*d**2*e/3
 + c**2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*b*d*e**2/7 + 6*a*c*d**2*e/7 + 3*
b**2*d**2*e/7 + 2*b*c*d**3/7) + x**5*(3*a**2*d*e**2/5 + 6*a*b*d**2*e/5 + 2
*a*c*d**3/5 + b**2*d**3/5) + x**3*(a**2*d**2*e + 2*a*b*d**3/3)
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.98

$$\int (d+ex^2)^3 (a+bx^2+cx^4)^2 \, dx = \frac{1}{15}c^2e^3x^{15} + \frac{1}{13}(3c^2de^2 + 2bce^3)x^{13} + \frac{1}{11}(3c^2d^2e + 6bcde^2 + (b^2+2ac)e^3)x^{11} + \frac{1}{9}(c^2d^3 + 6bcd^2e + 2abe^3 + 3(b^2+2ac)de^2)x^9 + \frac{1}{7}(2bcd^3 + 6abde^2 + a^2e^3 + 3(b^2+2ac)d^2e)x^7 + a^2d^3x + \frac{1}{5}(6abd^2e + 3a^2de^2 + (b^2+2ac)d^3)x^5 + \frac{1}{3}(2abd^3 + 3a^2d^2e)x^3$$

input
```
integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/15*c^2*e^3*x^15 + 1/13*(3*c^2*d*e^2 + 2*b*c*e^3)*x^13 + 1/11*(3*c^2*d^2*
e + 6*b*c*d*e^2 + (b^2 + 2*a*c)*e^3)*x^11 + 1/9*(c^2*d^3 + 6*b*c*d^2*e + 2
*a*b*e^3 + 3*(b^2 + 2*a*c)*d*e^2)*x^9 + 1/7*(2*b*c*d^3 + 6*a*b*d*e^2 + a^2
*e^3 + 3*(b^2 + 2*a*c)*d^2*e)*x^7 + a^2*d^3*x + 1/5*(6*a*b*d^2*e + 3*a^2*d
*e^2 + (b^2 + 2*a*c)*d^3)*x^5 + 1/3*(2*a*b*d^3 + 3*a^2*d^2*e)*x^3
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx = & \frac{1}{15}c^2e^3x^{15} + \frac{3}{13}c^2de^2x^{13} + \frac{2}{13}bce^3x^{13} + \frac{3}{11}c^2d^2ex^{11} \\ & + \frac{6}{11}bcde^2x^{11} + \frac{1}{11}b^2e^3x^{11} + \frac{2}{11}ace^3x^{11} + \frac{1}{9}c^2d^3x^9 \\ & + \frac{2}{3}bcd^2ex^9 + \frac{1}{3}b^2de^2x^9 + \frac{2}{3}acde^2x^9 + \frac{2}{9}abe^3x^9 \\ & + \frac{2}{7}bcd^3x^7 + \frac{3}{7}b^2d^2ex^7 + \frac{6}{7}acd^2ex^7 + \frac{6}{7}abde^2x^7 \\ & + \frac{1}{7}a^2e^3x^7 + \frac{1}{5}b^2d^3x^5 + \frac{2}{5}acd^3x^5 + \frac{6}{5}abd^2ex^5 \\ & + \frac{3}{5}a^2de^2x^5 + \frac{2}{3}abd^3x^3 + a^2d^2ex^3 + a^2d^3x\end{aligned}$$

input
```
integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/15*c^2*e^3*x^15 + 3/13*c^2*d*e^2*x^13 + 2/13*b*c*e^3*x^13 + 3/11*c^2*d^2
*e*x^11 + 6/11*b*c*d*e^2*x^11 + 1/11*b^2*e^3*x^11 + 2/11*a*c*e^3*x^11 + 1/
9*c^2*d^3*x^9 + 2/3*b*c*d^2*e*x^9 + 1/3*b^2*d*e^2*x^9 + 2/3*a*c*d*e^2*x^9
+ 2/9*a*b*e^3*x^9 + 2/7*b*c*d^3*x^7 + 3/7*b^2*d^2*e*x^7 + 6/7*a*c*d^2*e*x^
7 + 6/7*a*b*d*e^2*x^7 + 1/7*a^2*e^3*x^7 + 1/5*b^2*d^3*x^5 + 2/5*a*c*d^3*x^
5 + 6/5*a*b*d^2*e*x^5 + 3/5*a^2*d*e^2*x^5 + 2/3*a*b*d^3*x^3 + a^2*d^2*e*x^
3 + a^2*d^3*x
```

3.252.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.99

$$\int (d+ex^2)^3 (a+bx^2+cx^4)^2 \, dx = x^7 \left(\frac{a^2 e^3}{7} + \frac{6\,a\,b\,d\,e^2}{7} + \frac{6\,c\,a\,d^2\,e}{7} + \frac{3\,b^2\,d^2\,e}{7} + \frac{2\,c\,b\,d^3}{7}\right) + x^9 \left(\frac{b^2\,d\,e^2}{3} + \frac{2\,b\,c\,d^2\,e}{3} + \frac{2\,a\,b\,e^3}{9} + \frac{c^2\,d^3}{9} + \frac{2\,a\,c\,d\,e^2}{3}\right) + x^5 \left(\frac{3\,a^2\,d\,e^2}{5} + \frac{6\,a\,b\,d^2\,e}{5} + \frac{2\,c\,a\,d^3}{5} + \frac{b^2\,d^3}{5}\right) + x^{11} \left(\frac{b^2\,e^3}{11} + \frac{6\,b\,c\,d\,e^2}{11} + \frac{3\,c^2\,d^2\,e}{11} + \frac{2\,a\,c\,e^3}{11}\right) + a^2\,d^3\,x + \frac{c^2\,e^3\,x^{15}}{15} + \frac{a\,d^2\,x^3\,(3\,a\,e + 2\,b\,d)}{3} + \frac{c\,e^2\,x^{13}\,(2\,b\,e + 3\,c\,d)}{13}$$

input
```
int((d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^7*((a^2*e^3)/7 + (3*b^2*d^2*e)/7 + (2*b*c*d^3)/7 + (6*a*b*d*e^2)/7 + (6*
a*c*d^2*e)/7) + x^9*((c^2*d^3)/9 + (b^2*d*e^2)/3 + (2*a*b*e^3)/9 + (2*a*c*
d*e^2)/3 + (2*b*c*d^2*e)/3) + x^5*((b^2*d^3)/5 + (3*a^2*d*e^2)/5 + (2*a*c*
d^3)/5 + (6*a*b*d^2*e)/5) + x^11*((b^2*e^3)/11 + (3*c^2*d^2*e)/11 + (2*a*c
*e^3)/11 + (6*b*c*d*e^2)/11) + a^2*d^3*x + (c^2*e^3*x^15)/15 + (a*d^2*x^3*
(3*a*e + 2*b*d))/3 + (c*e^2*x^13*(2*b*e + 3*c*d))/13
```

3.253 $\int (d+ex^2)^2 (a+bx^2+cx^4)^2 \, dx$

3.253.1 Optimal result

Integrand size = 24, antiderivative size = 155

$$\begin{aligned}\int (d+ex^2)^2 (a+bx^2+cx^4)^2 \, dx = {} & a^2d^2x+\frac{2}{3}ad(bd+ae)x^3 \\ & +\frac{1}{5}\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)x^5 \\ & +\frac{2}{7}\left(bcd^2+b^2de+2acde+abe^2\right)x^7 \\ & +\frac{1}{9}\left(c^2d^2+b^2e^2+2ce(2bd+ae)\right)x^9 \\ & +\frac{2}{11}ce(cd+be)x^{11}+\frac{1}{13}c^2e^2x^{13}\end{aligned}$$

output
```
a^2*d^2*x+2/3*a*d*(a*e+b*d)*x^3+1/5*(b^2*d^2+4*a*b*d*e+a*(a*e^2+2*c*d^2))*
x^5+2/7*(a*b*e^2+2*a*c*d*e+b^2*d*e+b*c*d^2)*x^7+1/9*(c^2*d^2+b^2*e^2+2*c*e
*(a*e+2*b*d))*x^9+2/11*c*e*(b*e+c*d)*x^11+1/13*c^2*e^2*x^13
```

3.253.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx = a^2d^2x+\frac{2}{3}ad(bd+ae)x^3+\frac{1}{5}\left(b^2d^2+2acd^2+4abde+a^2e^2\right)x^5 + \frac{2}{7}\left(bcd^2+b^2de+2acde+abe^2\right)x^7 + \frac{1}{9}\left(c^2d^2+4bcde+b^2e^2+2ace^2\right)x^9 + \frac{2}{11}ce(cd+be)x^{11}+\frac{1}{13}c^2e^2x^{13}$$

input
```
Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 2*a*c*d^2 + 4*a*b*d*e
+ a^2*e^2)*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 +
((c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 2*a*c*e^2)*x^9)/9 + (2*c*e*(c*d + b*e)*x
^11)/11 + (c^2*e^2*x^13)/13
```

3.253.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx$$

$$\Big\downarrow \text{1467}$$

$$\int \left(a^2d^2+x^8\left(2ce(ae+2bd)+b^2e^2+c^2d^2\right)+2x^6\left(abe^2+2acde+b^2de+bcd^2\right)+x^4\left(4abde+a\left(ae^2+2cd^2\right)+b^2\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$a^2d^2x+\frac{1}{9}x^9\left(2ce(ae+2bd)+b^2e^2+c^2d^2\right)+\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right)+ \frac{1}{5}x^5\left(4abde+a\left(ae^2+2cd^2\right)+b^2d^2\right)+\frac{2}{3}adx^3(ae+bd)+\frac{2}{11}cex^{11}(be+cd)+\frac{1}{13}c^2e^2x^{13}$$

input
```
Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 4*a*b*d*e + a*(2*c*d^2
 + a*e^2))*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 +
((c^2*d^2 + b^2*e^2 + 2*c*e*(2*b*d + a*e))*x^9)/9 + (2*c*e*(c*d + b*e)*x^1
1)/11 + (c^2*e^2*x^13)/13
```

3.253.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.253.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00

method	result
default	$\frac{c^2e^2x^{13}}{13}+\frac{(2bc\,e^2+2ed\,c^2)x^{11}}{11}+\frac{(c^2d^2+4bcde+e^2(2ac+b^2))x^9}{9}+\frac{(2bc\,d^2+2ed(2ac+b^2)+2ab\,e^2)x^7}{7}+\frac{(d^2(2ac+b^2)+4}{5}$
norman	$\frac{c^2e^2x^{13}}{13}+\left(\frac{2}{11}bc\,e^2+\frac{2}{11}ed\,c^2\right)x^{11}+\left(\frac{2}{9}e^2ac+\frac{1}{9}b^2e^2+\frac{4}{9}bcde+\frac{1}{9}c^2d^2\right)x^9+\left(\frac{2}{7}ab\,e^2+\frac{4}{7}acde+\right.$
gosper	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}x^{11}bc\,e^2+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9b^2e^2+\frac{4}{9}x^9bcde+\frac{1}{9}x^9c^2d^2+\frac{2}{7}x^7ab\,e^2+$
risch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}x^{11}bc\,e^2+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9b^2e^2+\frac{4}{9}x^9bcde+\frac{1}{9}x^9c^2d^2+\frac{2}{7}x^7ab\,e^2+$
parallelrisch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}x^{11}bc\,e^2+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9e^2ac+\frac{1}{9}x^9b^2e^2+\frac{4}{9}x^9bcde+\frac{1}{9}x^9c^2d^2+\frac{2}{7}x^7ab\,e^2+$

input
```
int((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*c^2*e^2*x^13+1/11*(2*b*c*e^2+2*c^2*d*e)*x^11+1/9*(c^2*d^2+4*b*c*d*e+e
^2*(2*a*c+b^2))*x^9+1/7*(2*b*c*d^2+2*e*d*(2*a*c+b^2)+2*a*b*e^2)*x^7+1/5*(d
^2*(2*a*c+b^2)+4*a*b*d*e+e^2*a^2)*x^5+1/3*(2*a^2*d*e+2*a*b*d^2)*x^3+a^2*d^
2*x
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\int (d+ex^2)^2 (a+bx^2+cx^4)^2 \, dx = \frac{1}{13} c^2 e^2 x^{13} + \frac{2}{11} (c^2 de + bce^2) x^{11} + \frac{1}{9} (c^2 d^2 + 4\,bcde + (b^2 + 2\,ac) e^2) x^9 + \frac{2}{7} (bcd^2 + abe^2 + (b^2 + 2\,ac) de) x^7 + \frac{1}{5} (4\,abde + a^2 e^2 + (b^2 + 2\,ac) d^2) x^5 + a^2 d^2 x + \frac{2}{3} (abd^2 + a^2 de) x^3$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/13*c^2*e^2*x^13 + 2/11*(c^2*d*e + b*c*e^2)*x^11 + 1/9*(c^2*d^2 + 4*b*c*d
*e + (b^2 + 2*a*c)*e^2)*x^9 + 2/7*(b*c*d^2 + a*b*e^2 + (b^2 + 2*a*c)*d*e)*
x^7 + 1/5*(4*a*b*d*e + a^2*e^2 + (b^2 + 2*a*c)*d^2)*x^5 + a^2*d^2*x + 2/3*
(a*b*d^2 + a^2*d*e)*x^3
```

3.253.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.24

$$\int (d+ex^2)^2 (a+bx^2+cx^4)^2 \, dx = a^2 d^2 x + \frac{c^2 e^2 x^{13}}{13} + x^{11} \cdot \left(\frac{2bce^2}{11} + \frac{2c^2 de}{11}\right) + x^9 \cdot \left(\frac{2ace^2}{9} + \frac{b^2 e^2}{9} + \frac{4bcde}{9} + \frac{c^2 d^2}{9}\right) + x^7 \cdot \left(\frac{2abe^2}{7} + \frac{4acde}{7} + \frac{2b^2 de}{7} + \frac{2bcd^2}{7}\right) + x^5 \left(\frac{a^2 e^2}{5} + \frac{4abde}{5} + \frac{2acd^2}{5} + \frac{b^2 d^2}{5}\right) + x^3 \cdot \left(\frac{2a^2 de}{3} + \frac{2abd^2}{3}\right)$$

input
```
integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*d**2*x + c**2*e**2*x**13/13 + x**11*(2*b*c*e**2/11 + 2*c**2*d*e/11) +
 x**9*(2*a*c*e**2/9 + b**2*e**2/9 + 4*b*c*d*e/9 + c**2*d**2/9) + x**7*(2*a
*b*e**2/7 + 4*a*c*d*e/7 + 2*b**2*d*e/7 + 2*b*c*d**2/7) + x**5*(a**2*e**2/5
 + 4*a*b*d*e/5 + 2*a*c*d**2/5 + b**2*d**2/5) + x**3*(2*a**2*d*e/3 + 2*a*b*
d**2/3)
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{13}\,c^2e^2x^{13} + \frac{2}{11}\,\left(c^2de+bce^2\right)x^{11} + \frac{1}{9}\,\left(c^2d^2+4\,bcde+\left(b^2+2\,ac\right)e^2\right)x^9 + \frac{2}{7}\,\left(bcd^2+abe^2+\left(b^2+2\,ac\right)de\right)x^7 + \frac{1}{5}\,\left(4\,abde+a^2e^2+\left(b^2+2\,ac\right)d^2\right)x^5 + a^2d^2x + \frac{2}{3}\,\left(abd^2+a^2de\right)x^3$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/13*c^2*e^2*x^13 + 2/11*(c^2*d*e + b*c*e^2)*x^11 + 1/9*(c^2*d^2 + 4*b*c*d
*e + (b^2 + 2*a*c)*e^2)*x^9 + 2/7*(b*c*d^2 + a*b*e^2 + (b^2 + 2*a*c)*d*e)*
x^7 + 1/5*(4*a*b*d*e + a^2*e^2 + (b^2 + 2*a*c)*d^2)*x^5 + a^2*d^2*x + 2/3*
(a*b*d^2 + a^2*d*e)*x^3
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.17

$$\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{13}\,c^2e^2x^{13} + \frac{2}{11}\,c^2dex^{11} + \frac{2}{11}\,bce^2x^{11} + \frac{1}{9}\,c^2d^2x^9 + \frac{4}{9}\,bcdex^9 + \frac{1}{9}\,b^2e^2x^9 + \frac{2}{9}\,ace^2x^9 + \frac{2}{7}\,bcd^2x^7 + \frac{2}{7}\,b^2dex^7 + \frac{4}{7}\,acdex^7 + \frac{2}{7}\,abe^2x^7 + \frac{1}{5}\,b^2d^2x^5 + \frac{2}{5}\,acd^2x^5 + \frac{4}{5}\,abdex^5 + \frac{1}{5}\,a^2e^2x^5 + \frac{2}{3}\,abd^2x^3 + \frac{2}{3}\,a^2dex^3 + a^2d^2x$$

input
```
integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/13*c^2*e^2*x^13 + 2/11*c^2*d*e*x^11 + 2/11*b*c*e^2*x^11 + 1/9*c^2*d^2*x^
9 + 4/9*b*c*d*e*x^9 + 1/9*b^2*e^2*x^9 + 2/9*a*c*e^2*x^9 + 2/7*b*c*d^2*x^7
+ 2/7*b^2*d*e*x^7 + 4/7*a*c*d*e*x^7 + 2/7*a*b*e^2*x^7 + 1/5*b^2*d^2*x^5 +
2/5*a*c*d^2*x^5 + 4/5*a*b*d*e*x^5 + 1/5*a^2*e^2*x^5 + 2/3*a*b*d^2*x^3 + 2/
3*a^2*d*e*x^3 + a^2*d^2*x
```

3.253.9 Mupad [B] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int (d+ex^2)^2 (a+bx^2+cx^4)^2 \, dx = x^5 \left(\frac{a^2 e^2}{5}+\frac{4 a b d e}{5}+\frac{2 c a d^2}{5}+\frac{b^2 d^2}{5}\right) + x^9 \left(\frac{b^2 e^2}{9}+\frac{4 b c d e}{9}+\frac{c^2 d^2}{9}+\frac{2 a c e^2}{9}\right) + x^7 \left(\frac{2 b^2 d e}{7}+\frac{2 c b d^2}{7}+\frac{2 a b e^2}{7}+\frac{4 a c d e}{7}\right) + a^2 d^2 x + \frac{c^2 e^2 x^{13}}{13}+\frac{2 a d x^3 (a e+b d)}{3}+\frac{2 c e x^{11} (b e+c d)}{11}$$

input
```
int((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^5*((a^2*e^2)/5 + (b^2*d^2)/5 + (2*a*c*d^2)/5 + (4*a*b*d*e)/5) + x^9*((b^
2*e^2)/9 + (c^2*d^2)/9 + (2*a*c*e^2)/9 + (4*b*c*d*e)/9) + x^7*((2*a*b*e^2)
/7 + (2*b*c*d^2)/7 + (2*b^2*d*e)/7 + (4*a*c*d*e)/7) + a^2*d^2*x + (c^2*e^2
*x^13)/13 + (2*a*d*x^3*(a*e + b*d))/3 + (2*c*e*x^11*(b*e + c*d))/11
```

3.254 $\int (d+ex^2)(a+bx^2+cx^4)^2 \, dx$

3.254.1 Optimal result

Integrand size = 22, antiderivative size = 96

$$\int (d+ex^2)(a+bx^2+cx^4)^2 \, dx = a^2dx + \frac{1}{3}a(2bd+ae)x^3 + \frac{1}{5}(b^2d+2acd+2abe)x^5 + \frac{1}{7}(2bcd+b^2e+2ace)x^7 + \frac{1}{9}c(cd+2be)x^9 + \frac{1}{11}c^2ex^{11}$$

output
```
a^2*d*x+1/3*a*(a*e+2*b*d)*x^3+1/5*(2*a*b*e+2*a*c*d+b^2*d)*x^5+1/7*(2*a*c*e
+b^2*e+2*b*c*d)*x^7+1/9*c*(2*b*e+c*d)*x^9+1/11*c^2*e*x^11
```

3.254.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int (d+ex^2)(a+bx^2+cx^4)^2 \, dx = a^2dx + \frac{1}{3}a(2bd+ae)x^3 + \frac{1}{5}(b^2d+2acd+2abe)x^5 + \frac{1}{7}(2bcd+b^2e+2ace)x^7 + \frac{1}{9}c(cd+2be)x^9 + \frac{1}{11}c^2ex^{11}$$

input
```
Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 +
((2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^1
1)/11
```

3.254.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)\left(a+bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \text{1467}$$

$$\int \left(a^2d+x^6\left(2ace+b^2e+2bcd\right)+x^4\left(2abe+2acd+b^2d\right)+ax^2(ae+2bd)+cx^8(2be+cd)+c^2ex^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2dx+\frac{1}{7}x^7\left(2ace+b^2e+2bcd\right)+\frac{1}{5}x^5\left(2abe+2acd+b^2d\right)+\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

input
```
Int[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 +
((2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^1
1)/11
```

3.254.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.254.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

method	result
default	$\frac{c^2 e\,x^{11}}{11}+\frac{(2ebc+c^2d)x^9}{9}+\frac{(2bcd+e(2ac+b^2))x^7}{7}+\frac{(d(2ac+b^2)+2abe)x^5}{5}+\frac{(e\,a^2+2dab)x^3}{3}+a^2dx$
norman	$\frac{c^2 e\,x^{11}}{11}+\left(\frac{2}{9}ebc+\frac{1}{9}c^2d\right)x^9+\left(\frac{2}{7}ace+\frac{1}{7}b^2e+\frac{2}{7}bcd\right)x^7+\left(\frac{2}{5}abe+\frac{2}{5}acd+\frac{1}{5}b^2d\right)x^5+\left(\frac{1}{3}e\,a^2+\right.$
gosper	$\frac{1}{11}c^2e\,x^{11}+\frac{2}{9}x^9ebc+\frac{1}{9}c^2d\,x^9+\frac{2}{7}ace\,x^7+\frac{1}{7}x^7b^2e+\frac{2}{7}x^7bcd+\frac{2}{5}x^5abe+\frac{2}{5}acd\,x^5+\frac{1}{5}x^5b^2d+$
risch	$\frac{1}{11}c^2e\,x^{11}+\frac{2}{9}x^9ebc+\frac{1}{9}c^2d\,x^9+\frac{2}{7}ace\,x^7+\frac{1}{7}x^7b^2e+\frac{2}{7}x^7bcd+\frac{2}{5}x^5abe+\frac{2}{5}acd\,x^5+\frac{1}{5}x^5b^2d+$
parallelrisch	$\frac{1}{11}c^2e\,x^{11}+\frac{2}{9}x^9ebc+\frac{1}{9}c^2d\,x^9+\frac{2}{7}ace\,x^7+\frac{1}{7}x^7b^2e+\frac{2}{7}x^7bcd+\frac{2}{5}x^5abe+\frac{2}{5}acd\,x^5+\frac{1}{5}x^5b^2d+$

input
```
int((e*x^2+d)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*c^2*e*x^11+1/9*(2*b*c*e+c^2*d)*x^9+1/7*(2*b*c*d+e*(2*a*c+b^2))*x^7+1/
5*(d*(2*a*c+b^2)+2*a*b*e)*x^5+1/3*(a^2*e+2*a*b*d)*x^3+a^2*d*x
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{11}\,c^2ex^{11}+\frac{1}{9}\left(c^2d+2\,bce\right)x^9+\frac{1}{7}\left(2\,bcd+\left(b^2+2\,ac\right)e\right)x^7+\frac{1}{5}\left(2\,abe+\left(b^2+2\,ac\right)d\right)x^5+a^2dx+\frac{1}{3}\left(2\,abd+a^2e\right)x^3$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/11*c^2*e*x^11 + 1/9*(c^2*d + 2*b*c*e)*x^9 + 1/7*(2*b*c*d + (b^2 + 2*a*c)
*e)*x^7 + 1/5*(2*a*b*e + (b^2 + 2*a*c)*d)*x^5 + a^2*d*x + 1/3*(2*a*b*d + a
^2*e)*x^3
```

3.254.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int (d+ex^2) (a+bx^2+cx^4)^2 \, dx = a^2dx + \frac{c^2ex^{11}}{11} + x^9 \cdot \left(\frac{2bce}{9} + \frac{c^2d}{9}\right) + x^7 \cdot \left(\frac{2ace}{7} + \frac{b^2e}{7} + \frac{2bcd}{7}\right) + x^5 \cdot \left(\frac{2abe}{5} + \frac{2acd}{5} + \frac{b^2d}{5}\right) + x^3\left(\frac{a^2e}{3} + \frac{2abd}{3}\right)$$

input
```
integrate((e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*d*x + c**2*e*x**11/11 + x**9*(2*b*c*e/9 + c**2*d/9) + x**7*(2*a*c*e/7
 + b**2*e/7 + 2*b*c*d/7) + x**5*(2*a*b*e/5 + 2*a*c*d/5 + b**2*d/5) + x**3*
(a**2*e/3 + 2*a*b*d/3)
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int (d+ex^2) (a+bx^2+cx^4)^2 \, dx = \frac{1}{11} c^2ex^{11} + \frac{1}{9} (c^2d + 2bce)x^9 + \frac{1}{7} (2bcd + (b^2 + 2ac)e)x^7 + \frac{1}{5} (2abe + (b^2 + 2ac)d)x^5 + a^2dx + \frac{1}{3} (2abd + a^2e)x^3$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/11*c^2*e*x^11 + 1/9*(c^2*d + 2*b*c*e)*x^9 + 1/7*(2*b*c*d + (b^2 + 2*a*c)
*e)*x^7 + 1/5*(2*a*b*e + (b^2 + 2*a*c)*d)*x^5 + a^2*d*x + 1/3*(2*a*b*d + a
^2*e)*x^3
```

3.254.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int (d+ex^2) (a+bx^2+cx^4)^2 \, dx = \frac{1}{11} c^2ex^{11} + \frac{1}{9} c^2dx^9 + \frac{2}{9} bcex^9 + \frac{2}{7} bcdx^7 + \frac{1}{7} b^2ex^7 + \frac{2}{7} acex^7 + \frac{1}{5} b^2dx^5 + \frac{2}{5} acdx^5 + \frac{2}{5} abex^5 + \frac{2}{3} abdx^3 + \frac{1}{3} a^2ex^3 + a^2dx$$

input
```
integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/11*c^2*e*x^11 + 1/9*c^2*d*x^9 + 2/9*b*c*e*x^9 + 2/7*b*c*d*x^7 + 1/7*b^2*
e*x^7 + 2/7*a*c*e*x^7 + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*e*x^5 + 2/
3*a*b*d*x^3 + 1/3*a^2*e*x^3 + a^2*d*x
```

3.254.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int (d+ex^2)\left(a+bx^2+cx^4\right)^2\,dx = x^5\left(\frac{d\,b^2}{5}+\frac{2\,a\,e\,b}{5}+\frac{2\,a\,c\,d}{5}\right) + x^7\left(\frac{e\,b^2}{7}+\frac{2\,c\,d\,b}{7}+\frac{2\,a\,c\,e}{7}\right)+x^3\left(\frac{e\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right) + x^9\left(\frac{d\,c^2}{9}+\frac{2\,b\,e\,c}{9}\right)+\frac{c^2\,e\,x^{11}}{11}+a^2\,d\,x$$

input
```
int((d + e*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^5*((b^2*d)/5 + (2*a*b*e)/5 + (2*a*c*d)/5) + x^7*((b^2*e)/7 + (2*a*c*e)/7
 + (2*b*c*d)/7) + x^3*((a^2*e)/3 + (2*a*b*d)/3) + x^9*((c^2*d)/9 + (2*b*c*
e)/9) + (c^2*e*x^11)/11 + a^2*d*x
```

3.255 $\int (a+bx^2+cx^4)^2\,dx$

3.255.1 Optimal result

Integrand size = 14, antiderivative size = 49

$$\int (a+bx^2+cx^4)^2\,dx = a^2x+\frac{2}{3}abx^3+\frac{1}{5}(b^2+2ac)\,x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

3.255.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int (a+bx^2+cx^4)^2\,dx = a^2x+\frac{2}{3}abx^3+\frac{1}{5}(b^2+2ac)\,x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/
9
```

3.255.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^2 \, dx$$

$$\Big\downarrow \text{1403}$$

$$\int \left(a^2+b^2x^4\left(\frac{2ac}{b^2}+1\right)+2abx^2+2bcx^6+c^2x^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

input
```
Int[(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/
9
```

3.255.3.1 Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.255.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$a^2x+\frac{2ab\,x^3}{3}+\frac{(2ac+b^2)x^5}{5}+\frac{2bc\,x^7}{7}+\frac{c^2x^9}{9}$	42
norman	$\frac{c^2x^9}{9}+\frac{2bc\,x^7}{7}+\left(\frac{2ac}{5}+\frac{b^2}{5}\right)x^5+\frac{2ab\,x^3}{3}+a^2x$	43
gosper	$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{2}{3}ab\,x^3+a^2x$	44
risch	$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{2}{3}ab\,x^3+a^2x$	44
parallelrisch	$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{2}{3}ab\,x^3+a^2x$	44

input
```
int((c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,\left(b^2+2\,ac\right)x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.255.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \left(a+bx^2+cx^4\right)^2\,dx=a^2x+\frac{2abx^3}{3}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}+x^5\cdot\left(\frac{2ac}{5}+\frac{b^2}{5}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/
5)
```

3.255.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int (a + bx^2 + cx^4)^2 \, dx = \frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + a^2x + \frac{2}{15}(3\,cx^5 + 5\,bx^3)a$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)
*a
```

3.255.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int (a + bx^2 + cx^4)^2 \, dx = \frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{2}{3}abx^3 + a^2x$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*
x
```

3.255.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int (a + bx^2 + cx^4)^2 \, dx = a^2\,x + x^5\left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + \frac{c^2\,x^9}{9} + \frac{2\,a\,b\,x^3}{3} + \frac{2\,b\,c\,x^7}{7}$$

input
```
int((a + b*x^2 + c*x^4)^2,x)
```

output
```
a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7
)/7
```

3.256 $\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx$

3.256.1 Optimal result

Integrand size = 24, antiderivative size = 143

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx = -\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)x}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^3}{3e^3} - \frac{c(cd-2be)x^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{\left(cd^2-bde+ae^2\right)^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

output
```
-(-b*e+c*d)*(c*d^2-e*(-2*a*e+b*d))*x/e^4+1/3*(c^2*d^2+b^2*e^2-2*c*e*(-a*e+
b*d))*x^3/e^3-1/5*c*(-2*b*e+c*d)*x^5/e^2+1/7*c^2*x^7/e+(a*e^2-b*d*e+c*d^2)
^2*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx = \frac{(-cd+be)\left(cd^2-bde+2ae^2\right)x}{e^4}+\frac{\left(c^2d^2-2bcde+b^2e^2+2ace^2\right)x^3}{3e^3} + \frac{c(-cd+2be)x^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{\left(cd^2-bde+ae^2\right)^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{9/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```

output
```
((-(c*d) + b*e)*(c*d^2 - b*d*e + 2*a*e^2)*x)/e^4 + ((c^2*d^2 - 2*b*c*d*e +
 b^2*e^2 + 2*a*c*e^2)*x^3)/(3*e^3) + (c*(-(c*d) + 2*b*e)*x^5)/(5*e^2) + (c
^2*x^7)/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(S
qrt[d]*e^(9/2))
```

3.256.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx$$

$$\Big\downarrow \; 1467$$

$$\int \left(\frac{a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+c^2d^4}{e^4\,(d+ex^2)}+\frac{x^2\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{e^3}-\frac{(cd-be)\left(cd^2-e\right.}{e^4}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-bde+cd^2\right)^2}{\sqrt{d}e^{9/2}}+\frac{x^3\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{3e^3}-\frac{x(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}-\frac{cx^5(cd-2be)}{5e^2}+\frac{c^2x^7}{7e}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```

output
```
-((((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^2*e^2 - 2
*c*e*(b*d - a*e))*x^3)/(3*e^3) - (c*(c*d - 2*b*e)*x^5)/(5*e^2) + (c^2*x^7)
/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*
e^(9/2))
```

3.256.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.256.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

method	result
default	$\frac{\frac{c^2x^7e^3}{7}+\frac{\left((be-cd)c\,e^2+e^3bc\right)x^5}{5}+\frac{\left((be-cd)b\,e^2+ec\left(2a\,e^2-bde+c\,d^2\right)\right)x^3}{3}+(be-cd)\left(2a\,e^2-bde+c\,d^2\right)x}{e^4}+\frac{(a^2e^4-2abd\,e^3+2ac\,d^2e^2+b^2d}{e}$
risch	$\frac{c^2x^7}{7e}+\frac{2x^5bc}{5e}-\frac{c^2d\,x^5}{5e^2}+\frac{x^3b^2}{3e}-\frac{2x^3dbc}{3e^2}+\frac{2ca\,x^3}{3e}+\frac{c^2d^2x^3}{3e^3}+\frac{2abx}{e}-\frac{2cadx}{e^2}-\frac{b^2dx}{e^2}+\frac{2bc\,d^2x}{e^3}-\frac{c^2d^3x}{e^4}-\frac{\ln(ex}{2}$

input
```
int((c*x^4+b*x^2+a)^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(1/7*c^2*x^7*e^3+1/5*((b*e-c*d)*c*e^2+e^3*b*c)*x^5+1/3*((b*e-c*d)*b*
e^2+e*c*(2*a*e^2-b*d*e+c*d^2))*x^3+(b*e-c*d)*(2*a*e^2-b*d*e+c*d^2)*x)+(a^2
*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/e^4/(e*d)^
(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx$$
$$=\left[\frac{30\,c^2de^4x^7-42\,(c^2d^2e^3-2\,bcde^4)x^5+70\,(c^2d^3e^2-2\,bcd^2e^3+(b^2+2\,ac)de^4)x^3-105\,(c^2d^4-2\,bcd^3e-}{}\right.$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/210*(30*c^2*d*e^4*x^7 - 42*(c^2*d^2*e^3 - 2*b*c*d*e^4)*x^5 + 70*(c^2*d^
3*e^2 - 2*b*c*d^2*e^3 + (b^2 + 2*a*c)*d*e^4)*x^3 - 105*(c^2*d^4 - 2*b*c*d^
3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*sqrt(-d*e)*log((e*x^2
 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 210*(c^2*d^4*e - 2*b*c*d^3*e^2 - 2*a
*b*d*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)/(d*e^5), 1/105*(15*c^2*d*e^4*x^7 - 21
*(c^2*d^2*e^3 - 2*b*c*d*e^4)*x^5 + 35*(c^2*d^3*e^2 - 2*b*c*d^2*e^3 + (b^2
+ 2*a*c)*d*e^4)*x^3 + 105*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 +
 (b^2 + 2*a*c)*d^2*e^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 105*(c^2*d^4*e -
 2*b*c*d^3*e^2 - 2*a*b*d*e^4 + (b^2 + 2*a*c)*d^2*e^3)*x)/(d*e^5)]
```

3.256.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(133) = 266$.

Time = 0.49 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.59

$$
\begin{aligned}
&\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx \\
&= \frac{c^2x^7}{7e} + x^5\cdot\left(\frac{2bc}{5e} - \frac{c^2d}{5e^2}\right) + x^3\cdot\left(\frac{2ac}{3e} + \frac{b^2}{3e} - \frac{2bcd}{3e^2} + \frac{c^2d^2}{3e^3}\right) \\
&\quad + x\left(\frac{2ab}{e} - \frac{2acd}{e^2} - \frac{b^2d}{e^2} + \frac{2bcd^2}{e^3} - \frac{c^2d^3}{e^4}\right) \\
&\quad - \frac{\sqrt{-\frac{1}{de^9}}(ae^2-bde+cd^2)^2\log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}(ae^2-bde+cd^2)^2}{a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+c^2d^4}+x\right)}{2} \\
&\quad + \frac{\sqrt{-\frac{1}{de^9}}(ae^2-bde+cd^2)^2\log\left(\frac{de^4\sqrt{-\frac{1}{de^9}}(ae^2-bde+cd^2)^2}{a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+c^2d^4}+x\right)}{2}
\end{aligned}
$$

input
```
integrate((c*x**4+b*x**2+a)**2/(e*x**2+d),x)
```

output
```
c**2*x**7/(7*e) + x**5*(2*b*c/(5*e) - c**2*d/(5*e**2)) + x**3*(2*a*c/(3*e)
 + b**2/(3*e) - 2*b*c*d/(3*e**2) + c**2*d**2/(3*e**3)) + x*(2*a*b/e - 2*a*
c*d/e**2 - b**2*d/e**2 + 2*b*c*d**2/e**3 - c**2*d**3/e**4) - sqrt(-1/(d*e*
*9))*(a*e**2 - b*d*e + c*d**2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 -
b*d*e + c*d**2)**2/(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2
*e**2 - 2*b*c*d**3*e + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 - b*d
*e + c*d**2)**2*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c*d**2)**2/
(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*
e + c**2*d**4) + x)/2
```

3.256.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.256.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx = \frac{(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^4} + \frac{15\,c^2e^6x^7-21\,c^2de^5x^5+42\,bce^6x^5+35\,c^2d^2e^4x^3-70\,bcde^5x^3+35\,b^2e^6x^3+70\,ace^6x^3-105\,c^2d^3e^3x+}{105\,e^7}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="giac")
```

output
```
(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e
^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^4) + 1/105*(15*c^2*e^6*x^7 - 21*c^2
*d*e^5*x^5 + 42*b*c*e^6*x^5 + 35*c^2*d^2*e^4*x^3 - 70*b*c*d*e^5*x^3 + 35*b
^2*e^6*x^3 + 70*a*c*e^6*x^3 - 105*c^2*d^3*e^3*x + 210*b*c*d^2*e^4*x - 105*
b^2*d*e^5*x - 210*a*c*d*e^5*x + 210*a*b*e^6*x)/e^7
```

3.256.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx$$
$$= x^3\left(\frac{b^2+2\,a\,c}{3\,e}+\frac{d\left(\frac{c^2\,d}{e^2}-\frac{2\,b\,c}{e}\right)}{3\,e}\right) - x\left(\frac{d\left(\frac{b^2+2\,a\,c}{e}+\frac{d\left(\frac{c^2\,d}{e^2}-\frac{2\,b\,c}{e}\right)}{e}\right)}{e}-\frac{2\,a\,b}{e}\right) - x^5\left(\frac{c^2\,d}{5\,e^2}-\frac{2\,b\,c}{5\,e}\right)+\frac{c^2\,x^7}{7\,e} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x\,(c\,d^2-b\,d\,e+a\,e^2)^2}{\sqrt{d}\,(a^2\,e^4-2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2-2\,b\,c\,d^3\,e+c^2\,d^4)}\right)\,(c\,d^2-b\,d\,e+a\,e^2)^2}{\sqrt{d}\,e^{9/2}}$$

input

```
int((a + b*x^2 + c*x^4)^2/(d + e*x^2),x)
```

output

```
x^3*((2*a*c + b^2)/(3*e) + (d*((c^2*d)/e^2 - (2*b*c)/e))/(3*e)) - x*((d*((
2*a*c + b^2)/e + (d*((c^2*d)/e^2 - (2*b*c)/e))/e))/e - (2*a*b)/e) - x^5*((
c^2*d)/(5*e^2) - (2*b*c)/(5*e)) + (c^2*x^7)/(7*e) + (atan((e^(1/2)*x*(a*e^
2 + c*d^2 - b*d*e)^2)/(d^(1/2)*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*
e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2)))*(a*e^2 + c*d^2 - b*d*e)^2)/(d^(1/2)*e
^(9/2))
```

3.257 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx$

3.257.1 Optimal result

Integrand size = 24, antiderivative size = 166

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx = \frac{(3c^2d^2+b^2e^2-2ce(2bd-ae))\,x}{e^4} - \frac{2c(cd-be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2-bde+ae^2)^2\,x}{2de^4\,(d+ex^2)} - \frac{(cd^2-bde+ae^2)\,(7cd^2-e(3bd+ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

output
```
(3*c^2*d^2+b^2*e^2-2*c*e*(-a*e+2*b*d))*x/e^4-2/3*c*(-b*e+c*d)*x^3/e^3+1/5*
c^2*x^5/e^2+1/2*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)-1/2*(a*e^2-b*d*e+c
*d^2)*(7*c*d^2-e*(a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(9/2)
```

3.257.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx = \frac{(3c^2d^2+b^2e^2+2ce(-2bd+ae))\,x}{e^4} + \frac{2c(-cd+be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+e(-bd+ae))^2\,x}{2de^4\,(d+ex^2)} - \frac{(7c^2d^4+2cd^2e(-5bd+3ae)-e^2(-3b^2d^2+2abde+a^2e^2))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

output
```
((3*c^2*d^2 + b^2*e^2 + 2*c*e*(-2*b*d + a*e))*x)/e^4 + (2*c*(-(c*d) + b*e)
*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(2*d*
e^4*(d + e*x^2)) - ((7*c^2*d^4 + 2*c*d^2*e*(-5*b*d + 3*a*e) - e^2*(-3*b^2*
d^2 + 2*a*b*d*e + a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2
))
```

3.257.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1471, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{1471}$$

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{2de^4\left(d+ex^2\right)}-\frac{\int \frac{-\frac{2c^2dx^6}{e}+\frac{2cd(cd-2be)x^4}{e^2}-\frac{2d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{c^2d^4-2ce(bd-ae)d^2+e^2\left(b^2d^2-2abed-a^2e^2\right)}{e^4}}{ex^2+d}dx}{2d}$$

$$\Big\downarrow \text{2341}$$

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{2de^4\left(d+ex^2\right)}-\frac{\int \left(-\frac{2c^2dx^4}{e^2}+\frac{4cd(cd-be)x^2}{e^3}-\frac{2d\left(3c^2d^2+b^2e^2-2ce(2bd-ae)\right)}{e^4}+\frac{7c^2d^4-10bced^3+3b^2e^2d^2+6ace^2d^2-2abe^3d-a^2e^4}{e^4(ex^2+d)}\right)dx}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{2de^4\left(d+ex^2\right)}-\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-3bde+7cd^2\right)\left(ae^2-bde+cd^2\right)}{\sqrt{d}e^{9/2}}-\frac{2dx\left(-2ce(2bd-ae)+b^2e^2+3c^2d^2\right)}{e^4}+\frac{4cdx^3(cd-be)}{3e^3}-\frac{2c^2dx^5}{5e^2}}{2d}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((-2*d*(3*c^2*d^2 +
b^2*e^2 - 2*c*e*(2*b*d - a*e))*x)/e^4 + (4*c*d*(c*d - b*e)*x^3)/(3*e^3) -
(2*c^2*d*x^5)/(5*e^2) + ((7*c*d^2 - 3*b*d*e - a*e^2)*(c*d^2 - b*d*e + a*e^
2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2)))/(2*d)
```

3.257.3.1 Defintions of rubi rules used

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.257.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.28

method	result
default	$\frac{\frac{1}{5}e^2x^5c^2+\frac{2}{3}bc\,e^2x^3-\frac{2}{3}c^2de\,x^3+2e^2acx+b^2e^2x-4bcdex+3c^2d^2x}{e^4}+\frac{\frac{\left(a^2e^4-2abd\,e^3+2ac\,d^2e^2+b^2d^2e^2-2bc\,d^3e+c^2d^4\right)x}{2d\left(e\,x^2+d\right)}+\frac{\left(a^2e^4+2abd\,e^3\right.}{}}{e^4}$
risch	$\frac{c^2x^5}{5e^2}+\frac{2bc\,x^3}{3e^2}-\frac{2c^2d\,x^3}{3e^3}+\frac{2cax}{e^2}+\frac{b^2x}{e^2}-\frac{4bcdx}{e^3}+\frac{3c^2d^2x}{e^4}+\frac{(a^2e^4-2abd\,e^3+2ac\,d^2e^2+b^2d^2e^2-2bc\,d^3e+c^2d^4)x}{2d\,e^4(e\,x^2+d)}-\frac{\ln(e}{}$

input
```
int((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(1/5*e^2*x^5*c^2+2/3*b*c*e^2*x^3-2/3*c^2*d*e*x^3+2*e^2*a*c*x+b^2*e^2
*x-4*b*c*d*e*x+3*c^2*d^2*x)+1/e^4*(1/2*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+
b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/d*x/(e*x^2+d)+1/2*(a^2*e^4+2*a*b*d*e^3-6*
a*c*d^2*e^2-3*b^2*d^2*e^2+10*b*c*d^3*e-7*c^2*d^4)/d/(e*d)^(1/2)*arctan(e*x
/(e*d)^(1/2)))
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.61

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx$$

$$= \left[\frac{12\,c^2d^2e^4x^7 - 4\,(7\,c^2d^3e^3 - 10\,bcd^2e^4)x^5 + 20\,(7\,c^2d^4e^2 - 10\,bcd^3e^3 + 3\,(b^2+2\,ac)d^2e^4)x^3 + 15\,(7\,c^2d^5 -}{}\right.$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*c^2*d^2*e^4*x^7 - 4*(7*c^2*d^3*e^3 - 10*b*c*d^2*e^4)*x^5 + 20*(7
*c^2*d^4*e^2 - 10*b*c*d^3*e^3 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^3 + 15*(7*c^2*d
^5 - 10*b*c*d^4*e - 2*a*b*d^2*e^3 - a^2*d*e^4 + 3*(b^2 + 2*a*c)*d^3*e^2 +
(7*c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b*d*e^4 - a^2*e^5 + 3*(b^2 + 2*a*c)*d^
2*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 30*
(7*c^2*d^5*e - 10*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + 3*(b^2 + 2*a*c
)*d^3*e^3)*x)/(d^2*e^6*x^2 + d^3*e^5), 1/30*(6*c^2*d^2*e^4*x^7 - 2*(7*c^2*
d^3*e^3 - 10*b*c*d^2*e^4)*x^5 + 10*(7*c^2*d^4*e^2 - 10*b*c*d^3*e^3 + 3*(b^
2 + 2*a*c)*d^2*e^4)*x^3 - 15*(7*c^2*d^5 - 10*b*c*d^4*e - 2*a*b*d^2*e^3 - a
^2*d*e^4 + 3*(b^2 + 2*a*c)*d^3*e^2 + (7*c^2*d^4*e - 10*b*c*d^3*e^2 - 2*a*b
*d*e^4 - a^2*e^5 + 3*(b^2 + 2*a*c)*d^2*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e
)*x/d) + 15*(7*c^2*d^5*e - 10*b*c*d^4*e^2 - 2*a*b*d^2*e^4 + a^2*d*e^5 + 3*
(b^2 + 2*a*c)*d^3*e^3)*x)/(d^2*e^6*x^2 + d^3*e^5)]
```

3.257.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(156) = 312$.

Time = 1.17 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.92

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx = \frac{c^2x^5}{5e^2} + x^3\cdot\left(\frac{2bc}{3e^2}-\frac{2c^2d}{3e^3}\right) + x\left(\frac{2ac}{e^2}+\frac{b^2}{e^2}-\frac{4bcd}{e^3}+\frac{3c^2d^2}{e^4}\right) + \frac{x(a^2e^4-2abde^3+2acd^2e^2+b^2d^2e^2-2bcd^3e+c^2d^4)}{2d^2e^4+2de^5x^2} - \frac{\sqrt{-\frac{1}{d^3e^9}}\,(ae^2-bde+cd^2)\,(ae^2+3bde-7cd^2)\log\left(-\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}(ae^2-bde+cd^2)(ae^2+3bde-7cd^2)}{a^2e^4+2abde^3-6acd^2e^2-3b^2d^2e^2+10bcd^3e-7c^2d^4}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^9}}\,(ae^2-bde+cd^2)\,(ae^2+3bde-7cd^2)\log\left(\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}(ae^2-bde+cd^2)(ae^2+3bde-7cd^2)}{a^2e^4+2abde^3-6acd^2e^2-3b^2d^2e^2+10bcd^3e-7c^2d^4}+x\right)}{4}$$

input
```
integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**2,x)
```

output
```
c**2*x**5/(5*e**2) + x**3*(2*b*c/(3*e**2) - 2*c**2*d/(3*e**3)) + x*(2*a*c/
e**2 + b**2/e**2 - 4*b*c*d/e**3 + 3*c**2*d**2/e**4) + x*(a**2*e**4 - 2*a*b
*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4)/(2*
d**2*e**4 + 2*d*e**5*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2
)*(a*e**2 + 3*b*d*e - 7*c*d**2)*log(-d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**
2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)/(a**2*e**4 + 2*a*b*d*e**
3 - 6*a*c*d**2*e**2 - 3*b**2*d**2*e**2 + 10*b*c*d**3*e - 7*c**2*d**4) + x)
/4 + sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*
c*d**2)*log(d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e*
*2 + 3*b*d*e - 7*c*d**2)/(a**2*e**4 + 2*a*b*d*e**3 - 6*a*c*d**2*e**2 - 3*b
**2*d**2*e**2 + 10*b*c*d**3*e - 7*c**2*d**4) + x)/4
```

3.257.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.257.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx = -\frac{(7\,c^2d^4-10\,bcd^3e+3\,b^2d^2e^2+6\,acd^2e^2-2\,abde^3-a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^4} + \frac{c^2d^4x-2\,bcd^3ex+b^2d^2e^2x+2\,acd^2e^2x-2\,abde^3x+a^2e^4x}{2\,(ex^2+d)de^4} + \frac{3\,c^2e^8x^5-10\,c^2de^7x^3+10\,bce^8x^3+45\,c^2d^2e^6x-60\,bcde^7x+15\,b^2e^8x+30\,ace^8x}{15\,e^{10}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/2*(7*c^2*d^4 - 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 - 2*a*b*d*e
^3 - a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d*e^4) + 1/2*(c^2*d^4*x - 2
*b*c*d^3*e*x + b^2*d^2*e^2*x + 2*a*c*d^2*e^2*x - 2*a*b*d*e^3*x + a^2*e^4*x
)/((e*x^2 + d)*d*e^4) + 1/15*(3*c^2*e^8*x^5 - 10*c^2*d*e^7*x^3 + 10*b*c*e^
8*x^3 + 45*c^2*d^2*e^6*x - 60*b*c*d*e^7*x + 15*b^2*e^8*x + 30*a*c*e^8*x)/e
^10
```

3.257.9 Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx = x\left(\frac{b^2+2\,a\,c}{e^2}+\frac{2\,d\left(\frac{2\,c^2\,d}{e^3}-\frac{2\,b\,c}{e^2}\right)}{e}-\frac{c^2\,d^2}{e^4}\right)-x^3\left(\frac{2\,c^2\,d}{3\,e^3}-\frac{2\,b\,c}{3\,e^2}\right) + \frac{c^2\,x^5}{5\,e^2}+\frac{x\,(a^2\,e^4-2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2-2\,b\,c\,d^3\,e+c^2\,d^4)}{2\,d\,(e^5\,x^2+d\,e^4)} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,(c\,d^2-b\,d\,e+a\,e^2)\,(-7\,c\,d^2+3\,b\,d\,e+a\,e^2)}{\sqrt{d}\,(a^2\,e^4+2\,a\,b\,d\,e^3-6\,a\,c\,d^2\,e^2-3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-7\,c^2\,d^4)}\right)\,(c\,d^2-b\,d\,e+a\,e^2)\,(-7\,c\,d^2+3\,b\,d\,e+a\,e^2)}{2\,d^{3/2}\,e^{9/2}}$$

input
```
int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x)
```

output
```
x*((2*a*c + b^2)/e^2 + (2*d*((2*c^2*d)/e^3 - (2*b*c)/e^2))/e - (c^2*d^2)/e
^4) - x^3*((2*c^2*d)/(3*e^3) - (2*b*c)/(3*e^2)) + (c^2*x^5)/(5*e^2) + (x*(
a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^
2))/(2*d*(d*e^4 + e^5*x^2)) + (atan((e^(1/2)*x*(a*e^2 + c*d^2 - b*d*e)*(a*
e^2 - 7*c*d^2 + 3*b*d*e))/(d^(1/2)*(a^2*e^4 - 7*c^2*d^4 - 3*b^2*d^2*e^2 +
2*a*b*d*e^3 + 10*b*c*d^3*e - 6*a*c*d^2*e^2)))*(a*e^2 + c*d^2 - b*d*e)*(a*e
^2 - 7*c*d^2 + 3*b*d*e))/(2*d^(3/2)*e^(9/2))
```

3.257. $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx$

3.258 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx$

3.258.1 Optimal result

Integrand size = 24, antiderivative size = 201

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx &= -\frac{c(3cd-2be)x}{e^4}+\frac{c^2x^3}{3e^3}+\frac{(cd^2-bde+ae^2)^2x}{4de^4(d+ex^2)^2}\\ &\quad-\frac{(13cd^2-5bde-3ae^2)(cd^2-bde+ae^2)x}{8d^2e^4(d+ex^2)}\\ &\quad+\frac{(35c^2d^4-6cd^2e(5bd-ae)+e^2(3b^2d^2+2abde+3a^2e^2))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}\end{aligned}$$

output
```
-c*(-2*b*e+3*c*d)*x/e^4+1/3*c^2*x^3/e^3+1/4*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/
(e*x^2+d)^2-1/8*(-3*a*e^2-5*b*d*e+13*c*d^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/
(e*x^2+d)+1/8*(35*c^2*d^4-6*c*d^2*e*(-a*e+5*b*d)+e^2*(3*a^2*e^2+2*a*b*d*e+
3*b^2*d^2))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(9/2)
```

3.258.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx\\&=\frac{c(-3cd+2be)x}{e^4}+\frac{c^2x^3}{3e^3}+\frac{(cd^2+e(-bd+ae))^2x}{4de^4(d+ex^2)^2}\\&\quad-\frac{(13c^2d^4-2cd^2e(9bd-5ae)+e^2(5b^2d^2-2abde-3a^2e^2))x}{8d^2e^4(d+ex^2)}\\&\quad+\frac{(35c^2d^4+6cd^2e(-5bd+ae)+e^2(3b^2d^2+2abde+3a^2e^2))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}\end{aligned}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```

output
```
(c*(-3*c*d + 2*b*e)*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + e*(-(b*d) + a*e
))^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c^2*d^4 - 2*c*d^2*e*(9*b*d - 5*a*e)
 + e^2*(5*b^2*d^2 - 2*a*b*d*e - 3*a^2*e^2))*x)/(8*d^2*e^4*(d + e*x^2)) + (
(35*c^2*d^4 + 6*c*d^2*e*(-5*b*d + a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^
2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))
```

3.258.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1471, 2345, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{1471}$$

$$\frac{x(ae^2-bde+cd^2)^2}{4de^4(d+ex^2)^2}-\frac{\int \frac{-\frac{4c^2dx^6}{e}+\frac{4cd(cd-2be)x^4}{e^2}-\frac{4d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{\left(cd^2-e(bd-3ae)\right)\left(cd^2-e(bd+ae)\right)}{e^4}}{(ex^2+d)^2}dx}{4d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{x(ae^2-bde+cd^2)^2}{4de^4(d+ex^2)^2}-\frac{\frac{x(-3ae^2-5bde+13cd^2)(ae^2-bde+cd^2)}{2de^4(d+ex^2)}-\frac{\int\frac{\frac{8c^2d^2x^4}{e^2}-\frac{16cd^2(cd-be)x^2}{e^3}+\frac{11c^2d^4-2ce(7bd-3ae)d^2+e^2(3b^2d^2+2abed+3a^2e^2)}{e^4}}{ex^2+d}dx}{2d}}{4d}}{}$$

$\Big\downarrow$ 1467

$$\frac{x(ae^2-bde+cd^2)^2}{4de^4(d+ex^2)^2}-\frac{\frac{x(-3ae^2-5bde+13cd^2)(ae^2-bde+cd^2)}{2de^4(d+ex^2)}-\frac{\int\left(\frac{8c^2x^2d^2}{e^3}-\frac{8c(3cd-2be)d^2}{e^4}+\frac{35c^2d^4-30bced^3+3b^2e^2d^2+6ace^2d^2+2abe^3d+3a^2e^4}{e^4(ex^2+d)}\right)dx}{2d}}{4d}$$

$\Big\downarrow$ 2009

$$\frac{x(ae^2-bde+cd^2)^2}{4de^4(d+ex^2)^2}-\frac{\frac{x(-3ae^2-5bde+13cd^2)(ae^2-bde+cd^2)}{2de^4(d+ex^2)}-\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2(3a^2e^2+2abde+3b^2d^2)-6cd^2e(5bd-ae)+35c^2d^4\right)}{\sqrt{d}e^{9/2}}-\frac{8cd^2x(3cd-2be)}{e^4}+\frac{8c^2d^2x^3}{3e^3}}{2d}}{4d}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)^2*x)/(4*d*e^4*(d + e*x^2)^2) - (((13*c*d^2 - 5*b*
d*e - 3*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^4*(d + e*x^2)) - ((-8*c*d
^2*(3*c*d - 2*b*e)*x)/e^4 + (8*c^2*d^2*x^3)/(3*e^3) + ((35*c^2*d^4 - 6*c*d
^2*e*(5*b*d - a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2))*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(9/2)))/(2*d))/(4*d)
```

3.258.3.1 Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

3.258. $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx$

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.258.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.18

method	result
default	$\frac{c\left(\frac{1}{3}c\,x^3e+2bex-3cdx\right)}{e^4}+\frac{\frac{\frac{e\left(3a^2e^4+2abd\,e^3-10ac\,d^2e^2-5b^2d^2e^2+18bc\,d^3e-13c^2d^4\right)x^3}{8d^2}+\frac{\left(5a^2e^4-2abd\,e^3-6ac\,d^2e^2-3b^2d^2e^2+14bc\,d^3e-11\cdots\right.}{8d}}{\left(e\,x^2+d\right)^2}\cdots}{e^4}$
risch	$\frac{c^2x^3}{3e^3}+\frac{2cbx}{e^3}-\frac{3c^2dx}{e^4}+\frac{\frac{e\left(3a^2e^4+2abd\,e^3-10ac\,d^2e^2-5b^2d^2e^2+18bc\,d^3e-13c^2d^4\right)x^3}{8d^2}+\frac{\left(5a^2e^4-2abd\,e^3-6ac\,d^2e^2-3b^2d^2e^2+14bc\,d^3e-\cdots\right.}{8d}}{e^4\left(e\,x^2+d\right)^2}$

input
```
int((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c/e^4*(1/3*c*x^3*e+2*b*e*x-3*c*d*x)+1/e^4*((1/8*e*(3*a^2*e^4+2*a*b*d*e^3-1
0*a*c*d^2*e^2-5*b^2*d^2*e^2+18*b*c*d^3*e-13*c^2*d^4)/d^2*x^3+1/8*(5*a^2*e^
4-2*a*b*d*e^3-6*a*c*d^2*e^2-3*b^2*d^2*e^2+14*b*c*d^3*e-11*c^2*d^4)/d*x)/(e
*x^2+d)^2+1/8*(3*a^2*e^4+2*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2-30*b*c*d^
3*e+35*c^2*d^4)/d^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.258.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(185) = 370$.

Time = 0.26 (sec) , antiderivative size = 794, normalized size of antiderivative = 3.95

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx$$

$$= \left[\frac{16\,c^2d^3e^4x^7 - 16\,(7\,c^2d^4e^3 - 6\,bcd^3e^4)x^5 - 2\,(175\,c^2d^5e^2 - 150\,bcd^4e^3 - 6\,abd^2e^5 - 9\,a^2de^6 + 15\,(b^2 + 2\,a}{} \right.$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*c^2*d^3*e^4*x^7 - 16*(7*c^2*d^4*e^3 - 6*b*c*d^3*e^4)*x^5 - 2*(17
5*c^2*d^5*e^2 - 150*b*c*d^4*e^3 - 6*a*b*d^2*e^5 - 9*a^2*d*e^6 + 15*(b^2 +
2*a*c)*d^3*e^4)*x^3 - 3*(35*c^2*d^6 - 30*b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2
*d^2*e^4 + 3*(b^2 + 2*a*c)*d^4*e^2 + (35*c^2*d^4*e^2 - 30*b*c*d^3*e^3 + 2*
a*b*d*e^5 + 3*a^2*e^6 + 3*(b^2 + 2*a*c)*d^2*e^4)*x^4 + 2*(35*c^2*d^5*e - 3
0*b*c*d^4*e^2 + 2*a*b*d^2*e^4 + 3*a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*e^3)*x^2
)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(35*c^2*d^6
*e - 30*b*c*d^5*e^2 + 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 + 3*(b^2 + 2*a*c)*d^4*
e^3)*x)/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5), 1/24*(8*c^2*d^3*e^4*x^7 -
 8*(7*c^2*d^4*e^3 - 6*b*c*d^3*e^4)*x^5 - (175*c^2*d^5*e^2 - 150*b*c*d^4*e^
3 - 6*a*b*d^2*e^5 - 9*a^2*d*e^6 + 15*(b^2 + 2*a*c)*d^3*e^4)*x^3 + 3*(35*c^
2*d^6 - 30*b*c*d^5*e + 2*a*b*d^3*e^3 + 3*a^2*d^2*e^4 + 3*(b^2 + 2*a*c)*d^4
*e^2 + (35*c^2*d^4*e^2 - 30*b*c*d^3*e^3 + 2*a*b*d*e^5 + 3*a^2*e^6 + 3*(b^2
 + 2*a*c)*d^2*e^4)*x^4 + 2*(35*c^2*d^5*e - 30*b*c*d^4*e^2 + 2*a*b*d^2*e^4
+ 3*a^2*d*e^5 + 3*(b^2 + 2*a*c)*d^3*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x
/d) - 3*(35*c^2*d^6*e - 30*b*c*d^5*e^2 + 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 + 3
*(b^2 + 2*a*c)*d^4*e^3)*x)/(d^3*e^7*x^4 + 2*d^4*e^6*x^2 + d^5*e^5)]
```

3.258.6 Sympy [A] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx = \frac{c^2x^3}{3e^3} + x\left(\frac{2bc}{e^3} - \frac{3c^2d}{e^4}\right) - \frac{\sqrt{-\frac{1}{d^5e^9}}\cdot(3a^2e^4+2abde^3+6acd^2e^2+3b^2d^2e^2-30bcd^3e+35c^2d^4)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^9}}\cdot(3a^2e^4+2abde^3+6acd^2e^2+3b^2d^2e^2-30bcd^3e+35c^2d^4)\log\left(d^3e^4\sqrt{-\frac{1}{d^5e^9}}+x\right)}{16} + \frac{x^3\cdot(3a^2e^5+2abde^4-10acd^2e^3-5b^2d^2e^3+18bcd^3e^2-13c^2d^4e)+x(5a^2de^4-2abd^2e^3-6acd^3e^2-3b}{8d^4e^4+16d^3e^5x^2+8d^2e^6x^4}$$

input
```
integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**3,x)
```

output
```
c**2*x**3/(3*e**3) + x*(2*b*c/e**3 - 3*c**2*d/e**4) - sqrt(-1/(d**5*e**9))
*(3*a**2*e**4 + 2*a*b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e**2 - 30*b*c
*d**3*e + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + sqrt
(-1/(d**5*e**9))*(3*a**2*e**4 + 2*a*b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2*d*
*2*e**2 - 30*b*c*d**3*e + 35*c**2*d**4)*log(d**3*e**4*sqrt(-1/(d**5*e**9))
 + x)/16 + (x**3*(3*a**2*e**5 + 2*a*b*d*e**4 - 10*a*c*d**2*e**3 - 5*b**2*d
**2*e**3 + 18*b*c*d**3*e**2 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 2*a*b*d
**2*e**3 - 6*a*c*d**3*e**2 - 3*b**2*d**3*e**2 + 14*b*c*d**4*e - 11*c**2*d*
*5))/(8*d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)
```

3.258.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx$$
$$= \frac{(35\,c^2d^4 - 30\,bcd^3e + 3\,b^2d^2e^2 + 6\,acd^2e^2 + 2\,abde^3 + 3\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,\sqrt{de}d^2e^4}$$
$$- \frac{13\,c^2d^4ex^3 - 18\,bcd^3e^2x^3 + 5\,b^2d^2e^3x^3 + 10\,acd^2e^3x^3 - 2\,abde^4x^3 - 3\,a^2e^5x^3 + 11\,c^2d^5x - 14\,bcd^4ex + 3}{8\,(ex^2+d)^2d^2e^4}$$
$$+ \frac{c^2e^6x^3 - 9\,c^2de^5x + 6\,bce^6x}{3\,e^9}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
1/8*(35*c^2*d^4 - 30*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 2*a*b*d*e
^3 + 3*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2*e^4) - 1/8*(13*c^2*d^
4*e*x^3 - 18*b*c*d^3*e^2*x^3 + 5*b^2*d^2*e^3*x^3 + 10*a*c*d^2*e^3*x^3 - 2*
a*b*d*e^4*x^3 - 3*a^2*e^5*x^3 + 11*c^2*d^5*x - 14*b*c*d^4*e*x + 3*b^2*d^3*
e^2*x + 6*a*c*d^3*e^2*x + 2*a*b*d^2*e^3*x - 5*a^2*d*e^4*x)/((e*x^2 + d)^2*
d^2*e^4) + 1/3*(c^2*e^6*x^3 - 9*c^2*d*e^5*x + 6*b*c*e^6*x)/e^9
```

3.258.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx = \frac{c^2\,x^3}{3\,e^3} - x\left(\frac{3\,c^2\,d}{e^4} - \frac{2\,b\,c}{e^3}\right)$$
$$- \frac{\frac{x\,\left(-5\,a^2\,e^4+2\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2-14\,b\,c\,d^3\,e+11\,c^2\,d^4\right)}{8\,d} - \frac{x^3\,\left(3\,a^2\,e^5+2\,a\,b\,d\,e^4-10\,a\,c\,d^2\,e^3-5\,b^2\,d^2\,e^3+18\,b\,c\,d^3\,e^2-13\,c^2\,d^4\,e\right)}{8\,d^2}}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4}$$
$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,a^2\,e^4+2\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2-30\,b\,c\,d^3\,e+35\,c^2\,d^4\right)}{8\,d^{5/2}\,e^{9/2}}$$

input
```
int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x)
```

output
```
(c^2*x^3)/(3*e^3) - x*((3*c^2*d)/e^4 - (2*b*c)/e^3) - ((x*(11*c^2*d^4 - 5*
a^2*e^4 + 3*b^2*d^2*e^2 + 2*a*b*d*e^3 - 14*b*c*d^3*e + 6*a*c*d^2*e^2))/(8*
d) - (x^3*(3*a^2*e^5 - 13*c^2*d^4*e - 5*b^2*d^2*e^3 + 2*a*b*d*e^4 - 10*a*c
*d^2*e^3 + 18*b*c*d^3*e^2))/(8*d^2))/(d^2*e^4 + e^6*x^4 + 2*d*e^5*x^2) + (
atan((e^(1/2)*x)/d^(1/2))*(3*a^2*e^4 + 35*c^2*d^4 + 3*b^2*d^2*e^2 + 2*a*b*
d*e^3 - 30*b*c*d^3*e + 6*a*c*d^2*e^2))/(8*d^(5/2)*e^(9/2))
```

3.258. $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx$

3.259 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx$

3.259.1 Optimal result

Integrand size = 24, antiderivative size = 250

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{c^2x}{e^4}+\frac{(cd^2-bde+ae^2)^2x}{6de^4(d+ex^2)^3}-\frac{(19cd^2-7bde-5ae^2)(cd^2-bde+ae^2)x}{24d^2e^4(d+ex^2)^2}+\frac{(29c^2d^4-2cd^2e(11bd-ae)+e^2(b^2d^2+2abde+5a^2e^2))x}{16d^3e^4(d+ex^2)}-\frac{(35c^2d^4-2cd^2e(5bd+ae)-e^2(b^2d^2+2abde+5a^2e^2))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}$$

output
```
c^2*x/e^4+1/6*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)^3-1/24*(-5*a*e^2-7*b
*d*e+19*c*d^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/(e*x^2+d)^2+1/16*(29*c^2*d^4-
2*c*d^2*e*(-a*e+11*b*d)+e^2*(5*a^2*e^2+2*a*b*d*e+b^2*d^2))*x/d^3/e^4/(e*x^
2+d)-1/16*(35*c^2*d^4-2*c*d^2*e*(a*e+5*b*d)-e^2*(5*a^2*e^2+2*a*b*d*e+b^2*d
^2))*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(9/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx &= \frac{c^2x}{e^4}+\frac{(cd^2+e(-bd+ae))^2\,x}{6de^4\,(d+ex^2)^3}\\&\quad-\frac{(19c^2d^4+2cd^2e(-13bd+7ae)+e^2(7b^2d^2-2abde-5a^2e^2))\,x}{24d^2e^4\,(d+ex^2)^2}\\&\quad+\frac{(29c^2d^4+2cd^2e(-11bd+ae)+e^2(b^2d^2+2abde+5a^2e^2))\,x}{16d^3e^4\,(d+ex^2)}\\&\quad-\frac{(35c^2d^4-2cd^2e(5bd+ae)-e^2(b^2d^2+2abde+5a^2e^2))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}\end{aligned}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```

output
```
(c^2*x)/e^4 + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(6*d*e^4*(d + e*x^2)^3) - (
(19*c^2*d^4 + 2*c*d^2*e*(-13*b*d + 7*a*e) + e^2*(7*b^2*d^2 - 2*a*b*d*e - 5
*a^2*e^2))*x)/(24*d^2*e^4*(d + e*x^2)^2) + ((29*c^2*d^4 + 2*c*d^2*e*(-11*b
*d + a*e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x
^2)) - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e +
 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

3.259.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1471, 2345, 27, 1471, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx$$

$$\Big\downarrow \; 1471$$

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}-\frac{\int \frac{-\frac{6c^2dx^6}{e}+\frac{6cd(cd-2be)x^4}{e^2}-\frac{6d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{c^2d^4-2ce(bd-ae)d^2+e^2\left(b^2d^2-2abed-5a^2e^2\right)}{e^4}}{\left(ex^2+d\right)^3}dx}{6d}$$

$\big\downarrow$ 2345

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}-\frac{\frac{x\left(-5ae^2-7bde+19cd^2\right)\left(ae^2-bde+cd^2\right)}{4de^4\left(d+ex^2\right)^2}-\frac{\int \frac{3\left(\frac{8c^2d^2x^4}{e^2}-\frac{16cd^2(cd-be)x^2}{e^3}+\frac{5c^2d^4-2ce(3bd-ae)d^2+e^2\left(b^2d^2+2abed+5a^2e^2\right)}{e^4}\right)}{\left(ex^2+d\right)^2}dx}{4d}}{6d}$$

$\big\downarrow$ 27

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}-\frac{\frac{x\left(-5ae^2-7bde+19cd^2\right)\left(ae^2-bde+cd^2\right)}{4de^4\left(d+ex^2\right)^2}-\frac{3\int \frac{\frac{8c^2d^2x^4}{e^2}-\frac{16cd^2(cd-be)x^2}{e^3}+\frac{5c^2d^4-2ce(3bd-ae)d^2+e^2\left(b^2d^2+2abed+5a^2e^2\right)}{e^4}}{\left(ex^2+d\right)^2}dx}{4d}}{6d}$$

$\big\downarrow$ 1471

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}-\frac{\frac{x\left(-5ae^2-7bde+19cd^2\right)\left(ae^2-bde+cd^2\right)}{4de^4\left(d+ex^2\right)^2}-\frac{3\left(\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(11bd-ae)+29c^2d^4\right)}{2de^4\left(d+ex^2\right)}-\frac{\int \frac{19c^2d^4-16c^2ex^2d^3-2ce(5bd+ae)d^2-e^2\left(b^2d^2+\cdots\right.}{e^4\left(ex^2+d\right)}\cdots}{2d}\cdots\right.}{4d}\cdots}{6d}$$

$\big\downarrow$ 27

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}-\frac{\frac{x\left(-5ae^2-7bde+19cd^2\right)\left(ae^2-bde+cd^2\right)}{4de^4\left(d+ex^2\right)^2}-\frac{3\left(\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(11bd-ae)+29c^2d^4\right)}{2de^4\left(d+ex^2\right)}-\frac{\int \frac{19c^2d^4-16c^2ex^2d^3-2ce(5bd+ae)d^2-e^2\left(b^2d^2+\cdots\right.}{ex^2+d}\cdots}{2de^4}\cdots\right.}{4d}\cdots}{6d}$$

$\big\downarrow$ 299

3.259. $\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^4}\,dx$

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}-\frac{\frac{x\left(-5ae^2-7bde+19cd^2\right)\left(ae^2-bde+cd^2\right)}{4de^4\left(d+ex^2\right)^2}-\frac{3\left(\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(11bd-ae)+29c^2d^4\right)}{2de^4\left(d+ex^2\right)}-\frac{\left(-e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(ae+5bd)+35c^2\ldots\right.}{2de^4}\right.}{4d}}{6d}$$

$$\Big\downarrow \; 218$$

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{6de^4\left(d+ex^2\right)^3}-\frac{\frac{x\left(-5ae^2-7bde+19cd^2\right)\left(ae^2-bde+cd^2\right)}{4de^4\left(d+ex^2\right)^2}-\frac{3\left(\frac{x\left(e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(11bd-ae)+29c^2d^4\right)}{2de^4\left(d+ex^2\right)}-\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-e^2\left(5a^2e^2+2abde+b^2d^2\right)-2cd^2e(\ldots\right.}{\sqrt{d}\sqrt{e}}}{2de^4}\right.}{4d}}{6d}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) - (((19*c*d^2 - 7*b*
d*e - 5*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(4*d*e^4*(d + e*x^2)^2) - (3*(((
29*c^2*d^4 - 2*c*d^2*e*(11*b*d - a*e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e
^2))*x)/(2*d*e^4*(d + e*x^2)) - (-16*c^2*d^3*x + ((35*c^2*d^4 - 2*c*d^2*e*
(5*b*d + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/
Sqrt[d]])/(Sqrt[d]*Sqrt[e]))/(2*d*e^4)))/(4*d))/(6*d)
```

3.259.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.259.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.14

method	result
default	$\frac{c^2x}{e^4}+\frac{\frac{\frac{e^2\left(5a^2e^4+2abd\,e^3+2ac\,d^2e^2+b^2d^2e^2-22bc\,d^3e+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4+2abd\,e^3-2ac\,d^2e^2-b^2d^2e^2-10bc\,d^3e+17c^2d^4\right)x^3}{6d^2}+\frac{\left(11a^2e^4-2\cdots\right.}{\cdots}}{\left(e\,x^2+d\right)^3}\cdots}{e^4}$
risch	$\frac{c^2x}{e^4}+\frac{\frac{e^2\left(5a^2e^4+2abd\,e^3+2ac\,d^2e^2+b^2d^2e^2-22bc\,d^3e+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4+2abd\,e^3-2ac\,d^2e^2-b^2d^2e^2-10bc\,d^3e+17c^2d^4\right)x^3}{6d^2}+\frac{\left(11a^2e^4-2\cdots\right.}{\cdots}}{e^4\left(e\,x^2+d\right)^3}$

input
```
int((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

output
```
c^2*x/e^4+1/e^4*((1/16*e^2*(5*a^2*e^4+2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^
2-22*b*c*d^3*e+29*c^2*d^4)/d^3*x^5+1/6*e*(5*a^2*e^4+2*a*b*d*e^3-2*a*c*d^2*
e^2-b^2*d^2*e^2-10*b*c*d^3*e+17*c^2*d^4)/d^2*x^3+1/16*(11*a^2*e^4-2*a*b*d*
e^3-2*a*c*d^2*e^2-b^2*d^2*e^2-10*b*c*d^3*e+19*c^2*d^4)/d*x)/(e*x^2+d)^3+1/
16*(5*a^2*e^4+2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2+10*b*c*d^3*e-35*c^2*d^
4)/d^3/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.259.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(234) = 468$.

Time = 0.27 (sec) , antiderivative size = 1016, normalized size of antiderivative = 4.06

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx$$

$$= \left[\frac{96\,c^2d^4e^4x^7+6\,(77\,c^2d^5e^3-22\,bcd^4e^4+2\,abd^2e^6+5\,a^2de^7+(b^2+2\,ac)d^3e^5)x^5+16\,(35\,c^2d^6e^2-10\,bcd\right.$$

input

```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="fricas")
```

output

```
[1/96*(96*c^2*d^4*e^4*x^7 + 6*(77*c^2*d^5*e^3 - 22*b*c*d^4*e^4 + 2*a*b*d^2
*e^6 + 5*a^2*d*e^7 + (b^2 + 2*a*c)*d^3*e^5)*x^5 + 16*(35*c^2*d^6*e^2 - 10*
b*c*d^5*e^3 + 2*a*b*d^3*e^5 + 5*a^2*d^2*e^6 - (b^2 + 2*a*c)*d^4*e^4)*x^3 +
 3*(35*c^2*d^7 - 10*b*c*d^6*e - 2*a*b*d^4*e^3 - 5*a^2*d^3*e^4 - (b^2 + 2*a
*c)*d^5*e^2 + (35*c^2*d^4*e^3 - 10*b*c*d^3*e^4 - 2*a*b*d*e^6 - 5*a^2*e^7 -
 (b^2 + 2*a*c)*d^2*e^5)*x^6 + 3*(35*c^2*d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d
^2*e^5 - 5*a^2*d*e^6 - (b^2 + 2*a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6*e - 10*b
*c*d^5*e^2 - 2*a*b*d^3*e^4 - 5*a^2*d^2*e^5 - (b^2 + 2*a*c)*d^4*e^3)*x^2)*s
qrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^7*e
 - 10*b*c*d^6*e^2 - 2*a*b*d^4*e^4 + 11*a^2*d^3*e^5 - (b^2 + 2*a*c)*d^5*e^3)
*x)/(d^4*e^8*x^6 + 3*d^5*e^7*x^4 + 3*d^6*e^6*x^2 + d^7*e^5), 1/48*(48*c^2*
d^4*e^4*x^7 + 3*(77*c^2*d^5*e^3 - 22*b*c*d^4*e^4 + 2*a*b*d^2*e^6 + 5*a^2*d
*e^7 + (b^2 + 2*a*c)*d^3*e^5)*x^5 + 8*(35*c^2*d^6*e^2 - 10*b*c*d^5*e^3 + 2
*a*b*d^3*e^5 + 5*a^2*d^2*e^6 - (b^2 + 2*a*c)*d^4*e^4)*x^3 - 3*(35*c^2*d^7
 - 10*b*c*d^6*e - 2*a*b*d^4*e^3 - 5*a^2*d^3*e^4 - (b^2 + 2*a*c)*d^5*e^2 + (
35*c^2*d^4*e^3 - 10*b*c*d^3*e^4 - 2*a*b*d*e^6 - 5*a^2*e^7 - (b^2 + 2*a*c)*
d^2*e^5)*x^6 + 3*(35*c^2*d^5*e^2 - 10*b*c*d^4*e^3 - 2*a*b*d^2*e^5 - 5*a^2*
d*e^6 - (b^2 + 2*a*c)*d^3*e^4)*x^4 + 3*(35*c^2*d^6*e - 10*b*c*d^5*e^2 - 2*
a*b*d^3*e^4 - 5*a^2*d^2*e^5 - (b^2 + 2*a*c)*d^4*e^3)*x^2)*sqrt(d*e)*arctan
(sqrt(d*e)*x/d) + 3*(35*c^2*d^7*e - 10*b*c*d^6*e^2 - 2*a*b*d^4*e^4 + 11...
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**4,x)
```

output
```
Timed out
```

3.259.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx$$
$$= \frac{c^2x}{e^4} - \frac{(35\,c^2d^4 - 10\,bcd^3e - b^2d^2e^2 - 2\,acd^2e^2 - 2\,abde^3 - 5\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16\,\sqrt{de}d^3e^4}$$
$$+ \frac{87\,c^2d^4e^2x^5 - 66\,bcd^3e^3x^5 + 3\,b^2d^2e^4x^5 + 6\,acd^2e^4x^5 + 6\,abde^5x^5 + 15\,a^2e^6x^5 + 136\,c^2d^5ex^3 - 80\,bcd^4e^2}{}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="giac")
```

output
```
c^2*x/e^4 - 1/16*(35*c^2*d^4 - 10*b*c*d^3*e - b^2*d^2*e^2 - 2*a*c*d^2*e^2
- 2*a*b*d*e^3 - 5*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^3*e^4) + 1/4
8*(87*c^2*d^4*e^2*x^5 - 66*b*c*d^3*e^3*x^5 + 3*b^2*d^2*e^4*x^5 + 6*a*c*d^2
*e^4*x^5 + 6*a*b*d*e^5*x^5 + 15*a^2*e^6*x^5 + 136*c^2*d^5*e*x^3 - 80*b*c*d
^4*e^2*x^3 - 8*b^2*d^3*e^3*x^3 - 16*a*c*d^3*e^3*x^3 + 16*a*b*d^2*e^4*x^3 +
 40*a^2*d*e^5*x^3 + 57*c^2*d^6*x - 30*b*c*d^5*e*x - 3*b^2*d^4*e^2*x - 6*a*
c*d^4*e^2*x - 6*a*b*d^3*e^3*x + 33*a^2*d^2*e^4*x)/((e*x^2 + d)^3*d^3*e^4)
```

3.259.9 Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.23

$$\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^4} \, dx$$
$$= \frac{\frac{x^5 \left(5\,a^2\,e^6+2\,a\,b\,d\,e^5+2\,a\,c\,d^2\,e^4+b^2\,d^2\,e^4-22\,b\,c\,d^3\,e^3+29\,c^2\,d^4\,e^2\right)}{16\,d^3} - \frac{x\,\left(-11\,a^2\,e^4+2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-19\,c^2\,d^4\right)}{16\,d} + \frac{x^3}{}}{d^3\,e^4+3\,d^2\,e^5\,x^2+3\,d\,e^6\,x^4+e^7\,x^6}$$
$$+ \frac{c^2\,x}{e^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(5\,a^2\,e^4+2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-35\,c^2\,d^4\right)}{16\,d^{7/2}\,e^{9/2}}$$

input
```
int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x)
```

output
```
((x^5*(5*a^2*e^6 + b^2*d^2*e^4 + 29*c^2*d^4*e^2 + 2*a*b*d*e^5 + 2*a*c*d^2*
e^4 - 22*b*c*d^3*e^3))/(16*d^3) - (x*(b^2*d^2*e^2 - 19*c^2*d^4 - 11*a^2*e^
4 + 2*a*b*d*e^3 + 10*b*c*d^3*e + 2*a*c*d^2*e^2))/(16*d) + (x^3*(5*a^2*e^5
+ 17*c^2*d^4*e - b^2*d^2*e^3 + 2*a*b*d*e^4 - 2*a*c*d^2*e^3 - 10*b*c*d^3*e^
2))/(6*d^2))/(d^3*e^4 + e^7*x^6 + 3*d*e^6*x^4 + 3*d^2*e^5*x^2) + (c^2*x)/e
^4 + (atan((e^(1/2)*x)/d^(1/2))*(5*a^2*e^4 - 35*c^2*d^4 + b^2*d^2*e^2 + 2*
a*b*d*e^3 + 10*b*c*d^3*e + 2*a*c*d^2*e^2))/(16*d^(7/2)*e^(9/2))
```

3.260 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$

3.260.1 Optimal result

Integrand size = 24, antiderivative size = 317

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx &= \frac{(cd^2-bde+ae^2)^2 x}{8de^4(d+ex^2)^4} - \frac{(25cd^2-9bde-7ae^2)(cd^2-bde+ae^2)x}{48d^2e^4(d+ex^2)^3} \\ &+ \frac{(163c^2d^4-2cd^2e(59bd-3ae)+e^2(3b^2d^2+10abde+35a^2e^2))x}{192d^3e^4(d+ex^2)^2} \\ &- \frac{(93c^2d^4-2cd^2e(5bd+3ae)-e^2(3b^2d^2+10abde+35a^2e^2))x}{128d^4e^4(d+ex^2)} \\ &+ \frac{(35c^2d^4+2cd^2e(5bd+3ae)+e^2(3b^2d^2+10abde+35a^2e^2))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}}\end{aligned}$$

output
```
1/8*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)^4-1/48*(-7*a*e^2-9*b*d*e+25*c*
d^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/(e*x^2+d)^3+1/192*(163*c^2*d^4-2*c*d^2*
e*(-3*a*e+59*b*d)+e^2*(35*a^2*e^2+10*a*b*d*e+3*b^2*d^2))*x/d^3/e^4/(e*x^2+
d)^2-1/128*(93*c^2*d^4-2*c*d^2*e*(3*a*e+5*b*d)-e^2*(35*a^2*e^2+10*a*b*d*e+
3*b^2*d^2))*x/d^4/e^4/(e*x^2+d)+1/128*(35*c^2*d^4+2*c*d^2*e*(3*a*e+5*b*d)+
e^2*(35*a^2*e^2+10*a*b*d*e+3*b^2*d^2))*arctan(x*e^(1/2)/d^(1/2))/d^(9/2)/e
^(9/2)
```

3.260.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$$
$$= \frac{\frac{48d^{7/2}\sqrt{e}(cd^2+e(-bd+ae))^2x}{(d+ex^2)^4} - \frac{8d^{5/2}\sqrt{e}(25c^2d^4+2cd^2e(-17bd+9ae)+e^2(9b^2d^2-2abde-7a^2e^2))x}{(d+ex^2)^3} + \frac{2d^{3/2}\sqrt{e}(163c^2d^4+2cd^2e(-59bd+3a}{(d+ex}}{}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```

output
```
((48*d^(7/2)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))^2*x)/(d + e*x^2)^4 - (8*d^
(5/2)*Sqrt[e]*(25*c^2*d^4 + 2*c*d^2*e*(-17*b*d + 9*a*e) + e^2*(9*b^2*d^2 -
 2*a*b*d*e - 7*a^2*e^2))*x)/(d + e*x^2)^3 + (2*d^(3/2)*Sqrt[e]*(163*c^2*d^
4 + 2*c*d^2*e*(-59*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2
))*x)/(d + e*x^2)^2 - (3*Sqrt[d]*Sqrt[e]*(93*c^2*d^4 - 2*c*d^2*e*(5*b*d +
3*a*e) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(d + e*x^2) + 3*(35
*c^2*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^
2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(384*d^(9/2)*e^(9/2))
```

3.260.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1471, 2345, 1471, 27, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$$
$$\Big\downarrow \ 1471$$
$$\frac{x(ae^2-bde+cd^2)^2}{8de^4(d+ex^2)^4} - \frac{\int \frac{-\frac{8c^2dx^6}{e}+\frac{8cd(cd-2be)x^4}{e^2}-\frac{8d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{c^2d^4-2ce(bd-ae)d^2+e^2\left(b^2d^2-2abed-7a^2e^2\right)}{e^4}}{(ex^2+d)^4}dx}{8d}$$
$$\Big\downarrow \ 2345$$

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}-\frac{\frac{x\left(-7ae^2-9bde+25cd^2\right)\left(ae^2-bde+cd^2\right)}{6de^4\left(d+ex^2\right)^3}-\frac{\int\frac{\frac{48c^2d^2x^4}{e^2}-\frac{96cd^2(cd-be)x^2}{e^3}+\frac{19c^2d^4-2ce(11bd-3ae)d^2+e^2\left(3b^2d^2+10abed+35a^2e^2\right)}{e^4}}{\left(ex^2+d\right)^3}dx}{6d}}{8d}$$

$\downarrow$ 1471

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}-\frac{\frac{x\left(-7ae^2-9bde+25cd^2\right)\left(ae^2-bde+cd^2\right)}{6de^4\left(d+ex^2\right)^3}-\frac{\frac{x\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(59bd-3ae)+163c^2d^4\right)}{4de^4\left(d+ex^2\right)^2}-\frac{\int\frac{3\left(29c^2d^4-64c^2ex^2d^3-2ce(5bd+3ae)d^2-e^2\left(3\right.\right.}{e^4\left(ex^2+d\right)^2}}{4d}}{6d}}{8d}$$

$\downarrow$ 27

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}-\frac{\frac{x\left(-7ae^2-9bde+25cd^2\right)\left(ae^2-bde+cd^2\right)}{6de^4\left(d+ex^2\right)^3}-\frac{\frac{x\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(59bd-3ae)+163c^2d^4\right)}{4de^4\left(d+ex^2\right)^2}-\frac{3\int\frac{29c^2d^4-64c^2ex^2d^3-2ce(5bd+3ae)d^2-e^2\left(3b\right.}{\left(ex^2+d\right)^2}}{4de^4}}{6d}}{8d}$$

$\downarrow$ 298

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}-\frac{\frac{x\left(-7ae^2-9bde+25cd^2\right)\left(ae^2-bde+cd^2\right)}{6de^4\left(d+ex^2\right)^3}-\frac{\frac{x\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(59bd-3ae)+163c^2d^4\right)}{4de^4\left(d+ex^2\right)^2}-3\left(\frac{x\left(-e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(3ae+\right.}{2d\left(d+ex^2\right)}\right.}{6d}}{8d}$$

$\downarrow$ 218

$$\frac{x\left(ae^2-bde+cd^2\right)^2}{8de^4\left(d+ex^2\right)^4}-\frac{\frac{x\left(-7ae^2-9bde+25cd^2\right)\left(ae^2-bde+cd^2\right)}{6de^4\left(d+ex^2\right)^3}-\frac{\frac{x\left(e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(59bd-3ae)+163c^2d^4\right)}{4de^4\left(d+ex^2\right)^2}-3\left(\frac{x\left(-e^2\left(35a^2e^2+10abde+3b^2d^2\right)-2cd^2e(3ae+\right.}{2d\left(d+ex^2\right)}\right.}{6d}}{8d}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```

3.260. $\int\frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^5}\,dx$

output
```
((c*d^2 - b*d*e + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - (((25*c*d^2 - 9*b*
d*e - 7*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^4*(d + e*x^2)^3) - (((163
*c^2*d^4 - 2*c*d^2*e*(59*b*d - 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a
^2*e^2))*x)/(4*d*e^4*(d + e*x^2)^2) - (3*(((93*c^2*d^4 - 2*c*d^2*e*(5*b*d
+ 3*a*e) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(2*d*(d + e*x^2))
 - ((35*c^2*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e
+ 35*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e])))/(4*d*e^4
))/(6*d))/(8*d)
```

3.260.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.260. $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$

3.260.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.09

method	result
default	$\frac{\frac{\left(35a^{2}e^{4}+10abd\,e^{3}+6ac\,d^{2}e^{2}+3b^{2}d^{2}e^{2}+10bc\,d^{3}e-93c^{2}d^{4}\right)x^{7}}{128d^{4}e}+\frac{\left(385a^{2}e^{4}+110abd\,e^{3}+66ac\,d^{2}e^{2}+33b^{2}d^{2}e^{2}-146bc\,d^{3}e-511c^{2}d^{4}\right)x^{5}}{384d^{3}e^{2}}+\frac{\left(511a^{2}e^{4}+\right.}{}}{\left(e\,x^{2}+d\right)^{4}}$
risch	$\frac{\frac{\left(35a^{2}e^{4}+10abd\,e^{3}+6ac\,d^{2}e^{2}+3b^{2}d^{2}e^{2}+10bc\,d^{3}e-93c^{2}d^{4}\right)x^{7}}{128d^{4}e}+\frac{\left(385a^{2}e^{4}+110abd\,e^{3}+66ac\,d^{2}e^{2}+33b^{2}d^{2}e^{2}-146bc\,d^{3}e-511c^{2}d^{4}\right)x^{5}}{384d^{3}e^{2}}+\frac{\left(511a^{2}e^{4}+\right.}{}}{\left(e\,x^{2}+d\right)^{4}}$

input
```
int((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x,method=_RETURNVERBOSE)
```

output
```
(1/128*(35*a^2*e^4+10*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2+10*b*c*d^3*e-9
3*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+110*a*b*d*e^3+66*a*c*d^2*e^2+33*b^
2*d^2*e^2-146*b*c*d^3*e-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4+146*a*
b*d*e^3-66*a*c*d^2*e^2-33*b^2*d^2*e^2-110*b*c*d^3*e-385*c^2*d^4)/d^2/e^3*x
^3+1/128*(93*a^2*e^4-10*a*b*d*e^3-6*a*c*d^2*e^2-3*b^2*d^2*e^2-10*b*c*d^3*e
-35*c^2*d^4)/e^4/d*x)/(e*x^2+d)^4+1/128*(35*a^2*e^4+10*a*b*d*e^3+6*a*c*d^2
*e^2+3*b^2*d^2*e^2+10*b*c*d^3*e+35*c^2*d^4)/d^4/e^4/(e*d)^(1/2)*arctan(e*x
/(e*d)^(1/2))
```

3.260.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(299) = 598$.

Time = 0.29 (sec) , antiderivative size = 1266, normalized size of antiderivative = 3.99

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\left(d+ex^2\right)^5}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="fricas")
```

output
```
[-1/768*(6*(93*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e^
8 - 3*(b^2 + 2*a*c)*d^3*e^6)*x^7 + 2*(511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 -
110*a*b*d^3*e^6 - 385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + 2*(385
*c^2*d^7*e^2 + 110*b*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b
^2 + 2*a*c)*d^5*e^4)*x^3 + 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 +
 35*a^2*d^4*e^4 + 3*(b^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e
^5 + 10*a*b*d*e^7 + 35*a^2*e^8 + 3*(b^2 + 2*a*c)*d^2*e^6)*x^8 + 4*(35*c^2*
d^5*e^3 + 10*b*c*d^4*e^4 + 10*a*b*d^2*e^6 + 35*a^2*d*e^7 + 3*(b^2 + 2*a*c)
*d^3*e^5)*x^6 + 6*(35*c^2*d^6*e^2 + 10*b*c*d^5*e^3 + 10*a*b*d^3*e^5 + 35*a
^2*d^2*e^6 + 3*(b^2 + 2*a*c)*d^4*e^4)*x^4 + 4*(35*c^2*d^7*e + 10*b*c*d^6*e
^2 + 10*a*b*d^4*e^4 + 35*a^2*d^3*e^5 + 3*(b^2 + 2*a*c)*d^5*e^3)*x^2)*sqrt(
-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(35*c^2*d^8*e + 10
*b*c*d^7*e^2 + 10*a*b*d^5*e^4 - 93*a^2*d^4*e^5 + 3*(b^2 + 2*a*c)*d^6*e^3)*
x)/(d^5*e^9*x^8 + 4*d^6*e^8*x^6 + 6*d^7*e^7*x^4 + 4*d^8*e^6*x^2 + d^9*e^5)
, -1/384*(3*(93*c^2*d^5*e^4 - 10*b*c*d^4*e^5 - 10*a*b*d^2*e^7 - 35*a^2*d*e
^8 - 3*(b^2 + 2*a*c)*d^3*e^6)*x^7 + (511*c^2*d^6*e^3 + 146*b*c*d^5*e^4 - 1
10*a*b*d^3*e^6 - 385*a^2*d^2*e^7 - 33*(b^2 + 2*a*c)*d^4*e^5)*x^5 + (385*c^
2*d^7*e^2 + 110*b*c*d^6*e^3 - 146*a*b*d^4*e^5 - 511*a^2*d^3*e^6 + 33*(b^2
+ 2*a*c)*d^5*e^4)*x^3 - 3*(35*c^2*d^8 + 10*b*c*d^7*e + 10*a*b*d^5*e^3 + 35
*a^2*d^4*e^4 + 3*(b^2 + 2*a*c)*d^6*e^2 + (35*c^2*d^4*e^4 + 10*b*c*d^3*e...
```

3.260.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^5} \, dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**5,x)
```

output
```
Timed out
```

3.260.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$$
$$= \frac{(35\,c^2d^4+10\,bcd^3e+3\,b^2d^2e^2+6\,acd^2e^2+10\,abde^3+35\,a^2e^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{128\,\sqrt{de}d^4e^4}$$
$$- \frac{279\,c^2d^4e^3x^7-30\,bcd^3e^4x^7-9\,b^2d^2e^5x^7-18\,acd^2e^5x^7-30\,abde^6x^7-105\,a^2e^7x^7+511\,c^2d^5e^2x^5+146}{}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="giac")
```

output
```
1/128*(35*c^2*d^4 + 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 10*a*b*
d*e^3 + 35*a^2*e^4)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^4*e^4) - 1/384*(279
*c^2*d^4*e^3*x^7 - 30*b*c*d^3*e^4*x^7 - 9*b^2*d^2*e^5*x^7 - 18*a*c*d^2*e^5
*x^7 - 30*a*b*d*e^6*x^7 - 105*a^2*e^7*x^7 + 511*c^2*d^5*e^2*x^5 + 146*b*c*
d^4*e^3*x^5 - 33*b^2*d^3*e^4*x^5 - 66*a*c*d^3*e^4*x^5 - 110*a*b*d^2*e^5*x^
5 - 385*a^2*d*e^6*x^5 + 385*c^2*d^6*e*x^3 + 110*b*c*d^5*e^2*x^3 + 33*b^2*d
^4*e^3*x^3 + 66*a*c*d^4*e^3*x^3 - 146*a*b*d^3*e^4*x^3 - 511*a^2*d^2*e^5*x^
3 + 105*c^2*d^7*x + 30*b*c*d^6*e*x + 9*b^2*d^5*e^2*x + 18*a*c*d^5*e^2*x +
30*a*b*d^4*e^3*x - 279*a^2*d^3*e^4*x)/((e*x^2 + d)^4*d^4*e^4)
```

3.260.9 Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(35\,a^2\,e^4+10\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e+35\,c^2\,d^4)}{128\,d^{9/2}\,e^{9/2}}$$
$$-\frac{\frac{x\,(-93\,a^2\,e^4+10\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e+35\,c^2\,d^4)}{128\,d\,e^4}-\frac{x^7\,(35\,a^2\,e^4+10\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-93\,c^2\,d^4)}{128\,d^4\,e}}{d^4+4\,d^3\,e\,x^2+}$$

input
```
int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x)
```

output
```
(atan((e^(1/2)*x)/d^(1/2))*(35*a^2*e^4 + 35*c^2*d^4 + 3*b^2*d^2*e^2 + 10*a
*b*d*e^3 + 10*b*c*d^3*e + 6*a*c*d^2*e^2))/(128*d^(9/2)*e^(9/2)) - ((x*(35*
c^2*d^4 - 93*a^2*e^4 + 3*b^2*d^2*e^2 + 10*a*b*d*e^3 + 10*b*c*d^3*e + 6*a*c
*d^2*e^2))/(128*d*e^4) - (x^7*(35*a^2*e^4 - 93*c^2*d^4 + 3*b^2*d^2*e^2 + 1
0*a*b*d*e^3 + 10*b*c*d^3*e + 6*a*c*d^2*e^2))/(128*d^4*e) + (x^3*(385*c^2*d
^4 - 511*a^2*e^4 + 33*b^2*d^2*e^2 - 146*a*b*d*e^3 + 110*b*c*d^3*e + 66*a*c
*d^2*e^2))/(384*d^2*e^3) - (x^5*(385*a^2*e^4 - 511*c^2*d^4 + 33*b^2*d^2*e^
2 + 110*a*b*d*e^3 - 146*b*c*d^3*e + 66*a*c*d^2*e^2))/(384*d^3*e^2))/(d^4 +
 e^4*x^8 + 4*d^3*e*x^2 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4)
```

3.261 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$

3.261.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx = \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

output
```
c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*a
rctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx = \frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)x}{2de^2(d+ex^2)} - \frac{(3cd^2-bde-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

output
```
(c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2
- b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

3.261.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1471, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{1471}$$

$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{\int \frac{cd^2-2cex^2d-e(bd+ae)}{e^2(ex^2+d)}dx}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{\int \frac{cd^2-2cex^2d-e(bd+ae)}{ex^2+d}dx}{2de^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{(3cd^2-e(ae+bd))\int \frac{1}{ex^2+d}dx-2cdx}{2de^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3cd^2-e(ae+bd))}{\sqrt{d}\sqrt{e}}-2cdx}{2de^2}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - (-2*c*d*x + ((3*c*d^2
- e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]))/(2*d*e^2)
```

3.261.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.261.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
default	$\frac{cx}{e^2}+\frac{\frac{\left(a\,e^2-bde+c\,d^2\right)x}{2d\left(e\,x^2+d\right)}+\frac{\left(a\,e^2+bde-3c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^2}$
risch	$\frac{cx}{e^2}+\frac{(a\,e^2-bde+c\,d^2)x}{2d\,e^2(e\,x^2+d)}-\frac{\ln(ex+\sqrt{-ed})a}{4\sqrt{-ed}\,d}-\frac{\ln(ex+\sqrt{-ed})b}{4e\sqrt{-ed}}+\frac{3d\ln(ex+\sqrt{-ed})c}{4e^2\sqrt{-ed}}+\frac{\ln(-ex+\sqrt{-ed})a}{4\sqrt{-ed}\,d}+\frac{\ln(-ex+\sqrt{-ed})}{4e\sqrt{-ed}}$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*
d^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx$$
$$= \left[\frac{4\,cd^2e^2x^3 + (3\,cd^3 - bd^2e - ade^2 + (3\,cd^2e - bde^2 - ae^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right) + 2\,(3\,cd^3e - bd^2e}{4\,(d^2e^4x^2+d^3e^3)} \right.$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^
2 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) +
 2*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*
d^2*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)
*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*
x)/(d^2*e^4*x^2 + d^3*e^3)]
```

3.261.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(75) = 150.

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.84

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} + \frac{x(ae^2-bde+cd^2)}{2d^2e^2+2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

output
```
c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqr
t(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3
*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2
*e**2*sqrt(-1/(d**3*e**5)) + x)/4
```

3.261.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} - \frac{(3\,cd^2-bde-ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^2} + \frac{cd^2x-bdex+ae^2x}{2\,(ex^2+d)de^2}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
c*x/e^2 - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d
*e^2) + 1/2*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*d*e^2)
```

3.261.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{c\,x}{e^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2+b\,d\,e+a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

output
```
(c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/
2)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.262 $\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx$

3.262.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx=\frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)}-\frac{(3cd^2-e(bd+ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

output
```
c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*a
rctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx=\frac{cx}{e^2}+\frac{(cd^2-bde+ae^2)\,x}{2de^2(d+ex^2)}-\frac{(3cd^2-bde-ae^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

input
```
Integrate[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
```

output
```
(c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2
- b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

3.262.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2087, 1471, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}dx$$
$$\Big\downarrow \text{ 2087}$$
$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}dx$$
$$\Big\downarrow \text{ 1471}$$
$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{\int \frac{cd^2-2cex^2d-e(bd+ae)}{e^2(ex^2+d)}dx}{2d}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{\int \frac{cd^2-2cex^2d-e(bd+ae)}{ex^2+d}dx}{2de^2}$$
$$\Big\downarrow \text{ 299}$$
$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{(3cd^2-e(ae+bd))\int \frac{1}{ex^2+d}dx-2cdx}{2de^2}$$
$$\Big\downarrow \text{ 218}$$
$$\frac{x(ae^2-bde+cd^2)}{2de^2(d+ex^2)}-\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(3cd^2-e(ae+bd))}{\sqrt{d}\sqrt{e}}-2cdx}{2de^2}$$

input
```
Int[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - (-2*c*d*x + ((3*c*d^2
- e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]))/(2*d*e^2)
```

3.262.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2087
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

3.262.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
default	$\frac{cx}{e^{2}}+\frac{\frac{\left(a\,e^{2}-bde+c\,d^{2}\right)x}{2d\left(e\,x^{2}+d\right)}+\frac{\left(a\,e^{2}+bde-3c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\sqrt{ed}}}{e^{2}}$
risch	$\frac{cx}{e^{2}}+\frac{(a\,e^{2}-bde+c\,d^{2})x}{2d\,e^{2}(e\,x^{2}+d)}-\frac{\ln(ex+\sqrt{-ed})a}{4\sqrt{-ed}\,d}-\frac{\ln(ex+\sqrt{-ed})b}{4e\sqrt{-ed}}+\frac{3d\ln(ex+\sqrt{-ed})c}{4e^{2}\sqrt{-ed}}+\frac{\ln(-ex+\sqrt{-ed})a}{4\sqrt{-ed}\,d}+\frac{\ln(-ex+\sqrt{-ed})}{4e\sqrt{-ed}}$

input
```
int((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*
d^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx$$
$$= \left[\frac{4\,cd^2e^2x^3 + (3\,cd^3 - bd^2e - ade^2 + (3\,cd^2e - bde^2 - ae^3)x^2)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right) + 2\,(3\,cd^3e - bd^2e}{4\,(d^2e^4x^2 + d^3e^3)}\right.$$

input
```
integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^
2 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) +
 2*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*
d^2*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)
*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*
x)/(d^2*e^4*x^2 + d^3*e^3)]
```

3.262.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(75) = 150$.

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.84

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx = \frac{cx}{e^2} + \frac{x(ae^2-bde+cd^2)}{2d^2e^2+2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}(ae^2+bde-3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}}+x\right)}{4}$$

input
```
integrate((a+x**2*(c*x**2+b))/(e*x**2+d)**2,x)
```

output
```
c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqr
t(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3
*e**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2
*e**2*sqrt(-1/(d**3*e**5)) + x)/4
```

3.262.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + x^2(b + cx^2)}{(d + ex^2)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{a + x^2(b + cx^2)}{(d + ex^2)^2} \, dx = \frac{cx}{e^2} - \frac{(3\,cd^2 - bde - ae^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^2} + \frac{cd^2x - bdex + ae^2x}{2\,(ex^2 + d)de^2}$$

input
```
integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
c*x/e^2 - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d
*e^2) + 1/2*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^2 + d)*d*e^2)
```

3.262.9 Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx = \frac{c\,x}{e^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2+b\,d\,e+a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{2\,d\,\left(e^3\,x^2+d\,e^2\right)}$$

input
```
int((a + x^2*(b + c*x^2))/(d + e*x^2)^2,x)
```

output
```
(c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/
2)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.263 $\int \frac{\left(d+ex^2\right)^4}{a+bx^2+cx^4}\,dx$

3.263.1 Optimal result

Integrand size = 24, antiderivative size = 459

$$\int \frac{\left(d+ex^2\right)^4}{a+bx^2+cx^4}\,dx = \frac{e^2\left(6c^2d^2+b^2e^2-ce(4bd+ae)\right)x}{c^3}+\frac{e^3(4cd-be)x^3}{3c^2}+\frac{e^4x^5}{5c}$$
$$+\frac{\left(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)+\frac{2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2d^2+6abde+a^2e^2\right)}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)-\frac{2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2d^2+6abde+a^2e^2\right)}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{7/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
e^2*(6*c^2*d^2+b^2*e^2-c*e*(a*e+4*b*d))*x/c^3+1/3*e^3*(-b*e+4*c*d)*x^3/c^2
+1/5*e^4*x^5/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*
(e*(-b*e+2*c*d)*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))+(2*c^4*d^4+b^4*e^4-4*b
^2*c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*
b^2*d^2))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e*(-b*e+2*c*d
)*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))+(-2*c^4*d^4-b^4*e^4+4*b^2*c*e^3*(a*e
+b*d)+4*c^3*d^2*e*(3*a*e+b*d)-2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*d^2))/(-4
*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))\,x}{c^3} + \frac{e^3(4cd-be)x^3}{3c^2} + \frac{e^4x^5}{5c}$$
$$+ \frac{\left(2c^4d^4+b^3\left(b-\sqrt{b^2-4ac}\right)e^4+4c^3d^2e\left(-bd+\sqrt{b^2-4ac}d-3ae\right)+2bce^3\left(-2b^2d+2b\sqrt{b^2-4ac}d-2\cdots\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-\cdots}}$$
$$- \frac{\left(2c^4d^4+b^3\left(b+\sqrt{b^2-4ac}\right)e^4-4c^3d^2e\left(bd+\sqrt{b^2-4ac}d+3ae\right)-2bce^3\left(2b^2d+a\sqrt{b^2-4ac}e+2b\left(\sqrt{\cdots}\right.\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4\cdots}}$$

input
```
Integrate[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

output
```
(e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)
*x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((2*c^4*d^4 + b^3*(b - Sqrt[b^2 - 4*a*c]
)*e^4 + 4*c^3*d^2*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*b*c*e^3*(-2
*b^2*d + 2*b*Sqrt[b^2 - 4*a*c]*d - 2*a*b*e + a*Sqrt[b^2 - 4*a*c]*e) + 2*c^
2*e^2*(3*b^2*d^2 - 3*b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + a*e*(-2*Sqrt[b^2
- 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c
^4*d^4 + b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 4*c^3*d^2*e*(b*d + Sqrt[b^2 - 4
*a*c]*d + 3*a*e) - 2*b*c*e^3*(2*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 2*b*(Sqrt[
b^2 - 4*a*c]*d + a*e)) + 2*c^2*e^2*(3*b^2*d^2 + a*e*(2*Sqrt[b^2 - 4*a*c]*d
 + a*e) + 3*b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

3.263.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 1484}$$

$$\int \left(\frac{e^2\left(-ce(ae+4bd)+b^2e^2+6c^2d^2\right)}{c^3}+\frac{ex^2(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)-ab^2e^4+ace^3(ae+4bd}{c^3\left(a+bx^2+cx^4\right)}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}+e(2cd-be)\left(-2ce(ae+bd)\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)-\frac{2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3a}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

$$\frac{e^2x\left(-ce(ae+4bd)+b^2e^2+6c^2d^2\right)}{c^3}+\frac{e^3x^3(4cd-be)}{3c^2}+\frac{e^4x^5}{5c}$$

input
```
Int[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

output
```
(e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)
*x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 -
 2*c*e*(b*d + a*e)) + (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c
^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e)*(2*c^2*d^2
 + b^2*e^2 - 2*c*e*(b*d + a*e)) - (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d
+ a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^
2*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.263.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.263. $\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx$

3.263.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.49

method	result
risch	$\frac{e^4x^5}{5c} - \frac{e^4b\,x^3}{3c^2} + \frac{4d\,e^3x^3}{3c} - \frac{e^4ax}{c^2} + \frac{e^4b^2x}{c^3} - \frac{4e^3bdx}{c^2} + \frac{6e^2d^2x}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)} \frac{\left(e\left(2abc\,e^3-4a\,c^2d\,e^2-b^3e^3 \cdots\right.\right.}{\cdots}}{\cdots}$
default	$-\frac{e^2\left(-\frac{1}{5}e^2x^5c^2+\frac{1}{3}bc\,e^2x^3-\frac{4}{3}c^2de\,x^3+e^2acx-b^2e^2x+4bcdex-6c^2d^2x\right)}{c^3} + \frac{\left(2abc\,e^4\sqrt{-4ac+b^2}-4a\,c^2d\,e^3\sqrt{-4ac+b^2}-b^3e^4\sqrt{-4ac+b^2}+ \cdots\right.}{\cdots}$

input
```
int((e*x^2+d)^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*e^4*x^5/c-1/3*e^4/c^2*b*x^3+4/3*d*e^3*x^3/c-e^4/c^2*a*x+e^4/c^3*b^2*x-
4*e^3/c^2*b*d*x+6*e^2/c*d^2*x+1/2/c^3*sum((e*(2*a*b*c*e^3-4*a*c^2*d*e^2-b^
3*e^3+4*b^2*c*d*e^2-6*b*c^2*d^2*e+4*c^3*d^3)*_R^2+a^2*c*e^4-a*b^2*e^4+4*a*
b*c*d*e^3-6*a*c^2*d^2*e^2+c^3*d^4)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4
*c+_Z^2*b+a))
```

3.263.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16218 vs. 2(421) = 842.

Time = 270.80 (sec) , antiderivative size = 16218, normalized size of antiderivative = 35.33

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.263.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**4/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.263.7 Maxima [F]

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/15*(3*c^2*e^4*x^5 + 5*(4*c^2*d*e^3 - b*c*e^4)*x^3 + 15*(6*c^2*d^2*e^2 -
4*b*c*d*e^3 + (b^2 - a*c)*e^4)*x)/c^3 + integrate((c^3*d^4 - 6*a*c^2*d^2*e
^2 + 4*a*b*c*d*e^3 - (a*b^2 - a^2*c)*e^4 + (4*c^3*d^3*e - 6*b*c^2*d^2*e^2
+ 4*(b^2*c - a*c^2)*d*e^3 - (b^3 - 2*a*b*c)*e^4)*x^2)/(c*x^4 + b*x^2 + a),
 x)/c^3
```

3.263.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9306 vs. $2(421) = 842$.

Time = 1.30 (sec) , antiderivative size = 9306, normalized size of antiderivative = 20.27

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/8*(4*(2*b^4*c^5 - 16*a*b^2*c^6 + 32*a^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*c^6 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6)*c^2*d^3*
e - 6*(2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)
*c^2*d^2*e^2 + 4*(2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 9*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 24*sqrt(2)...
```

3.263.9 Mupad [B] (verification not implemented)

Time = 10.92 (sec) , antiderivative size = 29551, normalized size of antiderivative = 64.38

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^4/(a + b*x^2 + c*x^4),x)
```

output
```
x*((b*((b*e^4)/c^2 - (4*d*e^3)/c))/c - (a*e^4)/c^2 + (6*d^2*e^2)/c) - x^3*
((b*e^4)/(3*c^2) - (4*d*e^3)/(3*c)) + atan(((((16*a*c^8*d^4 + 16*a^3*c^6*e
^4 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2
*e^2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5
 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4
*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e
^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c
^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a
^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6
*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*
a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 -
1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-
(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*
a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*
d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e
^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 +
 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 3
04*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*
c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^
3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3...
```

3.263. $\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx$

3.264 $\int \frac{(d+ex^2)^3}{a+bx^2+cx^4} dx$

3.264.1 Optimal result

Integrand size = 24, antiderivative size = 316

$$\begin{aligned}&\int \frac{(d+ex^2)^3}{a+bx^2+cx^4} dx\\&=\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}\\&+\frac{\left(e(3c^2d^2+b^2e^2-ce(3bd+ae))+\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\\&+\frac{\left(e(3c^2d^2+b^2e^2-ce(3bd+ae))-\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
e^2*(-b*e+3*c*d)*x/c^2+1/3*e^3*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*(e*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))+(-b*e+2*c*d)*
(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(
-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2))*(e*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))-(-b*e+2*c*d)*(c^2*d^2+b^2
*e^2-c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

3.264.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx$$
$$= \frac{6\sqrt{c}e^2(3cd-be)x+2c^{3/2}e^3x^3+\frac{3\sqrt{2}\left(2c^3d^3+b^2\left(-b+\sqrt{b^2-4ac}\right)e^3+3c^2de\left(-bd+\sqrt{b^2-4ac}d-2ae\right)+ce^2\left(3b^2d-3b\sqrt{b^2-4ac}d+3abe\cdots\right.\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{\cdots}$$

input
```
Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

output
```
(6*Sqrt[c]*e^2*(3*c*d - b*e)*x + 2*c^(3/2)*e^3*x^3 + (3*Sqrt[2]*(2*c^3*d^3
 + b^2*(-b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(-(b*d) + Sqrt[b^2 - 4*a*c
]*d - 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e - a*Sqrt
[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(-2*c^3*d^3
+ b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d +
 2*a*e) - c*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*
d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

3.264.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1484}$$

$$\int \left(\frac{ex^2\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)+abe^3-3acde^2+c^2d^3}{c^2\left(a+bx^2+cx^4\right)}+\frac{e^2(3cd-be)}{c^2}+\frac{e^3x^2}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e(-ce(ae+3bd)+b^2e^2+3c^2d^2)+\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e(-ce(ae+3bd)+b^2e^2+3c^2d^2)-\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{e^2x(3cd-be)}{c^2}+\frac{e^3x^3}{3c}$$

input
```
Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

output
```
(e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^3)/(3*c) + ((e*(3*c^2*d^2 + b^2*e^2 - c
*e*(3*b*d + a*e)) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(3*c^2*d^2 + b^2*e
^2 - c*e*(3*b*d + a*e)) - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3
*a*e)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.264.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.264.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.41

method	result
risch	$\frac{e^3x^3}{3c}-\frac{e^3bx}{c^2}+\frac{3d\,e^2x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(e\left(-e^2ac+b^2e^2-3bcde+3c^2d^2\right)\_R^2+e^3ab-3acd\,e^2+c^2d^3\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c^2}$
default	$-\frac{e^2\left(-\frac{1}{3}c\,x^3e+bex-3cdx\right)}{c^2}+\frac{\frac{\left(-e^3ac\sqrt{-4ac+b^2}+b^2e^3\sqrt{-4ac+b^2}-3d\,e^2bc\sqrt{-4ac+b^2}+3d^2e\,c^2\sqrt{-4ac+b^2}-3abc\,e^3+6a\,c^2d\,e^2+b^3e^3-3b^2cd}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{}$

input
```
int((e*x^2+d)^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*e^3*x^3/c-e^3/c^2*b*x+3*d*e^2*x/c+1/2/c^2*sum((e*(-a*c*e^2+b^2*e^2-3*b
*c*d*e+3*c^2*d^2)*_R^2+e^3*a*b-3*a*c*d*e^2+c^2*d^3)/(2*_R^3*c+_R*b)*ln(x-_
R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.264.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9584 vs. 2(280) = 560.

Time = 44.92 (sec) , antiderivative size = 9584, normalized size of antiderivative = 30.33

$$\int \frac{\left(d+ex^2\right)^3}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.264.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**3/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.264.7 Maxima [F]

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^3}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/3*(c*e^3*x^3 + 3*(3*c*d*e^2 - b*e^3)*x)/c^2 - integrate(-(c^2*d^3 - 3*a*
c*d*e^2 + a*b*e^3 + (3*c^2*d^2*e - 3*b*c*d*e^2 + (b^2 - a*c)*e^3)*x^2)/(c*
x^4 + b*x^2 + a), x)/c^2
```

3.264.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6418 vs. $2(280) = 560$.

Time = 1.14 (sec) , antiderivative size = 6418, normalized size of antiderivative = 20.31

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/8*(3*(2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d^2*
e - 3*(2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c
^2*d*e^2 + (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - ...
```

3.264.9 Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 17954, normalized size of antiderivative = 56.82

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^3/(a + b*x^2 + c*x^4),x)
```

output
```
atan(((((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3
 - 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c
^6)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*
e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2
*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c
- b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 +
120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(
4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^
2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d
^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e
^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2
0*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c -
 b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c
*d*e^5*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^
6)))^(1/2))/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(
1/2) + a*b^4*e^6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3
*e^6 + 48*a^2*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*
e^6*(-(4*a*c - b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b
^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^
3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e...
```

3.264. $\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx$

3.265 $\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx$

3.265.1 Optimal result

Integrand size = 24, antiderivative size = 238

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \frac{e^2x}{c} + \frac{\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
e^2*x/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e*(-b*
e+2*c*d)+(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(3/2)*2
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*(e*(-b*e+2*c*d)+(-2*c^2*d^2-b^2*e^2+2*c*e*(a*e+b*d))/
(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.265.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \frac{2\sqrt{c}e^2x + \frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{\right.\right.}}{\sqrt{b^2-4ac}\sqrt{b}}}{2c^{3/2}}$$

input
```
Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```

output
```
(2*Sqrt[c]*e^2*x + (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2
*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (
Sqrt[2]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2
 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

3.265.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1484}$$

$$\int \left(\frac{-ae^2+ex^2(2cd-be)+cd^2}{c(a+bx^2+cx^4)}+\frac{e^2}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{e^2x}{c}$$

input
```
Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```

output
```
(e^2*x)/c + ((e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e) - (2*c
^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

3.265.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.265.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.32

method	result
risch	$\frac{e^2 x}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(e(-be+2cd)\_R^2-a e^2+c d^2\right) \ln\left(x-\_R\right)}{2c\_R^3+\_R b}}{2c}$
default	$\frac{e^2 x}{c} + \frac{\left(-b e^2\sqrt{-4ac+b^2}+2dce\sqrt{-4ac+b^2}+2e^2ac-b^2e^2+2bcde-2c^2d^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{\left(-b e^2\sqrt{-4ac+b^2}+\right.}{}$

input
```
int((e*x^2+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
e^2*x/c+1/2/c*sum((e*(-b*e+2*c*d)*_R^2-a*e^2+c*d^2)/(2*_R^3*c+_R*b)*ln(x-_
R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.265.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4690 vs. $2(204) = 408$.

Time = 4.22 (sec) , antiderivative size = 4690, normalized size of antiderivative = 19.71

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(2*e^2*x - sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^
2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c
^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*
a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2
- 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*
b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))
*log(2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)
*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)
*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*
c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^6 -
 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 -
 (a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3*b*
c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 - ((a*b^3*c^4 - 4*a^2
*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^4
)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3
*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c
^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a
^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e
+ 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*
e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*...
```

3.265.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.265.7 Maxima [F]

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \int \frac{(ex^2+d)^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
e^2*x/c - integrate(-(c*d^2 - a*e^2 + (2*c*d*e - b*e^2)*x^2)/(c*x^4 + b*x^
2 + a), x)/c
```

3.265.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4110 vs. $2(204) = 408$.

Time = 0.91 (sec) , antiderivative size = 4110, normalized size of antiderivative = 17.27

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
e^2*x/c + 1/8*(2*(2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*
c^2*d*e - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2
*e^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c^4 - 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sq...
```

3.265.9 Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 9600, normalized size of antiderivative = 40.34

$$\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^2/(a + b*x^2 + c*x^4),x)
```

output
```
atan((((((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/
c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(
4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e
^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^
3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3
*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6
*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*e
^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c -
 b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c -
 b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a
*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*
e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*
a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b
^2*c^4)))^(1/2) - (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2
*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 +
 12*a*b*c^2*d*e^3))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2
)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3
*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^
3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a...
```

3.265. $\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx$

3.266 $\int \frac{d+ex^2}{a+bx^2+cx^4} \, dx$

3.266.1 Optimal result

Integrand size = 22, antiderivative size = 174

$$\int \frac{d+ex^2}{a+bx^2+cx^4} \, dx = \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*d)
/(-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arct
an(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a*c+
b^2)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.266.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\int \frac{d+ex^2}{a+bx^2+cx^4} \, dx = \frac{\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
```

output
```
(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b
^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/S
qrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

3.266.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
```

output
```
((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e
- (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

3.266.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.266.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.26

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(\_R^2 e+d\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$
default	$4c\left(\frac{\left(e\sqrt{-4ac+b^2}+be-2cd\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\sqrt{2}\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$

input
```
int((e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2*e+d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.266.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1525 vs. $2(140) = 280$.

Time = 0.36 (sec) , antiderivative size = 1525, normalized size of antiderivative = 8.76

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)
*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b
^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x +
sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4
*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2
 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e
^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b
^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(b*c*d
^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*
(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x - sqrt(1/2)*((b^2*c - 4*a*c^
2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2
*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 -
 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)
*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b
^2*c - 4*a^2*c^2))) + 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 -
 (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c
^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(c^2*d^4 - b*c*d^3*e + a*b
*d*e^3 - a^2*e^4)*x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)
*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt...
```

3.266.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.266.7 Maxima [F]

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx = \int \frac{ex^2+d}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

3.266.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. 2(140) = 280.

Time = 0.82 (sec) , antiderivative size = 1402, normalized size of antiderivative = 8.06

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^
2 - 4*a*c)*b*c^2)*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arctan(2*sqrt(1/2)*x/sqrt
((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^
2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt...
```

3.266.9 Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 4109, normalized size of antiderivative = 23.61

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)/(a + b*x^2 + c*x^4),x)
```

output
```
- atan((((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^
3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*
a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c
^2 + a*b^4*c)))^(1/2) - 4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b
*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3
- 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*
e^2 - 4*b*c^2*d*e))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*
d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*
a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(
1/2)*1i + ((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 -
4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2
*c^2 + a*b^4*c)))^(1/2) + 4*b^2*c^2*d - 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(
-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^
3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*
c*e^2 - 4*b*c^2*d*e))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*
c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 1
6*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c...
```

3.267 $\int \frac{1}{a+bx^2+cx^4} \, dx$

3.267.1 Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{a+bx^2+cx^4} \, dx = \frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4
*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*2^(1/2)*c^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

3.267.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+bx^2+cx^4} \, dx = \frac{\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/
Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

3.267.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1406, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 1406}$$

$$\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]])
```

3.267.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

3.267.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$	38
default	$4c\left(-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$	117

input
```
int(1/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.267.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(114) = 228.

Time = 0.28 (sec) , antiderivative size = 613, normalized size of antiderivative = 4.09

$$\int \frac{1}{a+bx^2+cx^4}\,dx = -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx + \sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx - \sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx + \sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx - \sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

3.267. $\int \frac{1}{a+bx^2+cx^4}\,dx$

output
```
-1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^
2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqr
t(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)
)/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^
2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c -
(a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/
sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a
*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sq
rt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-
(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*
sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4
*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2
*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*
b^2 - 4*a^2*c)))
```

3.267.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

$$\int \frac{1}{a+bx^2+cx^4}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3}{c}\right.\right.\right.$$

input
```
integrate(1/(c*x**4+b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a
*b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b
**3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

3.267.7 Maxima [F]

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \int \frac{1}{cx^4+bx^2+a}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^4 + b*x^2 + a), x)
```

3.267.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. 2(114) = 228.

Time = 0.58 (sec) , antiderivative size = 1026, normalized size of antiderivative = 6.84

$$\int \frac{1}{a+bx^2+cx^4}\,dx$$
$$= \frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^3c-2\,b^4c+16\,\sqrt{2}\sqrt{\cdots}\right.}{\cdots}$$
$$+\frac{\left(\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^4-8\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}ab^2c-2\,\sqrt{2}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^3c+2\,b^4c+16\,\sqrt{2}\cdots\right.}{\cdots}$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2
 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*
a^2*c^3)*abs(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2...
```

3.267.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.09

$$\int \frac{1}{a+bx^2+cx^4}\,dx =$$

$$-\operatorname{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right.$$

$$-\operatorname{atan}\left(\frac{b^4\,x\,1\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,x}{4\,a\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-}\right.$$

input
```
int(1/(a + b*x^2 + c*x^4),x)
```

output
```
- atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 -
64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))
^(1/2) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(
b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8
*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*
b^2*c*x*8i)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^
3*c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a
*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b
^2*c))^(1/2)*2i
```

3.268 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.268.1 Optimal result

Integrand size = 24, antiderivative size = 254

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}$$

output
```
e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/d^(1/2)-1/2*arctan(x
*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*
a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1
/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e
+2*c*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

3.268.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{\sqrt{c}\left(-2cd+be+\sqrt{b^2-4ace}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{\sqrt{c}\left(2cd-be+\sqrt{b^2-4ace}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}$$

input
```
Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2
- 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 -
 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2
)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2
))
```

3.268.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \; 1484$$

$$\int \left(\frac{e^2}{(d+ex^2)(ae^2-bde+cd^2)} + \frac{-be+cd-cex^2}{(a+bx^2+cx^4)(ae^2-bde+cd^2)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

input
```
Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^
2 - b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqr
t[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

3.268.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.268.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.85

method	result
default	$\frac{4c\left(\frac{\left(be-2cd-e\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-e\sqrt{-4ac+b^2}-be+2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a\,e^2-bde+c\,d^2}+\frac{e^2\operatorname{arct}}{(a\,e^2-bd}$
risch	Expression too large to display

input
```
int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/(a*e^2-b*d*e+c*d^2)*c*(1/8*(b*e-2*c*d-e*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+e^2/(a*e^2-b*d*e+c*d^2)/(e*d)^(1/2)*arct
an(e*x/(e*d)^(1/2))
```

3.268.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7995 vs. $2(210) = 420$.

Time = 14.27 (sec) , antiderivative size = 16013, normalized size of antiderivative = 63.04

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.268.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.268.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.268.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7664 vs. 2(210) = 420.

Time = 1.78 (sec) , antiderivative size = 7664, normalized size of antiderivative = 30.17

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
e^2*arctan(e*x/sqrt(d*e))/((c*d^2 - b*d*e + a*e^2)*sqrt(d*e)) + 1/8*(2*(2*
b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^5 -
2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e + 4
*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^3*e^2 - (
2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +...
```

3.268.9 Mupad [B] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 23640, normalized size of antiderivative = 93.07

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

output

```
atan((((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d
^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 1
2*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16
*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^
2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5
*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 +
16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 +
16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4
- 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3
*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-
(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*
b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c
^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*
d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2
*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e
- 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^
3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 + b
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12...
```

3.268. $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

3.269 $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)}\,dx$

3.269.1 Optimal result

Integrand size = 24, antiderivative size = 429

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)}\,dx &= \frac{e^2x}{2d(cd^2-bde+ae^2)(d+ex^2)} \\ &+ \frac{\sqrt{c}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} \\ &- \frac{\sqrt{c}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} \\ &+ \frac{e^{3/2}(2cd-be)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)^2} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(cd^2-bde+ae^2)}\end{aligned}$$

output
```
1/2*e^2*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)+1/2*e^(3/2)*arctan(x*e^(1/2)/d^(
1/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)+e^(3/2)*(-b*e+2*c*d)*arctan(x*e^(1/2)/d^
(1/2))/(a*e^2-b*d*e+c*d^2)^2/d^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-2*c*
e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(-4*a*c+b^
2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c
*e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(-4*a*c+b
^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.269.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)}\,dx$$
$$= \frac{\frac{e^2(cd^2+e(-bd+ae))x}{d(d+ex^2)} + \frac{\sqrt{2}\sqrt{c}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-2c^2d^2+b\left(-b+\sqrt{b\right.\right.}{}}{2(cd^2+e(-bd+ae))^2}$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

output
```
((e^2*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^2)) + (Sqrt[2]*Sqrt[c]*(2*
c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d
 + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-2*c^2*d^2 + b
*(-b + Sqrt[b^2 - 4*a*c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]
*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (e^(3/2)*(5*c*d^2 + e*(-3*b*d + a*e))*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2))/(2*(c*d^2 + e*(-(b*d) + a*e))^2)
```

3.269.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \text{1484}$$
$$\int \left(\frac{-ce(ae+2bd)+b^2e^2-cex^2(2cd-be)+c^2d^2}{(a+bx^2+cx^4)(ae^2-bde+cd^2)^2} - \frac{e^2(be-2cd)}{(d+ex^2)(ae^2-bde+cd^2)^2} + \frac{e^2}{(d+ex^2)^2(ae^2-bde+cd^2)}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)^2}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)^2}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(2cd-be)}{\sqrt{d}\,(ae^2-bde+cd^2)^2}+\frac{e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\,(ae^2-bde+cd^2)}+\frac{e^2x}{2d\,(d+ex^2)\,(ae^2-bde+cd^2)}$$

input
```
Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

output
```
(e^2*x)/(2*d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + (Sqrt[c]*(2*c^2*d^2 +
b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2
- 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (Sqrt[c]
*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*
c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)
^2) + (e^(3/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2
- b*d*e + a*e^2)^2) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*
d^2 - b*d*e + a*e^2))
```

3.269.3.1 Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.269.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.80

method	result
default	$\frac{4c\left(\frac{\left(b\,e^{2}\sqrt{-4ac+b^{2}}-2dce\sqrt{-4ac+b^{2}}+2e^{2}ac-b^{2}e^{2}+2bcde-2c^{2}d^{2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\left(b\,e^{2}\sqrt{-4ac+b^{2}}-2dce\sqrt{-4ac+b^{2}}-2e^{2}a\cdots}{8\sqrt{-4a\cdots}}\right.}{\left(a\,e^{2}-bde+c\,d^{2}\right)^{2}}$
risch	Expression too large to display

input
```
int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/(a*e^2-b*d*e+c*d^2)^2*c*(1/8*(b*e^2*(-4*a*c+b^2)^(1/2)-2*d*c*e*(-4*a*c+b
^2)^(1/2)+2*e^2*a*c-b^2*e^2+2*b*c*d*e-2*c^2*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))-1/8*(b*e^2*(-4*a*c+b^2)^(1/2)-2*d*c*e*(-4*a*c+b^2)^(1/2)-2*e
^2*a*c+b^2*e^2-2*b*c*d*e+2*c^2*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)))+e^2/(a*e^2-b*d*e+c*d^2)^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*
(a*e^2-3*b*d*e+5*c*d^2)/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))
```

3.269.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.269.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e x^2\right)^2\left(a+b x^2+c x^4\right)} dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.269.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(d+e x^2\right)^2\left(a+b x^2+c x^4\right)} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.269.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2394 vs. $2(366) = 732$.

Time = 1.83 (sec) , antiderivative size = 2394, normalized size of antiderivative = 5.58

$$\int \frac{1}{\left(d+e x^2\right)^2\left(a+b x^2+c x^4\right)} dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*e^2*x/((c*d^3 - b*d^2*e + a*d*e^2)*(e*x^2 + d)) + 1/2*(sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 2*b^4*c
^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^2*c^4 + 16*a*b^2*c^4 + 2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*c^5 - 32*a^2*c^5 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b*c^4 + 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4 - 2*(b^2
 - 4*a*c)*b*c^4)*arctan(2*sqrt(1/2)*x/sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e + b^
3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^4 + sqrt((b*c^2*d^4
- 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^
4)^2 - 4*(a*c^2*d^4 - 2*a*b*c*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*
a^2*b*d*e^3 + a^3*e^4)*(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*
d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^
2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/(2*(a*b^4*c^2 - 8*a
^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^
5)*d^2*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3...
```

3.269.9 Mupad [B] (verification not implemented)

Time = 11.41 (sec) , antiderivative size = 91169, normalized size of antiderivative = 212.52

$$\int \frac{1}{(d+ex^2)^2 (a+bx^2+cx^4)} \, dx = \text{Too large to display}$$

input
```
int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x)
```

output
```
(atan(((((x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^
8*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 -
48*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*
d*e^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^
2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2
+ 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^
4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^
6*e^4 - 12*a^2*b*c*d^5*e^5)) - (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 -
 8*a^5*c^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*
d^6*e^7 + 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 -
 240*b^3*c^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6
*c^4*d^4*e^9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^
4*e^9 + 672*a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*
c^5*d^2*e^11 + 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e
^12 - 1984*a*b^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4
*e^9 + 160*a*b^5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*
e^8 + 40*a^2*b^5*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^
12)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d
^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d
^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b...
```

3.269. $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)}\,dx$

3.270 $\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx$

3.270.1 Optimal result

Integrand size = 24, antiderivative size = 563

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \frac{x\left(c\left(b^2d^3-2ad(cd^2-3ae^2)-\frac{abe(3cd^2+ae^2)}{c}\right)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2))\,x^2\right)}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$-\frac{\left(ab^3e^3+6ac\left(2cd+\sqrt{b^2-4ac}\,e\right)(cd^2+ae^2)-b^2\left(c^2d^3-3acde^2+a\sqrt{b^2-4ac}\,e^3\right)-bc\left(ae^2\left(3\sqrt{b^2-4a\right.\right.\right.}{2\sqrt{2}ac^{3/2}\,(b^2-4ac)^{3/2}\,\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\left(ab^3e^3+6ac\left(2cd-\sqrt{b^2-4ac}\,e\right)(cd^2+ae^2)-b^2\left(c^2d^3-3acde^2-a\sqrt{b^2-4ac}\,e^3\right)+bc\left(cd^2\left(\sqrt{b^2-4ac\right.\right.\right.}{2\sqrt{2}ac^{3/2}\,(b^2-4ac)^{3/2}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x*(c*(b^2*d^3-2*a*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2)/c)-(a*b^2*e
^3+2*a*c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*x^2)/a/c/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*
(a*b^3*e^3+6*a*c*(a*e^2+c*d^2)*(2*c*d+e*(-4*a*c+b^2)^(1/2))-b^2*(c^2*d^3-3
*a*c*d*e^2+a*e^3*(-4*a*c+b^2)^(1/2))-b*c*(c*d^2*(12*a*e+d*(-4*a*c+b^2)^(1/
2))+a*e^2*(8*a*e+3*d*(-4*a*c+b^2)^(1/2))))/a/c^(3/2)/(-4*a*c+b^2)^(3/2)*2^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2))*(a*b^3*e^3+6*a*c*(a*e^2+c*d^2)*(2*c*d-e*(-4*a*c+b^2)^(
1/2))-b^2*(c^2*d^3-3*a*c*d*e^2-a*e^3*(-4*a*c+b^2)^(1/2))+b*c*(c*d^2*(-12*a
*e+d*(-4*a*c+b^2)^(1/2))+a*e^2*(-8*a*e+3*d*(-4*a*c+b^2)^(1/2))))/a/c^(3/2)
/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.270.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2\sqrt{c}x\left(b^2(cd^3-ae^3x^2)+b(-a^2e^3+c^2d^3x^2-3acde(d-ex^2))+2ac(ae^2(3d+ex^2)-cd^2(d+3ex^2))\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(-ab^3e^3-6ac\left(2cd+\sqrt{b^2-4ac}e\right)(cd^2+a\ldots}{\ldots}}{\ldots}$$

input
```
Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*Sqrt[c]*x*(b^2*(c*d^3 - a*e^3*x^2) + b*(-(a^2*e^3) + c^2*d^3*x^2 - 3*a
*c*d*e*(d - e*x^2)) + 2*a*c*(a*e^2*(3*d + e*x^2) - c*d^2*(d + 3*e*x^2))))/
((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*e^3) - 6*a*c*(2*c*
d + Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*
Sqrt[b^2 - 4*a*c]*e^3) + b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^
2*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*(a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^
2*(-(c^2*d^3) + 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[
b^2 - 4*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]))/(4*a*c^(3/2))
```

3.270.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 521, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1517, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 1517$$

$$\frac{x\left(c\left(-\frac{abe(ae^2+3cd^2)}{c}-2ad(cd^2-3ae^2)+b^2d^3\right)-x^2\left(ab^2e^3-bcd(3ae^2+cd^2)+2ace(3cd^2-ae^2)\right)\right)}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{\int -\frac{b^2d^3-6a(cd^2+ae^2)d+\left(\frac{ab^2e^3}{c}-6a(cd^2+ae^2)e+b(cd^3+3ae^2d)\right)x^2+\frac{abe\left(3cd^2+ae^2\right)}{c}}{cx^4+bx^2+a}dx}{2a\,(b^2-4ac)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b^2d^3-6a(cd^2+ae^2)d+\left(\frac{ab^2e^3}{c}-6a(cd^2+ae^2)e+b(cd^3+3ae^2d)\right)x^2+\frac{abe\left(3cd^2+ae^2\right)}{c}}{cx^4+bx^2+a}dx}{2a\,(b^2-4ac)}+\frac{x\left(c\left(-\frac{abe(ae^2+3cd^2)}{c}-2ad(cd^2-3ae^2)+b^2d^3\right)-x^2\left(ab^2e^3-bcd(3ae^2+cd^2)+2ace(3cd^2-ae^2)\right)\right)}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{1}{2}\left(\frac{ab^2e^3}{c}-\frac{ab^3e^3-b^2cd(cd^2-3ae^2)-4abce(2ae^2+3cd^2)+12ac^2d(ae^2+cd^2)}{c\sqrt{b^2-4ac}}+b(3ade^2+cd^3)-6ae(ae^2+cd^2)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-\right.}}$$

$$\frac{x\left(c\left(-\frac{abe(ae^2+3cd^2)}{c}-2ad(cd^2-3ae^2)+b^2d^3\right)-x^2\left(ab^2e^3-bcd(3ae^2+cd^2)+2ace(3cd^2-ae^2)\right)\right)}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2e^3}{c}-\frac{ab^3e^3-b^2cd\left(cd^2-3ae^2\right)-4abce\left(2ae^2+3cd^2\right)+12ac^2d\left(ae^2+cd^2\right)}{c\sqrt{b^2-4ac}}+b(3ade^2+cd^3)-6ae(ae^2+cd^2)\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{}}{\sqrt{\sqrt{b^2}}}\right.}{}}{2a\,(b^2-4ac)}$$

$$\frac{x\left(c\left(-\frac{abe(ae^2+3cd^2)}{c}-2ad(cd^2-3ae^2)+b^2d^3\right)-x^2\left(ab^2e^3-bcd(3ae^2+cd^2)+2ace(3cd^2-ae^2)\right)\right)}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

input
```
Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

3.270. $\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx$

output
```
(x*(c*(b^2*d^3 - 2*a*d*(c*d^2 - 3*a*e^2) - (a*b*e*(3*c*d^2 + a*e^2))/c) -
(a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^2))/(2
*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((((a*b^2*e^3)/c - 6*a*e*(c*d^2
+ a*e^2) + b*(c*d^3 + 3*a*d*e^2) - (a*b^3*e^3 - b^2*c*d*(c*d^2 - 3*a*e^2)
+ 12*a*c^2*d*(c*d^2 + a*e^2) - 4*a*b*c*e*(3*c*d^2 + 2*a*e^2))/(c*Sqrt[b^2
- 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (((a*b^2*e^3)/c - 6*a*e*(c*d^2 +
 a*e^2) + b*(c*d^3 + 3*a*d*e^2) + (a*b^3*e^3 - b^2*c*d*(c*d^2 - 3*a*e^2) +
 12*a*c^2*d*(c*d^2 + a*e^2) - 4*a*b*c*e*(3*c*d^2 + 2*a*e^2))/(c*Sqrt[b^2 -
 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

3.270.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.270. $\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2} dx$

3.270.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.57

method	result
risch	$\frac{-\frac{\left(2a^{2}c\,e^{3}-a\,b^{2}e^{3}+3abcd\,e^{2}-6a\,c^{2}d^{2}e+b\,c^{2}d^{3}\right)x^{3}}{2ac\left(4ac-b^{2}\right)}+\frac{\left(a^{2}b\,e^{3}-6a^{2}cd\,e^{2}+3abc\,d^{2}e+2a\,c^{2}d^{3}-b^{2}c\,d^{3}\right)x}{2c\left(4ac-b^{2}\right)a}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\left(\frac{(6a}{}\right.}{}$
default	$\frac{-\frac{\left(2a^{2}c\,e^{3}-a\,b^{2}e^{3}+3abcd\,e^{2}-6a\,c^{2}d^{2}e+b\,c^{2}d^{3}\right)x^{3}}{2ac\left(4ac-b^{2}\right)}+\frac{\left(a^{2}b\,e^{3}-6a^{2}cd\,e^{2}+3abc\,d^{2}e+2a\,c^{2}d^{3}-b^{2}c\,d^{3}\right)x}{2c\left(4ac-b^{2}\right)a}}{c\,x^{4}+b\,x^{2}+a}+\frac{\frac{\left(6a^{2}c\,e^{3}\sqrt{-4ac+b^{2}}-a\,b^{2}e^{3}\sqrt{-4ac+b^{2}}\right.}{}}{}$

input
```
int((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(2*a^2*c*e^3-a*b^2*e^3+3*a*b*c*d*e^2-6*a*c^2*d^2*e+b*c^2*d^3)/a/c/(4
*a*c-b^2)*x^3+1/2/c*(a^2*b*e^3-6*a^2*c*d*e^2+3*a*b*c*d^2*e+2*a*c^2*d^3-b^2
*c*d^3)/(4*a*c-b^2)/a*x)/(c*x^4+b*x^2+a)+1/4/a/c*sum(((6*a^2*c*e^3-a*b^2*e
^3-3*a*b*c*d*e^2+6*a*c^2*d^2*e-b*c^2*d^3)/(4*a*c-b^2)*_R^2-(a^2*b*e^3-6*a^
2*c*d*e^2+3*a*b*c*d^2*e-6*a*c^2*d^3+b^2*c*d^3)/(4*a*c-b^2))/(2*_R^3*c+_R*b
)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.270.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12117 vs. 2(507) = 1014.

Time = 107.28 (sec) , antiderivative size = 12117, normalized size of antiderivative = 21.52

$$\int \frac{\left(d+ex^2\right)^3}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.270.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.270.7 Maxima [F]

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(ex^2+d)^3}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*x
^3 - (3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3)*x
)/(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*
b*c^2)*x^2) - 1/2*integrate(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + a^2*b*e^3 +
(b^2*c - 6*a*c^2)*d^3 + (b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 + (a*b^
2 - 6*a^2*c)*e^3)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a^2*c^2)
```

3.270.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8992 vs. $2(507) = 1014$.

Time = 1.77 (sec) , antiderivative size = 8992, normalized size of antiderivative = 15.97

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c^2*d^3*x^3 - 6*a*c^2*d^2*e*x^3 + 3*a*b*c*d*e^2*x^3 - a*b^2*e^3*x^3
 + 2*a^2*c*e^3*x^3 + b^2*c*d^3*x - 2*a*c^2*d^3*x - 3*a*b*c*d^2*e*x + 6*a^2
*c*d*e^2*x - a^2*b*e^3*x)/((c*x^4 + b*x^2 + a)*(a*b^2*c - 4*a^2*c^2)) + 1/
16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^3 - 6*(2*a*b^2*c^4
 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^
2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^2*e + 3*(2*a*b^3*c^3 - 8*a^2*b
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 -
4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^2*d*e^2 + (2*a*b^4*c^2 - 20*a^2*b^2*
c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)...
```

3.270.9 Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 29030, normalized size of antiderivative = 51.56

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x)
```

output
```
- ((x^3*(b*c^2*d^3 - a*b^2*e^3 + 2*a^2*c*e^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e
^2))/(2*a*c*(4*a*c - b^2)) - (x*(2*a*c^2*d^3 + a^2*b*e^3 - b^2*c*d^3 - 6*a
^2*c*d*e^2 + 3*a*b*c*d^2*e))/(2*a*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) -
atan(((((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*a
^6*c^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c
^6*d^3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 -
3072*a^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a
^3*b^6*c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*
a^5*b^2*c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^
2*c^3)) - (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b
*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*
a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e
- 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a
^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^
6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b
^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7
*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b
^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824
*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4
+ 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*...
```

3.270. $\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx$

3.271 $\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx$

3.271.1 Optimal result

Integrand size = 24, antiderivative size = 386

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx = \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\left(bcd^2-4acde+abe^2+\frac{8abcde+b^2(cd^2-ae^2)-4ac(3cd^2+ae^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(bcd^2-4acde+abe^2-\frac{8abcde+b^2(cd^2-ae^2)-4ac(3cd^2+ae^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+c*d^2)+(a*b*e^2-4*a*c*d*e+b*c*d^2)*x^
2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2))*(b*c*d^2-4*a*c*d*e+a*b*e^2+(8*a*b*c*d*e+b^2*(-a*e^2+c*d
^2)-4*a*c*(a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*(b*c*d^2-4*a*c*d*e+a*b*e^2+(-8*a*b*c*d*e-b^2*(-a*e^2+c*d
^2)+4*a*c*(a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.271.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2x(b^2d^2+2a^2e^2+bcd^2x^2+abe(-2d+ex^2)-2acd(d+2ex^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(b^2(cd^2-ae^2)-4ac\left(3cd^2+e\left(\sqrt{b^2-4ac}d+ae\right)\right)+b\left(a\sqrt{b^2-4ac}e^2+cd\left(\sqrt{b^2-4\cdots}\right)\right)\right)\cdots}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\cdots}{\cdots}$$

input
```
Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*x*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^2 + a*b*e*(-2*d + e*x^2) - 2*a*c*d*
(d + 2*e*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d^2
 - a*e^2) - 4*a*c*(3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b*(a*Sqrt[b^
2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-(c*d^2) + a*e^2) + b*(a*Sqrt[b^2 - 4
*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e)) + 4*a*c*(3*c*d^2 + e*(-(Sqr
t[b^2 - 4*a*c]*d) + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

3.271.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1517, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 1517$$

$$\frac{x\left(x^2\left(abe^2-4acde+bcd^2\right)-2abde-2a\left(cd^2-ae^2\right)+b^2d^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{b^2d^2+2abed+\left(bcd^2-4aced+abe^2\right)x^2-2a\left(3cd^2+ae^2\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{b^2d^2+2abed+(bcd^2-4aced+abe^2)x^2-2a(3cd^2+ae^2)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x(x^2(abe^2-4acde+bcd^2)-2abde-2a(cd^2-ae^2)+b^2d^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$\Big\downarrow$ 1480

$$\frac{\frac{1}{2}\left(\frac{b^2(cd^2-ae^2)+8abcde-4ac(ae^2+3cd^2)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(-\frac{b^2(cd^2-ae^2)+8abcde-4ac(ae^2-}{\sqrt{b^2-4ac}}\right.}{2a(b^2-4ac)}$$
$$\frac{x(x^2(abe^2-4acde+bcd^2)-2abde-2a(cd^2-ae^2)+b^2d^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$\Big\downarrow$ 218

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd^2-ae^2)+8abcde-4ac(ae^2+3cd^2)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd^2-ae^2)+8abcde-4ac(ae^2}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2a(b^2-4ac)}$$
$$\frac{x(x^2(abe^2-4acde+bcd^2)-2abde-2a(cd^2-ae^2)+b^2d^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

input
```
Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b
*e^2)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (((b*c*d^2 - 4*a*c*d
*e + a*b*e^2 + (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a*e^2
))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d^2 - 4*a*c*d*e
 + a*b*e^2 - (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a*e^2))
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

3.271. $\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx$

3.271.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.271.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.58

method	result
risch	$\frac{-\frac{(ab\,e^2-4acde+bc\,d^2)x^3}{2a(4ac-b^2)}-\frac{(2e^2a^2-2abde-2d^2ac+b^2d^2)x}{2a(4ac-b^2)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{(ab\,e^2-4acde+bc\,d^2)\_R^2}{4ac-b^2}+\frac{2e^2a^2-2ab\cdots}{4a\cdots}\right.}{2c\_R^3+\_R\cdots}}{4a}$
default	$\frac{-\frac{(ab\,e^2-4acde+bc\,d^2)x^3}{2a(4ac-b^2)}-\frac{(2e^2a^2-2abde-2d^2ac+b^2d^2)x}{2a(4ac-b^2)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{(-ab\,e^2\sqrt{-4ac+b^2}+4acde\sqrt{-4ac+b^2}-bc\,d^2\sqrt{-4ac+b^2}-4a^2c\,e^2-a\,b^2e^2+\cdots}{8c\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-\cdots}}}\right.}{\cdots}$

input
```
int((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a*(a*b*e^2-4*a*c*d*e+b*c*d^2)/(4*a*c-b^2)*x^3-1/2*(2*a^2*e^2-2*a*b*d
*e-2*a*c*d^2+b^2*d^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/a*sum((-(a*b*e^
2-4*a*c*d*e+b*c*d^2)/(4*a*c-b^2)*_R^2+(2*a^2*e^2-2*a*b*d*e+6*a*c*d^2-b^2*d
^2)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.271.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7338 vs. $2(344) = 688$.

Time = 12.70 (sec) , antiderivative size = 7338, normalized size of antiderivative = 19.01

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.271.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.271.7 Maxima [F]

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(ex^2+d)^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^3 - (2*a*b*d*e - 2*a^2*e^2 - (b^2 -
 2*a*c)*d^2)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 -
4*a^2*b*c)*x^2) + 1/2*integrate((2*a*b*d*e - 2*a^2*e^2 + (b^2 - 6*a*c)*d^2
 + (b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4
*a^2*c)
```

3.271.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6390 vs. $2(344) = 688$.

Time = 1.38 (sec) , antiderivative size = 6390, normalized size of antiderivative = 16.55

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*d^2*x^3 - 4*a*c*d*e*x^3 + a*b*e^2*x^3 + b^2*d^2*x - 2*a*c*d^2*x -
 2*a*b*d*e*x + 2*a^2*e^2*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16
*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3
 - 2*(b^2 - 4*a*c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d^2 - 4*(2*a*b^2*c^3 - 8*a^2
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)
*a*c^3)*(a*b^2 - 4*a^2*c)^2*d*e + (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 -
 4*a^2*c)^2*e^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 14*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 2*a*b^6*c^2 + 64*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*...
```

3.271.9 Mupad [B] (verification not implemented)

Time = 11.08 (sec) , antiderivative size = 18785, normalized size of antiderivative = 48.67

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x)
```

output
```
atan(((((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*
b^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2
*e^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 3
2*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^
6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11*c*d^4 + a^3*
b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b
*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*
c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 +
 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^
5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d
^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*
c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3
*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*
e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*
a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^
9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4
+ 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^
7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a
^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2)
 - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^...
```

3.271. $\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx$

3.272 $\int \frac{d+ex^2}{(a+bx^2+cx^4)^2}\,dx$

3.272.1 Optimal result

Integrand size = 22, antiderivative size = 293

$$\int \frac{d+ex^2}{(a+bx^2+cx^4)^2}\,dx = \frac{x(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(bd-2ae+\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(bd-2ae-\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2
+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*
d-2*a*e+(4*a*b*e-12*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2))*c^(1/2)*(b*d-2*a*e+(-4*a*b*e+12*a*c*d-b^2*d)/(-4*a*c+b^2)^
(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.272.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.06

$$\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{\frac{2x\left(b^2d+b\left(-ae+cdx^2\right)-2ac\left(d+ex^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}d+4ae\right)-2a\left(6cd+\sqrt{b^2-4ac}e\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2d+12\right.}{}}{4a}$$

input
```
Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*x*(b^2*d + b*(-(a*e) + c*d*x^2) - 2*a*c*(d + e*x^2)))/((b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 - 4*a*c]*d + 4
*a*e) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - 4*a*b*
e - 2*a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

3.272.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \text{ 1492}$$

$$\frac{x\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{db^2+aeb+c(bd-2ae)x^2-6acd}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{db^2+aeb+c(bd-2ae)x^2-6acd}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{1}{2}c\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + ((Sqrt[c]*(b*d - 2*a*e + (b^2*d - 12*a*c*d + 4*a*b*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*e - (b^2*d -
 12*a*c*d + 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 -
4*a*c))
```

3.272.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.272.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\frac{c(2ae-bd)x^3}{2a\left(4ac-b^2\right)}+\frac{\left(abe+2acd-b^2d\right)x}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{c(2ae-bd)\_R^2}{4ac-b^2}-\frac{abe-6acd+b^2d}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{4a}$
default	$16c^2\left(\frac{\frac{\left(-d\sqrt{-4ac+b^2}+2ae-bd\right)\sqrt{-4ac+b^2}\,x}{16ac\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}+\frac{\left(12\sqrt{-4ac+b^2}\,acd-3\sqrt{-4ac+b^2}\,b^2d-8a^2ce-6a\,b^2e+28abcd-3b^3d\right)\left(\sqrt{-4ac+b^2}-2b\right)\sqrt{2}\,\text{arcta}}{16a\left(4ac+3b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{4c(4ac-b^2)\sqrt{-4ac+b^2}}\right.$

input
```
int((e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*c*(2*a*e-b*d)/a/(4*a*c-b^2)*x^3+1/2*(a*b*e+2*a*c*d-b^2*d)/a/(4*a*c-b^
2)*x)/(c*x^4+b*x^2+a)+1/4/a*sum((c*(2*a*e-b*d)/(4*a*c-b^2)*_R^2-(a*b*e-6*a
*c*d+b^2*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+
a))
```

3.272.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4573 vs. $2(251) = 502$.

Time = 1.98 (sec) , antiderivative size = 4573, normalized size of antiderivative = 15.61

$$\int \frac{d+ex^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(2*(b*c*d - 2*a*c*e)*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2
*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3
*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4
*a^3*b*d*e^3 + a^4*e^4 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 -
9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2
- 64*a^6*c^3))*log(-((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4 - (3*b^5
*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*e - 3*(3*a*b^4*c - 28*a^2*b^2*c^2)*
d^2*e^2 - (9*a^2*b^3*c - 20*a^3*b*c^2)*d*e^3 - (3*a^3*b^2*c + 4*a^4*c^2)*e
^4)*x + 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c
^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112*a^4
*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3)*d
*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 - ((a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*a
^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + a^4*e^4
 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(
a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)))*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2
*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a...
```

3.272.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.272.7 Maxima [F]

$$\int \frac{d+ex^2}{(a+bx^2+cx^4)^2}\,dx = \int \frac{ex^2+d}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b*c*d - 2*a*c*e)*x^3 - (a*b*e - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^
2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - 1/2*integrate(
-(a*b*e + (b*c*d - 2*a*c*e)*x^2 + (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)
/(a*b^2 - 4*a^2*c)
```

3.272.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4425 vs. 2(251) = 502.

Time = 1.32 (sec) , antiderivative size = 4425, normalized size of antiderivative = 15.10

$$\int \frac{d+ex^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*d*x^3 - 2*a*c*e*x^3 + b^2*d*x - 2*a*c*d*x - a*b*e*x)/((c*x^4 + b*
x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*
d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c
^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*e + 2*(sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c +
64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*
a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)...
```

3.272.9 Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 12350, normalized size of antiderivative = 42.15

$$\int \frac{d + ex^2}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((d + e*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
atan(((((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^
4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 1
6*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2)) - (x*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2
)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a
^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 +
 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^
9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b
^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*
e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*
a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*
a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 -
768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 +
a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9
)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*
b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2
*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)
^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a
^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280...
```

3.272. $\int \frac{d+ex^2}{(a+bx^2+cx^4)^2}\,dx$

3.273 $\int \frac{1}{(a+bx^2+cx^4)^2} dx$

3.273.1 Optimal result

Integrand size = 14, antiderivative size = 252

$$\int \frac{1}{(a+bx^2+cx^4)^2} dx = \frac{x(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1
/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^
2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*ar
ctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b
*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2)
```

3.273.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{2x(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

output
```
((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

3.273.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1405, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{1}{2}c\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-2),x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((Sq
rt[c]*(b + (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
c]*(b - (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2
- 4*a*c))
```

3.273.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.273.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{bc\,x^3}{2a\left(4ac-b^2\right)}+\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{bc\_R^2}{4ac-b^2}+\frac{6ac-b^2}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{4a}$
default	$16c^2\left(-\frac{-\frac{\left(-b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16a\,c^2\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}-\frac{\left(b^2-12ac+b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{16ac\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{4\sqrt{-4ac+b^2}\,(4ac-b^2)}-\frac{\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16a\,c^2\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}\right.$

input
```
int(1/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a*b*c/(4*a*c-b^2)*x^3+1/2*(2*a*c-b^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+
a)+1/4/a*sum((-b*c/(4*a*c-b^2)*_R^2+(6*a*c-b^2)/(4*a*c-b^2))/(2*_R^3*c+_R*
b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

3.273.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. 2(206) = 412.

Time = 0.41 (sec) , antiderivative size = 2309, normalized size of antiderivative = 9.16

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
+ (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b
^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c +
81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*
b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b
^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^
2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*
c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2
)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15
*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a
^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*
b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 -
12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^
2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 6
72*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 -
 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/...
```

3.273.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.273.7 Maxima [F]

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \int \frac{1}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a
^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c
*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

3.273.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. 2(206) = 412.

Time = 0.61 (sec) , antiderivative size = 2682, normalized size of antiderivative = 10.64

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) -
1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 -
 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*
b*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2
 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 - 2*(sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 6...
```

3.273.9 Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 6404, normalized size of antiderivative = 25.41

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3
 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 384
0*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a
*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24
*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 -
 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2
*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^
5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^
6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^
4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 +
 b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*
c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5
)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^...
```

3.273. $\int \frac{1}{(a+bx^2+cx^4)^2}\,dx$

3.274 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx$

3.274.1 Optimal result

Integrand size = 24, antiderivative size = 660

$$\begin{aligned}\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx &= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)x^2)}{2a(b^2-4ac)(cd^2-bde+ae^2)(a+bx^2+cx^4)} \\
&- \frac{\sqrt{c}e^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} \\
&+ \frac{\sqrt{c}\left(bcd-b^2e+2ace+\frac{b^2cd-12ac^2d-b^3e+8abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} \\
&- \frac{\sqrt{c}e^2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)^2} \\
&+ \frac{\sqrt{c}\left(bcd-b^2e+2ace-\frac{b^2cd-12ac^2d-b^3e+8abce}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} \\
&+ \frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)^2}\end{aligned}$$

output
```
1/2*x*(b^2*c*d-2*a*c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^2)/a/(-
4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)+e^(7/2)*arctan(x*e^(1/2)/d^
(1/2))/(a*e^2-b*d*e+c*d^2)^2/d^(1/2)-1/2*e^2*arctan(x*2^(1/2)*c^(1/2)/(b-(
-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(a*e^
2-b*d*e+c*d^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)
*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*d-b^2*e+2*a*c*e+(8*a*b
*c*e-12*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/(a*e^2-b
*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*e^2*arctan(x*2^(1/2)*
c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e+2*c*d)/(-4*a*c+b^2)
^(1/2))/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arc
tan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*d-b^2*e+2
*a*c*e+(-8*a*b*c*e+12*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c
+b^2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.274.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.07

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2(cd^2+e(-bd+ae))x(b^3e-bc(3ae+cdx^2)+2ac^2(d-ex^2)+b^2c(-d+ex^2))}{a(-b^2+4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^4de^2+2ac\left(-6c^2d^3+5a\sqrt{b^2-4ac}e^3+cde\left(\sqrt{b^2-4ac}d-14ae\right.\right.\right.}{}}{}$$

input
```
Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((2*(c*d^2 + e*(-(b*d) + a*e))*x*(b^3*e - b*c*(3*a*e + c*d*x^2) + 2*a*c^2*
(d - e*x^2) + b^2*c*(-d + e*x^2)))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4))
+ (Sqrt[2]*Sqrt[c]*(b^4*d*e^2 + 2*a*c*(-6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c]*
e^3 + c*d*e*(Sqrt[b^2 - 4*a*c]*d - 14*a*e)) + b^3*e*(-2*c*d^2 + e*(Sqrt[b^
2 - 4*a*c]*d - 3*a*e)) + b^2*(c^2*d^3 - 3*a*Sqrt[b^2 - 4*a*c]*e^3 - c*d*e*
(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + b*c*(a*e^2*(-(Sqrt[b^2 - 4*a*c]*d) + 16
*a*e) + c*d^2*(Sqrt[b^2 - 4*a*c]*d + 20*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^4*d*e^2) - b^2*(c^2*d^3 + 3*a*Sqrt[b^2 - 4
*a*c]*e^3 + c*d*e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e)) + b^3*e*(2*c*d^2 + e*(S
qrt[b^2 - 4*a*c]*d + 3*a*e)) + 2*a*c*(6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c]*e^
3 + c*d*e*(Sqrt[b^2 - 4*a*c]*d + 14*a*e)) + b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*
d - 20*a*e) - a*e^2*(Sqrt[b^2 - 4*a*c]*d + 16*a*e)))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]) + (4*e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d])/(4*(c*d^2
 + e*(-(b*d) + a*e))^2)
```

3.274.3 Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{1567}$$

$$\int \left(-\frac{e^2(be-cd+cex^2)}{(a+bx^2+cx^4)(ae^2-bde+cd^2)^2}+\frac{-be+cd-cex^2}{(a+bx^2+cx^4)^2(ae^2-bde+cd^2)}+\frac{e^4}{(d+ex^2)(ae^2-bde+cd^2)^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c}e^2\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)^2}-\frac{\sqrt{c}e^2\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)^2}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{8abce-12ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)}{2\sqrt{2}a\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{8abce-12ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)}{2\sqrt{2}a\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)^2}+\frac{x\left(cx^2\left(2ace+b^2(-e)+bcd\right)+3abce-2ac^2d+b^3(-e)+b^2cd\right)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)\,(ae^2-bde+cd^2)}$$

input
```
Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*
x^2))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x^2 + c*x^4)) - (S
qrt[c]*e^2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^
2 - b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e + (b^2*c*d - 12*
a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*e^2*(e + (2*c*d - b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt[c]*
(b*c*d - b^2*e + 2*a*c*e - (b^2*c*d - 12*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^
2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^
2)^2)
```

3.274.3.1 Defintions of rubi rules used

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.274.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\frac{c\left(2a^{2}c\,e^{3}-a\,b^{2}e^{3}-abcd\,e^{2}+2a\,c^{2}d^{2}e+b^{3}d\,e^{2}-2b^{2}c\,d^{2}e+b\,c^{2}d^{3}\right)x^{3}}{2a\left(4ac-b^{2}\right)}+\frac{\left(3a^{2}b\,e^{3}c-2a^{2}c^{2}d\,e^{2}-a\,b^{3}e^{3}-2a\,b^{2}cd\,e^{2}+5ab\,c^{2}d^{2}e-2a\,c^{3}d^{3}+b^{4}d\,e^{2}-2}{2a\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}$
risch	Expression too large to display

input
```
int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*e^2-b*d*e+c*d^2)^2*((1/2*c*(2*a^2*c*e^3-a*b^2*e^3-a*b*c*d*e^2+2*a*c^
2*d^2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)/a/(4*a*c-b^2)*x^3+1/2*(3*a^2*b*
c*e^3-2*a^2*c^2*d*e^2-a*b^3*e^3-2*a*b^2*c*d*e^2+5*a*b*c^2*d^2*e-2*a*c^3*d^
3+b^4*d*e^2-2*b^3*c*d^2*e+b^2*c^2*d^3)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/
a/(4*a*c-b^2)*c*(1/8*(10*a^2*c*e^3*(-4*a*c+b^2)^(1/2)-3*a*b^2*e^3*(-4*a*c+
b^2)^(1/2)-a*b*c*d*e^2*(-4*a*c+b^2)^(1/2)+2*a*c^2*d^2*e*(-4*a*c+b^2)^(1/2)
+b^3*d*e^2*(-4*a*c+b^2)^(1/2)-2*b^2*c*d^2*e*(-4*a*c+b^2)^(1/2)+b*c^2*d^3*(
-4*a*c+b^2)^(1/2)-16*a^2*b*e^3*c+28*a^2*c^2*d*e^2+3*a*b^3*e^3+3*a*b^2*c*d*
e^2-20*a*b*c^2*d^2*e+12*a*c^3*d^3-b^4*d*e^2+2*b^3*c*d^2*e-b^2*c^2*d^3)/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(10*a^2*c*e^3*(-4*a*c+b^2)^(1/2)-3
*a*b^2*e^3*(-4*a*c+b^2)^(1/2)-a*b*c*d*e^2*(-4*a*c+b^2)^(1/2)+2*a*c^2*d^2*e
*(-4*a*c+b^2)^(1/2)+b^3*d*e^2*(-4*a*c+b^2)^(1/2)-2*b^2*c*d^2*e*(-4*a*c+b^2
)^(1/2)+b*c^2*d^3*(-4*a*c+b^2)^(1/2)+16*a^2*b*e^3*c-28*a^2*c^2*d*e^2-3*a*b
^3*e^3-3*a*b^2*c*d*e^2+20*a*b*c^2*d^2*e-12*a*c^3*d^3+b^4*d*e^2-2*b^3*c*d^2
*e+b^2*c^2*d^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))+e^4/(a*e^2-b*d
*e+c*d^2)^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.274.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.274.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.274.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\left(a+bx^2+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.274.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37254 vs. 2(574) = 1148.

Time = 7.05 (sec) , antiderivative size = 37254, normalized size of antiderivative = 56.45

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
e^4*arctan(e*x/sqrt(d*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^
2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(d*e)) + 1/16*((2*a^2*b^7*c^8 - 40*a^3*
b^5*c^9 + 224*a^4*b^3*c^10 - 384*a^5*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^8 + 192*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^9 - 48*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^10 - 2*(b^2 - 4*a*c)*a^2*b^5*c^
8 + 32*(b^2 - 4*a*c)*a^3*b^3*c^9 - 96*(b^2 - 4*a*c)*a^4*b*c^10)*d^11 - 2*(
6*a^2*b^8*c^7 - 116*a^3*b^6*c^8 + 640*a^4*b^4*c^9 - 1088*a^5*b^2*c^10 - 3*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 58
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 + 6
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 3
20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 -
 92*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c...
```

3.274.9 Mupad [B] (verification not implemented)

Time = 15.37 (sec) , antiderivative size = 237586, normalized size of antiderivative = 359.98

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x)
```

output
```
- atan(((((((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10
*c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^1
1*b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 91
7504*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e
^8 + 10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a
^12*c^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^
3 - 78848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^
2*b^12*c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8
*e^8 + 4608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^
17*c^2*d^5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e
^3 + 684544*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776
*a^3*b^10*c^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*
d^8*e^8 + 155136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504
*a^3*b^15*c^3*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*
d^14*e^2 + 864256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 +
5181440*a^4*b^7*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4
*b^9*c^8*d^9*e^7 + 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7
*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960
*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*
d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^...
```

3.274. $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx$

3.275 $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2}\,dx$

3.275.1 Optimal result

Integrand size = 24, antiderivative size = 1077

$$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2}\,dx = \frac{e^4x}{2d(cd^2-bde+ae^2)^2(d+ex^2)}$$
$$+\frac{x(abce(2cd-be)+(b^2-2ac)(c^2d^2+b^2e^2-ce(2bd+ae))-c(2b^2cde-4ac^2de-b^3e^2-bc(cd^2-3ae^{}}{2a(b^2-4ac)(cd^2-bde+ae^2)^2(a+bx^2+cx^4)}$$
$$+\frac{\sqrt{2}\sqrt{c}e^2\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bde+ae^2)^3}$$
$$+\frac{\sqrt{c}\left(b^4e^2-b^3e\left(2cd-\sqrt{b^2-4ac}e\right)-4ac^2\left(3cd^2-e\left(\sqrt{b^2-4ac}d+3ae\right)\right)+b^2c\left(cd^2-e\left(2\sqrt{b^2-4ac}d+\right.\right.\right.}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}(cd^{}}$$
$$-\frac{\sqrt{2}\sqrt{c}e^2\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}(cd^2-bde+ae^2)^3}$$
$$-\frac{\sqrt{c}\left(b^4e^2-b^3e\left(2cd+\sqrt{b^2-4ac}e\right)+bc\left(3a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-16ae\right)\right)+b^2c\left(cd^2+e\left(2\sqrt{b^{}}\right.\right.\right.}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}(cd^{}}$$
$$+\frac{2e^{7/2}(2cd-be)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(cd^2-bde+ae^2)^3}+\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(cd^2-bde+ae^2)^2}$$

output
```
1/2*e^4*x/d/(a*e^2-b*d*e+c*d^2)^2/(e*x^2+d)+1/2*x*(a*b*c*e*(-b*e+2*c*d)+(-
2*a*c+b^2)*(c^2*d^2+b^2*e^2-c*e*(a*e+2*b*d))-c*(2*b^2*c*d*e-4*a*c^2*d*e-b^
3*e^2-b*c*(-3*a*e^2+c*d^2))*x^2)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x
^4+b*x^2+a)+1/2*e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2-b*d*e+c*d
^2)^2+2*e^(7/2)*(-b*e+2*c*d)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)
^3/d^(1/2)+e^2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1
/2)*c^(1/2)*(3*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-c*e*(3*b*d+a*e+2*d*(-4
*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c
^(1/2)*(b^4*e^2-b^3*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-4*a*c^2*(3*c*d^2-e*(3*a
*e+d*(-4*a*c+b^2)^(1/2)))-b*c*(3*a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(16*a*e+d*(-
4*a*c+b^2)^(1/2)))+b^2*c*(c*d^2-e*(9*a*e+2*d*(-4*a*c+b^2)^(1/2))))/a/(-4*a
*c+b^2)^(3/2)/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-e
^2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)*
(3*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-c*e*(3*b*d+a*e-2*d*(-4*a*c+b^2)^(1
/2)))/(a*e^2-b*d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2
)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^4*
e^2-b^3*e*(2*c*d+e*(-4*a*c+b^2)^(1/2))+b*c*(3*a*e^2*(-4*a*c+b^2)^(1/2)-c*d
*(-16*a*e+d*(-4*a*c+b^2)^(1/2)))-4*a*c^2*(3*c*d^2+e*(-3*a*e+d*(-4*a*c+b^2)
^(1/2)))+b^2*c*(c*d^2+e*(-9*a*e+2*d*(-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2...
```

3.275.2 Mathematica [A] (verified)

Time = 3.32 (sec) , antiderivative size = 1020, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{2e^4x}{d(cd^2+e(-bd+ae))^2(d+ex^2)} - \frac{2x(b^4e^2+b^3ce(-2d+ex^2)+2ac^2(ae^2-cd(d-2ex^2))+b^2c(-4ae^2+cd(d-2ex^2))+bc^2(cd^2x^2-3a}{a(-b^2+4ac)(cd^2+e(-bd+ae))^2(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^5de^3+b^3e\left(cd-\sqrt{b^2-4ac}e\right)(3cd^2+5ae^2)+b^4e^2\left(-3cd^2+e\left(\sqrt{b^2-4ac}d-5ae\right)\right)-4ac^2\left(-3\right.\right.}{} - \frac{\sqrt{2}\sqrt{c}\left(b^5de^3+b^3e\left(cd+\sqrt{b^2-4ac}e\right)(3cd^2+5ae^2)-b^2c\left(c^2d^4+ae^3\left(7\sqrt{b^2-4ac}d-29ae\right)+3cd^2e(\sqrt{\right.\right.}{} + \frac{2e^{7/2}(9cd^2+e(-5bd+ae))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2+e(-bd+ae))^3}\right)$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((2*e^4*x)/(d*(c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x^2)) - (2*x*(b^4*e^2 +
b^3*c*e*(-2*d + e*x^2) + 2*a*c^2*(a*e^2 - c*d*(d - 2*e*x^2)) + b^2*c*(-4*a
*e^2 + c*d*(d - 2*e*x^2)) + b*c^2*(c*d^2*x^2 - 3*a*e*(-2*d + e*x^2))))/(a*
(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*(a + b*x^2 + c*x^4)) + (Sqrt[2
]*Sqrt[c]*(b^5*d*e^3 + b^3*e*(c*d - Sqrt[b^2 - 4*a*c]*e)*(3*c*d^2 + 5*a*e^
2) + b^4*e^2*(-3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - 5*a*e)) - 4*a*c^2*(-3*c^
2*d^4 + c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^3*(9*Sqrt[b^2 - 4*a*c
]*d + 7*a*e)) - b*c*(-19*a^2*Sqrt[b^2 - 4*a*c]*e^4 + 2*a*c*d*e^2*(-3*Sqrt[
b^2 - 4*a*c]*d + 26*a*e) + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d + 28*a*e)) + b^2*c
*(-(c^2*d^4) + 3*c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*e^3*(7*Sqrt[b^2
 - 4*a*c]*d + 29*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b
*d - a*e))^3) - (Sqrt[2]*Sqrt[c]*(b^5*d*e^3 + b^3*e*(c*d + Sqrt[b^2 - 4*a*
c]*e)*(3*c*d^2 + 5*a*e^2) - b^2*c*(c^2*d^4 + a*e^3*(7*Sqrt[b^2 - 4*a*c]*d
- 29*a*e) + 3*c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - b^4*e^2*(3*c*d^2 +
e*(Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + 4*a*c^2*(3*c^2*d^4 + a*e^3*(9*Sqrt[b^2
- 4*a*c]*d - 7*a*e) + c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)) + b*c*(-19*a
^2*Sqrt[b^2 - 4*a*c]*e^4 + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d - 28*a*e) - 2*a*c*
d*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 26*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*...
```

3.275.3 Rubi [A] (verified)

Time = 7.65 (sec) , antiderivative size = 1077, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{1567}$$

$$\int \left(\frac{e^2(-ce(ae+5bd)+2b^2e^2-2cex^2(2cd-be)+3c^2d^2)}{(a+bx^2+cx^4)(ae^2-bde+cd^2)^3}+\frac{-ce(ae+2bd)+b^2e^2-cex^2(2cd-be)+c^2d^2}{(a+bx^2+cx^4)^2(ae^2-bde+cd^2)^2}-\frac{}{(a}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{xe^4}{2d(cd^2-bed+ae^2)^2(ex^2+d)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{2d^{3/2}(cd^2-bed+ae^2)^2}+\frac{2(2cd-be)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)e^{7/2}}{\sqrt{d}(cd^2-bed+ae^2)^3}+$$
$$\frac{\sqrt{2}\sqrt{c}\left(3c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-ce\left(3bd+2\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)e^2}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bed+ae^2)^3}-$$
$$\frac{\sqrt{2}\sqrt{c}\left(3c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-ce\left(3bd-2\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)e^2}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}(cd^2-bed+ae^2)^3}+$$
$$\frac{\sqrt{c}\left(e^2b^4-e\left(2cd-\sqrt{b^2-4ac}e\right)b^3+c\left(cd^2-e\left(2\sqrt{b^2-4ac}d+9ae\right)\right)b^2-c\left(3a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d+\right.\right.\right.}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bed}$$
$$\frac{\sqrt{c}\left(e^2b^4-e\left(2cd+\sqrt{b^2-4ac}e\right)b^3+c\left(cd^2+e\left(2\sqrt{b^2-4ac}d-9ae\right)\right)b^2+c\left(3a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{b^2-4ac}d-\right.\right.\right.}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}(cd^2-bed}$$
$$\frac{x\left(-c(-e^2b^3+2cdeb^2-c(cd^2-3ae^2)b-4ac^2de)x^2+abce(2cd-be)+(b^2-2ac)(c^2d^2+b^2e^2-ce(2bd+ae))\right.}{2a(b^2-4ac)(cd^2-bed+ae^2)^2(cx^4+bx^2+a)}$$

input
```
Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(e^4*x)/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) + (x*(a*b*c*e*(2*c*d -
 b*e) + (b^2 - 2*a*c)*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e)) - c*(2*b^2*c
*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^2))/(2*a*(b^2 - 4*
a*c)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*e^2
*(3*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*
a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^3) +
(Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) - 4*a*c^2*(3*c*d^2
 - e*(Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + b^2*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c
]*d + 9*a*e)) - b*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d
+ 16*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sq
rt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a
*e^2)^2) - (Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
 - c*e*(3*b*d - 2*Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
]*(c*d^2 - b*d*e + a*e^2)^3) - (Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d + Sqrt[b^2
 - 4*a*c]*e) + b*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d -
 16*a*e)) + b^2*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 9*a*e)) - 4*a*c^2*(3
*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - 3*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqr...
```

3.275. $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2}\,dx$

3.275.3.1 Defintions of rubi rules used

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.275.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 1250, normalized size of antiderivative = 1.16

method	result	size
default	Expression too large to display	1250
risch	Expression too large to display	79373

input
```
int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*e^2-b*d*e+c*d^2)^3*((-1/2*c*(3*a^2*b*c*e^4-4*a^2*c^2*d*e^3-a*b^3*e^4
-a*b^2*c*d*e^3+6*a*b*c^2*d^2*e^2-4*a*c^3*d^3*e+b^4*d*e^3-3*b^3*c*d^2*e^2+3
*b^2*c^2*d^3*e-b*c^3*d^4)/a/(4*a*c-b^2)*x^3+1/2*(2*a^3*c^2*e^4-4*a^2*b^2*c
*e^4+4*a^2*b*c^2*d*e^3+a*b^4*e^4+2*a*b^3*c*d*e^3-9*a*b^2*c^2*d^2*e^2+8*a*b
*c^3*d^3*e-2*a*c^4*d^4-b^5*d*e^3+3*b^4*c*d^2*e^2-3*b^3*c^2*d^3*e+b^2*c^3*d
^4)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/a/(4*a*c-b^2)*c*(1/8*(-19*a^2*b*c*e
^4*(-4*a*c+b^2)^(1/2)+36*a^2*c^2*d*e^3*(-4*a*c+b^2)^(1/2)+5*a*b^3*e^4*(-4*
a*c+b^2)^(1/2)-7*b^2*c*d*e^3*a*(-4*a*c+b^2)^(1/2)-6*b*c^2*d^2*e^2*a*(-4*a*
c+b^2)^(1/2)+4*c^3*d^3*e*a*(-4*a*c+b^2)^(1/2)-b^4*d*e^3*(-4*a*c+b^2)^(1/2)
+3*b^3*c*d^2*e^2*(-4*a*c+b^2)^(1/2)-3*b^2*c^2*d^3*e*(-4*a*c+b^2)^(1/2)+b*c
^3*d^4*(-4*a*c+b^2)^(1/2)-28*a^3*c^2*e^4+29*a^2*b^2*c*e^4-52*a^2*b*c^2*d*e
^3+48*a^2*c^3*d^2*e^2-5*a*b^4*e^4+5*a*b^3*c*d*e^3+12*a*b^2*c^2*d^2*e^2-28*
a*b*c^3*d^3*e+12*a*c^4*d^4+b^5*d*e^3-3*b^4*c*d^2*e^2+3*b^3*c^2*d^3*e-b^2*c
^3*d^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-19*a^2*b*c*e^4*(-4*a*
c+b^2)^(1/2)+36*a^2*c^2*d*e^3*(-4*a*c+b^2)^(1/2)+5*a*b^3*e^4*(-4*a*c+b^2)^
(1/2)-7*b^2*c*d*e^3*a*(-4*a*c+b^2)^(1/2)-6*b*c^2*d^2*e^2*a*(-4*a*c+b^2)^(1
/2)+4*c^3*d^3*e*a*(-4*a*c+b^2)^(1/2)-b^4*d*e^3*(-4*a*c+b^2)^(1/2)+3*b^3*c*
d^2*e^2*(-4*a*c+b^2)^(1/2)-3*b^2*c^2*d^3*e*(-4*a*c+b^2)^(1/2)+b*c^3*d^4*(-
4*a*c+b^2)^(1/2)+28*a^3*c^2*e^4-29*a^2*b^2*c*e^4+52*a^2*b*c^2*d*e^3-48*...
```

3.275.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.275.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

3.275.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.275.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65158 vs. 2(954) = 1908.

Time = 10.36 (sec) , antiderivative size = 65158, normalized size of antiderivative = 60.50

$$\int \frac{1}{(d+ex^2)^2 (a+bx^2+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/16*((2*a^2*b^7*c^11 - 40*a^3*b^5*c^12 + 224*a^4*b^3*c^13 - 384*a^5*b*c^1
4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^9
+ 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^1
0 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^
10 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3
*c^11 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^4*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
5*c^11 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5
*b*c^12 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^2*c^12 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^3*c^12 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^4*b*c^13 - 2*(b^2 - 4*a*c)*a^2*b^5*c^11 + 32*(b^2 - 4*a*c)*a^3*b^3*c^12
 - 96*(b^2 - 4*a*c)*a^4*b*c^13)*d^16 - (18*a^2*b^8*c^10 - 344*a^3*b^6*c^11
 + 1888*a^4*b^4*c^12 - 3200*a^5*b^2*c^13 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^8 + 172*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^9 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^9 - 944*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^10 - 272*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^10 - 9*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^10 + 1600*sqrt(2)*sqrt(...
```

3.275.9 Mupad [B] (verification not implemented)

Time = 17.55 (sec) , antiderivative size = 97073, normalized size of antiderivative = 90.13

$$\int \frac{1}{(d+ex^2)^2 (a+bx^2+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x)
```

output
```
symsum(log(root(128723189760*a^14*b^4*c^9*d^13*e^14*z^6 + 128723189760*a^1
2*b^4*c^11*d^17*e^10*z^6 - 8432455680*a^11*b^12*c^4*d^11*e^16*z^6 - 843245
5680*a^7*b^12*c^8*d^19*e^8*z^6 + 12673351680*a^11*b^11*c^5*d^12*e^15*z^6 +
 12673351680*a^8*b^11*c^8*d^18*e^9*z^6 - 72637480960*a^12*b^9*c^6*d^12*e^1
5*z^6 - 72637480960*a^9*b^9*c^9*d^18*e^9*z^6 - 21048344576*a^9*b^12*c^6*d^
15*e^12*z^6 - 16609443840*a^17*b^3*c^7*d^8*e^19*z^6 - 16609443840*a^10*b^3
*c^14*d^22*e^5*z^6 + 145332633600*a^13*b^5*c^9*d^14*e^13*z^6 + 14533263360
0*a^12*b^5*c^10*d^16*e^11*z^6 + 123740356608*a^14*b^5*c^8*d^12*e^15*z^6 +
123740356608*a^11*b^5*c^11*d^18*e^9*z^6 + 3460300800*a^17*b^5*c^5*d^6*e^21
*z^6 + 3460300800*a^8*b^5*c^14*d^24*e^3*z^6 - 7751073792*a^15*b^7*c^5*d^8*
e^19*z^6 - 7751073792*a^8*b^7*c^12*d^22*e^5*z^6 + 12041846784*a^14*b^7*c^6
*d^10*e^17*z^6 + 12041846784*a^9*b^7*c^11*d^20*e^7*z^6 - 325545099264*a^14
*b^3*c^10*d^14*e^13*z^6 - 325545099264*a^13*b^3*c^11*d^16*e^11*z^6 - 33305
39520*a^13*b^10*c^4*d^9*e^18*z^6 - 3330539520*a^7*b^10*c^10*d^21*e^6*z^6 +
 157789716480*a^12*b^7*c^8*d^14*e^13*z^6 + 157789716480*a^11*b^7*c^9*d^16*
e^11*z^6 + 37492359168*a^11*b^10*c^6*d^13*e^14*z^6 + 37492359168*a^9*b^10*
c^8*d^17*e^10*z^6 + 301989888*a^8*b^3*c^16*d^26*e*z^6 - 7266631680*a^17*b^
4*c^6*d^7*e^20*z^6 - 7266631680*a^9*b^4*c^14*d^23*e^4*z^6 - 201326592*a^20
*b*c^6*d^4*e^23*z^6 - 188743680*a^7*b^5*c^15*d^26*e*z^6 + 45747339264*a^13
*b^8*c^6*d^11*e^16*z^6 + 45747339264*a^9*b^8*c^10*d^19*e^8*z^6 - 746127...
```

3.275. $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2}\,dx$

3.276 $\int \left(d+e x^2\right)^{5/2} \left(a+b x^2+c x^4\right) \, dx$

3.276.1 Optimal result

Integrand size = 24, antiderivative size = 215

$$\int \left(d+e x^2\right)^{5/2} \left(a+b x^2+c x^4\right) \, dx = \frac{d^2\left(3cd^2-10bde+80ae^2\right)x\sqrt{d+ex^2}}{256e^2} + \frac{d\left(3cd^2-10bde+80ae^2\right)x\left(d+ex^2\right)^{3/2}}{384e^2} + \frac{\left(3cd^2-10bde+80ae^2\right)x\left(d+ex^2\right)^{5/2}}{480e^2} - \frac{(3cd-10be)x\left(d+ex^2\right)^{7/2}}{80e^2} + \frac{cx^3\left(d+ex^2\right)^{7/2}}{10e} + \frac{d^3\left(3cd^2-10bde+80ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{256e^{5/2}}$$

output
```
1/384*d*(80*a*e^2-10*b*d*e+3*c*d^2)*x*(e*x^2+d)^(3/2)/e^2+1/480*(80*a*e^2-
10*b*d*e+3*c*d^2)*x*(e*x^2+d)^(5/2)/e^2-1/80*(-10*b*e+3*c*d)*x*(e*x^2+d)^(
7/2)/e^2+1/10*c*x^3*(e*x^2+d)^(7/2)/e+1/256*d^3*(80*a*e^2-10*b*d*e+3*c*d^2
)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/256*d^2*(80*a*e^2-10*b*d*e+
3*c*d^2)*x*(e*x^2+d)^(1/2)/e^2
```

3.276.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.84

$$\int \left(d+ex^2\right)^{5/2} \left(a+bx^2+cx^4\right) dx = \frac{\sqrt{e}x\sqrt{d+ex^2}(c(-45d^4+30d^3ex^2+744d^2e^2x^4+1008de^3x^6+384e^4x^8)+10e(8ae(33d^2+26de$$

input
```
Integrate[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```

output
```
(Sqrt[e]*x*Sqrt[d + e*x^2]*(c*(-45*d^4 + 30*d^3*e*x^2 + 744*d^2*e^2*x^4 +
1008*d*e^3*x^6 + 384*e^4*x^8) + 10*e*(8*a*e*(33*d^2 + 26*d*e*x^2 + 8*e^2*x
^4) + b*(15*d^3 + 118*d^2*e*x^2 + 136*d*e^2*x^4 + 48*e^3*x^6))) - 15*(3*c*
d^5 - 10*d^3*e*(b*d - 8*a*e))*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(3840*e
^(5/2))
```

3.276.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1473, 299, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{5/2} \left(a+bx^2+cx^4\right) dx$$

$$\Big\downarrow \ 1473$$

$$\frac{\int \left(ex^2+d\right)^{5/2} \left(10ae-(3cd-10be)x^2\right) dx}{10e}+\frac{cx^3 \left(d+ex^2\right)^{7/2}}{10e}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{(3cd^2-10e(bd-8ae)) \int \left(ex^2+d\right)^{5/2} dx}{8e}-\frac{x \left(d+ex^2\right)^{7/2} (3cd-10be)}{8e}}{10e}+\frac{cx^3 \left(d+ex^2\right)^{7/2}}{10e}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{(3cd^2-10e(bd-8ae)) \left(\frac{5}{6}d\int \left(ex^2+d\right)^{3/2} dx+\frac{1}{6}x \left(d+ex^2\right)^{5/2}\right)}{8e}-\frac{x \left(d+ex^2\right)^{7/2} (3cd-10be)}{8e}}{10e}+\frac{cx^3 \left(d+ex^2\right)^{7/2}}{10e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(3cd^2-10e(bd-8ae)\right)\left(\frac{5}{6}d\left(\frac{3}{4}d\int\sqrt{ex^2+d}dx+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)+\frac{1}{6}x\left(d+ex^2\right)^{5/2}\right)}{8e}-\frac{x\left(d+ex^2\right)^{7/2}(3cd-10be)}{8e}}{10e}+\frac{cx^3\left(d+ex^2\right)^{7/2}}{10e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(3cd^2-10e(bd-8ae)\right)\left(\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{1}{2}d\int\frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)+\frac{1}{6}x\left(d+ex^2\right)^{5/2}\right)}{8e}-\frac{x\left(d+ex^2\right)^{7/2}(3cd-10be)}{8e}}{10e}+\frac{cx^3\left(d+ex^2\right)^{7/2}}{10e}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\left(3cd^2-10e(bd-8ae)\right)\left(\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{1}{2}d\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)+\frac{1}{6}x\left(d+ex^2\right)^{5/2}\right)}{8e}-\frac{x\left(d+ex^2\right)^{7/2}(3cd-10be)}{8e}}{10e}+\frac{cx^3\left(d+ex^2\right)^{7/2}}{10e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)+\frac{1}{6}x\left(d+ex^2\right)^{5/2}\right)\left(3cd^2-10e(bd-8ae)\right)}{8e}-\frac{x\left(d+ex^2\right)^{7/2}(3cd-10be)}{8e}}{10e}+\frac{cx^3\left(d+ex^2\right)^{7/2}}{10e}$$

input
```
Int[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```

output
```
(c*x^3*(d + e*x^2)^(7/2))/(10*e) + (-1/8*((3*c*d - 10*b*e)*x*(d + e*x^2)^(
7/2))/e + ((3*c*d^2 - 10*e*(b*d - 8*a*e))*((x*(d + e*x^2)^(5/2))/6 + (5*d*
((x*(d + e*x^2)^(3/2))/4 + (3*d*((x*Sqrt[d + e*x^2])/2 + (d*ArcTanh[(Sqrt[
e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/4))/6))/(8*e))/(10*e)
```

3.276. $\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx$

3.276.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

3.276.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{\frac{5d^3\left(a e^2-\frac{1}{8}bde+\frac{3}{80}c d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e x^2+d}}{x\sqrt{e}}\right)}{16}+\frac{11\left(d^2\left(\frac{31}{110}c x^4+\frac{59}{132}b x^2+a\right)e^{\frac{5}{2}}+\frac{26d\left(\frac{63}{130}c x^4+\frac{17}{26}b x^2+a\right)x^2 e^{\frac{7}{2}}}{33}+\frac{8\left(\frac{3}{5}c x^4+\frac{3}{4}b x^2+\right.}{33}\right.}{16}}{e^{\frac{5}{2}}}$
risch	$\frac{x(384e^4c x^8+480e^4b x^6+1008d e^3c x^6+640a e^4x^4+1360bd e^3x^4+744c d^2e^2x^4+2080d e^3a x^2+1180e^2d^2b x^2+30d^3ec x^2+264}{3840e^2}$
default	$a\left(\frac{x(e x^2+d)^{\frac{5}{2}}}{6}+\frac{5d\left(\frac{x(e x^2+d)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right)}{4}\right)}{6}\right)+c\left(\frac{x^3(e x^2+d)^{\frac{7}{2}}}{10e}-\frac{3d\left(\frac{x(e x^2+d)}{8e}\right.}{}\right.$

input

```
int((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output

```
11/16/e^(5/2)*(5/11*d^3*(a*e^2-1/8*b*d*e+3/80*c*d^2)*arctanh((e*x^2+d)^(1/
2)/x/e^(1/2))+(d^2*(31/110*c*x^4+59/132*b*x^2+a)*e^(5/2)+26/33*d*(63/130*c
*x^4+17/26*b*x^2+a)*x^2*e^(7/2)+8/33*(3/5*c*x^4+3/4*b*x^2+a)*x^4*e^(9/2)+5
/88*((1/5*c*x^2+b)*e^(3/2)-3/10*c*d*e^(1/2))*d^3)*(e*x^2+d)^(1/2)*x)
```

3.276. $\int (d+ex^2)^{5/2}(a+bx^2+cx^4)\,dx$

3.276.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.72

$$\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx = \left[\frac{15\left(3\,cd^5-10\,bd^4e+80\,ad^3e^2\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(384\,ce^5x^9+48\left(21\,c\right.\right.}{}\right.$$

$$-\frac{15\left(3\,cd^5-10\,bd^4e+80\,ad^3e^2\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\left(384\,ce^5x^9+48\left(21\,cde^4+10\,be^5\right)x^7+8\left(93\,cd^2e^3\right.\right.}{3}$$

input
```
integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/7680*(15*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*sqrt(e)*log(-2*e*x^2 - 2
*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(384*c*e^5*x^9 + 48*(21*c*d*e^4 + 10*b
*e^5)*x^7 + 8*(93*c*d^2*e^3 + 170*b*d*e^4 + 80*a*e^5)*x^5 + 10*(3*c*d^3*e^
2 + 118*b*d^2*e^3 + 208*a*d*e^4)*x^3 - 15*(3*c*d^4*e - 10*b*d^3*e^2 - 176*
a*d^2*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/3840*(15*(3*c*d^5 - 10*b*d^4*e + 80
*a*d^3*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (384*c*e^5*x^9 +
 48*(21*c*d*e^4 + 10*b*e^5)*x^7 + 8*(93*c*d^2*e^3 + 170*b*d*e^4 + 80*a*e^5
)*x^5 + 10*(3*c*d^3*e^2 + 118*b*d^2*e^3 + 208*a*d*e^4)*x^3 - 15*(3*c*d^4*e
 - 10*b*d^3*e^2 - 176*a*d^2*e^3)*x)*sqrt(e*x^2 + d))/e^3]
```

3.276.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(209) = 418$.

Time = 0.52 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.04

$$\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx = \begin{cases} \sqrt{d+ex^2}\left(\frac{ce^2x^9}{10} + \frac{x^7\left(be^3+\frac{21cde^2}{10}\right)}{8e} + \frac{x^5\left(ae^3+3bde^2+3cd^2e-\frac{7d\left(be^3+\frac{21cde^2}{10}\right)}{8e}\right)}{6e} + \frac{x^3\cdot\left(3ade^2+3bd^2e+cd^3-\frac{5d\left(ae\right.}{}\right.}{} \right. \\ d^{\frac{5}{2}}\left(ax+\frac{bx^3}{3}+\frac{cx^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**(5/2)*(c*x**4+b*x**2+a),x)
```

output
```
Piecewise((sqrt(d + e*x**2)*(c*e**2*x**9/10 + x**7*(b*e**3 + 21*c*d*e**2/1
0)/(8*e) + x**5*(a*e**3 + 3*b*d*e**2 + 3*c*d**2*e - 7*d*(b*e**3 + 21*c*d*e
**2/10)/(8*e))/(6*e) + x**3*(3*a*d*e**2 + 3*b*d**2*e + c*d**3 - 5*d*(a*e**
3 + 3*b*d*e**2 + 3*c*d**2*e - 7*d*(b*e**3 + 21*c*d*e**2/10)/(8*e))/(6*e))/
(4*e) + x*(3*a*d**2*e + b*d**3 - 3*d*(3*a*d*e**2 + 3*b*d**2*e + c*d**3 - 5
*d*(a*e**3 + 3*b*d*e**2 + 3*c*d**2*e - 7*d*(b*e**3 + 21*c*d*e**2/10)/(8*e)
)/(6*e))/(4*e))/(2*e)) + (a*d**3 - d*(3*a*d**2*e + b*d**3 - 3*d*(3*a*d*e**
2 + 3*b*d**2*e + c*d**3 - 5*d*(a*e**3 + 3*b*d*e**2 + 3*c*d**2*e - 7*d*(b*e
**3 + 21*c*d*e**2/10)/(8*e))/(6*e))/(4*e))/(2*e))*Piecewise((log(2*sqrt(e)
*sqrt(d + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2), Tru
e)), Ne(e, 0)), (d**(5/2)*(a*x + b*x**3/3 + c*x**5/5), True))
```

3.276. $\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx$

3.276.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{5/2} \left(a+bx^2+cx^4\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.91

$$\int \left(d+ex^2\right)^{5/2} \left(a+bx^2+cx^4\right)\,dx = \frac{1}{3840}\left(2\left(4\left(6\left(8\,ce^2x^2+\frac{21\,cde^9+10\,be^{10}}{e^8}\right)x^2+\frac{93\,cd^2e^8+170\,bde^9+80\,ae^{10}}{e^8}\right)x^2+\frac{5\,(3\,cd^3 \cdots\right.\right.\right. - \frac{\left(3\,cd^5-10\,bd^4e+80\,ad^3e^2\right)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{256\,e^{\frac{5}{2}}}$$

input
```
integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*c*e^2*x^2 + (21*c*d*e^9 + 10*b*e^10)/e^8)*x^2 + (93*c*d
^2*e^8 + 170*b*d*e^9 + 80*a*e^10)/e^8)*x^2 + 5*(3*c*d^3*e^7 + 118*b*d^2*e^
8 + 208*a*d*e^9)/e^8)*x^2 - 15*(3*c*d^4*e^6 - 10*b*d^3*e^7 - 176*a*d^2*e^8
)/e^8)*sqrt(e*x^2 + d)*x - 1/256*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*log
(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^{5/2} \left(a+bx^2+cx^4\right) dx = \int (e\,x^2+d)^{5/2} \left(c\,x^4+b\,x^2+a\right) dx$$

input
```
int((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x)
```

output
```
int((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4), x)
```

3.277 $\int (d+ex^2)^{3/2} (a+bx^2+cx^4)\, dx$

3.277.1 Optimal result

Integrand size = 24, antiderivative size = 175

$$\int (d+ex^2)^{3/2} (a+bx^2+cx^4)\, dx = \frac{d(3cd^2-8bde+48ae^2)\, x\sqrt{d+ex^2}}{128e^2} + \frac{(3cd^2-8bde+48ae^2)\, x(d+ex^2)^{3/2}}{192e^2} - \frac{(3cd-8be)x(d+ex^2)^{5/2}}{48e^2} + \frac{cx^3(d+ex^2)^{5/2}}{8e} + \frac{d^2(3cd^2-8bde+48ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{128e^{5/2}}$$

output
```
1/192*(48*a*e^2-8*b*d*e+3*c*d^2)*x*(e*x^2+d)^(3/2)/e^2-1/48*(-8*b*e+3*c*d)
*x*(e*x^2+d)^(5/2)/e^2+1/8*c*x^3*(e*x^2+d)^(5/2)/e+1/128*d^2*(48*a*e^2-8*b
*d*e+3*c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/128*d*(48*a*e^2
-8*b*d*e+3*c*d^2)*x*(e*x^2+d)^(1/2)/e^2
```

3.277.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

$$\int (d+ex^2)^{3/2} (a+bx^2+cx^4)\, dx = \frac{\sqrt{e}x\sqrt{d+ex^2}(c(-9d^3+6d^2ex^2+72de^2x^4+48e^3x^6)+8e(6ae(5d+2ex^2)+b(3d^2+14dex^2+}{384e^{5/2}}$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
```

output
```
(Sqrt[e]*x*Sqrt[d + e*x^2]*(c*(-9*d^3 + 6*d^2*e*x^2 + 72*d*e^2*x^4 + 48*e^
3*x^6) + 8*e*(6*a*e*(5*d + 2*e*x^2) + b*(3*d^2 + 14*d*e*x^2 + 8*e^2*x^4)))
 - 3*(3*c*d^4 - 8*d^2*e*(b*d - 6*a*e))*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]]
)/(384*e^(5/2))
```

3.277.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1473, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right) \, dx$$

$$\Big\downarrow \text{1473}$$

$$\frac{\int \left(ex^2+d\right)^{3/2} \left(8ae-(3cd-8be)x^2\right) dx}{8e}+\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\left(3cd^2-8e(bd-6ae)\right) \int \left(ex^2+d\right)^{3/2}dx}{6e}-\frac{x\left(d+ex^2\right)^{5/2}(3cd-8be)}{6e}}{8e}+\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(3cd^2-8e(bd-6ae)\right) \left(\frac{3}{4}d\int \sqrt{ex^2+d}dx+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{6e}-\frac{x\left(d+ex^2\right)^{5/2}(3cd-8be)}{6e}}{8e}+\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(3cd^2-8e(bd-6ae)\right) \left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{6e}-\frac{x\left(d+ex^2\right)^{5/2}(3cd-8be)}{6e}}{8e}+\frac{cx^3 \left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(3cd^2-8e(bd-6ae))\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)}{6e}-\frac{x\left(d+ex^2\right)^{5/2}(3cd-8be)}{6e}}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(\frac{3}{4}d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)(3cd^2-8e(bd-6ae))}{6e}-\frac{x\left(d+ex^2\right)^{5/2}(3cd-8be)}{6e}}{8e}+\frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}$$

input
```
Int[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
```

output
```
(c*x^3*(d + e*x^2)^(5/2))/(8*e) + (-1/6*((3*c*d - 8*b*e)*x*(d + e*x^2)^(5/
2))/e + ((3*c*d^2 - 8*e*(b*d - 6*a*e))*((x*(d + e*x^2)^(3/2))/4 + (3*d*((x
*Sqrt[d + e*x^2])/2 + (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])
))/4))/(6*e))/(8*e)
```

3.277.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

3.277.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{3\left(a\,e^{2}-\frac{1}{6}bde+\frac{1}{16}c\,d^{2}\right)d^{2}\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)}{8}+\frac{5\left(d\left(\frac{3}{10}c\,x^{4}+\frac{7}{15}b\,x^{2}+a\right)e^{\frac{5}{2}}+\frac{2x^{2}\left(\frac{1}{2}c\,x^{4}+\frac{2}{3}b\,x^{2}+a\right)e^{\frac{7}{2}}}{5}+\frac{\left(\left(\frac{c\,x^{2}}{4}+b\right)e^{\frac{3}{2}}-\frac{3cd\sqrt{e}}{8}\right)d^{2}\cdots}{10}\cdots\right.}{8}\cdots}{e^{\frac{5}{2}}}$
risch	$\frac{x\left(48e^{3}c\,x^{6}+64e^{3}b\,x^{4}+72d\,e^{2}c\,x^{4}+96a\,e^{3}x^{2}+112d\,e^{2}b\,x^{2}+6d^{2}e\,x^{2}c+240d\,e^{2}a+24d^{2}eb-9d^{3}c\right)\sqrt{e\,x^{2}+d}}{384e^{2}}+\frac{d^{2}\left(48a\,e^{2}-8bde\cdots\right.}{\cdots}$
default	$a\left(\frac{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right)}{4}\right)+c\left(\frac{x^{3}\left(e\,x^{2}+d\right)^{\frac{5}{2}}}{8e}-\frac{3d\left(\frac{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}{6e}-\frac{d\left(\frac{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4}\cdots\right.}{\cdots}\right.}{\cdots}\right.$

input
```
int((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
5/8*(3/5*(a*e^2-1/6*b*d*e+1/16*c*d^2)*d^2*arctanh((e*x^2+d)^(1/2)/x/e^(1/2
))+(d*(3/10*c*x^4+7/15*b*x^2+a)*e^(5/2)+2/5*x^2*(1/2*c*x^4+2/3*b*x^2+a)*e^
(7/2)+1/10*((1/4*c*x^2+b)*e^(3/2)-3/8*c*d*e^(1/2))*d^2)*(e*x^2+d)^(1/2)*x)
/e^(5/2)
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.74

$$\int \left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)\,dx = \left[\frac{3\left(3\,cd^4-8\,bd^3e+48\,ad^2e^2\right)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\left(48\,ce^4x^7+8\left(9\,cde^3+\right.\right.}{768\,e^3}\right.$$

$$-\frac{3\left(3\,cd^4-8\,bd^3e+48\,ad^2e^2\right)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\left(48\,ce^4x^7+8\left(9\,cde^3+8\,be^4\right)x^5+2\left(3\,cd^2e^2+56\,bde\right.\right.}{384\,e^3}$$

input
```
integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/768*(3*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) + 2*(48*c*e^4*x^7 + 8*(9*c*d*e^3 + 8*b*e^4)*x
^5 + 2*(3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*x^3 - 3*(3*c*d^3*e - 8*b*d^2*
e^2 - 80*a*d*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/384*(3*(3*c*d^4 - 8*b*d^3*e
+ 48*a*d^2*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (48*c*e^4*x^
7 + 8*(9*c*d*e^3 + 8*b*e^4)*x^5 + 2*(3*c*d^2*e^2 + 56*b*d*e^3 + 48*a*e^4)*
x^3 - 3*(3*c*d^3*e - 8*b*d^2*e^2 - 80*a*d*e^3)*x)*sqrt(e*x^2 + d))/e^3]
```

3.277.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.57

$$\int \left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)\,dx = \begin{cases} \sqrt{d+ex^2}\left(\frac{cex^7}{8}+\frac{x^5\left(be^2+\frac{9cde}{8}\right)}{6e}+\frac{x^3\left(ae^2+2bde+cd^2-\frac{5d\left(be^2+\frac{9cde}{8}\right)}{6e}\right)}{4e}+\frac{x\left(2ade+bd^2-\frac{3d\left(ae^2+2bde+cd^2-\frac{5d\left(be^2\cdots\right)}{6\cdots}\right)}{4e}\cdots\right)}{2e}\cdots\right. \\ d^{\frac{3}{2}}\left(ax+\frac{bx^3}{3}+\frac{cx^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**(3/2)*(c*x**4+b*x**2+a),x)
```

output
```
Piecewise((sqrt(d + e*x**2)*(c*e*x**7/8 + x**5*(b*e**2 + 9*c*d*e/8)/(6*e)
+ x**3*(a*e**2 + 2*b*d*e + c*d**2 - 5*d*(b*e**2 + 9*c*d*e/8)/(6*e))/(4*e)
+ x*(2*a*d*e + b*d**2 - 3*d*(a*e**2 + 2*b*d*e + c*d**2 - 5*d*(b*e**2 + 9*c
*d*e/8)/(6*e))/(4*e))/(2*e)) + (a*d**2 - d*(2*a*d*e + b*d**2 - 3*d*(a*e**2
 + 2*b*d*e + c*d**2 - 5*d*(b*e**2 + 9*c*d*e/8)/(6*e))/(4*e))/(2*e))*Piecew
ise((log(2*sqrt(e)*sqrt(d + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)
/sqrt(e*x**2), True)), Ne(e, 0)), (d**(3/2)*(a*x + b*x**3/3 + c*x**5/5), T
rue))
```

3.277.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

$$\int \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right) \, dx = \frac{1}{384} \left(2 \left(4 \left(6\, cex^2 + \frac{9\, cde^6 + 8\, be^7}{e^6}\right) x^2 + \frac{3\, cd^2e^5 + 56\, bde^6 + 48\, ae^7}{e^6}\right) x^2 - \frac{3\, (3\, cd^3e^4 - 8\, bd^2e^5}{e^6} \right. - \frac{(3\, cd^4 - 8\, bd^3e + 48\, ad^2e^2) \log \left(\left|-\sqrt{e}x + \sqrt{ex^2+d}\right|\right)}{128\, e^{\frac{5}{2}}}$$

input
```
integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*c*e*x^2 + (9*c*d*e^6 + 8*b*e^7)/e^6)*x^2 + (3*c*d^2*e^5 + 5
6*b*d*e^6 + 48*a*e^7)/e^6)*x^2 - 3*(3*c*d^3*e^4 - 8*b*d^2*e^5 - 80*a*d*e^6
)/e^6)*sqrt(e*x^2 + d)*x - 1/128*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*log(
abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right) \, dx = \int {\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right) \,d x$$

input
```
int((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x)
```

output
```
int((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4), x)
```

3.278 $\int \sqrt{d+ex^2}(a+bx^2+cx^4)\,dx$

3.278.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx = \frac{(cd^2-2bde+8ae^2)\,x\sqrt{d+ex^2}}{16e^2} - \frac{(cd-2be)x(d+ex^2)^{3/2}}{8e^2} + \frac{cx^3(d+ex^2)^{3/2}}{6e} + \frac{d(cd^2-2bde+8ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{5/2}}$$

output
```
-1/8*(-2*b*e+c*d)*x*(e*x^2+d)^(3/2)/e^2+1/6*c*x^3*(e*x^2+d)^(3/2)/e+1/16*d
*(8*a*e^2-2*b*d*e+c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/16*(
8*a*e^2-2*b*d*e+c*d^2)*x*(e*x^2+d)^(1/2)/e^2
```

3.278.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx = \frac{x\sqrt{d+ex^2}(-3cd^2+6bde+24ae^2+2cdex^2+12be^2x^2+8ce^2x^4)}{48e^2} - \frac{d(cd^2-2bde+8ae^2)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{16e^{5/2}}$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

output
```
(x*Sqrt[d + e*x^2]*(-3*c*d^2 + 6*b*d*e + 24*a*e^2 + 2*c*d*e*x^2 + 12*b*e^2
*x^2 + 8*c*e^2*x^4))/(48*e^2) - (d*(c*d^2 - 2*b*d*e + 8*a*e^2)*Log[-(Sqrt[
e]*x) + Sqrt[d + e*x^2]])/(16*e^(5/2))
```

3.278.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1473, 27, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx$$

$$\Big\downarrow \; 1473$$

$$\frac{\int 3\sqrt{ex^2+d}\left(2ae-(cd-2be)x^2\right)dx}{6e}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sqrt{ex^2+d}\left(2ae-(cd-2be)x^2\right)dx}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{\left(8ae^2-2bde+cd^2\right)\int \sqrt{ex^2+d}dx}{4e}-\frac{x\left(d+ex^2\right)^{3/2}(cd-2be)}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{\left(8ae^2-2bde+cd^2\right)\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}-\frac{x\left(d+ex^2\right)^{3/2}(cd-2be)}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\left(8ae^2-2bde+cd^2\right)\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)}{4e}-\frac{x\left(d+ex^2\right)^{3/2}(cd-2be)}{4e}}{2e}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)(8ae^2-2bde+cd^2)}{4e}-\frac{x(d+ex^2)^{3/2}(cd-2be)}{4e}}{2e}+\frac{cx^3(d+ex^2)^{3/2}}{6e}$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

output
```
(c*x^3*(d + e*x^2)^(3/2))/(6*e) + (-1/4*((c*d - 2*b*e)*x*(d + e*x^2)^(3/2)
)/e + ((c*d^2 - 2*b*d*e + 8*a*e^2)*((x*Sqrt[d + e*x^2])/2 + (d*ArcTanh[(Sq
rt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e])))/(4*e))/(2*e)
```

3.278.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

3.278.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{d\left(a\,e^{2}-\frac{1}{4}bde+\frac{1}{8}c\,d^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)+\sqrt{e\,x^{2}+d}\left(\left(\frac{1}{3}c\,x^{4}+\frac{1}{2}b\,x^{2}+a\right)e^{\frac{5}{2}}+\frac{d\left(\left(\frac{c\,x^{2}}{3}+b\right)e^{\frac{3}{2}}-\frac{cd\sqrt{e}}{2}\right)}{4}\right)x}{2e^{\frac{5}{2}}}$
risch	$\frac{x\left(8c\,e^{2}x^{4}+12b\,e^{2}x^{2}+2de\,x^{2}c+24a\,e^{2}+6bde-3c\,d^{2}\right)\sqrt{e\,x^{2}+d}}{48e^{2}}+\frac{d\left(8a\,e^{2}-2bde+c\,d^{2}\right)\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{16e^{\frac{5}{2}}}$
default	$a\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right)+c\left(\frac{x^{3}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{6e}-\frac{d\left(\frac{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4e}-\frac{d\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right)}{4e}\right.}{2e}\right.$

input
```
int((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*(a*e^2-1/4*b*d*e+1/8*c*d^2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+(e*x
^2+d)^(1/2)*((1/3*c*x^4+1/2*b*x^2+a)*e^(5/2)+1/4*d*((1/3*c*x^2+b)*e^(3/2)-
1/2*c*d*e^(1/2)))*x)/e^(5/2)
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.76

$$\int \sqrt{d+ex^2}(a+bx^2+cx^4)\,dx$$
$$= \left[\frac{3(cd^3-2bd^2e+8ade^2)\sqrt{e}\log\left(-2ex^2-2\sqrt{ex^2+d}\sqrt{e}x-d\right)+2(8ce^3x^5+2(cde^2+6be^3)x^3-3(cd}{96e^3}\right.$$
$$-\frac{3(cd^3-2bd^2e+8ade^2)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(8ce^3x^5+2(cde^2+6be^3)x^3-3(cd^2e-2bde^2-8a}{48e^3}$$

input
```
integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/96*(3*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x
^2 + d)*sqrt(e)*x - d) + 2*(8*c*e^3*x^5 + 2*(c*d*e^2 + 6*b*e^3)*x^3 - 3*(c
*d^2*e - 2*b*d*e^2 - 8*a*e^3)*x)*sqrt(e*x^2 + d))/e^3, -1/48*(3*(c*d^3 - 2
*b*d^2*e + 8*a*d*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (8*c*e
^3*x^5 + 2*(c*d*e^2 + 6*b*e^3)*x^3 - 3*(c*d^2*e - 2*b*d*e^2 - 8*a*e^3)*x)*
sqrt(e*x^2 + d))/e^3]
```

3.278.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.16

$$\int \sqrt{d+ex^2}(a+bx^2+cx^4)\,dx$$
$$= \begin{cases} \sqrt{d+ex^2}\left(\frac{cx^5}{6}+\frac{x^3\left(be+\frac{cd}{6}\right)}{4e}+\frac{x\left(ae+bd-\frac{3d\left(be+\frac{cd}{6}\right)}{4e}\right)}{2e}\right)+\left(ad-\frac{d\left(ae+bd-\frac{3d\left(be+\frac{cd}{6}\right)}{4e}\right)}{2e}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{e}\sqrt{d+ex^2}+2ex\right)}{\sqrt{e}} \\ \frac{x\log(x)}{\sqrt{ex^2}}\end{cases}\right. \\ \sqrt{d}\left(ax+\frac{bx^3}{3}+\frac{cx^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**(1/2)*(c*x**4+b*x**2+a),x)
```

output
```
Piecewise((sqrt(d + e*x**2)*(c*x**5/6 + x**3*(b*e + c*d/6)/(4*e) + x*(a*e
+ b*d - 3*d*(b*e + c*d/6)/(4*e))/(2*e)) + (a*d - d*(a*e + b*d - 3*d*(b*e +
 c*d/6)/(4*e))/(2*e))*Piecewise((log(2*sqrt(e)*sqrt(d + e*x**2) + 2*e*x)/s
qrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2), True)), Ne(e, 0)), (sqrt(d)*(a*
x + b*x**3/3 + c*x**5/5), True))
```

3.278.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx = \frac{1}{48}\left(2\left(4\,cx^2+\frac{cde^3+6\,be^4}{e^4}\right)x^2-\frac{3\left(cd^2e^2-2\,bde^3-8\,ae^4\right)}{e^4}\right)\sqrt{ex^2+d}x - \frac{\left(cd^3-2\,bd^2e+8\,ade^2\right)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{16\,e^{\frac{5}{2}}}$$

input
```
integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/48*(2*(4*c*x^2 + (c*d*e^3 + 6*b*e^4)/e^4)*x^2 - 3*(c*d^2*e^2 - 2*b*d*e^3
 - 8*a*e^4)/e^4)*sqrt(e*x^2 + d)*x - 1/16*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*
log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx=\int \sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)\,dx$$

input
```
int((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4),x)
```

output
```
int((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4), x)
```

3.279 $\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}} \, dx$

3.279.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}} \, dx = -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}+\frac{(3cd^2-4bde+8ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{5/2}}$$

output
```
1/8*(8*a*e^2-4*b*d*e+3*c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)-1
/8*(-4*b*e+3*c*d)*x*(e*x^2+d)^(1/2)/e^2+1/4*c*x^3*(e*x^2+d)^(1/2)/e
```

3.279.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}} \, dx = \frac{\sqrt{d+ex^2}(-3cdx+4bex+2cex^3)}{8e^2}+\frac{(3cd^2-4bde+8ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{-\sqrt{d}+\sqrt{d+ex^2}}\right)}{4e^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(-3*c*d*x + 4*b*e*x + 2*c*e*x^3))/(8*e^2) + ((3*c*d^2 - 4
*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/(-Sqrt[d] + Sqrt[d + e*x^2])])/(4*e^
(5/2))
```

3.279.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1473, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \; 1473$$

$$\frac{\int \frac{4ae-(3cd-4be)x^2}{\sqrt{ex^2+d}}dx}{4e}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{(3cd^2-4e(bd-2ae))\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}-\frac{x\sqrt{d+ex^2}(3cd-4be)}{2e}}{4e}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{(3cd^2-4e(bd-2ae))\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}-\frac{x\sqrt{d+ex^2}(3cd-4be)}{2e}}{4e}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(3cd^2-4e(bd-2ae))}{2e^{3/2}}-\frac{x\sqrt{d+ex^2}(3cd-4be)}{2e}}{4e}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

input
```
Int[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]
```

output
```
(c*x^3*Sqrt[d + e*x^2])/(4*e) + (-1/2*((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/
e + ((3*c*d^2 - 4*e*(b*d - 2*a*e))*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(
2*e^(3/2)))/(4*e)
```

3.279.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

3.279.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x\left(2c\,x^{2}e+4be-3cd\right)\sqrt{e\,x^{2}+d}}{8e^{2}}+\frac{\left(8a\,e^{2}-4bde+3c\,d^{2}\right)\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{8e^{\frac{5}{2}}}$
pseudoelliptic	$\frac{\left(a\,e^{2}-\frac{1}{2}bde+\frac{3}{8}c\,d^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)+\frac{\left(\left(\frac{c\,x^{2}}{2}+b\right)e^{\frac{3}{2}}-\frac{3cd\sqrt{e}}{4}\right)\sqrt{e\,x^{2}+d}\,x}{2}}{e^{\frac{5}{2}}}$
default	$\frac{a\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{\sqrt{e}}+c\left(\frac{x^{3}\sqrt{e\,x^{2}+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)+b\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{\right.}\right.}{2e^{\frac{3}{2}}}\right.$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*c*e*x^2+4*b*e-3*c*d)*(e*x^2+d)^(1/2)/e^2+1/8*(8*a*e^2-4*b*d*e+3*c
*d^2)/e^(5/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.79

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{(3\,cd^2-4\,bde+8\,ae^2)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+2\,(2\,ce^2x^3-(3\,cde-4\,be^2)x)\sqrt{ex^2+d}}{16\,e^3},\right.$$
$$\left.-\frac{(3\,cd^2-4\,bde+8\,ae^2)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(2\,ce^2x^3-(3\,cde-4\,be^2)x)\sqrt{ex^2+d}}{8\,e^3}\right]$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*c*d^2 - 4*b*d*e + 8*a*e^2)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 +
 d)*sqrt(e)*x - d) + 2*(2*c*e^2*x^3 - (3*c*d*e - 4*b*e^2)*x)*sqrt(e*x^2 +
d))/e^3, -1/8*((3*c*d^2 - 4*b*d*e + 8*a*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sq
rt(e*x^2 + d)) - (2*c*e^2*x^3 - (3*c*d*e - 4*b*e^2)*x)*sqrt(e*x^2 + d))/e^
3]
```

3.279.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx$$
$$= \begin{cases} \left(a-\frac{d\left(b-\frac{3cd}{4e}\right)}{2e}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{e}\sqrt{d+ex^2}+2ex\right)}{\sqrt{e}} & \text{for } d\neq 0\\ \frac{x\log(x)}{\sqrt{ex^2}} & \text{otherwise}\end{cases}\right)+\sqrt{d+ex^2}\left(\frac{cx^3}{4e}+\frac{x\left(b-\frac{3cd}{4e}\right)}{2e}\right) & \text{for } e\neq 0\\ \frac{ax+\frac{bx^3}{3}+\frac{cx^5}{5}}{\sqrt{d}} & \text{otherwise}\end{cases}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

output
```
Piecewise(((a - d*(b - 3*c*d/(4*e))/(2*e))*Piecewise((log(2*sqrt(e)*sqrt(d
 + e*x**2) + 2*e*x)/sqrt(e), Ne(d, 0)), (x*log(x)/sqrt(e*x**2), True)) + s
qrt(d + e*x**2)*(c*x**3/(4*e) + x*(b - 3*c*d/(4*e))/(2*e)), Ne(e, 0)), ((a
*x + b*x**3/3 + c*x**5/5)/sqrt(d), True))
```

3.279.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx = \frac{1}{8}\sqrt{ex^2+d}\left(\frac{2\,cx^2}{e} - \frac{3\,cde-4\,be^2}{e^3}\right)x - \frac{(3\,cd^2-4\,bde+8\,ae^2)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{8\,e^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(e*x^2 + d)*(2*c*x^2/e - (3*c*d*e - 4*b*e^2)/e^3)*x - 1/8*(3*c*d^2
 - 4*b*d*e + 8*a*e^2)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx=\int \frac{c\,x^4+b\,x^2+a}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^(1/2), x)
```

3.280 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx$

3.280.1 Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx=\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}-\frac{(3cd-2be)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}$$

output
```
-1/2*(-2*b*e+3*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+(a+d*(-b*e+
c*d)/e^2)*x/d/(e*x^2+d)^(1/2)+1/2*c*x*(e*x^2+d)^(1/2)/e^2
```

3.280.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx=\frac{x(3cd^2-2bde+2ae^2+cdex^2)}{2de^2\sqrt{d+ex^2}}+\frac{(3cd-2be)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{2e^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

output
```
(x*(3*c*d^2 - 2*b*d*e + 2*a*e^2 + c*d*e*x^2))/(2*d*e^2*Sqrt[d + e*x^2]) +
((3*c*d - 2*b*e)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(2*e^(5/2))
```

3.280.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1471, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{x(ae^2-bde+cd^2)}{de^2\sqrt{d+ex^2}}-\frac{\int \frac{d(-cex^2+cd-be)}{e^2\sqrt{ex^2+d}}dx}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x(ae^2-bde+cd^2)}{de^2\sqrt{d+ex^2}}-\frac{\int \frac{-cex^2+cd-be}{\sqrt{ex^2+d}}dx}{e^2}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{x(ae^2-bde+cd^2)}{de^2\sqrt{d+ex^2}}-\frac{\frac{1}{2}(3cd-2be)\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{1}{2}cx\sqrt{d+ex^2}}{e^2}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{x(ae^2-bde+cd^2)}{de^2\sqrt{d+ex^2}}-\frac{\frac{1}{2}(3cd-2be)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{1}{2}cx\sqrt{d+ex^2}}{e^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x(ae^2-bde+cd^2)}{de^2\sqrt{d+ex^2}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)(3cd-2be)}{2\sqrt{e}}-\frac{1}{2}cx\sqrt{d+ex^2}}{e^2}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(d*e^2*Sqrt[d + e*x^2]) - (-1/2*(c*x*Sqrt[d +
e*x^2]) + ((3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[e
]))/e^2
```

3.280.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.280.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{\sqrt{e\,x^{2}+d}\,d\left(be-\frac{3cd}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{e\,x^{2}+d}}{x\sqrt{e}}\right)+\left(-\left(-\frac{c\,x^{2}}{2}+b\right)d\,e^{\frac{3}{2}}+\frac{3c\,d^{2}\sqrt{e}}{2}+a\,e^{\frac{5}{2}}\right)x}{\sqrt{e\,x^{2}+d}\,e^{\frac{5}{2}}\,d}$
risch	$\frac{cx\sqrt{e\,x^{2}+d}}{2e^{2}}+\frac{\frac{2a\,e^{2}x}{d\sqrt{e\,x^{2}+d}}-\frac{cdx}{\sqrt{e\,x^{2}+d}}+\left(2b\,e^{2}-3dce\right)\left(-\frac{x}{e\sqrt{e\,x^{2}+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{e^{\frac{3}{2}}}\right)}{2e^{2}}$
default	$\frac{ax}{d\sqrt{e\,x^{2}+d}}+c\left(\frac{x^{3}}{2e\sqrt{e\,x^{2}+d}}-\frac{3d\left(-\frac{x}{e\sqrt{e\,x^{2}+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{e^{\frac{3}{2}}}\right)}{2e}\right)+b\left(-\frac{x}{e\sqrt{e\,x^{2}+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{e^{\frac{3}{2}}}\right.$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x^2+d)^(1/2)/e^(5/2)*((e*x^2+d)^(1/2)*d*(b*e-3/2*c*d)*arctanh((e*x^2+
d)^(1/2)/x/e^(1/2))+(-(-1/2*c*x^2+b)*d*e^(3/2)+3/2*c*d^2*e^(1/2)+a*e^(5/2)
)*x)/d
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.80

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx = \left[-\frac{(3\,cd^3-2\,bd^2e+(3\,cd^2e-2\,bde^2)x^2)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\,(}{4\,(de^4x^2+d^2e^3)}\right.$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((3*c*d^3 - 2*b*d^2*e + (3*c*d^2*e - 2*b*d*e^2)*x^2)*sqrt(e)*log(-2*
e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(c*d*e^2*x^3 + (3*c*d^2*e - 2
*b*d*e^2 + 2*a*e^3)*x)*sqrt(e*x^2 + d))/(d*e^4*x^2 + d^2*e^3), 1/2*((3*c*d
^3 - 2*b*d^2*e + (3*c*d^2*e - 2*b*d*e^2)*x^2)*sqrt(-e)*arctan(sqrt(-e)*x/s
qrt(e*x^2 + d)) + (c*d*e^2*x^3 + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*x)*sqrt
(e*x^2 + d))/(d*e^4*x^2 + d^2*e^3)]
```

3.280.6 Sympy [A] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx = \frac{ax}{d^{\frac{3}{2}}\sqrt{1+\frac{ex^2}{d}}} + b\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{\frac{3}{2}}} - \frac{x}{\sqrt{d}e\sqrt{1+\frac{ex^2}{d}}}\right) + c\left(\frac{3\sqrt{d}x}{2e^2\sqrt{1+\frac{ex^2}{d}}} - \frac{3d\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{d}e\sqrt{1+\frac{ex^2}{d}}}\right)$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(3/2),x)
```

output
```
a*x/(d**(3/2)*sqrt(1 + e*x**2/d)) + b*(asinh(sqrt(e)*x/sqrt(d))/e**(3/2) -
 x/(sqrt(d)*e*sqrt(1 + e*x**2/d))) + c*(3*sqrt(d)*x/(2*e**2*sqrt(1 + e*x**
2/d)) - 3*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(5/2)) + x**3/(2*sqrt(d)*e*sqrt
(1 + e*x**2/d)))
```

3.280.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx = \frac{\left(\frac{cx^2}{e}+\frac{3\,cd^2e-2\,bde^2+2\,ae^3}{de^3}\right)x}{2\,\sqrt{ex^2+d}}+\frac{(3\,cd-2\,be)\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{2\,e^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
1/2*(c*x^2/e + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)/(d*e^3))*x/sqrt(e*x^2 + d
) + 1/2*(3*c*d - 2*b*e)*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(5/2)
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx = \int \frac{c\,x^4+b\,x^2+a}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2), x)
```

3.281 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx$

3.281.1 Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx=\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d(d+ex^2)^{3/2}}-\frac{(4cd^2-e(bd+2ae))\,x}{3d^2e^2\sqrt{d+ex^2}}+\frac{c\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

output
```
1/3*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^(3/2)+c*arctanh(x*e^(1/2)/(e*x^2+d)
^(1/2))/e^(5/2)-1/3*(4*c*d^2-e*(2*a*e+b*d))*x/d^2/e^2/(e*x^2+d)^(1/2)
```

3.281.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx=\frac{-cd^2x(3d+4ex^2)+e^2x(3ad+bdx^2+2aex^2)}{3d^2e^2(d+ex^2)^{3/2}}-\frac{c\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)}{e^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

output
```
(-(c*d^2*x*(3*d + 4*e*x^2)) + e^2*x*(3*a*d + b*d*x^2 + 2*a*e*x^2))/(3*d^2*
e^2*(d + e*x^2)^(3/2)) - (c*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/e^(5/2)
```

3.281.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1471, 25, 27, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{1471}$$

$$\frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^2)^{3/2}}-\frac{\int -\frac{3cdx^2+e\left(2a-\frac{d(cd-be)}{e^2}\right)}{e(ex^2+d)^{3/2}}dx}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{cd^2}{e}+3cx^2d+bd+2ae}{e(ex^2+d)^{3/2}}dx}{3d}+\frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-\frac{cd^2}{e}+3cx^2d+bd+2ae}{(ex^2+d)^{3/2}}dx}{3de}+\frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{3cd\int \frac{1}{\sqrt{ex^2+d}}dx}{e}+\frac{x\left(2ae+bd-\frac{4cd^2}{e}\right)}{d\sqrt{d+ex^2}}}{3de}+\frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3cd\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}+\frac{x\left(2ae+bd-\frac{4cd^2}{e}\right)}{d\sqrt{d+ex^2}}}{3de}+\frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{x\left(2ae+bd-\frac{4cd^2}{e}\right)}{d\sqrt{d+ex^2}}+\frac{3cd\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}}{3de}+\frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^2)^{3/2}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

output
```
((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^2)^(3/2)) + (((b*d - (4*c*d^
2)/e + 2*a*e)*x)/(d*Sqrt[d + e*x^2]) + (3*c*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]])/e^(3/2))/(3*d*e)
```

3.281.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.281.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{(e\,x^2+d)^{\frac{3}{2}} c\,d^2 \operatorname{arctanh}\left(\frac{\sqrt{e\,x^2+d}}{x\sqrt{e}}\right)+x\left(d\left(\frac{b\,x^2}{3}+a\right)e^{\frac{5}{2}}-\frac{4c\,d^2 e^{\frac{3}{2}} x^2}{3}-c\,d^3\sqrt{e}+\frac{2a\,e^{\frac{7}{2}} x^2}{3}\right)}{e^{\frac{5}{2}}(e\,x^2+d)^{\frac{3}{2}} d^2}$
default	$a\left(\frac{x}{3d(e\,x^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{e\,x^2+d}}\right)+c\left(-\frac{x^3}{3e(e\,x^2+d)^{\frac{3}{2}}}+\frac{-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}}{e}\right)+b\left(-\frac{x}{2e(e\,x^2+d)}\right.$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^(5/2)/(e*x^2+d)^(3/2)*((e*x^2+d)^(3/2)*c*d^2*arctanh((e*x^2+d)^(1/2)/x
/e^(1/2))+x*(d*(1/3*b*x^2+a)*e^(5/2)-4/3*c*d^2*e^(3/2)*x^2-c*d^3*e^(1/2)+2
/3*a*e^(7/2)*x^2))/d^2
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.86

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx = \left[\frac{3\,(cd^2e^2x^4+2\,cd^3ex^2+cd^4)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\,((4\,cd^2e^2-}{6\,(d^2e^5x^4+2\,d^3e^4x^2+d^4e^3)}\right.$$

$$-\frac{3\,(cd^2e^2x^4+2\,cd^3ex^2+cd^4)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+((4\,cd^2e^2-bde^3-2\,ae^4)x^3+3\,(cd^3e-ade^3)x)\sqrt{ex^2}}{3\,(d^2e^5x^4+2\,d^3e^4x^2+d^4e^3)}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(e)*log(-2*e*x^2 - 2*s
qrt(e*x^2 + d)*sqrt(e)*x - d) - 2*((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^3 +
 3*(c*d^3*e - a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 +
d^4*e^3), -1/3*(3*(c*d^2*e^2*x^4 + 2*c*d^3*e*x^2 + c*d^4)*sqrt(-e)*arctan(
sqrt(-e)*x/sqrt(e*x^2 + d)) + ((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^3 + 3*(
c*d^3*e - a*d*e^3)*x)*sqrt(e*x^2 + d))/(d^2*e^5*x^4 + 2*d^3*e^4*x^2 + d^4*
e^3)]
```

3.281.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(94) = 188.

Time = 6.51 (sec) , antiderivative size = 450, normalized size of antiderivative = 4.46

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx = a\left(\frac{3dx}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}} + \frac{2ex^3}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}}\right) + \frac{bx^3}{3d^{\frac{5}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{3}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}} + c\left(\frac{3d^{\frac{39}{2}}e^{11}\sqrt{1+\frac{ex^2}{d}}\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}} + \frac{3d^{\frac{37}{2}}e^{12}x^2\sqrt{1+\frac{ex^2}{d}}\operatorname{asinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}} - \frac{3d^{19}e^{\frac{23}{2}}x}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}} - \frac{4d^{18}e^{\frac{25}{2}}x^3}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^2}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^2\sqrt{1+\frac{ex^2}{d}}}\right)$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(5/2),x)
```

output
```
a*(3*d*x/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 + e*x**
2/d)) + 2*e*x**3/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1
 + e*x**2/d))) + b*x**3/(3*d**(5/2)*sqrt(1 + e*x**2/d) + 3*d**(3/2)*e*x**2
*sqrt(1 + e*x**2/d)) + c*(3*d**(39/2)*e**11*sqrt(1 + e*x**2/d)*asinh(sqrt(
e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(
29/2)*x**2*sqrt(1 + e*x**2/d)) + 3*d**(37/2)*e**12*x**2*sqrt(1 + e*x**2/d)
*asinh(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d*
*(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 3*d**19*e**(23/2)*x/(3*d**(39
/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x
**2/d)) - 4*d**18*e**(25/2)*x**3/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d)
 + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)))
```

3.281.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx = -\frac{x\left(\frac{(4\,cd^2e^2-bde^3-2\,ae^4)x^2}{d^2e^3}+\frac{3\,(cd^3e-ade^3)}{d^2e^3}\right)}{3\,(ex^2+d)^{\frac{3}{2}}}-\frac{c\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{e^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
-1/3*x*((4*c*d^2*e^2 - b*d*e^3 - 2*a*e^4)*x^2/(d^2*e^3) + 3*(c*d^3*e - a*d
*e^3)/(d^2*e^3))/(e*x^2 + d)^(3/2) - c*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d
)))/e^(5/2)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx = \int \frac{c\,x^4+b\,x^2+a}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2), x)
```

3.282 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx$

3.282.1 Optimal result

Integrand size = 24, antiderivative size = 86

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx=\frac{ax}{d(d+ex^2)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2(d+ex^2)^{5/2}}+\frac{(3cd^2+2e(bd+4ae))\,x^5}{15d^3(d+ex^2)^{5/2}}$$

output
```
a*x/d/(e*x^2+d)^(5/2)+1/3*(4*a*e+b*d)*x^3/d^2/(e*x^2+d)^(5/2)+1/15*(3*c*d^
2+2*e*(4*a*e+b*d))*x^5/d^3/(e*x^2+d)^(5/2)
```

3.282.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx=\frac{15ad^2x+5bd^2x^3+20adex^3+3cd^2x^5+2bdex^5+8ae^2x^5}{15d^3(d+ex^2)^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

output
```
(15*a*d^2*x + 5*b*d^2*x^3 + 20*a*d*e*x^3 + 3*c*d^2*x^5 + 2*b*d*e*x^5 + 8*a
*e^2*x^5)/(15*d^3*(d + e*x^2)^(5/2))
```

3.282.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1469, 2075, 362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{7/2}} \, dx$$

$$\Big\downarrow \text{1469}$$

$$\frac{\int \frac{x^2\left(4ae+d\left(cx^2+b\right)\right)}{\left(ex^2+d\right)^{7/2}}dx}{d}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{x^2\left(cdx^2+bd+4ae\right)}{\left(ex^2+d\right)^{7/2}}dx}{d}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{1}{5}\left(\frac{8ae}{d}+2b+\frac{3cd}{e}\right)\int \frac{x^2}{\left(ex^2+d\right)^{5/2}}dx+\frac{x^3\left(4ae+bd-\frac{cd^2}{e}\right)}{5d\left(d+ex^2\right)^{5/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{x^3\left(4ae+bd-\frac{cd^2}{e}\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{x^3\left(\frac{8ae}{d}+2b+\frac{3cd}{e}\right)}{15d\left(d+ex^2\right)^{3/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

output
```
(a*x)/(d*(d + e*x^2)^(5/2)) + (((b*d - (c*d^2)/e + 4*a*e)*x^3)/(5*d*(d + e
*x^2)^(5/2)) + ((2*b + (3*c*d)/e + (8*a*e)/d)*x^3)/(15*d*(d + e*x^2)^(3/2)
))/d
```

3.282.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1469
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d
 + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*
a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4
*p + 2*q + 1, 0]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

3.282.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{5}c\,x^4+\frac{1}{3}b\,x^2+a\right)d^2+\frac{4e\left(\frac{b\,x^2}{10}+a\right)x^2d}{3}+\frac{8a\,e^2x^4}{15}\right)}{\left(e\,x^2+d\right)^{\frac{5}{2}}d^3}$
gosper	$\frac{x\left(8a\,e^2x^4+2bde\,x^4+3c\,d^2x^4+20ade\,x^2+5b\,d^2x^2+15a\,d^2\right)}{15\left(e\,x^2+d\right)^{\frac{5}{2}}d^3}$
trager	$\frac{x\left(8a\,e^2x^4+2bde\,x^4+3c\,d^2x^4+20ade\,x^2+5b\,d^2x^2+15a\,d^2\right)}{15\left(e\,x^2+d\right)^{\frac{5}{2}}d^3}$
default	$a\left(\frac{x}{5d\left(e\,x^2+d\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d\left(e\,x^2+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^2\sqrt{e\,x^2+d}}}{d}\right)+c\left(-\frac{x^3}{2e\left(e\,x^2+d\right)^{\frac{5}{2}}}+\frac{3d\left(-\frac{x}{4e\left(e\,x^2+d\right)^{\frac{5}{2}}}+\frac{d\left(\frac{x}{5d\left(e\,x^2+d\right)^{\frac{5}{2}}}+\cdots\right)}{\cdots}\right)}{2e}\cdots\right.$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x^2+d)^(5/2)*x*((1/5*c*x^4+1/3*b*x^2+a)*d^2+4/3*e*(1/10*b*x^2+a)*x^2*
d+8/15*a*e^2*x^4)/d^3
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx=\frac{\left((3\,cd^2+2\,bde+8\,ae^2)x^5+15\,ad^2x+5\,(bd^2+4\,ade)x^3\right)\sqrt{ex^2+d}}{15\,(d^3e^3x^6+3\,d^4e^2x^4+3\,d^5ex^2+d^6)}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
1/15*((3*c*d^2 + 2*b*d*e + 8*a*e^2)*x^5 + 15*a*d^2*x + 5*(b*d^2 + 4*a*d*e)
*x^3)*sqrt(e*x^2 + d)/(d^3*e^3*x^6 + 3*d^4*e^2*x^4 + 3*d^5*e*x^2 + d^6)
```

3.282.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(80) = 160$.

Time = 14.75 (sec) , antiderivative size = 639, normalized size of antiderivative = 7.43

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx = a\left(\frac{15d^5x}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}} + \frac{35d^4ex^3}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}} + \frac{28d^3e^2x^5}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}} + \frac{8d^2e^3x^7}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{15}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+45d^{\frac{13}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{11}{2}}e^3x^6\sqrt{1+\frac{ex^2}{d}}}\right) + b\left(\frac{5dx^3}{15d^{\frac{9}{2}}\sqrt{1+\frac{ex^2}{d}}+30d^{\frac{7}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{5}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}} + \frac{2ex^5}{15d^{\frac{9}{2}}\sqrt{1+\frac{ex^2}{d}}+30d^{\frac{7}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+15d^{\frac{5}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}}\right) + \frac{cx^5}{5d^{\frac{7}{2}}\sqrt{1+\frac{ex^2}{d}}+10d^{\frac{5}{2}}ex^2\sqrt{1+\frac{ex^2}{d}}+5d^{\frac{3}{2}}e^2x^4\sqrt{1+\frac{ex^2}{d}}}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(7/2),x)
```

output
```
a*(15*d**5*x/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1
 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e*
*3*x**6*sqrt(1 + e*x**2/d)) + 35*d**4*e*x**3/(15*d**(17/2)*sqrt(1 + e*x**2
/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt
(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 28*d**3*e**2
*x**5/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x*
*2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6
*sqrt(1 + e*x**2/d)) + 8*d**2*e**3*x**7/(15*d**(17/2)*sqrt(1 + e*x**2/d) +
 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 +
e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d))) + b*(5*d*x**3/(15*
d**(9/2)*sqrt(1 + e*x**2/d) + 30*d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d
**(5/2)*e**2*x**4*sqrt(1 + e*x**2/d)) + 2*e*x**5/(15*d**(9/2)*sqrt(1 + e*x
**2/d) + 30*d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**2*x**4*sqr
t(1 + e*x**2/d))) + c*x**5/(5*d**(7/2)*sqrt(1 + e*x**2/d) + 10*d**(5/2)*e*
x**2*sqrt(1 + e*x**2/d) + 5*d**(3/2)*e**2*x**4*sqrt(1 + e*x**2/d))
```

3.282.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(76) = 152$.

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.01

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx = -\frac{cx^3}{2\,(ex^2+d)^{\frac{5}{2}}e} + \frac{8\,ax}{15\,\sqrt{ex^2+d}d^3} + \frac{4\,ax}{15\,(ex^2+d)^{\frac{3}{2}}d^2} + \frac{ax}{5\,(ex^2+d)^{\frac{5}{2}}d} + \frac{cx}{10\,(ex^2+d)^{\frac{3}{2}}e^2} + \frac{cx}{5\,\sqrt{ex^2+d}de^2} - \frac{3\,cdx}{10\,(ex^2+d)^{\frac{5}{2}}e^2} - \frac{bx}{5\,(ex^2+d)^{\frac{5}{2}}e} + \frac{2\,bx}{15\,\sqrt{ex^2+d}d^2e} + \frac{bx}{15\,(ex^2+d)^{\frac{3}{2}}de}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
-1/2*c*x^3/((e*x^2 + d)^(5/2)*e) + 8/15*a*x/(sqrt(e*x^2 + d)*d^3) + 4/15*a
*x/((e*x^2 + d)^(3/2)*d^2) + 1/5*a*x/((e*x^2 + d)^(5/2)*d) + 1/10*c*x/((e*
x^2 + d)^(3/2)*e^2) + 1/5*c*x/(sqrt(e*x^2 + d)*d*e^2) - 3/10*c*d*x/((e*x^2
 + d)^(5/2)*e^2) - 1/5*b*x/((e*x^2 + d)^(5/2)*e) + 2/15*b*x/(sqrt(e*x^2 +
d)*d^2*e) + 1/15*b*x/((e*x^2 + d)^(3/2)*d*e)
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{\left(x^2\left(\frac{(3\,cd^2e^2+2\,bde^3+8\,ae^4)x^2}{d^3e^2}+\frac{5\,(bd^2e^2+4\,ade^3)}{d^3e^2}\right)+\frac{15\,a}{d}\right)x}{15\,(ex^2+d)^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
1/15*(x^2*((3*c*d^2*e^2 + 2*b*d*e^3 + 8*a*e^4)*x^2/(d^3*e^2) + 5*(b*d^2*e^
2 + 4*a*d*e^3)/(d^3*e^2)) + 15*a/d)*x/(e*x^2 + d)^(5/2)
```

3.282.9 Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.55

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx = \frac{3\,c\,d^4\,x-6\,c\,d^3\,x\,(e\,x^2+d)-3\,b\,d^3\,e\,x+8\,a\,e^2\,x\,(e\,x^2+d)^2+3\,c\,d^2\,x\,(e\,x^2+d)^2+3}{15\,d^3\,e^2\,(e\,x^2+d}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x)
```

output
```
(3*c*d^4*x - 6*c*d^3*x*(d + e*x^2) - 3*b*d^3*e*x + 8*a*e^2*x*(d + e*x^2)^2
 + 3*c*d^2*x*(d + e*x^2)^2 + 3*a*d^2*e^2*x + 4*a*d*e^2*x*(d + e*x^2) + 2*b
*d*e*x*(d + e*x^2)^2 + b*d^2*e*x*(d + e*x^2))/(15*d^3*e^2*(d + e*x^2)^(5/2
))
```

3.283 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx$

3.283.1 Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{ax}{d(d+ex^2)^{7/2}} + \frac{(bd+6ae)x^3}{3d^2(d+ex^2)^{7/2}} + \frac{(3cd^2+4e(bd+6ae))x^5}{15d^3(d+ex^2)^{7/2}} + \frac{2e(3cd^2+4e(bd+6ae))x^7}{105d^4(d+ex^2)^{7/2}}$$

output
```
a*x/d/(e*x^2+d)^(7/2)+1/3*(6*a*e+b*d)*x^3/d^2/(e*x^2+d)^(7/2)+1/15*(3*c*d^
2+4*e*(6*a*e+b*d))*x^5/d^3/(e*x^2+d)^(7/2)+2/105*e*(3*c*d^2+4*e*(6*a*e+b*d
))*x^7/d^4/(e*x^2+d)^(7/2)
```

3.283.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{105ad^3x+35bd^3x^3+210ad^2ex^3+21cd^3x^5+28bd^2ex^5+168ade^2x^5+6cd^2ex^7+8bc}{105d^4(d+ex^2)^{7/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

output
```
(105*a*d^3*x + 35*b*d^3*x^3 + 210*a*d^2*e*x^3 + 21*c*d^3*x^5 + 28*b*d^2*e*
x^5 + 168*a*d*e^2*x^5 + 6*c*d^2*e*x^7 + 8*b*d*e^2*x^7 + 48*a*e^3*x^7)/(105
*d^4*(d + e*x^2)^(7/2))
```

3.283.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1469, 2075, 362, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{ 1469}$$

$$\frac{\int \frac{x^2(6ae+d(cx^2+b))}{(ex^2+d)^{9/2}}dx}{d}+\frac{ax}{d(d+ex^2)^{7/2}}$$

$$\Big\downarrow \text{ 2075}$$

$$\frac{\int \frac{x^2(cdx^2+bd+6ae)}{(ex^2+d)^{9/2}}dx}{d}+\frac{ax}{d(d+ex^2)^{7/2}}$$

$$\Big\downarrow \text{ 362}$$

$$\frac{\frac{1}{7}\left(\frac{4(6ae+bd)}{d}+\frac{3cd}{e}\right)\int \frac{x^2}{(ex^2+d)^{7/2}}dx+\frac{x^3\left(6ae+bd-\frac{cd^2}{e}\right)}{7d(d+ex^2)^{7/2}}}{d}+\frac{ax}{d(d+ex^2)^{7/2}}$$

$$\Big\downarrow \text{ 245}$$

$$\frac{\frac{1}{7}\left(\frac{4(6ae+bd)}{d}+\frac{3cd}{e}\right)\left(\frac{2e\int \frac{x^4}{(ex^2+d)^{7/2}}dx}{3d}+\frac{x^3}{3d(d+ex^2)^{5/2}}\right)+\frac{x^3\left(6ae+bd-\frac{cd^2}{e}\right)}{7d(d+ex^2)^{7/2}}}{d}+\frac{ax}{d(d+ex^2)^{7/2}}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{\frac{x^3\left(6ae+bd-\frac{cd^2}{e}\right)}{7d(d+ex^2)^{7/2}}+\frac{1}{7}\left(\frac{2ex^5}{15d^2(d+ex^2)^{5/2}}+\frac{x^3}{3d(d+ex^2)^{5/2}}\right)\left(\frac{4(6ae+bd)}{d}+\frac{3cd}{e}\right)}{d}+\frac{ax}{d(d+ex^2)^{7/2}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

output
```
(a*x)/(d*(d + e*x^2)^(7/2)) + (((b*d - (c*d^2)/e + 6*a*e)*x^3)/(7*d*(d + e
*x^2)^(7/2)) + (((3*c*d)/e + (4*(b*d + 6*a*e))/d)*(x^3/(3*d*(d + e*x^2)^(5
/2)) + (2*e*x^5)/(15*d^2*(d + e*x^2)^(5/2))))/7)/d
```

3.283.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1469
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d
 + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*
a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4
*p + 2*q + 1, 0]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

3.283. $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}} dx$

3.283.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{\left(\left(\frac{1}{5}c\,x^{4}+\frac{1}{3}b\,x^{2}+a\right)d^{3}+2e\left(\frac{1}{35}c\,x^{4}+\frac{2}{15}b\,x^{2}+a\right)x^{2}d^{2}+\frac{8\left(\frac{b\,x^{2}}{21}+a\right)e^{2}x^{4}d}{5}+\frac{16a\,e^{3}x^{6}}{35}\right)x}{\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
gosper	$\frac{x\left(48a\,e^{3}x^{6}+8bd\,e^{2}x^{6}+6c\,d^{2}e\,x^{6}+168ad\,e^{2}x^{4}+28b\,d^{2}e\,x^{4}+21c\,d^{3}x^{4}+210a\,d^{2}e\,x^{2}+35b\,d^{3}x^{2}+105d^{3}a\right)}{105\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
trager	$\frac{x\left(48a\,e^{3}x^{6}+8bd\,e^{2}x^{6}+6c\,d^{2}e\,x^{6}+168ad\,e^{2}x^{4}+28b\,d^{2}e\,x^{4}+21c\,d^{3}x^{4}+210a\,d^{2}e\,x^{2}+35b\,d^{3}x^{2}+105d^{3}a\right)}{105\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
default	$a\left(\frac{x}{7d\left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^{2}\sqrt{e\,x^{2}+d}}\right)}{7d}}{d}\right)+c\left(-\frac{x^{3}}{4e\left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{3d\left(-\frac{x}{6e\left(e\,x^{2}+d\right)^{\frac{7}{2}}}\right.}{}\right.$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((1/5*c*x^4+1/3*b*x^2+a)*d^3+2*e*(1/35*c*x^4+2/15*b*x^2+a)*x^2*d^2+8/5*(1/
21*b*x^2+a)*e^2*x^4*d+16/35*a*e^3*x^6)/(e*x^2+d)^(7/2)*x/d^4
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{(2\,(3\,cd^2e+4\,bde^2+24\,ae^3)x^7+7\,(3\,cd^3+4\,bd^2e+24\,ade^2)x^5+105\,ad^3x+35\,(bd^3}{105\,(d^4e^4x^8+4\,d^5e^3x^6+6\,d^6e^2x^4+4\,d^7ex^2+d^8)}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(2*(3*c*d^2*e + 4*b*d*e^2 + 24*a*e^3)*x^7 + 7*(3*c*d^3 + 4*b*d^2*e +
 24*a*d*e^2)*x^5 + 105*a*d^3*x + 35*(b*d^3 + 6*a*d^2*e)*x^3)*sqrt(e*x^2 +
d)/(d^4*e^4*x^8 + 4*d^5*e^3*x^6 + 6*d^6*e^2*x^4 + 4*d^7*e*x^2 + d^8)
```

3.283.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1989 vs. 2(119) = 238.

Time = 35.55 (sec) , antiderivative size = 1989, normalized size of antiderivative = 15.79

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(9/2),x)
```

output
```
a*(35*d**14*x/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt
(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2
)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d
) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*
sqrt(1 + e*x**2/d)) + 175*d**13*e*x**3/(35*d**(37/2)*sqrt(1 + e*x**2/d) +
210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 +
 e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e*
*4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) +
 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 371*d**12*e**2*x**5/(35*d**
(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*
d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 +
e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**
5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) +
 429*d**11*e**3*x**7/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x*
*2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d
**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e
*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6
*x**12*sqrt(1 + e*x**2/d)) + 286*d**10*e**4*x**9/(35*d**(37/2)*sqrt(1 + e*
x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**
4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525...
```

3.283.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(112) = 224$.

Time = 0.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.80

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx = -\frac{cx^3}{4\,(ex^2+d)^{\frac{7}{2}}e} + \frac{16\,ax}{35\,\sqrt{ex^2+d}d^4} + \frac{8\,ax}{35\,(ex^2+d)^{\frac{3}{2}}d^3} + \frac{6\,ax}{35\,(ex^2+d)^{\frac{5}{2}}d^2} + \frac{ax}{7\,(ex^2+d)^{\frac{7}{2}}d} + \frac{3\,cx}{140\,(ex^2+d)^{\frac{5}{2}}e^2} + \frac{2\,cx}{35\,\sqrt{ex^2+d}d^2e^2} + \frac{cx}{35\,(ex^2+d)^{\frac{3}{2}}de^2} - \frac{3\,cdx}{28\,(ex^2+d)^{\frac{7}{2}}e^2} - \frac{bx}{7\,(ex^2+d)^{\frac{7}{2}}e} + \frac{8\,bx}{105\,\sqrt{ex^2+d}d^3e} + \frac{4\,bx}{105\,(ex^2+d)^{\frac{3}{2}}d^2e} + \frac{bx}{35\,(ex^2+d)^{\frac{5}{2}}de}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

output
```
-1/4*c*x^3/((e*x^2 + d)^(7/2)*e) + 16/35*a*x/(sqrt(e*x^2 + d)*d^4) + 8/35*
a*x/((e*x^2 + d)^(3/2)*d^3) + 6/35*a*x/((e*x^2 + d)^(5/2)*d^2) + 1/7*a*x/(
(e*x^2 + d)^(7/2)*d) + 3/140*c*x/((e*x^2 + d)^(5/2)*e^2) + 2/35*c*x/(sqrt(
e*x^2 + d)*d^2*e^2) + 1/35*c*x/((e*x^2 + d)^(3/2)*d*e^2) - 3/28*c*d*x/((e*
x^2 + d)^(7/2)*e^2) - 1/7*b*x/((e*x^2 + d)^(7/2)*e) + 8/105*b*x/(sqrt(e*x^
2 + d)*d^3*e) + 4/105*b*x/((e*x^2 + d)^(3/2)*d^2*e) + 1/35*b*x/((e*x^2 + d
)^(5/2)*d*e)
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{2\left(3\,cd^2e^4+4\,bde^5+24\,ae^6\right)x^2}{d^4e^3}+\frac{7\left(3\,cd^3e^3+4\,bd^2e^4+24\,ade^5\right)}{d^4e^3}\right)+\frac{35\left(bd^3e^3+6\,ad^2e^4\right)}{d^4e^3}\right)x^2+\frac{105\,a}{d}\right)}{105\,(ex^2+d)^{\frac{7}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*(2*(3*c*d^2*e^4 + 4*b*d*e^5 + 24*a*e^6)*x^2/(d^4*e^3) + 7*(3*c
*d^3*e^3 + 4*b*d^2*e^4 + 24*a*d*e^5)/(d^4*e^3)) + 35*(b*d^3*e^3 + 6*a*d^2*
e^4)/(d^4*e^3))*x^2 + 105*a/d)*x/(e*x^2 + d)^(7/2)
```

3.283.9 Mupad [B] (verification not implemented)

Time = 7.88 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx = \frac{x\left(\frac{a}{7\,d}-\frac{d\left(\frac{b}{7\,d}-\frac{c}{7\,e}\right)}{e}\right)}{\left(e\,x^2+d\right)^{7/2}}-\frac{x\left(\frac{c}{5\,e^2}-\frac{-c\,d^2+b\,d\,e+6\,a\,e^2}{35\,d^2\,e^2}\right)}{\left(e\,x^2+d\right)^{5/2}} + \frac{x\left(3\,c\,d^2+4\,b\,d\,e+24\,a\,e^2\right)}{105\,d^3\,e^2\,\left(e\,x^2+d\right)^{3/2}}+\frac{x\left(6\,c\,d^2+8\,b\,d\,e+48\,a\,e^2\right)}{105\,d^4\,e^2\,\sqrt{e\,x^2+d}}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x)
```

output
```
(x*(a/(7*d) - (d*(b/(7*d) - c/(7*e)))/e))/(d + e*x^2)^(7/2) - (x*(c/(5*e^2
) - (6*a*e^2 - c*d^2 + b*d*e)/(35*d^2*e^2)))/(d + e*x^2)^(5/2) + (x*(24*a*
e^2 + 3*c*d^2 + 4*b*d*e))/(105*d^3*e^2*(d + e*x^2)^(3/2)) + (x*(48*a*e^2 +
 6*c*d^2 + 8*b*d*e))/(105*d^4*e^2*(d + e*x^2)^(1/2))
```

3.284 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx$

3.284.1 Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx = \frac{ax}{d\,(d+ex^2)^{9/2}} + \frac{(bd+8ae)x^3}{3d^2\,(d+ex^2)^{9/2}} + \frac{(cd^2+2e(bd+8ae))\,x^5}{5d^3\,(d+ex^2)^{9/2}} + \frac{4e(cd^2+2e(bd+8ae))\,x^7}{35d^4\,(d+ex^2)^{9/2}} + \frac{8e^2(cd^2+2e(bd+8ae))\,x^9}{315d^5\,(d+ex^2)^{9/2}}$$

output
```
a*x/d/(e*x^2+d)^(9/2)+1/3*(8*a*e+b*d)*x^3/d^2/(e*x^2+d)^(9/2)+1/5*(c*d^2+2
*e*(8*a*e+b*d))*x^5/d^3/(e*x^2+d)^(9/2)+4/35*e*(c*d^2+2*e*(8*a*e+b*d))*x^7
/d^4/(e*x^2+d)^(9/2)+8/315*e^2*(c*d^2+2*e*(8*a*e+b*d))*x^9/d^5/(e*x^2+d)^(
9/2)
```

3.284.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx = \frac{a(315d^4x+840d^3ex^3+1008d^2e^2x^5+576de^3x^7+128e^4x^9)+dx^3(cdx^2(63d^2+36dex}{315d^5\,(d+ex^2)^{9/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

output
```
(a*(315*d^4*x + 840*d^3*e*x^3 + 1008*d^2*e^2*x^5 + 576*d*e^3*x^7 + 128*e^4
*x^9) + d*x^3*(c*d*x^2*(63*d^2 + 36*d*e*x^2 + 8*e^2*x^4) + b*(105*d^3 + 12
6*d^2*e*x^2 + 72*d*e^2*x^4 + 16*e^3*x^6)))/(315*d^5*(d + e*x^2)^(9/2))
```

3.284.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1469, 2075, 362, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}dx$$

$$\Big\downarrow \text{1469}$$

$$\frac{\int \frac{x^2(8ae+d(cx^2+b))}{(ex^2+d)^{11/2}}dx}{d}+\frac{ax}{d(d+ex^2)^{9/2}}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{x^2(cdx^2+bd+8ae)}{(ex^2+d)^{11/2}}dx}{d}+\frac{ax}{d(d+ex^2)^{9/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{1}{3}\left(\frac{2(8ae+bd)}{d}+\frac{cd}{e}\right)\int \frac{x^2}{(ex^2+d)^{9/2}}dx+\frac{x^3\left(8ae+bd-\frac{cd^2}{e}\right)}{9d(d+ex^2)^{9/2}}}{d}+\frac{ax}{d(d+ex^2)^{9/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{1}{3}\left(\frac{2(8ae+bd)}{d}+\frac{cd}{e}\right)\left(\frac{4e\int \frac{x^4}{(ex^2+d)^{9/2}}dx}{3d}+\frac{x^3}{3d(d+ex^2)^{7/2}}\right)+\frac{x^3\left(8ae+bd-\frac{cd^2}{e}\right)}{9d(d+ex^2)^{9/2}}}{d}+\frac{ax}{d(d+ex^2)^{9/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{1}{3}\left(\frac{2(8ae+bd)}{d}+\frac{cd}{e}\right)\left(\frac{4e\left(\frac{2e\int \frac{x^6}{(ex^2+d)^{9/2}}dx}{5d}+\frac{x^5}{5d(d+ex^2)^{7/2}}\right)}{3d}+\frac{x^3}{3d(d+ex^2)^{7/2}}\right)+\frac{x^3\left(8ae+bd-\frac{cd^2}{e}\right)}{9d(d+ex^2)^{9/2}}}{d}+\frac{ax}{d(d+ex^2)^{9/2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{x^3\left(8ae+bd-\frac{cd^2}{e}\right)}{9d(d+ex^2)^{9/2}}+\frac{1}{3}\left(\frac{4e\left(\frac{2ex^7}{35d^2\left(d+ex^2\right)^{7/2}}+\frac{x^5}{5d\left(d+ex^2\right)^{7/2}}\right)}{3d}+\frac{x^3}{3d(d+ex^2)^{7/2}}\right)\left(\frac{2(8ae+bd)}{d}+\frac{cd}{e}\right)}{d}+\frac{ax}{d\left(d+ex^2\right)^{9/2}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

output
```
(a*x)/(d*(d + e*x^2)^(9/2)) + (((b*d - (c*d^2)/e + 8*a*e)*x^3)/(9*d*(d + e
*x^2)^(9/2)) + (((c*d)/e + (2*(b*d + 8*a*e))/d)*(x^3/(3*d*(d + e*x^2)^(7/2
)) + (4*e*(x^5/(5*d*(d + e*x^2)^(7/2)) + (2*e*x^7)/(35*d^2*(d + e*x^2)^(7/
2))))/(3*d)))/3)/d
```

3.284.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1469
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d
 + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*
a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4
*p + 2*q + 1, 0]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

3.284.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{\left(\left(\frac{1}{5}c\,x^{4}+\frac{1}{3}b\,x^{2}+a\right)d^{4}+\frac{8e\left(\frac{3}{70}c\,x^{4}+\frac{3}{20}b\,x^{2}+a\right)x^{2}d^{3}}{3}+\frac{16e^{2}\left(\frac{1}{126}c\,x^{4}+\frac{1}{14}b\,x^{2}+a\right)x^{4}d^{2}}{5}+\frac{64\left(\frac{b\,x^{2}}{36}+a\right)e^{3}x^{6}d}{35}+\frac{128a\,e^{4}x^{8}}{315}\right)x}{\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
gosper	$\frac{x\left(128a\,e^{4}x^{8}+16bd\,e^{3}x^{8}+8c\,d^{2}e^{2}x^{8}+576ad\,e^{3}x^{6}+72b\,d^{2}e^{2}x^{6}+36c\,d^{3}e\,x^{6}+1008a\,d^{2}e^{2}x^{4}+126b\,d^{3}e\,x^{4}+63c\,d^{4}x^{4}+840a\,d^{3}e\,x\right.}{315\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
trager	$\frac{x\left(128a\,e^{4}x^{8}+16bd\,e^{3}x^{8}+8c\,d^{2}e^{2}x^{8}+576ad\,e^{3}x^{6}+72b\,d^{2}e^{2}x^{6}+36c\,d^{3}e\,x^{6}+1008a\,d^{2}e^{2}x^{4}+126b\,d^{3}e\,x^{4}+63c\,d^{4}x^{4}+840a\,d^{3}e\,x\right.}{315\left(e\,x^{2}+d\right)^{\frac{9}{2}}d^{5}}$
default	$a\left(\frac{x}{9d\left(e\,x^{2}+d\right)^{\frac{9}{2}}}+\frac{\frac{8x}{63d\left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^{2}\sqrt{e\,x^{2}+d}}\right)}{7d}\right)}{9d}}{d}\right)+c\left(-\frac{x^{3}}{6e\left(e\,x^{2}+d\right)^{\frac{9}{2}}}+\right.$

3.284. $\int \frac{a+bx^{2}+cx^{4}}{\left(d+ex^{2}\right)^{11/2}}\,dx$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
((1/5*c*x^4+1/3*b*x^2+a)*d^4+8/3*e*(3/70*c*x^4+3/20*b*x^2+a)*x^2*d^3+16/5*
e^2*(1/126*c*x^4+1/14*b*x^2+a)*x^4*d^2+64/35*(1/36*b*x^2+a)*e^3*x^6*d+128/
315*a*e^4*x^8)/(e*x^2+d)^(9/2)*x/d^5
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx = \frac{(8\,(cd^2e^2+2\,bde^3+16\,ae^4)x^9+36\,(cd^3e+2\,bd^2e^2+16\,ade^3)x^7+315\,ad^4x+63\,(cd^4}{315\,(d^5e^5x^{10}+5\,d^6e^4x^8+10\,d^7e^3x^6+10\,d^8e^2x}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="fricas")
```

output
```
1/315*(8*(c*d^2*e^2 + 2*b*d*e^3 + 16*a*e^4)*x^9 + 36*(c*d^3*e + 2*b*d^2*e^
2 + 16*a*d*e^3)*x^7 + 315*a*d^4*x + 63*(c*d^4 + 2*b*d^3*e + 16*a*d^2*e^2)*
x^5 + 105*(b*d^4 + 8*a*d^3*e)*x^3)*sqrt(e*x^2 + d)/(d^5*e^5*x^10 + 5*d^6*e
^4*x^8 + 10*d^7*e^3*x^6 + 10*d^8*e^2*x^4 + 5*d^9*e*x^2 + d^10)
```

3.284.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5187 vs. $2(160) = 320$.

Time = 80.52 (sec) , antiderivative size = 5187, normalized size of antiderivative = 31.44

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(11/2),x)
```

output
```
a*(315*d**30*x/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e*x**2*s
qrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 37800*d
**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*sqrt(1 +
 e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*d**(59/
2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(1 + e*x
**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53/2)*e*
*9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x**2/d)
) + 2730*d**29*e*x**3/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e
*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) +
37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*s
qrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*
d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(
1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(5
3/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*
x**2/d)) + 10773*d**28*e**2*x**5/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*
d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e
*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*
e**4*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/
d) + 66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*
x**14*sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d...
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.70

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx =& -\frac{cx^3}{6\,(ex^2+d)^{\frac{9}{2}}e}+\frac{128\,ax}{315\,\sqrt{ex^2+d}d^5}\\ &+\frac{64\,ax}{315\,(ex^2+d)^{\frac{3}{2}}d^4}+\frac{16\,ax}{105\,(ex^2+d)^{\frac{5}{2}}d^3}+\frac{8\,ax}{63\,(ex^2+d)^{\frac{7}{2}}d^2}+\frac{ax}{9\,(ex^2+d)^{\frac{9}{2}}d}\\ &+\frac{cx}{126\,(ex^2+d)^{\frac{7}{2}}e^2}+\frac{8\,cx}{315\,\sqrt{ex^2+d}d^3e^2}+\frac{4\,cx}{315\,(ex^2+d)^{\frac{3}{2}}d^2e^2}\\ &+\frac{cx}{105\,(ex^2+d)^{\frac{5}{2}}de^2}-\frac{cdx}{18\,(ex^2+d)^{\frac{9}{2}}e^2}-\frac{bx}{9\,(ex^2+d)^{\frac{9}{2}}e}+\frac{16\,bx}{315\,\sqrt{ex^2+d}d^4e}\\ &+\frac{8\,bx}{315\,(ex^2+d)^{\frac{3}{2}}d^3e}+\frac{2\,bx}{105\,(ex^2+d)^{\frac{5}{2}}d^2e}+\frac{bx}{63\,(ex^2+d)^{\frac{7}{2}}de}\end{aligned}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="maxima")
```

output
```
-1/6*c*x^3/((e*x^2 + d)^(9/2)*e) + 128/315*a*x/(sqrt(e*x^2 + d)*d^5) + 64/
315*a*x/((e*x^2 + d)^(3/2)*d^4) + 16/105*a*x/((e*x^2 + d)^(5/2)*d^3) + 8/6
3*a*x/((e*x^2 + d)^(7/2)*d^2) + 1/9*a*x/((e*x^2 + d)^(9/2)*d) + 1/126*c*x/
((e*x^2 + d)^(7/2)*e^2) + 8/315*c*x/(sqrt(e*x^2 + d)*d^3*e^2) + 4/315*c*x/
((e*x^2 + d)^(3/2)*d^2*e^2) + 1/105*c*x/((e*x^2 + d)^(5/2)*d*e^2) - 1/18*c
*d*x/((e*x^2 + d)^(9/2)*e^2) - 1/9*b*x/((e*x^2 + d)^(9/2)*e) + 16/315*b*x/
(sqrt(e*x^2 + d)*d^4*e) + 8/315*b*x/((e*x^2 + d)^(3/2)*d^3*e) + 2/105*b*x/
((e*x^2 + d)^(5/2)*d^2*e) + 1/63*b*x/((e*x^2 + d)^(7/2)*d*e)
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx = \frac{\left(\left(\left(4\,x^2\left(\frac{2\,(cd^2e^6+2\,bde^7+16\,ae^8)x^2}{d^5e^4}+\frac{9\,(cd^3e^5+2\,bd^2e^6+16\,ade^7)}{d^5e^4}\right)+\frac{63\,(cd^4e^4+2\,bd^3e^5+16\,ad^2e^6)}{d^5e^4}\right)x^2}{315\,(ex^2+d)^{\frac{9}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="giac")
```

output
```
1/315*(((4*x^2*(2*(c*d^2*e^6 + 2*b*d*e^7 + 16*a*e^8)*x^2/(d^5*e^4) + 9*(c*
d^3*e^5 + 2*b*d^2*e^6 + 16*a*d*e^7)/(d^5*e^4)) + 63*(c*d^4*e^4 + 2*b*d^3*e
^5 + 16*a*d^2*e^6)/(d^5*e^4))*x^2 + 105*(b*d^4*e^4 + 8*a*d^3*e^5)/(d^5*e^4
))*x^2 + 315*a/d)*x/(e*x^2 + d)^(9/2)
```

3.284.9 Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx = \frac{x\left(\frac{a}{9\,d}-\frac{d\left(\frac{b}{9\,d}-\frac{c}{9\,e}\right)}{e}\right)}{(e\,x^2+d)^{9/2}} - \frac{x\left(\frac{c}{7\,e^2}-\frac{-c\,d^2+b\,d\,e+8\,a\,e^2}{63\,d^2\,e^2}\right)}{(e\,x^2+d)^{7/2}} + \frac{x\,(c\,d^2+2\,b\,d\,e+16\,a\,e^2)}{105\,d^3\,e^2\,(e\,x^2+d)^{5/2}} + \frac{x\,(4\,c\,d^2+8\,b\,d\,e+64\,a\,e^2)}{315\,d^4\,e^2\,(e\,x^2+d)^{3/2}} + \frac{x\,(8\,c\,d^2+16\,b\,d\,e+128\,a\,e^2)}{315\,d^5\,e^2\,\sqrt{e\,x^2+d}}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x)
```

output
```
(x*(a/(9*d) - (d*(b/(9*d) - c/(9*e)))/e))/(d + e*x^2)^(9/2) - (x*(c/(7*e^2
) - (8*a*e^2 - c*d^2 + b*d*e)/(63*d^2*e^2)))/(d + e*x^2)^(7/2) + (x*(16*a*
e^2 + c*d^2 + 2*b*d*e))/(105*d^3*e^2*(d + e*x^2)^(5/2)) + (x*(64*a*e^2 + 4
*c*d^2 + 8*b*d*e))/(315*d^4*e^2*(d + e*x^2)^(3/2)) + (x*(128*a*e^2 + 8*c*d
^2 + 16*b*d*e))/(315*d^5*e^2*(d + e*x^2)^(1/2))
```

3.284. $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx$

3.285 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx$

3.285.1 Optimal result

Integrand size = 24, antiderivative size = 210

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx = \frac{ax}{d(d+ex^2)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2(d+ex^2)^{11/2}} + \frac{(3cd^2+8e(bd+10ae))x^5}{15d^3(d+ex^2)^{11/2}} + \frac{2e(3cd^2+8e(bd+10ae))x^7}{35d^4(d+ex^2)^{11/2}} + \frac{8e^2(3cd^2+8e(bd+10ae))x^9}{315d^5(d+ex^2)^{11/2}} + \frac{16e^3(3cd^2+8e(bd+10ae))x^{11}}{3465d^6(d+ex^2)^{11/2}}$$

output
```
a*x/d/(e*x^2+d)^(11/2)+1/3*(10*a*e+b*d)*x^3/d^2/(e*x^2+d)^(11/2)+1/15*(3*c
*d^2+8*e*(10*a*e+b*d))*x^5/d^3/(e*x^2+d)^(11/2)+2/35*e*(3*c*d^2+8*e*(10*a*
e+b*d))*x^7/d^4/(e*x^2+d)^(11/2)+8/315*e^2*(3*c*d^2+8*e*(10*a*e+b*d))*x^9/
d^5/(e*x^2+d)^(11/2)+16/3465*e^3*(3*c*d^2+8*e*(10*a*e+b*d))*x^11/d^6/(e*x^
2+d)^(11/2)
```

3.285.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.80

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx = \frac{5a(693d^5x+2310d^4ex^3+3696d^3e^2x^5+3168d^2e^3x^7+1408de^4x^9+256e^5x^{11})+dx^3(}{}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

output
```
(5*a*(693*d^5*x + 2310*d^4*e*x^3 + 3696*d^3*e^2*x^5 + 3168*d^2*e^3*x^7 + 1
408*d*e^4*x^9 + 256*e^5*x^11) + d*x^3*(3*c*d*x^2*(231*d^3 + 198*d^2*e*x^2
+ 88*d*e^2*x^4 + 16*e^3*x^6) + b*(1155*d^4 + 1848*d^3*e*x^2 + 1584*d^2*e^2
*x^4 + 704*d*e^3*x^6 + 128*e^4*x^8)))/(3465*d^6*(d + e*x^2)^(11/2))
```

3.285.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1469, 2075, 362, 245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}dx$$

$$\Big\downarrow \text{1469}$$

$$\frac{\int \frac{x^2(10ae+d(cx^2+b))}{(ex^2+d)^{13/2}}dx}{d}+\frac{ax}{d(d+ex^2)^{11/2}}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{x^2(cdx^2+bd+10ae)}{(ex^2+d)^{13/2}}dx}{d}+\frac{ax}{d(d+ex^2)^{11/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{1}{11}\left(\frac{80ae}{d}+8b+\frac{3cd}{e}\right)\int \frac{x^2}{(ex^2+d)^{11/2}}dx+\frac{x^3\left(10ae+bd-\frac{cd^2}{e}\right)}{11d(d+ex^2)^{11/2}}}{d}+\frac{ax}{d(d+ex^2)^{11/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{1}{11}\left(\frac{80ae}{d}+8b+\frac{3cd}{e}\right)\left(\frac{2e\int \frac{x^4}{(ex^2+d)^{11/2}}dx}{d}+\frac{x^3}{3d(d+ex^2)^{9/2}}\right)+\frac{x^3\left(10ae+bd-\frac{cd^2}{e}\right)}{11d(d+ex^2)^{11/2}}}{d}+\frac{ax}{d(d+ex^2)^{11/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{1}{11}\left(\frac{80ae}{d}+8b+\frac{3cd}{e}\right)\left(\frac{2e\left(\frac{4e\int\frac{x^6}{\left(ex^2+d\right)^{11/2}}dx}{5d}+\frac{x^5}{5d\left(d+ex^2\right)^{9/2}}\right)}{d}+\frac{x^3}{3d(d+ex^2)^{9/2}}\right)+\frac{x^3\left(10ae+bd-\frac{cd^2}{e}\right)}{11d(d+ex^2)^{11/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \text{ 245}$$

$$\frac{\frac{1}{11}\left(\frac{80ae}{d}+8b+\frac{3cd}{e}\right)\left(\frac{2e\left(\frac{4e\left(\frac{2e\int\frac{x^8}{\left(ex^2+d\right)^{11/2}}dx}{7d}+\frac{x^7}{7d\left(d+ex^2\right)^{9/2}}\right)}{5d}+\frac{x^5}{5d\left(d+ex^2\right)^{9/2}}\right)}{d}+\frac{x^3}{3d(d+ex^2)^{9/2}}\right)+\frac{x^3\left(10ae+bd-\frac{cd^2}{e}\right)}{11d(d+ex^2)^{11/2}}}{d}+\frac{ax}{d\left(d+ex^2\right)^{11/2}}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{\frac{x^3\left(10ae+bd-\frac{cd^2}{e}\right)}{11d(d+ex^2)^{11/2}}+\frac{1}{11}\left(\frac{2e\left(\frac{4e\left(\frac{2ex^9}{63d^2\left(d+ex^2\right)^{9/2}}+\frac{x^7}{7d\left(d+ex^2\right)^{9/2}}\right)}{5d}+\frac{x^5}{5d\left(d+ex^2\right)^{9/2}}\right)}{d}+\frac{x^3}{3d(d+ex^2)^{9/2}}\right)\left(\frac{80ae}{d}+8b+\frac{3cd}{e}\right)}{d}+\frac{ax}{d\left(d+ex^2\right)^{11/2}}$$

input

```
Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

3.285. $\int\frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx$

output
```
(a*x)/(d*(d + e*x^2)^(11/2)) + (((b*d - (c*d^2)/e + 10*a*e)*x^3)/(11*d*(d
+ e*x^2)^(11/2)) + ((8*b + (3*c*d)/e + (80*a*e)/d)*(x^3/(3*d*(d + e*x^2)^(
9/2)) + (2*e*(x^5/(5*d*(d + e*x^2)^(9/2)) + (4*e*(x^7/(7*d*(d + e*x^2)^(9/
2)) + (2*e*x^9)/(63*d^2*(d + e*x^2)^(9/2))))/(5*d)))/d))/11)/d
```

3.285.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1469
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d
 + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*
a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4
*p + 2*q + 1, 0]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

3.285. $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx$

3.285.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{\left(\left(\frac{1}{5}c\,x^4+\frac{1}{3}b\,x^2+a\right)d^5+\frac{10e\left(\frac{9}{175}c\,x^4+\frac{4}{25}b\,x^2+a\right)x^2d^4}{3}+\frac{16e^2\left(\frac{1}{70}c\,x^4+\frac{3}{35}b\,x^2+a\right)x^4d^3}{3}+\frac{32e^3x^6\left(\frac{1}{330}c\,x^4+\frac{2}{45}b\,x^2+a\right)d^2}{7}+\frac{128e^4x^{\cdots}}{}\right.}{(e\,x^2+d)^{\frac{11}{2}}d^6}$
gosper	$\frac{x\left(1280a\,e^5x^{10}+128bd\,e^4x^{10}+48c\,d^2e^3x^{10}+7040ad\,e^4x^8+704b\,d^2e^3x^8+264c\,d^3e^2x^8+15840a\,d^2e^3x^6+1584b\,d^3e^2x^6+594c\,d\cdots\right.}{3465(e\,x^2+d)^{\frac{11}{2}}d^6}$
trager	$\frac{x\left(1280a\,e^5x^{10}+128bd\,e^4x^{10}+48c\,d^2e^3x^{10}+7040ad\,e^4x^8+704b\,d^2e^3x^8+264c\,d^3e^2x^8+15840a\,d^2e^3x^6+1584b\,d^3e^2x^6+594c\,d\cdots\right.}{3465(e\,x^2+d)^{\frac{11}{2}}d^6}$
default	$a\left(\frac{x}{11d(e\,x^2+d)^{\frac{11}{2}}}+\frac{\frac{10x}{99d(e\,x^2+d)^{\frac{9}{2}}}+\frac{10\left(\frac{8x}{63d(e\,x^2+d)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d(e\,x^2+d)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d(e\,x^2+d)^{\frac{3}{2}}}+\frac{8x}{15d^2\sqrt{e\,x^2+d}}\right)}{7d}\right)}{9d}\right)}{11d}}{d}\right)+$

3.285. $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx$

input
```
int((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
((1/5*c*x^4+1/3*b*x^2+a)*d^5+10/3*e*(9/175*c*x^4+4/25*b*x^2+a)*x^2*d^4+16/
3*e^2*(1/70*c*x^4+3/35*b*x^2+a)*x^4*d^3+32/7*e^3*x^6*(1/330*c*x^4+2/45*b*x
^2+a)*d^2+128/63*e^4*x^8*(1/55*b*x^2+a)*d+256/693*a*e^5*x^10)/(e*x^2+d)^(1
1/2)*x/d^6
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.07

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{13/2}}\,dx = \frac{\left(16\left(3\,cd^2e^3+8\,bde^4+80\,ae^5\right)x^{11}+88\left(3\,cd^3e^2+8\,bd^2e^3+80\,ade^4\right)x^9+198\left(3\,cd^4e\right.}{3465\left(d^6e^6x^{12}+6\,d^7e^5x^{10}+1\right.}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="fricas")
```

output
```
1/3465*(16*(3*c*d^2*e^3 + 8*b*d*e^4 + 80*a*e^5)*x^11 + 88*(3*c*d^3*e^2 + 8
*b*d^2*e^3 + 80*a*d*e^4)*x^9 + 198*(3*c*d^4*e + 8*b*d^3*e^2 + 80*a*d^2*e^3
)*x^7 + 3465*a*d^5*x + 231*(3*c*d^5 + 8*b*d^4*e + 80*a*d^3*e^2)*x^5 + 1155
*(b*d^5 + 10*a*d^4*e)*x^3)*sqrt(e*x^2 + d)/(d^6*e^6*x^12 + 6*d^7*e^5*x^10
+ 15*d^8*e^4*x^8 + 20*d^9*e^3*x^6 + 15*d^10*e^2*x^4 + 6*d^11*e*x^2 + d^12)
```

3.285.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11602 vs. 2(206) = 412.

Time = 154.15 (sec) , antiderivative size = 11602, normalized size of antiderivative = 55.25

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(13/2),x)
```

output
```
a*(693*d**55*x/(693*d**(123/2)*sqrt(1 + e*x**2/d) + 10395*d**(121/2)*e*x**
2*sqrt(1 + e*x**2/d) + 72765*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 315
315*d**(117/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 945945*d**(115/2)*e**4*x**8*
sqrt(1 + e*x**2/d) + 2081079*d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 34
68465*d**(111/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 4459455*d**(109/2)*e**7*x
**14*sqrt(1 + e*x**2/d) + 4459455*d**(107/2)*e**8*x**16*sqrt(1 + e*x**2/d)
 + 3468465*d**(105/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 2081079*d**(103/2)*e
**10*x**20*sqrt(1 + e*x**2/d) + 945945*d**(101/2)*e**11*x**22*sqrt(1 + e*x
**2/d) + 315315*d**(99/2)*e**12*x**24*sqrt(1 + e*x**2/d) + 72765*d**(97/2)
*e**13*x**26*sqrt(1 + e*x**2/d) + 10395*d**(95/2)*e**14*x**28*sqrt(1 + e*x
**2/d) + 693*d**(93/2)*e**15*x**30*sqrt(1 + e*x**2/d)) + 9240*d**54*e*x**3
/(693*d**(123/2)*sqrt(1 + e*x**2/d) + 10395*d**(121/2)*e*x**2*sqrt(1 + e*x
**2/d) + 72765*d**(119/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 315315*d**(117/2)
*e**3*x**6*sqrt(1 + e*x**2/d) + 945945*d**(115/2)*e**4*x**8*sqrt(1 + e*x**
2/d) + 2081079*d**(113/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 3468465*d**(111/
2)*e**6*x**12*sqrt(1 + e*x**2/d) + 4459455*d**(109/2)*e**7*x**14*sqrt(1 +
e*x**2/d) + 4459455*d**(107/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3468465*d**
(105/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 2081079*d**(103/2)*e**10*x**20*sqr
t(1 + e*x**2/d) + 945945*d**(101/2)*e**11*x**22*sqrt(1 + e*x**2/d) + 31531
5*d**(99/2)*e**12*x**24*sqrt(1 + e*x**2/d) + 72765*d**(97/2)*e**13*x**2...
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.60

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx = &-\frac{cx^3}{8\,(ex^2+d)^{\frac{11}{2}}e}+\frac{256\,ax}{693\,\sqrt{ex^2+d}d^6}+\frac{128\,ax}{693\,(ex^2+d)^{\frac{3}{2}}d^5}\\ &+\frac{32\,ax}{231\,(ex^2+d)^{\frac{5}{2}}d^4}+\frac{80\,ax}{693\,(ex^2+d)^{\frac{7}{2}}d^3}+\frac{10\,ax}{99\,(ex^2+d)^{\frac{9}{2}}d^2}+\frac{ax}{11\,(ex^2+d)^{\frac{11}{2}}d}\\ &+\frac{cx}{264\,(ex^2+d)^{\frac{9}{2}}e^2}+\frac{16\,cx}{1155\,\sqrt{ex^2+d}d^4e^2}+\frac{8\,cx}{1155\,(ex^2+d)^{\frac{3}{2}}d^3e^2}+\frac{2\,cx}{385\,(ex^2+d)^{\frac{5}{2}}d^2e^2}\\ &+\frac{cx}{231\,(ex^2+d)^{\frac{7}{2}}de^2}-\frac{3\,cdx}{88\,(ex^2+d)^{\frac{11}{2}}e^2}-\frac{bx}{11\,(ex^2+d)^{\frac{11}{2}}e}+\frac{128\,bx}{3465\,\sqrt{ex^2+d}d^5e}\\ &+\frac{64\,bx}{3465\,(ex^2+d)^{\frac{3}{2}}d^4e}+\frac{16\,bx}{1155\,(ex^2+d)^{\frac{5}{2}}d^3e}+\frac{8\,bx}{693\,(ex^2+d)^{\frac{7}{2}}d^2e}+\frac{bx}{99\,(ex^2+d)^{\frac{9}{2}}de}\end{aligned}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="maxima")
```

output
```
-1/8*c*x^3/((e*x^2 + d)^(11/2)*e) + 256/693*a*x/(sqrt(e*x^2 + d)*d^6) + 12
8/693*a*x/((e*x^2 + d)^(3/2)*d^5) + 32/231*a*x/((e*x^2 + d)^(5/2)*d^4) + 8
0/693*a*x/((e*x^2 + d)^(7/2)*d^3) + 10/99*a*x/((e*x^2 + d)^(9/2)*d^2) + 1/
11*a*x/((e*x^2 + d)^(11/2)*d) + 1/264*c*x/((e*x^2 + d)^(9/2)*e^2) + 16/115
5*c*x/(sqrt(e*x^2 + d)*d^4*e^2) + 8/1155*c*x/((e*x^2 + d)^(3/2)*d^3*e^2) +
 2/385*c*x/((e*x^2 + d)^(5/2)*d^2*e^2) + 1/231*c*x/((e*x^2 + d)^(7/2)*d*e^
2) - 3/88*c*d*x/((e*x^2 + d)^(11/2)*e^2) - 1/11*b*x/((e*x^2 + d)^(11/2)*e)
 + 128/3465*b*x/(sqrt(e*x^2 + d)*d^5*e) + 64/3465*b*x/((e*x^2 + d)^(3/2)*d
^4*e) + 16/1155*b*x/((e*x^2 + d)^(5/2)*d^3*e) + 8/693*b*x/((e*x^2 + d)^(7/
2)*d^2*e) + 1/99*b*x/((e*x^2 + d)^(9/2)*d*e)
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx = \frac{\left(\left(\left(2\left(4\,x^2\left(\frac{2\,(3\,cd^2e^8+8\,bde^9+80\,ae^{10})x^2}{d^6e^5}+\frac{11\,(3\,cd^3e^7+8\,bd^2e^8+80\,ade^9)}{d^6e^5}\right)+\frac{99\,(3\,cd^4e^6+8\,bd^3e^7+80\,ad}{d^6e^5}\right.\right.\right.\right.}{3465\,(ex^2+d}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="giac")
```

output
```
1/3465*((((2*(4*x^2*(2*(3*c*d^2*e^8 + 8*b*d*e^9 + 80*a*e^10)*x^2/(d^6*e^5)
+ 11*(3*c*d^3*e^7 + 8*b*d^2*e^8 + 80*a*d*e^9)/(d^6*e^5)) + 99*(3*c*d^4*e^6
 + 8*b*d^3*e^7 + 80*a*d^2*e^8)/(d^6*e^5))*x^2 + 231*(3*c*d^5*e^5 + 8*b*d^4
*e^6 + 80*a*d^3*e^7)/(d^6*e^5))*x^2 + 1155*(b*d^5*e^5 + 10*a*d^4*e^6)/(d^6
*e^5))*x^2 + 3465*a/d)*x/(e*x^2 + d)^(11/2)
```

3.285.9 Mupad [B] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx &= \frac{x\left(\frac{a}{11\,d}-\frac{d\left(\frac{b}{11\,d}-\frac{c}{11\,e}\right)}{e}\right)}{(e\,x^2+d)^{11/2}}-\frac{x\left(\frac{c}{9\,e^2}-\frac{-c\,d^2+b\,d\,e+10\,a\,e^2}{99\,d^2\,e^2}\right)}{(e\,x^2+d)^{9/2}}\\ &+\frac{x\,(3\,c\,d^2+8\,b\,d\,e+80\,a\,e^2)}{693\,d^3\,e^2\,(e\,x^2+d)^{7/2}}+\frac{x\,(6\,c\,d^2+16\,b\,d\,e+160\,a\,e^2)}{1155\,d^4\,e^2\,(e\,x^2+d)^{5/2}}\\ &+\frac{x\,(24\,c\,d^2+64\,b\,d\,e+640\,a\,e^2)}{3465\,d^5\,e^2\,(e\,x^2+d)^{3/2}}+\frac{x\,(48\,c\,d^2+128\,b\,d\,e+1280\,a\,e^2)}{3465\,d^6\,e^2\,\sqrt{e\,x^2+d}}\end{aligned}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x)
```

output
```
(x*(a/(11*d) - (d*(b/(11*d) - c/(11*e)))/e))/(d + e*x^2)^(11/2) - (x*(c/(9
*e^2) - (10*a*e^2 - c*d^2 + b*d*e)/(99*d^2*e^2)))/(d + e*x^2)^(9/2) + (x*(
80*a*e^2 + 3*c*d^2 + 8*b*d*e))/(693*d^3*e^2*(d + e*x^2)^(7/2)) + (x*(160*a
*e^2 + 6*c*d^2 + 16*b*d*e))/(1155*d^4*e^2*(d + e*x^2)^(5/2)) + (x*(640*a*e
^2 + 24*c*d^2 + 64*b*d*e))/(3465*d^5*e^2*(d + e*x^2)^(3/2)) + (x*(1280*a*e
^2 + 48*c*d^2 + 128*b*d*e))/(3465*d^6*e^2*(d + e*x^2)^(1/2))
```

3.285. $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx$

3.286 $\int (7+5x^2)^3 \sqrt{2+3x^2+x^4}\,dx$

3.286.1 Optimal result

Integrand size = 24, antiderivative size = 193

$$\int (7+5x^2)^3 \sqrt{2+3x^2+x^4}\,dx = \frac{577x(2+x^2)}{3\sqrt{2+3x^2+x^4}} + \frac{1}{21}x(2608+757x^2)\sqrt{2+3x^2+x^4} + \frac{275}{7}x(2+3x^2+x^4)^{3/2} + \frac{125}{9}x^3(2+3x^2+x^4)^{3/2} - \frac{577\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}} + \frac{2945\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{21\sqrt{2+3x^2+x^4}}$$

output
```
275/7*x*(x^4+3*x^2+2)^(3/2)+125/9*x^3*(x^4+3*x^2+2)^(3/2)+577/3*x*(x^2+2)/
(x^4+3*x^2+2)^(1/2)-577/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2
+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
+2945/21*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(
1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/21*x*(757*x^2+
2608)*(x^4+3*x^2+2)^(1/2)
```

3.286.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.63 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.62

$$\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\,dx = \frac{25548x+61214x^3+57312x^5+28496x^7+7725x^9+875x^{11}-12117i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\right|}{63\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(25548*x + 61214*x^3 + 57312*x^5 + 28496*x^7 + 7725*x^9 + 875*x^11 - (1211
7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (555
3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(63*S
qrt[2 + 3*x^2 + x^4])
```

3.286.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1518, 27, 2207, 1490, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+7\right)^3 \sqrt{x^4+3x^2+2}\,dx$$

$$\Big\downarrow \text{1518}$$

$$\frac{1}{9}\int 3\sqrt{x^4+3x^2+2}\left(825x^4+1955x^2+1029\right)dx+\frac{125}{9}\left(x^4+3x^2+2\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \sqrt{x^4+3x^2+2}\left(825x^4+1955x^2+1029\right)dx+\frac{125}{9}\left(x^4+3x^2+2\right)^{3/2}x^3$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{3}\left(\frac{1}{7}\int \left(3785x^2+5553\right)\sqrt{x^4+3x^2+2}dx+\frac{825}{7}x\left(x^4+3x^2+2\right)^{3/2}\right)+\frac{125}{9}\left(x^4+3x^2+2\right)^{3/2}x^3$$

$$\downarrow 1490$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{15}\int \frac{15(4039x^2+5890)}{\sqrt{x^4+3x^2+2}}dx+x\sqrt{x^4+3x^2+2}(757x^2+2608)\right)+\frac{825}{7}x(x^4+3x^2+2)^{3/2}\right)+\frac{125}{9}(x^4+3x^2+2)^{3/2}x^3$$

$$\downarrow 27$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\int \frac{4039x^2+5890}{\sqrt{x^4+3x^2+2}}dx+x\sqrt{x^4+3x^2+2}(757x^2+2608)\right)+\frac{825}{7}x(x^4+3x^2+2)^{3/2}\right)+\frac{125}{9}(x^4+3x^2+2)^{3/2}x^3$$

$$\downarrow 1503$$

$$\frac{1}{3}\left(\frac{1}{7}\left(5890\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+4039\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+x\sqrt{x^4+3x^2+2}(757x^2+2608)\right)+\frac{825}{7}x(x^4+3\right.$$
$$\frac{125}{9}(x^4+3x^2+2)^{3/2}x^3$$

$$\downarrow 1412$$

$$\frac{1}{3}\left(\frac{1}{7}\left(4039\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{2945\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+x\sqrt{x^4+3x^2+2}(757x^2+\right.\right.$$
$$\frac{125}{9}(x^4+3x^2+2)^{3/2}x^3$$

$$\downarrow 1455$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2945\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+4039\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E(\arctan(\right.}{\sqrt{x^4+3x^2+2}}\right.\right.\right.$$
$$\frac{125}{9}(x^4+3x^2+2)^{3/2}x^3$$

input
```
Int[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(125*x^3*(2 + 3*x^2 + x^4)^(3/2))/9 + ((825*x*(2 + 3*x^2 + x^4)^(3/2))/7 +
 (x*(2608 + 757*x^2)*Sqrt[2 + 3*x^2 + x^4] + 4039*((x*(2 + x^2))/Sqrt[2 +
3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcT
an[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (2945*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^
2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4])/7)/3
```

3.286. $\int (7+5x^2)^3\sqrt{2+3x^2+x^4}\,dx$

3.286.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.286. $\int (7+5x^2)^3 \sqrt{2+3x^2+x^4}\,dx$

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.286.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x(875x^6+5100x^4+11446x^2+12774)\sqrt{x^4+3x^2+2}}{63}-\frac{2945i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}}+\frac{577i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}}{2}\right.\right.}{6\sqrt{x^4+3x^2}}$
default	$\frac{4258x\sqrt{x^4+3x^2+2}}{21}-\frac{2945i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}}+\frac{577i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{6\sqrt{x^4+3x^2+2}}+\frac{1}{}$
elliptic	$\frac{4258x\sqrt{x^4+3x^2+2}}{21}-\frac{2945i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}}+\frac{577i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{6\sqrt{x^4+3x^2+2}}+\frac{1}{}$

input
```
int((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/63*x*(875*x^6+5100*x^4+11446*x^2+12774)*(x^4+3*x^2+2)^(1/2)-2945/21*I*2^
(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^
(1/2)*x,2^(1/2))+577/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(
1/2)))
```

3.286.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.33

$$\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\,dx$$
$$= \frac{-12117i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+29787i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+\left(875\,x^8+5100\,x^6+11446\,x^4+12774\,x^2+121\right.}{63\,x}$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/63*(-12117*I*x*elliptic_e(arcsin(I/x), 2) + 29787*I*x*elliptic_f(arcsin(
I/x), 2) + (875*x^8 + 5100*x^6 + 11446*x^4 + 12774*x^2 + 12117)*sqrt(x^4 +
 3*x^2 + 2))/x
```

3.286.6 Sympy [F]

$$\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(5x^2+7\right)^3\,dx$$

input
```
integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3, x)
```

3.286.7 Maxima [F]

$$\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^3\,dx$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

3.286.8 Giac [F]

$$\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^3 dx$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^3 \sqrt{2+3x^2+x^4}\,dx = \int \left(5\,x^2+7\right)^3 \sqrt{x^4+3\,x^2+2}\,dx$$

input
```
int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2), x)
```

3.287 $\int (7+5x^2)^2 \sqrt{2+3x^2+x^4}\,dx$

3.287.1 Optimal result

Integrand size = 24, antiderivative size = 168

$$\begin{aligned}\int (7+5x^2)^2 \sqrt{2+3x^2+x^4}\,dx &= \frac{31x(2+x^2)}{\sqrt{2+3x^2+x^4}} + \frac{1}{21}x(407+114x^2)\sqrt{2+3x^2+x^4} \\ &+ \frac{25}{7}x(2+3x^2+x^4)^{3/2} \\ &- \frac{31\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} \\ &+ \frac{472\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{21\sqrt{2+3x^2+x^4}}\end{aligned}$$

output
```
25/7*x*(x^4+3*x^2+2)^(3/2)+31*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-31*(x^2+1)^(3/
2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+
2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+472/21*(x^2+1)^(3/2)*(1/(x^2+1))^(1/
2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/
(x^4+3*x^2+2)^(1/2)+1/21*x*(114*x^2+407)*(x^4+3*x^2+2)^(1/2)
```

3.287.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.69 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.68

$$\int (7+5x^2)^2\sqrt{2+3x^2+x^4}\,dx$$
$$=\frac{1114x+2349x^3+1724x^5+564x^7+75x^9-651i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-293i\sqrt{1+x^2}\sqrt{}}{21\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(1114*x + 2349*x^3 + 1724*x^5 + 564*x^7 + 75*x^9 - (651*I)*Sqrt[1 + x^2]*S
qrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (293*I)*Sqrt[1 + x^2]*Sq
rt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

3.287.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1518, 1490, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)^2\sqrt{x^4+3x^2+2}\,dx$$
$$\Big\downarrow \text{1518}$$
$$\frac{1}{7}\int (190x^2+293)\sqrt{x^4+3x^2+2}dx+\frac{25}{7}x(x^4+3x^2+2)^{3/2}$$
$$\Big\downarrow \text{1490}$$
$$\frac{1}{7}\left(\frac{1}{15}\int \frac{5(651x^2+944)}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{3}x\sqrt{x^4+3x^2+2}(114x^2+407)\right)+\frac{25}{7}x(x^4+3x^2+2)^{3/2}$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{7}\left(\frac{1}{3}\int \frac{651x^2+944}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{3}x\sqrt{x^4+3x^2+2}(114x^2+407)\right)+\frac{25}{7}x(x^4+3x^2+2)^{3/2}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{7}\left(\frac{1}{3}\left(944\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+651\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{3}x\sqrt{x^4+3x^2+2}(114x^2+407)\right)+\frac{25}{7}x\left(x^4+3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{7}\left(\frac{1}{3}\left(651\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{472\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{1}{3}x\sqrt{x^4+3x^2+2}(114x^2\right.$$
$$\frac{25}{7}x\left(x^4+3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{7}\left(\frac{1}{3}\left(\frac{472\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+651\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E(\arctan(x)}{\sqrt{x^4+3x^2+2}}\right.\right.\right.$$
$$\frac{25}{7}x\left(x^4+3x^2+2\right)^{3/2}$$

input
```
Int[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(25*x*(2 + 3*x^2 + x^4)^(3/2))/7 + ((x*(407 + 114*x^2)*Sqrt[2 + 3*x^2 + x^
4])/3 + (651*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqr
t[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) +
 (472*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2
])/Sqrt[2 + 3*x^2 + x^4])/3)/7
```

3.287.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.287. $\int (7+5x^2)^2 \sqrt{2+3x^2+x^4}\, dx$

3.287.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.79

method	result
risch	$\frac{x(75x^4+339x^2+557)\sqrt{x^4+3x^2+2}}{21} - \frac{472i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}} + \frac{31i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2}\right.\right.}{2\sqrt{x^4+3x^2+2}}$
default	$\frac{557x\sqrt{x^4+3x^2+2}}{21} - \frac{472i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}} + \frac{31i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}} + \frac{25x^5}{}$
elliptic	$\frac{557x\sqrt{x^4+3x^2+2}}{21} - \frac{472i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{21\sqrt{x^4+3x^2+2}} + \frac{31i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}} + \frac{25x^5}{}$

input
```
int((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*x*(75*x^4+339*x^2+557)*(x^4+3*x^2+2)^(1/2)-472/21*I*2^(1/2)*(2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2)
)+31/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Ellipt
icF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.287.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

$$\int (7+5x^2)^2\sqrt{2+3x^2+x^4}\,dx = \frac{-651i\,xE\left(\arcsin\left(\frac{i}{x}\right)\mid 2\right)+1595i\,xF\left(\arcsin\left(\frac{i}{x}\right)\mid 2\right)+(75\,x^6+339\,x^4+557\,x^2+651)\sqrt{x^4+3\,x^2+2}}{21\,x}$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/21*(-651*I*x*elliptic_e(arcsin(I/x), 2) + 1595*I*x*elliptic_f(arcsin(I/x
), 2) + (75*x^6 + 339*x^4 + 557*x^2 + 651)*sqrt(x^4 + 3*x^2 + 2))/x
```

3.287.6 Sympy [F]

$$\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\, dx = \int \sqrt{\left(x^2+1\right)\left(x^2+2\right)}\left(5x^2+7\right)^2\, dx$$

input
```
integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2, x)
```

3.287.7 Maxima [F]

$$\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\, dx = \int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2\, dx$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

3.287.8 Giac [F]

$$\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\, dx = \int \sqrt{x^4+3\,x^2+2}\left(5\,x^2+7\right)^2\, dx$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\, dx = \int \left(5\,x^2+7\right)^2 \sqrt{x^4+3\,x^2+2}\, dx$$

input
```
int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2), x)
```

3.288 $\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx$

3.288.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx = \frac{5x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x(10+3x^2)\sqrt{2+3x^2+x^4} - \frac{5\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{11\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}$$

output
```
5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Elliptic
E(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+
2)^(1/2)+11/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/
2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/3*x*(3*x^
2+10)*(x^4+3*x^2+2)^(1/2)
```

3.288.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.60 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.73

$$\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx = \frac{20x+36x^3+19x^5+3x^7-15i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-7i\sqrt{1+x^2}\sqrt{2+x^2}\operatorname{EllipticF}\left(iar\right.}{3\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(20*x + 36*x^3 + 19*x^5 + 3*x^7 - (15*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellip
ticE[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elliptic
F[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

3.288.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1490, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)\sqrt{x^4+3x^2+2}\,dx$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{15}\int \frac{5(15x^2+22)}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{3}x\sqrt{x^4+3x^2+2}(3x^2+10)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{15x^2+22}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{3}x\sqrt{x^4+3x^2+2}(3x^2+10)$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{3}\left(22\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+15\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{3}x\sqrt{x^4+3x^2+2}(3x^2+10)$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{3}\left(15\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{11\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{1}{3}x\sqrt{x^4+3x^2+2}(3x^2+10)$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{3}\left(\frac{11\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+15\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}\right)\right)$$

$$\frac{1}{3}x\sqrt{x^4+3x^2+2}(3x^2+10)$$

input
```
Int[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(x*(10 + 3*x^2)*Sqrt[2 + 3*x^2 + x^4])/3 + (15*((x*(2 + x^2))/Sqrt[2 + 3*x
^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[
x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (11*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4])/3
```

3.288.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.288.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

method	result
risch	$\frac{x(3x^2+10)\sqrt{x^4+3x^2+2}}{3}-\frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$
default	$\frac{10x\sqrt{x^4+3x^2+2}}{3}-\frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}+x^3\sqrt{x^4}$
elliptic	$\frac{10x\sqrt{x^4+3x^2+2}}{3}-\frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}+x^3\sqrt{x^4}$

input
```
int((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(3*x^2+10)*(x^4+3*x^2+2)^(1/2)-11/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+5/2*I*2^(1/
2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1
/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.288.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.36

$$\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx = \frac{-15i\,xE\left(\arcsin\left(\frac{i}{x}\right)\mid 2\right)+37i\,xF\left(\arcsin\left(\frac{i}{x}\right)\mid 2\right)+(3\,x^4+10\,x^2+15)\sqrt{x^4+3\,x^2+2}}{3\,x}$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-15*I*x*elliptic_e(arcsin(I/x), 2) + 37*I*x*elliptic_f(arcsin(I/x), 2
) + (3*x^4 + 10*x^2 + 15)*sqrt(x^4 + 3*x^2 + 2))/x
```

3.288.6 Sympy [F]

$$\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx = \int \sqrt{(x^2+1)(x^2+2)}(5x^2+7)\,dx$$

input
```
integrate((5*x**2+7)*(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7), x)
```

3.288.7 Maxima [F]

$$\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}(5\,x^2+7)\,dx$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

3.288.8 Giac [F]

$$\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx=\int \sqrt{x^4+3\,x^2+2}\big(5\,x^2+7\big)\,dx$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx=\int \left(5\,x^2+7\right)\,\sqrt{x^4+3\,x^2+2}\,dx$$

input
```
int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2), x)
```

3.289 $\int \sqrt{2+3x^2+x^4}\,dx$

3.289.1 Optimal result

Integrand size = 14, antiderivative size = 141

$$\int \sqrt{2+3x^2+x^4}\,dx=\frac{x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\sqrt{2+3x^2+x^4}-\frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2+3x^2+x^4}}+\frac{2\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}$$

output
```
x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/
(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(
1/2)+2/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(
1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/3*x*(x^4+3*x^2
+2)^(1/2)
```

3.289.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.65 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \sqrt{2+3x^2+x^4}\,dx$$
$$=\frac{2x+3x^3+x^5-3i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right.}{3\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(2*x + 3*x^3 + x^5 - (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh
[x/Sqrt[2]], 2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt
[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

3.289.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1404, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^4+3x^2+2}\,dx$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{3x^2+4}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{3}\sqrt{x^4+3x^2+2}x$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{3}\left(4\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+3\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{3}\sqrt{x^4+3x^2+2}x$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{3}\left(3\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{1}{3}\sqrt{x^4+3x^2+2}x$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{3}\left(\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+3\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)\right)+\frac{1}{3}\sqrt{x^4+3x^2+2}x$$

input
```
Int[Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(x*Sqrt[2 + 3*x^2 + x^4])/3 + (3*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (S
qrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt
[2 + 3*x^2 + x^4]) + (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellipt
icF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4])/3
```

3.289.3.1 Defintions of rubi rules used

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.289.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x\sqrt{x^4+3x^2+2}}{3}-\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	121
risch	$\frac{x\sqrt{x^4+3x^2+2}}{3}-\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	121
elliptic	$\frac{x\sqrt{x^4+3x^2+2}}{3}-\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	121

input
```
int((x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(x^4+3*x^2+2)^(1/2)-2/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4
+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I*2^(1/2)*(2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2
))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.289.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.33

$$\int \sqrt{2+3x^2+x^4}\,dx = \frac{-3i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+7i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+\sqrt{x^4+3\,x^2+2}\,(x^2+3)}{3\,x}$$

input
```
integrate((x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-3*I*x*elliptic_e(arcsin(I/x), 2) + 7*I*x*elliptic_f(arcsin(I/x), 2)
+ sqrt(x^4 + 3*x^2 + 2)*(x^2 + 3))/x
```

3.289.6 Sympy [F]

$$\int \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3x^2+2}\,dx$$

input
```
integrate((x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(x**4 + 3*x**2 + 2), x)
```

3.289.7 Maxima [F]

$$\int \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

3.289.8 Giac [F]

$$\int \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+2}\,dx$$

input
```
int((3*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int((3*x^2 + x^4 + 2)^(1/2), x)
```

3.290 $\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2} \, dx$

3.290.1 Optimal result

Integrand size = 24, antiderivative size = 178

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2} \, dx = \frac{x(2+x^2)}{5\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{5\sqrt{2+3x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}}$$

output
```
1/5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+3/70*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ell
ipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(
x^4+3*x^2+2)^(1/2)-1/5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)
^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/
5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*(
(x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

3.290.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.53 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\left(35E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+21\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)-6\,\text{EllipticPi}\left(\frac{10}{7},i\text{arcsin}\right.\right.}{175\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

output
```
((-1/175*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(35*EllipticE[I*ArcSinh[x/Sqrt[2]]
, 2] + 21*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - 6*EllipticPi[10/7, I*ArcSin
h[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

3.290.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1522, 27, 1503, 1412, 1455, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+3x^2+2}}{5x^2+7}dx$$

$$\Big\downarrow \text{ 1522}$$

$$\frac{1}{10}\int \frac{2(x^2+1)}{\sqrt{x^4+3x^2+2}}dx-\frac{1}{10}\int -\frac{6(x^2+1)}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int \frac{x^2+1}{\sqrt{x^4+3x^2+2}}dx+\frac{3}{5}\int \frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{5}\left(\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{3}{5}\int \frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \; 1412$$

$$\frac{1}{5}\left(\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right)+\frac{3}{5}\int \frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \; 1455$$

$$\frac{3}{5}\int \frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx+\frac{1}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}+\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}\right)$$

$$\Big\downarrow \; 1786$$

$$\frac{3\sqrt{x^2+1}\sqrt{x^2+2}\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2(5x^2+7)}}dx}{5\sqrt{x^4+3x^2+2}}+\frac{1}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}+\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}\right)$$

$$\Big\downarrow \; 414$$

$$\frac{3(x^2+2)\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{35\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}+\frac{1}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}+\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}\right)$$

input
```
Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

output
```
((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(
1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + ((1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2
 + x^4]))/5 + (3*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(35*Sqrt[2]*Sq
rt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

3.290. $\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx$

3.290.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1522
```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/(2*e)   Int[(b - q + 2*c*x^2)/S
qrt[a + b*x^2 + c*x^4], x], x] - Simp[1/(2*e)   Int[(b*d - 2*a*e - d*q + (2
*c*d - b*e - e*q)*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.290. $\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx$

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

3.290.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{50\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{10\sqrt{x^4+3x^2+2}}+\frac{6i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{175\sqrt{x^4+3x^2+2}}$	138
elliptic	$-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{50\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{10\sqrt{x^4+3x^2+2}}+\frac{6i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{175\sqrt{x^4+3x^2+2}}$	138

input
```
int((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

output
```
-3/50*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
F(1/2*I*2^(1/2)*x,2^(1/2))-1/10*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x
^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+6/175*I*2^(1/2)*(1+1/
2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,
10/7,2^(1/2))
```

3.290.5 Fricas [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

3.290.6 Sympy [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{(x^2+1)\,(x^2+2)}}{5x^2+7}\,dx$$

input
```
integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7),x)
```

output
```
Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7), x)
```

3.290.7 Maxima [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

3.290.8 Giac [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

input
```
int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7),x)
```

output
```
int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7), x)
```

3.291 $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx$

3.291.1 Optimal result

Integrand size = 24, antiderivative size = 209

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx = -\frac{x(2+x^2)}{70\sqrt{2+3x^2+x^4}}+\frac{x\sqrt{2+3x^2+x^4}}{14(7+5x^2)} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{140\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{980\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

output
```
-1/70*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/1960*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+
1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2
+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/70*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellipt
icE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^
2+2)^(1/2)+3/280*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2)
,1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/14*x*(
x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

3.291.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx$$
$$= \frac{350x+525x^3+175x^5+35i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)\,E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-84i\sqrt{1+x^2}\sqrt{2+x^2}(7+5}{}$$

input
```
Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

output
```
(350*x + 525*x^3 + 175*x^5 + (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2
)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (84*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*
(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[
2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (5*I)*x^2*Sqrt[1 + x^
2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(2450*(7 + 5*x
^2)*Sqrt[2 + 3*x^2 + x^4])
```

3.291.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1554, 25, 2234, 25, 1503, 1412, 1455, 1538, 27, 1412, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+3x^2+2}}{(5x^2+7)^2}\,dx$$
$$\Big\downarrow \text{1554}$$
$$\frac{x\sqrt{x^4+3x^2+2}}{14\,(5x^2+7)}-\frac{1}{14}\int -\frac{2-x^4}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{14}\int \frac{2-x^4}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx+\frac{\sqrt{x^4+3x^2+2}x}{14\,(5x^2+7)}$$

$\Big\downarrow$ 2234

$$\frac{1}{14}\left(\frac{1}{25}\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx-\frac{1}{25}\int-\frac{7-5x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)}$$

$\Big\downarrow$ 25

$$\frac{1}{14}\left(\frac{1}{25}\int\frac{7-5x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{25}\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)}$$

$\Big\downarrow$ 1503

$$\frac{1}{14}\left(\frac{1}{25}\left(7\int\frac{1}{\sqrt{x^4+3x^2+2}}dx-5\int\frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{25}\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)}$$

$\Big\downarrow$ 1412

$$\frac{1}{14}\left(\frac{1}{25}\left(\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-5\int\frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{25}\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}d\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)}$$

$\Big\downarrow$ 1455

$$\frac{1}{14}\left(\frac{1}{25}\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx+\frac{1}{25}\left(\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-5\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\right.\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)}$$

$\Big\downarrow$ 1538

$$\frac{1}{14}\left(\frac{1}{25}\left(\frac{1}{2}\int\frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{4}\int\frac{2(x^2+1)}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{25}\left(\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}(\arctan(\right.\right.}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$
$$\frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)}$$

$\Big\downarrow$ 27

3.291. $\int\frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx$

$$\frac{1}{14}\left(\frac{1}{25}\left(\frac{1}{2}\int\frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{2}\int\frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{25}\left(\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x\right.}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)}$$

$\downarrow$ 1412

$$\frac{1}{14}\left(\frac{1}{25}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5}{2}\int\frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{25}\left(\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}}{\sqrt{2}\sqrt{x}}\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)}$$

$\downarrow$ 1786

$$\frac{1}{14}\left(\frac{1}{25}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{x^2+1}\sqrt{x^2+2}\int\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(5x^2+7)}dx}{2\sqrt{x^4+3x^2+2}}\right)+\frac{1}{25}\left(\frac{7(x^2+1)\sqrt{\frac{x}{x}}}{\sqrt{}}\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)}$$

$\downarrow$ 414

$$\frac{1}{14}\left(\frac{1}{25}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+2)\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}\right)+\frac{1}{25}\left(\frac{7(x^2+1)\sqrt{}}{}\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{14(5x^2+7)}$$

input
```
Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

output
```
(x*Sqrt[2 + 3*x^2 + x^4])/(14*(7 + 5*x^2)) + ((-5*((x*(2 + x^2))/Sqrt[2 +
3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcT
an[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (7*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2
)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]))/25 + (((1 +
 x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*Sqrt
[2 + 3*x^2 + x^4]) - (5*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(14*Sqr
t[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4]))/25)/14
```

3.291. $\int\frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx$

3.291.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.291. $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx$

rule 1538
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/(2*c*d - e*(b - q)))   I
nt[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[e/(2*c*d - e*(b - q))   Int[(b
- q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !LtQ[c, 0]
```

rule 1554
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(q +
 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*((a*(2*q + 3) +
2*b*(q + 2)*x^2 + c*(2*q + 5)*x^4)/Sqrt[a + b*x^2 + c*x^4]), x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && ILtQ[q, -1]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.291. $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2} \, dx$

3.291.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

method	result
default	$\frac{x\sqrt{x^4+3x^2+2}}{70x^2+98}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{175\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{140\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\right.}{2450\sqrt{x^4+3x^2+2}}$
elliptic	$\frac{x\sqrt{x^4+3x^2+2}}{70x^2+98}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{175\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{140\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\right.}{2450\sqrt{x^4+3x^2+2}}$
risch	$\frac{x\sqrt{x^4+3x^2+2}}{70x^2+98}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{100\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{140\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}}{2450}$

input
```
int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

output
```
1/14*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-3/175*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/140*I*2^
(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^
(1/2)*x,2^(1/2))-1/2450*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x
^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

3.291.5 Fricas [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

3.291.6 Sympy [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{(x^2+1)(x^2+2)}}{(5x^2+7)^2}\,dx$$

input
```
integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

output
```
Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**2, x)
```

3.291.7 Maxima [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

3.291.8 Giac [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{(5\,x^2+7)^2}\,dx$$

input
```
int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^2,x)
```

output
```
int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^2, x)
```

3.292 $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx$

3.292.1 Optimal result

Integrand size = 24, antiderivative size = 237

$$\begin{aligned}\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx = & -\frac{11x(2+x^2)}{11760\sqrt{2+3x^2+x^4}}+\frac{x\sqrt{2+3x^2+x^4}}{28\,(7+5x^2)^2} \\ & +\frac{11x\sqrt{2+3x^2+x^4}}{2352\,(7+5x^2)}+\frac{11(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{5880\sqrt{2}\sqrt{2+3x^2+x^4}} \\ & +\frac{81(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{2+3x^2+x^4}} \\ & -\frac{1201(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{164640\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

output
```
-11/11760*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1201/329280*(x^2+2)*(1/(x^2+1))^(1
/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^
2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+11/11760*(x^2+1)^(3/2)*(1/(x^2+1))
^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1
/2)/(x^4+3*x^2+2)^(1/2)+81/15680*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF
(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2
)^(1/2)+1/28*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+11/2352*x*(x^4+3*x^2+2)^(1/
2)/(5*x^2+7)
```

3.292.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.35 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx = \frac{\frac{14700x(2+3x^2+x^4)}{(7+5x^2)^2}+\frac{1925x(2+3x^2+x^4)}{7+5x^2}+385i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-434i\sqrt{1+x^2}\sqrt{2+x^2}\,\mathrm{Ell}}{411600\sqrt{2+3x^2+x}}$$

input
```
Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

output
```
((14700*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (1925*x*(2 + 3*x^2 + x^4))/(7
 + 5*x^2) + (385*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt
[2]], 2] - (434*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[
2]], 2] - (1201*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[
x/Sqrt[2]], 2])/(411600*Sqrt[2 + 3*x^2 + x^4])
```

3.292.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.28, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {1554, 25, 2210, 2234, 25, 1503, 1412, 1455, 1538, 27, 1412, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+3x^2+2}}{(5x^2+7)^3}dx$$

$$\Big\downarrow \text{ 1554}$$

$$\frac{x\sqrt{x^4+3x^2+2}}{28(5x^2+7)^2}-\frac{1}{28}\int -\frac{x^4+6x^2+6}{(5x^2+7)^2\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{28}\int \frac{x^4+6x^2+6}{(5x^2+7)^2\sqrt{x^4+3x^2+2}}dx+\frac{\sqrt{x^4+3x^2+2}\,x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{ 2210}$$

$$\frac{1}{28}\left(\frac{1}{84}\int\frac{-11x^4-14x^2+50}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx+\frac{11\sqrt{x^4+3x^2+2}x}{84(5x^2+7)}\right)+\frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2}$$

$\downarrow$ 2234

$$\frac{1}{28}\left(\frac{1}{84}\left(\frac{1201}{25}\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx-\frac{1}{25}\int-\frac{7-55x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{11\sqrt{x^4+3x^2+2}x}{84(5x^2+7)}\right)+\frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2}$$

$\downarrow$ 25

$$\frac{1}{28}\left(\frac{1}{84}\left(\frac{1}{25}\int\frac{7-55x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{1201}{25}\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{11\sqrt{x^4+3x^2+2}x}{84(5x^2+7)}\right)+\frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2}$$

$\downarrow$ 1503

$$\frac{1}{28}\left(\frac{1}{84}\left(\frac{1}{25}\left(7\int\frac{1}{\sqrt{x^4+3x^2+2}}dx-55\int\frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1201}{25}\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{11\sqrt{x}}{84}\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2}$$

$\downarrow$ 1412

$$\frac{1}{28}\left(\frac{1}{84}\left(\frac{1}{25}\left(\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-55\int\frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1201}{25}\int\frac{1}{(5x^2+7)\sqrt{x^4+}}\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2}$$

$\downarrow$ 1455

$$\frac{1}{28}\left(\frac{1}{84}\left(\frac{1201}{25}\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx+\frac{1}{25}\left(\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-55\left(\frac{x(x^2+}{\sqrt{x^4+3x^2}}\right.\right.\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2}$$

$\downarrow$ 1538

3.292. $\int\frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx$

$$\frac{1}{28}\left(\frac{1}{84}\left(\frac{1201}{25}\left(\frac{1}{2}\int\frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{4}\int\frac{2(x^2+1)}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{25}\left(\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(a\right.}{\sqrt{2}\sqrt{x^4+3x^2+}}\right.\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2}$$

$\big\downarrow$ 27

$$\frac{1}{28}\left(\frac{1}{84}\left(\frac{1201}{25}\left(\frac{1}{2}\int\frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{2}\int\frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{25}\left(\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(a\right.}{\sqrt{2}\sqrt{x^4+3x^2+}}\right.\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2}$$

$\big\downarrow$ 1412

$$\frac{1}{28}\left(\frac{1}{84}\left(\frac{1201}{25}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5}{2}\int\frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{25}\left(\frac{7(x^2+1)}{}\right.\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2}$$

$\big\downarrow$ 1786

$$\frac{1}{28}\left(\frac{1}{84}\left(\frac{1201}{25}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{x^2+1}\sqrt{x^2+2}\int\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(5x^2+7)}dx}{2\sqrt{x^4+3x^2+2}}\right)+\frac{1}{25}\left(\frac{7(x^2+}{}\right.\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2}$$

$\big\downarrow$ 414

$$\frac{1}{28}\left(\frac{1}{84}\left(\frac{1201}{25}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+2)\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}\right)+\frac{1}{25}\left(\frac{7(x^2}{}\right.\right.\right.$$
$$\frac{\sqrt{x^4+3x^2+2}x}{28(5x^2+7)^2}$$

input
```
Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

3.292. $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx$

output
```
(x*Sqrt[2 + 3*x^2 + x^4])/(28*(7 + 5*x^2)^2) + ((11*x*Sqrt[2 + 3*x^2 + x^4
])/(84*(7 + 5*x^2)) + ((-55*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2
]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 +
3*x^2 + x^4]) + (7*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x]
, 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]))/25 + (1201*(((1 + x^2)*Sqrt[(2 +
x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4
]) - (5*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(14*Sqrt[2]*Sqrt[(2 + x
^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])))/25)/84)/28
```

3.292.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1538
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/(2*c*d - e*(b - q)))   I
nt[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[e/(2*c*d - e*(b - q))   Int[(b
- q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !LtQ[c, 0]
```

rule 1554
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(q +
 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*((a*(2*q + 3) +
2*b*(q + 2)*x^2 + c*(2*q + 5)*x^4)/Sqrt[a + b*x^2 + c*x^4]), x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && ILtQ[q, -1]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

3.292. $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx$

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.292.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.77

method	result
risch	$\frac{\sqrt{x^4+3x^2+2}\,x(55x^2+161)}{2352(5x^2+7)^2}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{16800\sqrt{x^4+3x^2+2}}-\frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{23520\sqrt{x^4+3x^2+2}}-$
default	$\frac{x\sqrt{x^4+3x^2+2}}{28(5x^2+7)^2}+\frac{11x\sqrt{x^4+3x^2+2}}{2352(5x^2+7)}-\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{58800\sqrt{x^4+3x^2+2}}+\frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{23520\sqrt{x^4+3x^2+2}}-$ 1201i√
elliptic	$\frac{x\sqrt{x^4+3x^2+2}}{28(5x^2+7)^2}+\frac{11x\sqrt{x^4+3x^2+2}}{2352(5x^2+7)}-\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{58800\sqrt{x^4+3x^2+2}}+\frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{23520\sqrt{x^4+3x^2+2}}-$ 1201i√

input
```
int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2352*(x^4+3*x^2+2)^(1/2)*x*(55*x^2+161)/(5*x^2+7)^2-1/16800*I*2^(1/2)*(2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,
2^(1/2))-11/23520*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1
/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2))
)-1201/411600*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2
)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

3.292.5 Fricas [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{x^4+3\,x^2+2}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

3.292.6 Sympy [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx = \int \frac{\sqrt{(x^2+1)(x^2+2)}}{(5x^2+7)^3}\,dx$$

input
```
integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

output
```
Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**3, x)
```

3.292.7 Maxima [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

3.292.8 Giac [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3} \, dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{\left(5\,x^2+7\right)^3} \, dx$$

input
```
int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^3,x)
```

output
```
int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^3, x)
```

3.293 $\int (7+5x^2)^3 (2+3x^2+x^4)^{3/2} \, dx$

3.293.1 Optimal result

Integrand size = 24, antiderivative size = 219

$$\int (7+5x^2)^3 (2+3x^2+x^4)^{3/2} \, dx = \frac{20884x(2+x^2)}{65\sqrt{2+3x^2+x^4}} + \frac{x(1032541+297911x^2)\sqrt{2+3x^2+x^4}}{5005} + \frac{x(208212+65345x^2)(2+3x^2+x^4)^{3/2}}{3003} + \frac{3825}{143}x(2+3x^2+x^4)^{5/2} + \frac{125}{13}x^3(2+3x^2+x^4)^{5/2} - \frac{20884\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{65\sqrt{2+3x^2+x^4}} + \frac{1171349\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\text{EllipticF}\,(}{5005\sqrt{2+3x^2+x^4}}$$

output
```
1/3003*x*(65345*x^2+208212)*(x^4+3*x^2+2)^(3/2)+3825/143*x*(x^4+3*x^2+2)^(
5/2)+125/13*x^3*(x^4+3*x^2+2)^(5/2)+20884/65*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)
-20884/65*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^
(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1171349/5005*(x
^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/
2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/5005*x*(297911*x^2+103254
1)*(x^4+3*x^2+2)^(1/2)
```

3.293.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.59

$$\int (7+5x^2)^3 (2+3x^2+x^4)^{3/2}\,dx = \frac{13572486x+40493455x^3+54938052x^5+46218643x^7+25350660x^9+8705725x^{11}+170100}{}$$

input
```
Integrate[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(13572486*x + 40493455*x^3 + 54938052*x^5 + 46218643*x^7 + 25350660*x^9 +
8705725*x^11 + 1701000*x^13 + 144375*x^15 - (4824204*I)*Sqrt[1 + x^2]*Sqrt
[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (2203890*I)*Sqrt[1 + x^2]*S
qrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(15015*Sqrt[2 + 3*x^2 + x
^4])
```

3.293.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1518, 2207, 1490, 27, 1490, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)^3 (x^4+3x^2+2)^{3/2}\,dx$$

$$\Big\downarrow \text{1518}$$

$$\frac{1}{13}\int (x^4+3x^2+2)^{3/2}(3825x^4+8805x^2+4459)\,dx+\frac{125}{13}(x^4+3x^2+2)^{5/2}x^3$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{13}\left(\frac{1}{11}\int (28005x^2+41399)(x^4+3x^2+2)^{3/2}\,dx+\frac{3825}{11}x(x^4+3x^2+2)^{5/2}\right)+\frac{125}{13}(x^4+3x^2+2)^{5/2}x^3$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{21}\int 3\left(297911x^2+440778\right)\sqrt{x^4+3x^2+2}dx+\frac{1}{21}x\left(65345x^2+208212\right)\left(x^4+3x^2+2\right)^{3/2}\right)+\frac{3825}{11}x(\right.$$
$$\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\int \left(297911x^2+440778\right)\sqrt{x^4+3x^2+2}dx+\frac{1}{21}x\left(65345x^2+208212\right)\left(x^4+3x^2+2\right)^{3/2}\right)+\frac{3825}{11}x(x^4\right.$$
$$\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{1}{15}\int \frac{6\left(804034x^2+1171349\right)}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+2}\left(297911x^2+1032541\right)\right)+\frac{1}{21}x\left(65345x^2+208\right.\right.\right.$$
$$\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{2}{5}\int \frac{804034x^2+1171349}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+2}\left(297911x^2+1032541\right)\right)+\frac{1}{21}x\left(65345x^2+208212\right)\right.\right.$$
$$\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{2}{5}\left(1171349\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+804034\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{5}x\sqrt{x^4+3x^2+2}\left(297911x^2+1\right.\right.\right.\right.$$
$$\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{2}{5}\left(804034\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{1171349\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right)+\frac{1}{5}x\sqrt{x^4+3\right.\right.\right.\right.$$
$$\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3$$

$$\Big\downarrow \text{1455}$$

3.293. $\int \left(7+5x^2\right)^3\left(2+3x^2+x^4\right)^{3/2}\,dx$

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{2}{5}\left(\frac{1171349(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+804034\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2}{x^2}}}{\sqrt{x^4+}}\right.\right.\right.\right.\right.$$

$$\frac{125}{13}\left(x^4+3x^2+2\right)^{5/2}x^3$$

input
```
Int[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(125*x^3*(2 + 3*x^2 + x^4)^(5/2))/13 + ((3825*x*(2 + 3*x^2 + x^4)^(5/2))/1
1 + ((x*(208212 + 65345*x^2)*(2 + 3*x^2 + x^4)^(3/2))/21 + ((x*(1032541 +
297911*x^2)*Sqrt[2 + 3*x^2 + x^4])/5 + (2*(804034*((x*(2 + x^2))/Sqrt[2 +
3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcT
an[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (1171349*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])))/5)/7
)/11)/13
```

3.293.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.293. $\int (7+5x^2)^3\left(2+3x^2+x^4\right)^{3/2}\,dx$

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.293. $\int (7+5x^2)^3 (2+3x^2+x^4)^{3/2} \, dx$

3.293.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.68

method	result
risch	$\frac{x(144375x^{10}+1267875x^8+4613350x^6+8974860x^4+10067363x^2+6786243)\sqrt{x^4+3x^2+2}}{15015} - \frac{1171349i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\right.}{5005\sqrt{x^4+3x^2+2}}$
default	$\frac{598324x^5\sqrt{x^4+3x^2+2}}{1001} + \frac{10067363x^3\sqrt{x^4+3x^2+2}}{15015} + \frac{2262081x\sqrt{x^4+3x^2+2}}{5005} - \frac{1171349i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{5005\sqrt{x^4+3x^2+2}} +$
elliptic	$\frac{598324x^5\sqrt{x^4+3x^2+2}}{1001} + \frac{10067363x^3\sqrt{x^4+3x^2+2}}{15015} + \frac{2262081x\sqrt{x^4+3x^2+2}}{5005} - \frac{1171349i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{5005\sqrt{x^4+3x^2+2}} +$

input
```
int((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15015*x*(144375*x^10+1267875*x^8+4613350*x^6+8974860*x^4+10067363*x^2+67
86243)*(x^4+3*x^2+2)^(1/2)-1171349/5005*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+10442/65*I*2^
(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2
^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.293.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.33

$$\int (7+5x^2)^3 (2+3x^2+x^4)^{3/2}\,dx = \frac{-4824204i\,xE(\arcsin\left(\frac{i}{x}\right)\,|\,2) + 11852298i\,xF(\arcsin\left(\frac{i}{x}\right)\,|\,2) + (144375\,x^{12} + 1267875\,x^{10} + \cdots}{15015\,x}$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/15015*(-4824204*I*x*elliptic_e(arcsin(I/x), 2) + 11852298*I*x*elliptic_f
(arcsin(I/x), 2) + (144375*x^12 + 1267875*x^10 + 4613350*x^8 + 8974860*x^6
 + 10067363*x^4 + 6786243*x^2 + 4824204)*sqrt(x^4 + 3*x^2 + 2))/x
```

3.293.6 Sympy [F]

$$\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2} \, dx = \int \left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}} \left(5x^2+7\right)^3 \, dx$$

input
```
integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3, x)
```

3.293.7 Maxima [F]

$$\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2} \, dx = \int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^3 \, dx$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

3.293.8 Giac [F]

$$\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2} \, dx = \int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^3 \, dx$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int (7+5x^2)^3 (2+3x^2+x^4)^{3/2}\,dx = \int (5\,x^2+7)^3\,(x^4+3\,x^2+2)^{3/2}\,dx$$

input
```
int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2), x)
```

3.294 $\int (7+5x^2)^2 (2+3x^2+x^4)^{3/2} \, dx$

3.294.1 Optimal result

Integrand size = 24, antiderivative size = 198

$$\begin{aligned}\int (7+5x^2)^2 (2+3x^2+x^4)^{3/2} \, dx &= \frac{742x(2+x^2)}{15\sqrt{2+3x^2+x^4}} \\ &+ \frac{x(36783+10643x^2)\sqrt{2+3x^2+x^4}}{1155} + \frac{1}{693}x(7281+2240x^2)(2+3x^2+x^4)^{3/2} \\ &+ \frac{25}{11}x(2+3x^2+x^4)^{5/2} - \frac{742\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\middle|\frac{1}{2}\right)}{15\sqrt{2+3x^2+x^4}} \\ &+ \frac{13879\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{385\sqrt{2+3x^2+x^4}}\end{aligned}$$

output
```
1/693*x*(2240*x^2+7281)*(x^4+3*x^2+2)^(3/2)+25/11*x*(x^4+3*x^2+2)^(5/2)+74
2/15*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-742/15*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*
EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^
4+3*x^2+2)^(1/2)+13879/385*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^
2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2
)+1/1155*x*(10643*x^2+36783)*(x^4+3*x^2+2)^(1/2)
```

3.294.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.63

$$\int (7+5x^2)^2 (2+3x^2 +x^4)^{3/2} \, dx = \frac{429318x + 1160065x^3 + 1333551x^5 + 892084x^7 + 363480x^9 + 82075x^{11} + 7875x^{13} - 171402i}{3465\sqrt{2+}}$$

input
```
Integrate[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(429318*x + 1160065*x^3 + 1333551*x^5 + 892084*x^7 + 363480*x^9 + 82075*x^
11 + 7875*x^13 - (171402*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSin
h[x/Sqrt[2]], 2] - (78420*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSi
nh[x/Sqrt[2]], 2])/(3465*Sqrt[2 + 3*x^2 + x^4])
```

3.294.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1518, 1490, 1490, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)^2 (x^4+3x^2+2)^{3/2} \, dx$$

$$\Big\downarrow \text{1518}$$

$$\frac{1}{11} \int (320x^2+489) (x^4+3x^2+2)^{3/2} dx + \frac{25}{11} x (x^4+3x^2+2)^{5/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{11} \left(\frac{1}{21} \int (10643x^2+15684) \sqrt{x^4+3x^2+2} dx + \frac{1}{63} x (2240x^2+7281) (x^4+3x^2+2)^{3/2} \right) + \frac{25}{11} x (x^4+3x^2+2)^{5/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{11}\left(\frac{1}{21}\left(\frac{1}{15}\int \frac{6(28567x^2+41637)}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+2}(10643x^2+36783)\right)+\frac{1}{63}x(2240x^2+7281)\left(x^4+3x\right.\right.$$
$$\frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2}$$

$\Big\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{21}\left(\frac{2}{5}\int \frac{28567x^2+41637}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+2}(10643x^2+36783)\right)+\frac{1}{63}x(2240x^2+7281)\left(x^4+3x^2+2\right.\right.$$
$$\frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2}$$

$\Big\downarrow$ 1503

$$\frac{1}{11}\left(\frac{1}{21}\left(\frac{2}{5}\left(41637\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+28567\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{5}x\sqrt{x^4+3x^2+2}(10643x^2+36783)\right)\right.$$
$$\frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2}$$

$\Big\downarrow$ 1412

$$\frac{1}{11}\left(\frac{1}{21}\left(\frac{2}{5}\left(28567\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{41637(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right)+\frac{1}{5}x\sqrt{x^4+3x^2+2(\right.\right.$$
$$\frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2}$$

$\Big\downarrow$ 1455

$$\frac{1}{11}\left(\frac{1}{21}\left(\frac{2}{5}\left(\frac{41637(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+28567\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E(\text{ar}}{\sqrt{x^4+3x^2+}}\right.\right.\right.\right.$$
$$\frac{25}{11}x\left(x^4+3x^2+2\right)^{5/2}$$

input
```
Int[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(25*x*(2 + 3*x^2 + x^4)^(5/2))/11 + ((x*(7281 + 2240*x^2)*(2 + 3*x^2 + x^4
)^(3/2))/63 + ((x*(36783 + 10643*x^2)*Sqrt[2 + 3*x^2 + x^4])/5 + (2*(28567
*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/
(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (41637*(1 +
 x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2
 + 3*x^2 + x^4])))/5)/21)/11
```

3.294. $\int \left(7+5x^2\right)^2\left(2+3x^2+x^4\right)^{3/2}\,dx$

3.294.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.294. $\int (7+5x^2)^2 (2+3x^2+x^4)^{3/2}\,dx$

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.294.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x(7875x^8+58450x^6+172380x^4+258044x^2+214659)\sqrt{x^4+3x^2+2}}{3465} - \frac{13879i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{385\sqrt{x^4+3x^2+2}} + \frac{371i\sqrt{2}\,\sqrt{2x^2+4}}{}$
default	$\frac{11492x^5\sqrt{x^4+3x^2+2}}{231} + \frac{258044x^3\sqrt{x^4+3x^2+2}}{3465} + \frac{23851x\sqrt{x^4+3x^2+2}}{385} - \frac{13879i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{385\sqrt{x^4+3x^2+2}} + \frac{371i\sqrt{2}}{}$
elliptic	$\frac{11492x^5\sqrt{x^4+3x^2+2}}{231} + \frac{258044x^3\sqrt{x^4+3x^2+2}}{3465} + \frac{23851x\sqrt{x^4+3x^2+2}}{385} - \frac{13879i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{385\sqrt{x^4+3x^2+2}} + \frac{371i\sqrt{2}}{}$

input
```
int((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3465*x*(7875*x^8+58450*x^6+172380*x^4+258044*x^2+214659)*(x^4+3*x^2+2)^(
1/2)-13879/385*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)
*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+371/15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE
(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.294.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.34

$$\int (7+5x^2)^2 (2+3x^2+x^4)^{3/2}\,dx = \frac{-171402i\,xE(\arcsin\left(\frac{i}{x}\right)\,|2) + 421224i\,xF(\arcsin\left(\frac{i}{x}\right)\,|2) + (7875\,x^{10} + 58450\,x^8 + 172380\,x^6}{3465\,x}$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/3465*(-171402*I*x*elliptic_e(arcsin(I/x), 2) + 421224*I*x*elliptic_f(arc
sin(I/x), 2) + (7875*x^10 + 58450*x^8 + 172380*x^6 + 258044*x^4 + 214659*x
^2 + 171402)*sqrt(x^4 + 3*x^2 + 2))/x
```

3.294.6 Sympy [F]

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2} dx = \int \left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}} \left(5x^2+7\right)^2 dx$$

input
```
integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2, x)
```

3.294.7 Maxima [F]

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2} dx = \int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^2 dx$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

3.294.8 Giac [F]

$$\int \left(7+5x^2\right)^2 \left(2+3x^2+x^4\right)^{3/2} dx = \int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^2 dx$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int (7+5x^2)^2 (2+3x^2+x^4)^{3/2} \, dx = \int (5\,x^2+7)^2 \,(x^4+3\,x^2+2)^{3/2} \, dx$$

input
```
int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2), x)
```

3.295 $\int (7+5x^2)(2+3x^2+x^4)^{3/2} dx$

3.295.1 Optimal result

Integrand size = 22, antiderivative size = 179

$$\int (7+5x^2)(2+3x^2+x^4)^{3/2} dx = \frac{116x(2+x^2)}{15\sqrt{2+3x^2+x^4}} + \frac{1}{105}x(519+149x^2)\sqrt{2+3x^2+x^4} + \frac{1}{63}x(108+35x^2)(2+3x^2+x^4)^{3/2} - \frac{116\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{15\sqrt{2+3x^2+x^4}} + \frac{197\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{35\sqrt{2+3x^2+x^4}}$$

output
```
1/63*x*(35*x^2+108)*(x^4+3*x^2+2)^(3/2)+116/15*x*(x^2+2)/(x^4+3*x^2+2)^(1/
2)-116/15*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^
(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+197/35*(x^2+1)^
(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x
^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/105*x*(149*x^2+519)*(x^4+3*x^2+
2)^(1/2)
```

3.295.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.82 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int (7+5x^2)(2+3x^2+x^4)^{3/2}\,dx = \frac{5274x+12745x^3+12018x^5+5962x^7+1590x^9+175x^{11}-2436i\sqrt{1+x^2}\sqrt{2+x^2}E\Big(i\operatorname{arcsinh}}{315\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(5274*x + 12745*x^3 + 12018*x^5 + 5962*x^7 + 1590*x^9 + 175*x^11 - (2436*I
)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (1110*I
)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(315*Sqr
t[2 + 3*x^2 + x^4])
```

3.295.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1490, 1490, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)(x^4+3x^2+2)^{3/2}\,dx$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{21}\int (149x^2+222)\sqrt{x^4+3x^2+2}dx+\frac{1}{63}x(35x^2+108)(x^4+3x^2+2)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{21}\left(\frac{1}{15}\int \frac{6(406x^2+591)}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+2}(149x^2+519)\right)+ \frac{1}{63}x(35x^2+108)(x^4+3x^2+2)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\left(\frac{2}{5}\int \frac{406x^2+591}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+2}(149x^2+519)\right)+\frac{1}{63}x(35x^2+108)\left(x^4+3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{21}\left(\frac{2}{5}\left(591\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+406\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{5}x\sqrt{x^4+3x^2+2}(149x^2+519)\right)+\frac{1}{63}x(35x^2+108)\left(x^4+3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{ 1412}$$

$$\frac{1}{21}\left(\frac{2}{5}\left(406\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{591(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right)+\frac{1}{5}x\sqrt{x^4+3x^2+2}(149x^2+\right.$$
$$\frac{1}{63}x(35x^2+108)\left(x^4+3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{ 1455}$$

$$\frac{1}{21}\left(\frac{2}{5}\left(\frac{591(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+406\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\right|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}\right.\right.\right.$$
$$\frac{1}{63}x(35x^2+108)\left(x^4+3x^2+2\right)^{3/2}$$

input
```
Int[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(x*(108 + 35*x^2)*(2 + 3*x^2 + x^4)^(3/2))/63 + ((x*(519 + 149*x^2)*Sqrt[2
 + 3*x^2 + x^4])/5 + (2*(406*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[
2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 +
 3*x^2 + x^4]) + (591*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan
[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])))/5)/21
```

3.295. $\int (7+5x^2)\left(2+3x^2+x^4\right)^{3/2}\,dx$

3.295.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.295. $\int (7+5x^2) \left(2+3x^2+x^4\right)^{3/2} \, dx$

3.295.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x(175x^6+1065x^4+2417x^2+2637)\sqrt{x^4+3x^2+2}}{315}-\frac{197i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^4+3x^2+2}}+\frac{58i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{\right.\right.}}{15\sqrt{x^4+3x^2+2}}$
default	$\frac{71x^5\sqrt{x^4+3x^2+2}}{21}+\frac{2417x^3\sqrt{x^4+3x^2+2}}{315}+\frac{293x\sqrt{x^4+3x^2+2}}{35}-\frac{197i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^4+3x^2+2}}+\frac{58i\sqrt{2}\sqrt{2x^2+4}\sqrt{x}}{}$
elliptic	$\frac{71x^5\sqrt{x^4+3x^2+2}}{21}+\frac{2417x^3\sqrt{x^4+3x^2+2}}{315}+\frac{293x\sqrt{x^4+3x^2+2}}{35}-\frac{197i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^4+3x^2+2}}+\frac{58i\sqrt{2}\sqrt{2x^2+4}\sqrt{x}}{}$

input
```
int((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*x*(175*x^6+1065*x^4+2417*x^2+2637)*(x^4+3*x^2+2)^(1/2)-197/35*I*2^(1
/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1
/2)*x,2^(1/2))+58/15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)
^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/
2)))
```

3.295.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.35

$$\int (7+5x^2)\left(2+3x^2+x^4\right)^{3/2}\,dx=\frac{-2436i\,xE(\arcsin\left(\frac{i}{x}\right)\,|2)+5982i\,xF(\arcsin\left(\frac{i}{x}\right)\,|2)+(175\,x^8+1065\,x^6+2417\,x^4+2637\,x^2}{315\,x}$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(-2436*I*x*elliptic_e(arcsin(I/x), 2) + 5982*I*x*elliptic_f(arcsin(I
/x), 2) + (175*x^8 + 1065*x^6 + 2417*x^4 + 2637*x^2 + 2436)*sqrt(x^4 + 3*x
^2 + 2))/x
```

3.295.6 Sympy [F]

$$\int \left(7+5x^2\right) \left(2+3x^2+x^4\right)^{3/2} \, dx = \int \left(\left(x^2+1\right) \left(x^2+2\right)\right)^{\frac{3}{2}} \cdot \left(5x^2+7\right) \, dx$$

input
```
integrate((5*x**2+7)*(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7), x)
```

3.295.7 Maxima [F]

$$\int \left(7+5x^2\right) \left(2+3x^2+x^4\right)^{3/2} \, dx = \int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right) dx$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

3.295.8 Giac [F]

$$\int \left(7+5x^2\right) \left(2+3x^2+x^4\right)^{3/2} \, dx = \int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right) dx$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int (7+5x^2)\left(2+3x^2+x^4\right)^{3/2}\,dx = \int \left(5\,x^2+7\right)\,\left(x^4+3\,x^2+2\right)^{3/2}\,dx$$

input
```
int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2), x)
```

3.296 $\int (2+3x^2+x^4)^{3/2}\,dx$

3.296.1 Optimal result

Integrand size = 14, antiderivative size = 172

$$\int \left(2+3x^2+x^4\right)^{3/2}\,dx=\frac{6x(2+x^2)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}-\frac{6\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}}+\frac{31\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{35\sqrt{2+3x^2+x^4}}$$

output
```
1/7*x*(x^4+3*x^2+2)^(3/2)+6/5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-6/5*(x^2+1)^(3
/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2
+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+31/35*(x^2+1)^(3/2)*(1/(x^2+1))^(1/
2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/
(x^4+3*x^2+2)^(1/2)+1/35*x*(9*x^2+29)*(x^4+3*x^2+2)^(1/2)
```

3.296.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.66

$$\int (2+3x^2+x^4)^{3/2}\,dx = \frac{78x+165x^3+121x^5+39x^7+5x^9-42i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-20i\sqrt{1+x^2}}{35\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(78*x + 165*x^3 + 121*x^5 + 39*x^7 + 5*x^9 - (42*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (20*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

3.296.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1404, 1490, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^4+3x^2+2)^{3/2}\,dx$$

$$\Big\downarrow \text{1404}$$

$$\frac{3}{7}\int (3x^2+4)\sqrt{x^4+3x^2+2}dx+\frac{1}{7}x(x^4+3x^2+2)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{3}{7}\left(\frac{1}{15}\int \frac{2(21x^2+31)}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{15}x\sqrt{x^4+3x^2+2}(9x^2+29)\right)+\frac{1}{7}x(x^4+3x^2+2)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}\left(\frac{2}{15}\int \frac{21x^2+31}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{15}x\sqrt{x^4+3x^2+2}(9x^2+29)\right)+\frac{1}{7}x(x^4+3x^2+2)^{3/2}$$

$$\Big\downarrow \text{1503}$$

$$\frac{3}{7}\left(\frac{2}{15}\left(31\int\frac{1}{\sqrt{x^4+3x^2+2}}dx+21\int\frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{15}x\sqrt{x^4+3x^2+2}(9x^2+29)\right)+\frac{1}{7}x(x^4+3x^2+2)^{3/2}$$

$$\Big\downarrow \text{1412}$$

$$\frac{3}{7}\left(\frac{2}{15}\left(21\int\frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{31(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right)+\frac{1}{15}x\sqrt{x^4+3x^2+2}(9x^2+29)\right)+\frac{1}{7}x(x^4+3x^2+2)^{3/2}$$

$$\Big\downarrow \text{1455}$$

$$\frac{3}{7}\left(\frac{2}{15}\left(\frac{31(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+21\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right.\right.\right.$$
$$\frac{1}{7}x(x^4+3x^2+2)^{3/2}$$

input
```
Int[(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(x*(2 + 3*x^2 + x^4)^(3/2))/7 + (3*((x*(29 + 9*x^2)*Sqrt[2 + 3*x^2 + x^4])
/15 + (2*(21*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqr
t[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) +
 (31*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]))))/15))/7
```

3.296.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.296. $\int (2+3x^2+x^4)^{3/2} \, dx$

3.296.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x(5x^4+24x^2+39)\sqrt{x^4+3x^2+2}}{35} - \frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^4+3x^2+2}} + \frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right.}{5\sqrt{x^4+3x^2+2}}$
default	$\frac{x^5\sqrt{x^4+3x^2+2}}{7} + \frac{24x^3\sqrt{x^4+3x^2+2}}{35} + \frac{39x\sqrt{x^4+3x^2+2}}{35} - \frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^4+3x^2+2}} + \frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\right.}{5\sqrt{x^4}}$
elliptic	$\frac{x^5\sqrt{x^4+3x^2+2}}{7} + \frac{24x^3\sqrt{x^4+3x^2+2}}{35} + \frac{39x\sqrt{x^4+3x^2+2}}{35} - \frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^4+3x^2+2}} + \frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\right.}{5\sqrt{x^4}}$

input
```
int((x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x*(5*x^4+24*x^2+39)*(x^4+3*x^2+2)^(1/2)-31/35*I*2^(1/2)*(2*x^2+4)^(1/
2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+3/
5*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1
/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.296.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.34

$$\int \left(2+3x^2+x^4\right)^{3/2}\,dx = \frac{-42i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right) + 104i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right) + \left(5\,x^6+24\,x^4+39\,x^2+42\right)\sqrt{x^4+3\,x^2+2}}{35\,x}$$

input
```
integrate((x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/35*(-42*I*x*elliptic_e(arcsin(I/x), 2) + 104*I*x*elliptic_f(arcsin(I/x),
 2) + (5*x^6 + 24*x^4 + 39*x^2 + 42)*sqrt(x^4 + 3*x^2 + 2))/x
```

3.296.6 Sympy [F]

$$\int \left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3x^2+2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral((x**4 + 3*x**2 + 2)**(3/2), x)
```

3.296.7 Maxima [F]

$$\int \left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

3.296.8 Giac [F]

$$\int \left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \left(2+3x^2+x^4\right)^{3/2}\,dx=\int \left(x^4+3\,x^2+2\right)^{3/2}\,dx$$

input
```
int((3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int((3*x^2 + x^4 + 2)^(3/2), x)
```

3.297 $\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx$

3.297.1 Optimal result

Integrand size = 24, antiderivative size = 207

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \frac{24x(2+x^2)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x(11+3x^2)\sqrt{2+3x^2+x^4} - \frac{24\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}} + \frac{56\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{375\sqrt{2+3x^2+x^4}} - \frac{9\sqrt{2}(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

output
```
24/125*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-9/875*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+
1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2
+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-24/125*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Elli
pticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*
x^2+2)^(1/2)+56/375*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1
/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/75*
x*(3*x^2+11)*(x^4+3*x^2+2)^(1/2)
```

3.297.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.71

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \frac{3850x+6825x^3+3500x^5+525x^7-2520i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{}$$

input
```
Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

output
```
(3850*x + 6825*x^3 + 3500*x^5 + 525*x^7 - (2520*I)*Sqrt[1 + x^2]*Sqrt[2 +
x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (1022*I)*Sqrt[1 + x^2]*Sqrt[2 +
x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (108*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(13125*Sqrt[2 + 3*x^2 + x^4
])
```

3.297.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1529, 27, 1786, 414, 2207, 27, 2207, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4+3x^2+2)^{3/2}}{5x^2+7}\,dx$$

$$\Big\downarrow \text{1529}$$

$$\frac{1}{500}\int \frac{4(25x^6+115x^4+164x^2+74)}{\sqrt{x^4+3x^2+2}}dx-\frac{9}{125}\int \frac{2(x^2+1)}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{125}\int \frac{25x^6+115x^4+164x^2+74}{\sqrt{x^4+3x^2+2}}dx-\frac{18}{125}\int \frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{1786}$$

$$\frac{1}{125}\int \frac{25x^6+115x^4+164x^2+74}{\sqrt{x^4+3x^2+2}}dx-\frac{18\sqrt{x^2+1}\sqrt{x^2+2}\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(5x^2+7)}dx}{125\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{414}$$

$$\frac{1}{125}\int \frac{25x^6+115x^4+164x^2+74}{\sqrt{x^4+3x^2+2}}dx-\frac{9\sqrt{2}(x^2+2)\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{125}\left(\frac{1}{5}\int \frac{5(55x^4+134x^2+74)}{\sqrt{x^4+3x^2+2}}dx+5\sqrt{x^4+3x^2+2}x^3\right)-\frac{9\sqrt{2}(x^2+2)\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{125}\left(\int \frac{55x^4+134x^2+74}{\sqrt{x^4+3x^2+2}}dx+5\sqrt{x^4+3x^2+2}x^3\right)-\frac{9\sqrt{2}(x^2+2)\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{125}\left(\frac{1}{3}\int \frac{8(9x^2+14)}{\sqrt{x^4+3x^2+2}}dx+\frac{55}{3}\sqrt{x^4+3x^2+2}x+5\sqrt{x^4+3x^2+2}x^3\right)-\frac{9\sqrt{2}(x^2+2)\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{125}\left(\frac{8}{3}\int \frac{9x^2+14}{\sqrt{x^4+3x^2+2}}dx+\frac{55}{3}\sqrt{x^4+3x^2+2}x+5\sqrt{x^4+3x^2+2}x^3\right)-\frac{9\sqrt{2}(x^2+2)\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{125}\left(\frac{8}{3}\left(14\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+9\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{55}{3}\sqrt{x^4+3x^2+2}x+5\sqrt{x^4+3x^2+2}x^3\right)-\frac{9\sqrt{2}(x^2+2)\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1412}$$

3.297. $\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx$

$$\frac{1}{125}\left(\frac{8}{3}\left(9\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{7\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{55}{3}\sqrt{x^4+3x^2+2}x+5\sqrt{x^4}\right.$$

$$\frac{9\sqrt{2}(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \ 1455$$

$$\frac{1}{125}\left(\frac{8}{3}\left(\frac{7\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+9\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right.}{\sqrt{x^4+3x^2+2}}\right.\right.\right.$$

$$\frac{9\sqrt{2}(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

input
```
Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

output
```
((55*x*Sqrt[2 + 3*x^2 + x^4])/3 + 5*x^3*Sqrt[2 + 3*x^2 + x^4] + (8*(9*((x*
(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 +
x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (7*Sqrt[2]*(1 +
x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 +
 x^4]))/3)/125 - (9*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(87
5*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

3.297.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1529
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(2*c*d - e*(b + q)))*((c*d^2 -
 b*d*e + a*e^2)^(p - 1/2)/(4*c*e^(2*p)))   Int[(b - q + 2*c*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[1/(4*c*e^(2*p))   Int[(1/Sqrt[a
+ b*x^2 + c*x^4])*ExpandToSum[(4*c*e^(2*p)*(a + b*x^2 + c*x^4)^(p + 1/2) +
(2*c*d - e*(b + q))*(c*d^2 - b*d*e + a*e^2)^(p - 1/2)*(b - q + 2*c*x^2))/(d
 + e*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p - 1/2, 0] && PosQ[b^2 - 4*a*c] &
& PosQ[c/a]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

3.297. $\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx$

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.297.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.82

method	result
default	$\frac{x^{3}\sqrt{x^{4}+3x^{2}+2}}{25}+\frac{11x\sqrt{x^{4}+3x^{2}+2}}{75}-\frac{73i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{1875\sqrt{x^{4}+3x^{2}+2}}-\frac{12i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{125\sqrt{x^{4}+3x^{2}+2}}-\frac{36i\sqrt{2}}{}$
elliptic	$\frac{x^{3}\sqrt{x^{4}+3x^{2}+2}}{25}+\frac{11x\sqrt{x^{4}+3x^{2}+2}}{75}-\frac{73i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{1875\sqrt{x^{4}+3x^{2}+2}}-\frac{12i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{125\sqrt{x^{4}+3x^{2}+2}}-\frac{36i\sqrt{2}}{}$
risch	$\frac{x\left(3x^{2}+11\right)\sqrt{x^{4}+3x^{2}+2}}{75}-\frac{253i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{1875\sqrt{x^{4}+3x^{2}+2}}+\frac{12i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{125\sqrt{x^{4}+3x^{2}+2}}-$

input
```
int((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

output
```
1/25*x^3*(x^4+3*x^2+2)^(1/2)+11/75*x*(x^4+3*x^2+2)^(1/2)-73/1875*I*2^(1/2)
*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)
*x,2^(1/2))-12/125*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(
1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-36/4375*I*2^(1/2)*(1+1/2*x^2)^(1/2
)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2
))
```

3.297.5 Fricas [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+2)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

output
```
integral((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.297.6 Sympy [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{((x^2+1)\,(x^2+2))^{\frac{3}{2}}}{5x^2+7}\,dx$$

input
```
integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7),x)
```

output
```
Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7), x)
```

3.297.7 Maxima [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+2)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.297.8 Giac [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+2)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+2)^{3/2}}{5\,x^2+7}\,dx$$

input
```
int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7),x)
```

output
```
int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.298 $\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2} dx$

3.298.1 Optimal result

Integrand size = 24, antiderivative size = 222

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2} dx = \frac{9x(2+x^2)}{175\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175(7+5x^2)} - \frac{9\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{175\sqrt{2+3x^2+x^4}} + \frac{59(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{1050\sqrt{2+3x^2+x^4}} + \frac{9(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{2450\sqrt{2+3x^2+x^4}}$$

output

```
9/175*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-9/175*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*
EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^
4+3*x^2+2)^(1/2)+59/1050*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+
1)^(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)+9/2450
*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2
))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/75*x*(x^4+3*x^2+2)^(1/2
)-3/175*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

3.298.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \frac{2800x+6650x^3+5075x^5+1225x^7-945i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)\,E\Big(i\text{arcsinh}$$

input
```
Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

output
```
(2800*x + 6650*x^3 + 5075*x^5 + 1225*x^7 - (945*I)*Sqrt[1 + x^2]*Sqrt[2 +
x^2]*(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (182*I)*Sqrt[1 + x^2
]*Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (189*I)*S
qrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (13
5*I)*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]]
, 2])/(18375*(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])
```

3.298.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.50, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4+3x^2+2)^{3/2}}{(5x^2+7)^2}\,dx$$

$$\Big\downarrow \ 1556$$

$$\int \Bigg(\frac{x^4}{25\sqrt{x^4+3x^2+2}}+\frac{16x^2}{125\sqrt{x^4+3x^2+2}}-\frac{12}{625\,(5x^2+7)\,\sqrt{x^4+3x^2+2}}+\frac{36}{625\,(5x^2+7)^2\,\sqrt{x^4+3x^2+2}}+\frac{}{62}$$

$$\Big\downarrow \ 2009$$

$$\frac{44\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{1875\sqrt{x^4+3x^2+2}}+\frac{81(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{8750\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{9\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{175\sqrt{x^4+3x^2+2}}+\frac{3\sqrt{2}(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}-\frac{39(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{12250\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}-\frac{3\sqrt{x^4+3x^2+2}x}{175(5x^2+7)}+\frac{1}{75}\sqrt{x^4+3x^2+2}x+\frac{9(x^2+2)x}{175\sqrt{x^4+3x^2+2}}$$

input
```
Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

output
```
(9*x*(2 + x^2))/(175*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/75
 - (3*x*Sqrt[2 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) - (9*Sqrt[2]*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(175*Sqrt[2 + 3*x^2 + x
^4]) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/
(8750*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (44*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2
)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(1875*Sqrt[2 + 3*x^2 + x^4]) - (39
*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(12250*Sqrt[2]*Sqrt[(2 + x^2)/
(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4]) + (3*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, A
rcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

3.298.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.298.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.80

method	result
default	$-\frac{3x\sqrt{x^4+3x^2+2}}{175(5x^2+7)}+\frac{x\sqrt{x^4+3x^2+2}}{75}-\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2625\sqrt{x^4+3x^2+2}}-\frac{9i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{350\sqrt{x^4+3x^2+2}}+\frac{9i\sqrt{2}}{}$
elliptic	$-\frac{3x\sqrt{x^4+3x^2+2}}{175(5x^2+7)}+\frac{x\sqrt{x^4+3x^2+2}}{75}-\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2625\sqrt{x^4+3x^2+2}}-\frac{9i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{350\sqrt{x^4+3x^2+2}}+\frac{9i\sqrt{2}}{}$
risch	$\frac{\sqrt{x^4+3x^2+2}\,x(7x^2+8)}{525x^2+735}-\frac{23i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{750\sqrt{x^4+3x^2+2}}+\frac{9i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{350\sqrt{x^4+3x^2+2}}+\frac{9i}{}$

input
```
int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/175*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)+1/75*x*(x^4+3*x^2+2)^(1/2)-13/2625*
I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*
I*2^(1/2)*x,2^(1/2))-9/350*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*
x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+9/6125*I*2^(1/2)*(1+1/2*x^
2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7
,2^(1/2))
```

3.298.5 Fricas [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx=\int \frac{(x^4+3\,x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

output
```
integral((x^4 + 3*x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

3.298.6 Sympy [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{((x^2+1)(x^2+2))^{\frac{3}{2}}}{(5x^2+7)^2}\,dx$$

input
```
integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**2,x)
```

output
```
Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**2, x)
```

3.298.7 Maxima [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{(x^4+3\,x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.298.8 Giac [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{(x^4+3\,x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{(x^4+3\,x^2+2)^{3/2}}{(5\,x^2+7)^2}\,dx$$

input
```
int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^2,x)
```

output
```
int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.299 $\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx$

3.299.1 Optimal result

Integrand size = 24, antiderivative size = 231

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \frac{3x(2+x^2)}{392\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\,(7+5x^2)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\,(7+5x^2)} - \frac{3(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{196\sqrt{2+3x^2+x^4}} + \frac{5(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{784\sqrt{2+3x^2+x^4}} + \frac{141(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{27440\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

output
```
3/392*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+141/54880*(x^2+2)*(1/(x^2+1))^(1/2)*(x
^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(
x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-3/196*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*El
lipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+
2)^(1/2)+5/784*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1
/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)-3/350*x*(x^4+3*x
^2+2)^(1/2)/(5*x^2+7)^2+17/9800*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

3.299.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.39 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.75

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \frac{-\frac{588x(2+3x^2+x^4)}{(7+5x^2)^2}+\frac{119x(2+3x^2+x^4)}{7+5x^2}-525i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-4$$

input
```
Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

output
```
((-588*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (119*x*(2 + 3*x^2 + x^4))/(7 +
 5*x^2) - (525*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2
]], 2] - (406*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]
], 2] + (141*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/S
qrt[2]], 2])/(68600*Sqrt[2 + 3*x^2 + x^4])
```

3.299.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4+3x^2+2)^{3/2}}{(5x^2+7)^3}\,dx$$

$$\Big\downarrow \text{1556}$$

$$\int \left(\frac{x^2}{125\sqrt{x^4+3x^2+2}}-\frac{11}{625(5x^2+7)\sqrt{x^4+3x^2+2}}-\frac{12}{625(5x^2+7)^2\sqrt{x^4+3x^2+2}}+\frac{36}{625(5x^2+7)^3\sqrt{x^4+3}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{784\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{6\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{875\sqrt{x^4+3x^2+2}}-\frac{39(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{24500\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{141(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{27440\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}+\frac{17\sqrt{x^4+3x^2+2}x}{9800(5x^2+7)}-\frac{3\sqrt{x^4+3x^2+2}x}{350(5x^2+7)^2}+\frac{3(x^2+2)x}{392\sqrt{x^4+3x^2+2}}$$

input
```
Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

output
```
(3*x*(2 + x^2))/(392*Sqrt[2 + 3*x^2 + x^4]) - (3*x*Sqrt[2 + 3*x^2 + x^4])/
(350*(7 + 5*x^2)^2) + (17*x*Sqrt[2 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) - (3
9*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(24500*Sq
rt[2]*Sqrt[2 + 3*x^2 + x^4]) - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^
2)]*EllipticE[ArcTan[x], 1/2])/(875*Sqrt[2 + 3*x^2 + x^4]) + (5*(1 + x^2)*
Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[2 +
 3*x^2 + x^4]) + (141*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(27440*Sq
rt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

3.299.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.299.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.53 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.79

method	result
risch	$\frac{\sqrt{x^4+3x^2+2}\,x(17x^2+7)}{1960(5x^2+7)^2}-\frac{19i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2800\sqrt{x^4+3x^2+2}}+\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{784\sqrt{x^4+3x^2+2}}+$
default	$-\frac{3x\sqrt{x^4+3x^2+2}}{350(5x^2+7)^2}+\frac{17x\sqrt{x^4+3x^2+2}}{9800(5x^2+7)}-\frac{29i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9800\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{784\sqrt{x^4+3x^2+2}}+\frac{141i}{}$
elliptic	$-\frac{3x\sqrt{x^4+3x^2+2}}{350(5x^2+7)^2}+\frac{17x\sqrt{x^4+3x^2+2}}{9800(5x^2+7)}-\frac{29i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9800\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{784\sqrt{x^4+3x^2+2}}+\frac{141i}{}$

input
```
int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1960*(x^4+3*x^2+2)^(1/2)*x*(17*x^2+7)/(5*x^2+7)^2-19/2800*I*2^(1/2)*(2*x
^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^
(1/2))+3/784*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(
EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))+141
/68600*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Ellip
ticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

3.299.5 Fricas [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx=\int \frac{(x^4+3\,x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

output
```
integral((x^4 + 3*x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

3.299.6 Sympy [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{((x^2+1)(x^2+2))^{\frac{3}{2}}}{(5x^2+7)^3}\,dx$$

input
```
integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**3,x)
```

output
```
Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**3, x)
```

3.299.7 Maxima [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(x^4+3\,x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.299.8 Giac [F]

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(x^4+3\,x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx=\int \frac{(x^4+3\,x^2+2)^{3/2}}{(5\,x^2+7)^3}\,dx$$

input
```
int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^3,x)
```

output
```
int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.300 $\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$

3.300.1 Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \frac{135x(2+x^2)}{\sqrt{2+3x^2+x^4}} + 75x\sqrt{2+3x^2+x^4} + 25x^3\sqrt{2+3x^2+x^4} - \frac{135\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{193(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

output
```
135*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+193/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*El
lipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+
3*x^2+2)^(1/2)-135*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/
2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+75*x*(
x^4+3*x^2+2)^(1/2)+25*x^3*(x^4+3*x^2+2)^(1/2)
```

3.300.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{25x(6+11x^2+6x^4+x^6)-135i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-58i\sqrt{1+x^2}\sqrt{2+x^2}\operatorname{EllipticF}\left(\right.}{\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(25*x*(6 + 11*x^2 + 6*x^4 + x^6) - (135*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ell
ipticE[I*ArcSinh[x/Sqrt[2]], 2] - (58*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellip
ticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]
```

3.300.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1518, 27, 2207, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^3}{\sqrt{x^4+3x^2+2}}dx$$
$$\Big\downarrow \text{1518}$$
$$\frac{1}{5}\int \frac{5(225x^4+585x^2+343)}{\sqrt{x^4+3x^2+2}}dx+25\sqrt{x^4+3x^2+2}x^3$$
$$\Big\downarrow \text{27}$$
$$\int \frac{225x^4+585x^2+343}{\sqrt{x^4+3x^2+2}}dx+25\sqrt{x^4+3x^2+2}x^3$$
$$\Big\downarrow \text{2207}$$
$$\frac{1}{3}\int \frac{3(135x^2+193)}{\sqrt{x^4+3x^2+2}}dx+75\sqrt{x^4+3x^2+2}x+25\sqrt{x^4+3x^2+2}x^3$$

$$\Big\downarrow \; 27$$

$$\int \frac{135x^2+193}{\sqrt{x^4+3x^2+2}}dx+75\sqrt{x^4+3x^2+2}x+25\sqrt{x^4+3x^2+2}x^3$$

$$\Big\downarrow \; 1503$$

$$193\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+135\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+75\sqrt{x^4+3x^2+2}x+25\sqrt{x^4+3x^2+2}x^3$$

$$\Big\downarrow \; 1412$$

$$135\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{193(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+75\sqrt{x^4+3x^2+2}x+25\sqrt{x^4+3x^2+2}x^3$$

$$\Big\downarrow \; 1455$$

$$\frac{193(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+135\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)+75\sqrt{x^4+3x^2+2}x+25\sqrt{x^4+3x^2+2}x^3$$

input
```
Int[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
75*x*Sqrt[2 + 3*x^2 + x^4] + 25*x^3*Sqrt[2 + 3*x^2 + x^4] + 135*((x*(2 + x
^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*
EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (193*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^
4])
```

3.300. $\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$

3.300.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.300. $\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}} \, dx$

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.300.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.80

method	result
risch	$25x(x^2+3)\sqrt{x^4+3x^2+2}-\frac{193i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{135i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{}}{}\right.\right.}{2\sqrt{x^4+3x^2+2}}$
default	$-\frac{193i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+25x^3\sqrt{x^4+3x^2+2}+75x\sqrt{x^4+3x^2+2}+\frac{135i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\right.}{2\sqrt{x}}$
elliptic	$-\frac{193i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+25x^3\sqrt{x^4+3x^2+2}+75x\sqrt{x^4+3x^2+2}+\frac{135i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\right.}{2\sqrt{x}}$

input
```
int((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
25*x*(x^2+3)*(x^4+3*x^2+2)^(1/2)-193/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+135/2*I*2^(1/2
)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/
2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.300.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.34

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \frac{-135i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|2\right)+328i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|2\right)+5\left(5\,x^4+15\,x^2+27\right)\sqrt{x^4+3\,x^2+2}}{x}$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
(-135*I*x*elliptic_e(arcsin(I/x), 2) + 328*I*x*elliptic_f(arcsin(I/x), 2)
+ 5*(5*x^4 + 15*x^2 + 27)*sqrt(x^4 + 3*x^2 + 2))/x
```

3.300.6 Sympy [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5x^2+7)^3}{\sqrt{(x^2+1)(x^2+2)}}\,dx$$

input
```
integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral((5*x**2 + 7)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.300.7 Maxima [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

3.300.8 Giac [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.301 $\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

3.301.1 Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{20x(2+x^2)}{\sqrt{2+3x^2+x^4}} + \frac{25}{3}x\sqrt{2+3x^2+x^4} - \frac{20\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{97(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+3x^2+x^4}}$$

output
```
20*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+97/6*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Elli
pticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*
x^2+2)^(1/2)-20*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),
1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+25/3*x*(x
^4+3*x^2+2)^(1/2)
```

3.301.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{25x(2+3x^2+x^4)-60i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-37i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\text{arcsinh}\right.}{3\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(25*x*(2 + 3*x^2 + x^4) - (60*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*A
rcSinh[x/Sqrt[2]], 2] - (37*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*Arc
Sinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

3.301.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1518, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^2}{\sqrt{x^4+3x^2+2}}\,dx$$
$$\Big\downarrow \text{1518}$$
$$\frac{1}{3}\int \frac{60x^2+97}{\sqrt{x^4+3x^2+2}}dx+\frac{25}{3}\sqrt{x^4+3x^2+2}x$$
$$\Big\downarrow \text{1503}$$
$$\frac{1}{3}\left(97\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+60\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{25}{3}\sqrt{x^4+3x^2+2}x$$
$$\Big\downarrow \text{1412}$$
$$\frac{1}{3}\left(60\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{97(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right)+\frac{25}{3}\sqrt{x^4+3x^2+2}x$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{3}\left(\frac{97(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+60\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)\right)+\frac{25}{3}\sqrt{x^4+3x^2+2}x$$

input
```
Int[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(25*x*Sqrt[2 + 3*x^2 + x^4])/3 + (60*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4]
- (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/
Sqrt[2 + 3*x^2 + x^4]) + (97*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF
[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]))/3
```

3.301.3.1 Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.301. $\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.301.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{97 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{6 \sqrt{x^{4}+3 x^{2}+2}}+\frac{25 x \sqrt{x^{4}+3 x^{2}+2}}{3}+\frac{10 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)\right)}{\sqrt{x^{4}+3 x^{2}+2}}$	121
risch	$-\frac{97 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{6 \sqrt{x^{4}+3 x^{2}+2}}+\frac{25 x \sqrt{x^{4}+3 x^{2}+2}}{3}+\frac{10 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)\right)}{\sqrt{x^{4}+3 x^{2}+2}}$	121
elliptic	$-\frac{97 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{6 \sqrt{x^{4}+3 x^{2}+2}}+\frac{25 x \sqrt{x^{4}+3 x^{2}+2}}{3}+\frac{10 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)\right)}{\sqrt{x^{4}+3 x^{2}+2}}$	121

input
```
int((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-97/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
F(1/2*I*2^(1/2)*x,2^(1/2))+25/3*x*(x^4+3*x^2+2)^(1/2)+10*I*2^(1/2)*(2*x^2+
4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1
/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.301.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.35

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{-60i\,xE(\arcsin\left(\frac{i}{x}\right)\,|2)+157i\,xF(\arcsin\left(\frac{i}{x}\right)\,|2)+5\sqrt{x^4+3\,x^2+2}(5\,x^2+12)}{3\,x}$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-60*I*x*elliptic_e(arcsin(I/x), 2) + 157*I*x*elliptic_f(arcsin(I/x),
2) + 5*sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 12))/x
```

3.301.6 Sympy [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5x^2+7)^2}{\sqrt{(x^2+1)(x^2+2)}}\,dx$$

input
```
integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral((5*x**2 + 7)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.301.7 Maxima [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

3.301.8 Giac [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{{\left(5\,x^2+7\right)}^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.302 $\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx$

3.302.1 Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{5x(2+x^2)}{\sqrt{2+3x^2+x^4}} - \frac{5\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{7(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

output
```
5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+7/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellipt
icF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^
2+2)^(1/2)-5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2
*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

3.302.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\left(5E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+2\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)\right)}{\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(5*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] +
2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

3.302.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+7}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 1503}$$

$$7\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+5\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 1412}$$

$$5\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{ 1455}$$

$$\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+5\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)$$

input
```
Int[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
5*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (7*(1 + x^
2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 +
3*x^2 + x^4])
```

3.302.3.1 Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.302.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{7 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}+\frac{5 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}$	106
elliptic	$-\frac{7 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}+\frac{5 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}$	106

input
```
int((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-7/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF
(1/2*I*2^(1/2)*x,2^(1/2))+5/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4
+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2
)*x,2^(1/2)))
```

3.302.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.34

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{-5i\,xE\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+12i\,xF\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+5\sqrt{x^4+3\,x^2+2}}{x}$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
(-5*I*x*elliptic_e(arcsin(I/x), 2) + 12*I*x*elliptic_f(arcsin(I/x), 2) + 5
*sqrt(x^4 + 3*x^2 + 2))/x
```

3.302.6 Sympy [F]

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{5x^2+7}{\sqrt{(x^2+1)\,(x^2+2)}}\,dx$$

input
```
integrate((5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral((5*x**2 + 7)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.302.7 Maxima [F]

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

3.302.8 Giac [F]

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx=\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.303 $\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$

3.303.1 Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx=\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

output
```
1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))
*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

3.303.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx=-\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[1/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt
[2 + 3*x^2 + x^4]
```

3.303.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}}\,dx$$

$$\Big\downarrow \text{1412}$$

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

input
```
Int[1/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*S
qrt[2 + 3*x^2 + x^4])
```

3.303.3.1 Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.303.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}$	46
elliptic	$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}$	46

input
```
int(1/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF
(1/2*I*2^(1/2)*x,2^(1/2))
```

3.303.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx = -i\,F\left(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2\right)$$

input
```
integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-I*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2)
```

3.303.6 Sympy [F]

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3x^2+2}}\,dx$$

input
```
integrate(1/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(x**4 + 3*x**2 + 2), x)
```

3.303.7 Maxima [F]

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

3.303.8 Giac [F]

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int(1/(3*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int(1/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.304 $\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$

3.304.1 Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{5(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

output
```
-5/28*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2
/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/4*(x
^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/
2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

3.304.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticPi}\left(\frac{10}{7},i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{7\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
((-1/7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]
], 2])/Sqrt[2 + 3*x^2 + x^4]
```

3.304.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1538, 27, 1412, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{1538}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{4}\int \frac{2(x^2+1)}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{2}\int \frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{1412}$$

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5}{2}\int \frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{1786}$$

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{x^2+1}\sqrt{x^2+2}\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(5x^2+7)}dx}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{414}$$

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+2)\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

input
```
Int[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]
*Sqrt[2 + 3*x^2 + x^4]) - (5*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(1
4*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

3.304.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1538
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/(2*c*d - e*(b - q)))   I
nt[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[e/(2*c*d - e*(b - q))   Int[(b
- q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !LtQ[c, 0]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

3.304. $\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$

3.304.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{7\sqrt{x^4+3x^2+2}}$	47
elliptic	$-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{7\sqrt{x^4+3x^2+2}}$	47

input
```
int(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Ellipti
cPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

3.304.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^6 + 22*x^4 + 31*x^2 + 14), x)
```

3.304.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2+1)\,(x^2+2)}\,(5x^2+7)}\,dx$$

input
```
integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)), x)
```

3.304.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

3.304.8 Giac [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{(5\,x^2+7)\,\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)),x)
```

output
```
int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.305 $\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$

3.305.1 Optimal result

Integrand size = 24, antiderivative size = 209

$$\begin{aligned}\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx = {} & \frac{5x(2+x^2)}{84\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{84(7+5x^2)} \\ & -\frac{5(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{42\sqrt{2}\sqrt{2+3x^2+x^4}} \\ & +\frac{9(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{56\sqrt{2}\sqrt{2+3x^2+x^4}} \\ & -\frac{65(2+x^2)\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{1176\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

output
```
5/84*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-65/2352*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+
1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2
+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-5/84*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellipt
icE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^
2+2)^(1/2)+9/112*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2)
,1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-25/84*x*
(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

3.305.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.30 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{-350x-525x^3-175x^5-35i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)\,E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-14i\sqrt{1+x^2}\sqrt{2+x^2}(7+}{}$$

input
```
Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
(-350*x - 525*x^3 - 175*x^5 - (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^
2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]
*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (91*I)*Sqrt[1 + x^2]*Sqr
t[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (65*I)*x^2*Sqrt[1 +
 x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(588*(7 + 5
*x^2)*Sqrt[2 + 3*x^2 + x^4])
```

3.305.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1551, 2234, 27, 1503, 1412, 1455, 1538, 27, 1412, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^2\sqrt{x^4+3x^2+2}}\,dx$$
$$\Big\downarrow \text{1551}$$
$$\frac{1}{84}\int \frac{25x^4+70x^2+62}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx-\frac{25x\sqrt{x^4+3x^2+2}}{84\,(5x^2+7)}$$
$$\Big\downarrow \text{2234}$$
$$\frac{1}{84}\left(13\int \frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx-\frac{1}{25}\int -\frac{25(5x^2+7)}{\sqrt{x^4+3x^2+2}}dx\right)-\frac{25x\sqrt{x^4+3x^2+2}}{84\,(5x^2+7)}$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{84}\left(13\int \frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx+\int \frac{5x^2+7}{\sqrt{x^4+3x^2+2}}dx\right)-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{84}\left(7\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+5\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+13\int \frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{84}\left(5\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+13\int \frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx+\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right)-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{84}\left(13\int \frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx+\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+5\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2}{}\right.\right.$$
$$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}$$

$$\Big\downarrow \text{1538}$$

$$\frac{1}{84}\left(13\left(\frac{1}{2}\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{4}\int \frac{2(x^2+1)}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right.$$
$$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{84}\left(13\left(\frac{1}{2}\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{2}\int \frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right.$$
$$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}$$

$$\Big\downarrow \text{1412}$$

3.305. $\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$

$$\frac{1}{84}\left(13\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5}{2}\int\frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{Ellipt}}{\sqrt{2}\sqrt{x^4+3}}\right.$$
$$\frac{25x\sqrt{x^4+3x^2+2}}{84\left(5x^2+7\right)}$$

$\downarrow$ 1786

$$\frac{1}{84}\left(13\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{x^2+1}\sqrt{x^2+2}\int\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(5x^2+7)}dx}{2\sqrt{x^4+3x^2+2}}\right)+\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{E}}{\sqrt{2}\sqrt{x^4}}\right.$$
$$\frac{25x\sqrt{x^4+3x^2+2}}{84\left(5x^2+7\right)}$$

$\downarrow$ 414

$$\frac{1}{84}\left(\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+5\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)+1\right.$$
$$\frac{25x\sqrt{x^4+3x^2+2}}{84\left(5x^2+7\right)}$$

input
```
Int[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
(-25*x*Sqrt[2 + 3*x^2 + x^4])/(84*(7 + 5*x^2)) + (5*((x*(2 + x^2))/Sqrt[2
+ 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[Ar
cTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (7*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x
^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + 13*(((1
+ x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*Sqr
t[2 + 3*x^2 + x^4]) - (5*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(14*Sq
rt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])))/84
```

3.305.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.305. $\int\frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1538
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/(2*c*d - e*(b - q)))   I
nt[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[e/(2*c*d - e*(b - q))   Int[(b
- q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !LtQ[c, 0]
```

3.305. $\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.305.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

method	result
default	$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{84\sqrt{x^4+3x^2+2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{168\sqrt{x^4+3x^2+2}}-\frac{13i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\right.}{588\sqrt{x^4+3x^2+}}$
elliptic	$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{84\sqrt{x^4+3x^2+2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{168\sqrt{x^4+3x^2+2}}-\frac{13i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\right.}{588\sqrt{x^4+3x^2+}}$
risch	$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{24\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{168\sqrt{x^4+3x^2+2}}-\frac{13i\sqrt{2}\,\sqrt{}}{}$

input
```
int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-25/84*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-1/84*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-5/168*I*2
^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2
^(1/2)*x,2^(1/2))-13/588*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*
x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

3.305.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^8 + 145*x^6 + 309*x^4 + 287*x^2 + 98)
, x)
```

3.305.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2+1)\,(x^2+2)}\,(5x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2), x)
```

3.305.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

3.305.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^2 \sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^2 \sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{(5\,x^2+7)^2\,\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2)),x)
```

output
```
int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.306 $\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx$

3.306.1 Optimal result

Integrand size = 24, antiderivative size = 237

$$\begin{aligned}\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx &= \frac{65x(2+x^2)}{4704\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{168\left(7+5x^2\right)^2}\\ &\quad-\frac{325x\sqrt{2+3x^2+x^4}}{4704\left(7+5x^2\right)}\\ &\quad-\frac{65(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{2352\sqrt{2}\sqrt{2+3x^2+x^4}}\\ &\quad+\frac{631(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\mathrm{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{9408\sqrt{2}\sqrt{2+3x^2+x^4}}\\ &\quad-\frac{2525(2+x^2)\,\mathrm{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{65856\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

output
```
65/4704*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-2525/131712*(x^2+2)*(1/(x^2+1))^(1/2
)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+
2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-65/4704*(x^2+1)^(3/2)*(1/(x^2+1))^(1
/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)
/(x^4+3*x^2+2)^(1/2)+631/18816*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x
/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^
(1/2)-25/168*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2-325/4704*x*(x^4+3*x^2+2)^(1
/2)/(5*x^2+7)
```

3.306.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.78

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{-175x(238+487x^2+314x^4+65x^6)-455i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)^2\,E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+14i\sqrt{1+}}{32928\,(7}$$

input
```
Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
(-175*x*(238 + 487*x^2 + 314*x^4 + 65*x^6) - (455*I)*Sqrt[1 + x^2]*Sqrt[2
+ x^2]*(7 + 5*x^2)^2*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (14*I)*Sqrt[1 +
x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (505
*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticPi[10/7, I*ArcSinh[x
/Sqrt[2]], 2])/(32928*(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4])
```

3.306.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {1551, 2210, 27, 2234, 27, 1503, 1412, 1455, 1538, 27, 1412, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^3\sqrt{x^4+3x^2+2}}\,dx$$

$$\Big\downarrow \text{1551}$$

$$\frac{1}{168}\int \frac{-25x^4-10x^2+74}{(5x^2+7)^2\sqrt{x^4+3x^2+2}}dx-\frac{25x\sqrt{x^4+3x^2+2}}{168\,(5x^2+7)^2}$$

$$\Big\downarrow \text{2210}$$

$$\frac{1}{168}\left(\frac{1}{84}\int \frac{3(325x^4+770x^2+946)}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx-\frac{325x\sqrt{x^4+3x^2+2}}{28\,(5x^2+7)}\right)-\frac{25x\sqrt{x^4+3x^2+2}}{168\,(5x^2+7)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{168}\left(\frac{1}{28}\int\frac{325x^4+770x^2+946}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx-\frac{325x\sqrt{x^4+3x^2+2}}{28(5x^2+7)}\right)-\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}$$

$$\Big\downarrow \text{ 2234}$$

$$\frac{1}{168}\left(\frac{1}{28}\left(505\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx-\frac{1}{25}\int-\frac{25(65x^2+63)}{\sqrt{x^4+3x^2+2}}dx\right)-\frac{325x\sqrt{x^4+3x^2+2}}{28(5x^2+7)}\right)-\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{168}\left(\frac{1}{28}\left(505\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx+\int\frac{65x^2+63}{\sqrt{x^4+3x^2+2}}dx\right)-\frac{325x\sqrt{x^4+3x^2+2}}{28(5x^2+7)}\right)-\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{168}\left(\frac{1}{28}\left(63\int\frac{1}{\sqrt{x^4+3x^2+2}}dx+65\int\frac{x^2}{\sqrt{x^4+3x^2+2}}dx+505\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)-\frac{325x\sqrt{x^4+}}{28(5x^2}\right.$$
$$\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}$$

$$\Big\downarrow \text{ 1412}$$

$$\frac{1}{168}\left(\frac{1}{28}\left(65\int\frac{x^2}{\sqrt{x^4+3x^2+2}}dx+505\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx+\frac{63(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\operatorname{EllipticF}(\arctan(x)}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right.\right.$$
$$\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}$$

$$\Big\downarrow \text{ 1455}$$

$$\frac{1}{168}\left(\frac{1}{28}\left(505\int\frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx+\frac{63(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+65\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}\right.\right.\right.$$
$$\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}$$

$$\Big\downarrow \text{ 1538}$$

3.306. $\int\frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx$

$$\frac{1}{168}\left(\frac{1}{28}\left(505\left(\frac{1}{2}\int\frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{4}\int\frac{2(x^2+1)}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{63(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}(\text{arcta}}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right.\right.$$
$$\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}$$

$\downarrow$ 27

$$\frac{1}{168}\left(\frac{1}{28}\left(505\left(\frac{1}{2}\int\frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{2}\int\frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{63(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}(\text{arcta}}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right.\right.$$
$$\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}$$

$\downarrow$ 1412

$$\frac{1}{168}\left(\frac{1}{28}\left(505\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5}{2}\int\frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)+\frac{63(x^2+1)\sqrt{\frac{x^2}{x^2}}}{\sqrt{2}}\right.\right.$$
$$\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}$$

$\downarrow$ 1786

$$\frac{1}{168}\left(\frac{1}{28}\left(505\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{x^2+1}\sqrt{x^2+2}\int\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(5x^2+7)}dx}{2\sqrt{x^4+3x^2+2}}\right)+\frac{63(x^2+1)}{}\right.\right.$$
$$\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}$$

$\downarrow$ 414

$$\frac{1}{168}\left(\frac{1}{28}\left(\frac{63(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+65\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\middle|\frac{1}{2}\right.}{\sqrt{x^4+3x^2+2}}\right.\right.\right.$$
$$\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}$$

input
```
Int[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```

3.306. $\int\frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx$

output
```
(-25*x*Sqrt[2 + 3*x^2 + x^4])/(168*(7 + 5*x^2)^2) + ((-325*x*Sqrt[2 + 3*x^
2 + x^4])/(28*(7 + 5*x^2)) + (65*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (S
qrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt
[2 + 3*x^2 + x^4]) + (63*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[Arc
Tan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + 505*(((1 + x^2)*Sqrt[(2 +
x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4
]) - (5*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(14*Sqrt[2]*Sqrt[(2 + x
^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])))/28)/168
```

3.306.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.306. $\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx$

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1538
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/(2*c*d - e*(b - q)))   I
nt[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[e/(2*c*d - e*(b - q))   Int[(b
- q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !LtQ[c, 0]
```

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

3.306. $\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx$

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.306.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.48 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{25\sqrt{x^4+3x^2+2}\,x(65x^2+119)}{4704(5x^2+7)^2}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{448\sqrt{x^4+3x^2+2}}+\frac{65i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right.}{9408\sqrt{x^4+3x^2+2}}$
default	$-\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}-\frac{325x\sqrt{x^4+3x^2+2}}{4704(5x^2+7)}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4704\sqrt{x^4+3x^2+2}}-\frac{65i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9408\sqrt{x^4+3x^2+2}}-505$
elliptic	$-\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}-\frac{325x\sqrt{x^4+3x^2+2}}{4704(5x^2+7)}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4704\sqrt{x^4+3x^2+2}}-\frac{65i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9408\sqrt{x^4+3x^2+2}}-505$

input
```
int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-25/4704*(x^4+3*x^2+2)^(1/2)*x*(65*x^2+119)/(5*x^2+7)^2-3/448*I*2^(1/2)*(2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,
2^(1/2))+65/9408*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/
2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
-505/32928*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*E
llipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

3.306.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^10 + 900*x^8 + 2560*x^6 + 3598*x^4 +
 2499*x^2 + 686), x)
```

3.306.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2+1)(x^2+2)}\,(5x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3), x)
```

3.306.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

3.306.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^3 \sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{(5\,x^2+7)^3\,\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2)),x)
```

output
```
int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.307 $\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx$

3.307.1 Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{7679x(2+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{5000}{3}x\sqrt{2+3x^2+x^4} + 625x^3\sqrt{2+3x^2+x^4} - \frac{7679(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{15383(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+3x^2+x^4}}$$

output
```
7679/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/2*x*(179*x^2+115)/(x^4+3*x^2+2)^(1/
2)-7679/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^
(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+15383/6*(x^2+1)
^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((
x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+5000/3*x*(x^4+3*x^2+2)^(1/2)+625
*x^3*(x^4+3*x^2+2)^(1/2)
```

3.307.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.58

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{19655x+36963x^3+21250x^5+3750x^7-23037i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\right.}{6\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(19655*x + 36963*x^3 + 21250*x^5 + 3750*x^7 - (23037*I)*Sqrt[1 + x^2]*Sqrt
[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (7729*I)*Sqrt[1 + x^2]*Sqrt
[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(6*Sqrt[2 + 3*x^2 + x^4])
```

3.307.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1517, 25, 2207, 27, 2207, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^5}{(x^4+3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1517}$$

$$-\frac{1}{2}\int -\frac{6250x^6+25000x^4+35179x^2+16922}{\sqrt{x^4+3x^2+2}}dx-\frac{x(179x^2+115)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\int \frac{6250x^6+25000x^4+35179x^2+16922}{\sqrt{x^4+3x^2+2}}dx-\frac{x(179x^2+115)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{1}{2}\left(\frac{1}{5}\int \frac{5(10000x^4+27679x^2+16922)}{\sqrt{x^4+3x^2+2}}dx+1250\sqrt{x^4+3x^2+2}x^3\right)-\frac{x(179x^2+115)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\int \frac{10000x^4+27679x^2+16922}{\sqrt{x^4+3x^2+2}}dx+1250\sqrt{x^4+3x^2+2}x^3\right)-\frac{x(179x^2+115)}{2\sqrt{x^4+3x^2+2}}$$

↓ 2207

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{23037x^2+30766}{\sqrt{x^4+3x^2+2}}dx+\frac{10000}{3}\sqrt{x^4+3x^2+2}x+1250\sqrt{x^4+3x^2+2}x^3\right)-\frac{x(179x^2+115)}{2\sqrt{x^4+3x^2+2}}$$

↓ 1503

$$\frac{1}{2}\left(\frac{1}{3}\left(30766\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+23037\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{10000}{3}\sqrt{x^4+3x^2+2}x+1250\sqrt{x^4+3x^2+2}\right.$$
$$\frac{x(179x^2+115)}{2\sqrt{x^4+3x^2+2}}$$

↓ 1412

$$\frac{1}{2}\left(\frac{1}{3}\left(23037\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{15383\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{10000}{3}\sqrt{x^4+3x^2+2}\right.$$
$$\frac{x(179x^2+115)}{2\sqrt{x^4+3x^2+2}}$$

↓ 1455

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{15383\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+23037\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E(\text{arcta}}{\sqrt{x^4+3x^2+2}}\right.\right.\right.$$
$$\frac{x(179x^2+115)}{2\sqrt{x^4+3x^2+2}}$$

input
```
Int[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
-1/2*(x*(115 + 179*x^2))/Sqrt[2 + 3*x^2 + x^4] + ((10000*x*Sqrt[2 + 3*x^2
+ x^4])/3 + 1250*x^3*Sqrt[2 + 3*x^2 + x^4] + (23037*((x*(2 + x^2))/Sqrt[2
+ 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[Ar
cTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (15383*Sqrt[2]*(1 + x^2)*Sqrt[(2 +
 x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4])/3)/2
```

3.307. $\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx$

3.307.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.307. $\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx$

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.307.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.96 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

method	result
risch	$\frac{x(3750x^6+21250x^4+36963x^2+19655)}{6\sqrt{x^4+3x^2+2}} - \frac{15383i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}} + \frac{7679i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\right.}{4\sqrt{x^4+3x^2+2}}$
elliptic	$-\frac{2\left(\frac{179}{4}x^3+\frac{115}{4}x\right)}{\sqrt{x^4+3x^2+2}} + 625x^3\sqrt{x^4+3x^2+2} + \frac{5000x\sqrt{x^4+3x^2+2}}{3} - \frac{15383i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}} + \frac{7679i}{}$
default	$-\frac{33614\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}} - \frac{15383i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}} + \frac{7679i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$

input
```
int((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(3750*x^6+21250*x^4+36963*x^2+19655)/(x^4+3*x^2+2)^(1/2)-15383/6*I*2
^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2
^(1/2)*x,2^(1/2))+7679/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^
2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2
^(1/2)))
```

3.307.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.53

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{23037\,(i\,x^5+3i\,x^3+2i\,x)E(\arcsin\left(\frac{i}{x}\right)\,|\,2)+53803\,(-i\,x^5-3i\,x^3-2i\,x)F(\arcsin\left(\frac{i}{x}\right)\,|\,2)-2\,(1875\,x^8+}{6\,(x^5+3\,x^3+2\,x)}$$

input
```
integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(23037*(I*x^5 + 3*I*x^3 + 2*I*x)*elliptic_e(arcsin(I/x), 2) + 53803*(
-I*x^5 - 3*I*x^3 - 2*I*x)*elliptic_f(arcsin(I/x), 2) - 2*(1875*x^8 + 10625
*x^6 + 30000*x^4 + 44383*x^2 + 23037)*sqrt(x^4 + 3*x^2 + 2))/(x^5 + 3*x^3
+ 2*x)
```

3.307.6 Sympy [F]

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^5}{((x^2+1)(x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**5/(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**5/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

3.307.7 Maxima [F]

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.307.8 Giac [F]

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^5/(3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^5/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.308 $\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx$

3.308.1 Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{637x(2+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{x(145+113x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{625}{3}x\sqrt{2+3x^2+x^4} - \frac{637(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{1067\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}$$

output
```
637/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(113*x^2+145)/(x^4+3*x^2+2)^(1/2
)-637/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1
/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1067/3*(x^2+1)^(3
/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2
+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/3*x*(x^4+3*x^2+2)^(1/2)
```

3.308.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{2935x+4089x^3+1250x^5-1911i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-2357i}{6\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(2935*x + 4089*x^3 + 1250*x^5 - (1911*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellip
ticE[I*ArcSinh[x/Sqrt[2]], 2] - (2357*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellip
ticF[I*ArcSinh[x/Sqrt[2]], 2])/(6*Sqrt[2 + 3*x^2 + x^4])
```

3.308.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1517, 25, 2207, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^4}{(x^4+3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \ 1517$$

$$\frac{x(113x^2+145)}{2\sqrt{x^4+3x^2+2}}-\frac{1}{2}\int -\frac{1250x^4+3137x^2+2256}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\int \frac{1250x^4+3137x^2+2256}{\sqrt{x^4+3x^2+2}}dx+\frac{x(113x^2+145)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \ 2207$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1911x^2+4268}{\sqrt{x^4+3x^2+2}}dx+\frac{1250}{3}\sqrt{x^4+3x^2+2}x\right)+\frac{x(113x^2+145)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \ 1503$$

$$\frac{1}{2}\left(\frac{1}{3}\left(4268\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+1911\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1250}{3}\sqrt{x^4+3x^2+2}x\right)+\frac{x(113x^2+145)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \ 1412$$

$$\frac{1}{2}\left(\frac{1}{3}\left(1911\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{2134\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{1250}{3}\sqrt{x^4+3x^2+2}x\right)+\frac{x(113x^2+145)}{2\sqrt{x^4+3x^2+2}}$$

$\downarrow$ 1455

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{2134\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+1911\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E(\arctan(\right.\right.\right.}{\sqrt{x^4+3x^2+2}}$$

$$\frac{x(113x^2+145)}{2\sqrt{x^4+3x^2+2}}$$

input
```
Int[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(x*(145 + 113*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + ((1250*x*Sqrt[2 + 3*x^2 +
x^4])/3 + (1911*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*
Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]
) + (2134*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x],
 1/2])/Sqrt[2 + 3*x^2 + x^4])/3)/2
```

3.308.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.308. $\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx$

3.308.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

method	result
risch	$\frac{x(1250x^4+4089x^2+2935)}{6\sqrt{x^4+3x^2+2}} - \frac{1067i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}} + \frac{637i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right.}{4\sqrt{x^4+3x^2+2}}$
elliptic	$-\frac{2(-\frac{113}{4}x^3-\frac{145}{4}x)}{\sqrt{x^4+3x^2+2}} + \frac{625x\sqrt{x^4+3x^2+2}}{3} - \frac{1067i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}} + \frac{637i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right.\right.}{4\sqrt{x^4+3x^2+2}}$
default	$-\frac{4802(-\frac{3}{4}x^3-\frac{5}{4}x)}{\sqrt{x^4+3x^2+2}} - \frac{1067i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}} + \frac{637i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}} -$

input
```
int((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(1250*x^4+4089*x^2+2935)/(x^4+3*x^2+2)^(1/2)-1067/3*I*2^(1/2)*(2*x^2
+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1
/2))+637/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(El
lipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.308.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.56

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{1911\,(i\,x^5+3i\,x^3+2i\,x)E(\arcsin\left(\frac{i}{x}\right)\,|\,2)+6179\,(-i\,x^5-3i\,x^3-2i\,x)F(\arcsin\left(\frac{i}{x}\right)\,|\,2)-2\,(625\,x^6+300}{6\,(x^5+3\,x^3+2\,x)}$$

input
```
integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(1911*(I*x^5 + 3*I*x^3 + 2*I*x)*elliptic_e(arcsin(I/x), 2) + 6179*(-I
*x^5 - 3*I*x^3 - 2*I*x)*elliptic_f(arcsin(I/x), 2) - 2*(625*x^6 + 3000*x^4
 + 4334*x^2 + 1911)*sqrt(x^4 + 3*x^2 + 2))/(x^5 + 3*x^3 + 2*x)
```

3.308.6 Sympy [F]

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^4}{((x^2+1)(x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**4/(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**4/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

3.308.7 Maxima [F]

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.308.8 Giac [F]

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^4/(3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^4/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.309 $\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}} \, dx$

3.309.1 Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}} \, dx = \frac{x(5-11x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{261x(2+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{261(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{169(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

output
```
1/2*x*(-11*x^2+5)/(x^4+3*x^2+2)^(1/2)+261/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-
261/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2
))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+169/2*(x^2+1)^(3/2)
*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)
/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

3.309.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}} \, dx = -\frac{-5x+11x^3+261i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+77i\sqrt{1+x^2}\sqrt{2+x^2}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\right.}{2\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
-1/2*(-5*x + 11*x^3 + (261*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcS
inh[x/Sqrt[2]], 2] + (77*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSin
h[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]
```

3.309.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1517, 25, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^3}{(x^4+3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1517}$$

$$\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}}-\frac{1}{2}\int -\frac{261x^2+338}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{261x^2+338}{\sqrt{x^4+3x^2+2}}dx+\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{2}\left(338\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+261\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{2}\left(261\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{169\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{2}\left(\frac{169\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+261\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right.\right.$$

$$\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}}$$

input
```
Int[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(x*(5 - 11*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (261*((x*(2 + x^2))/Sqrt[2 +
3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcT
an[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (169*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2
)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4])/2
```

3.309.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.309.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{x\left(11x^{2}-5\right)}{2\sqrt{x^{4}+3x^{2}+2}}-\frac{169i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^{4}+3x^{2}+2}}+\frac{261i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^{4}+3x^{2}+2}}$
elliptic	$-\frac{2\left(\frac{11}{4}x^{3}-\frac{5}{4}x\right)}{\sqrt{x^{4}+3x^{2}+2}}-\frac{169i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^{4}+3x^{2}+2}}+\frac{261i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^{4}+3x^{2}+2}}$
default	$-\frac{686\left(-\frac{3}{4}x^{3}-\frac{5}{4}x\right)}{\sqrt{x^{4}+3x^{2}+2}}-\frac{169i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^{4}+3x^{2}+2}}+\frac{261i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^{4}+3x^{2}+2}}-\frac{25}{\sqrt{}}$

input
```
int((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(11*x^2-5)/(x^4+3*x^2+2)^(1/2)-169/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+261/4*I*2
^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*
2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.309.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.61

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = -\frac{261\,(i\,x^5+3i\,x^3+2i\,x)E(\arcsin\left(\frac{i}{x}\right)\,|2)+599\,(-i\,x^5-3i\,x^3-2i\,x)F(\arcsin\left(\frac{i}{x}\right)\,|2)-2\,(125\,x^4+394\,x}{2\,(x^5+3\,x^3+2\,x)}$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(261*(I*x^5 + 3*I*x^3 + 2*I*x)*elliptic_e(arcsin(I/x), 2) + 599*(-I*x
^5 - 3*I*x^3 - 2*I*x)*elliptic_f(arcsin(I/x), 2) - 2*(125*x^4 + 394*x^2 +
261)*sqrt(x^4 + 3*x^2 + 2))/(x^5 + 3*x^3 + 2*x)
```

3.309.6 Sympy [F]

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^3}{((x^2+1)\,(x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**3/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

3.309.7 Maxima [F]

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.309.8 Giac [F]

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.310 $\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx$

3.310.1 Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = -\frac{17x(2+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{17(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{6\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}$$

output
```
-17/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(17*x^2+25)/(x^4+3*x^2+2)^(1/2)+
17/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2)
)*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+6*(x^2+1)^(3/2)*(1/(
x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2
+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

3.310.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{25x+17x^3+17i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-41i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{El}}{2\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(25*x + 17*x^3 + (17*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/
Sqrt[2]], 2] - (41*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sq
rt[2]], 2])/(2*Sqrt[2 + 3*x^2 + x^4])
```

3.310.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1517, 25, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(5x^2+7\right)^2}{\left(x^4+3x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1517}$$

$$\frac{x\left(17x^2+25\right)}{2\sqrt{x^4+3x^2+2}}-\frac{1}{2}\int -\frac{24-17x^2}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{24-17x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{x\left(17x^2+25\right)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{2}\left(24\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-17\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{x\left(17x^2+25\right)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{2}\left(\frac{12\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}-17\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{x\left(17x^2+25\right)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{2}\left(\frac{12\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}-17\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)\right)$$
$$\frac{x(17x^2+25)}{2\sqrt{x^4+3x^2+2}}$$

input
```
Int[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(x*(25 + 17*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (-17*((x*(2 + x^2))/Sqrt[2 +
 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[Arc
Tan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (12*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2
)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4])/2
```

3.310.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.310.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

method	result
risch	$\frac{x(17x^2+25)}{2\sqrt{x^4+3x^2+2}}-\frac{6i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{17i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$
elliptic	$-\frac{2\left(-\frac{17}{4}x^3-\frac{25}{4}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{6i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{17i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$
default	$-\frac{98\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{6i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{17i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}-\frac{50(-\frac{3}{2}\cdots}{\sqrt{x^4+\cdots}}$

input
```
int((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(17*x^2+25)/(x^4+3*x^2+2)^(1/2)-6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-17/4*I*2^(1/2
)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/
2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.310.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.56

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = -\frac{17\,(i\,x^4+3i\,x^2+2i)E(\arcsin\left(\frac{1}{2}i\sqrt{2}x\right)\,|\,2)+31\,(i\,x^4+3i\,x^2+2i)F(\arcsin\left(\frac{1}{2}i\sqrt{2}x\right)\,|\,2)-2\sqrt{x^4+3\,x^2}}{4\,(x^4+3\,x^2+2)}$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(17*(I*x^4 + 3*I*x^2 + 2*I)*elliptic_e(arcsin(1/2*I*sqrt(2)*x), 2) +
31*(I*x^4 + 3*I*x^2 + 2*I)*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2) - 2*sqrt
(x^4 + 3*x^2 + 2)*(17*x^3 + 25*x))/(x^4 + 3*x^2 + 2)
```

3.310.6 Sympy [F]

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^2}{((x^2+1)(x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**2/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

3.310.7 Maxima [F]

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.310.8 Giac [F]

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.311 $\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx$

3.311.1 Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = -\frac{x(2+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{x(5+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

output
```
-1/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(x^2+5)/(x^4+3*x^2+2)^(1/2)+1/2*(
x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1
/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/2*(x^2+1)^(3/2)*(1/(x^2+
1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))
^(1/2)/(x^4+3*x^2+2)^(1/2)
```

3.311.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{5x+x^3+i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|\,2\right) - 3i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}}{2\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(5*x + x^3 + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]],
 2] - (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]
)/(2*Sqrt[2 + 3*x^2 + x^4])
```

3.311.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1492, 25, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+7}{(x^4+3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1492}$$

$$\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}-\frac{1}{2}\int -\frac{2-x^2}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{2-x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1412}$$

$$\frac{1}{2}\left(\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}-\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{2}\left(\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}-\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}\right)+\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}$$

input
```
Int[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(x*(5 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (-((x*(2 + x^2))/Sqrt[2 + 3*x^2
+ x^4]) + (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x]
, 1/2])/Sqrt[2 + 3*x^2 + x^4] + (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2
)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4])/2
```

3.311.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.311.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.87

method	result
risch	$\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$
elliptic	$-\frac{2\left(-\frac{1}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$
default	$-\frac{14\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}-\frac{10(x^3+\frac{3}{2}$

input
```
int((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^2+5)/(x^4+3*x^2+2)^(1/2)-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1
/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-1/4*I*2^(1/2)*(
2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*
x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.311.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.56

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{(-i\,x^4-3i\,x^2-2i)E(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\mid 2)-3\,(i\,x^4+3i\,x^2+2i)F(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right.}{4\,(x^4+3\,x^2+2)}$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*((-I*x^4 - 3*I*x^2 - 2*I)*elliptic_e(arcsin(1/2*I*sqrt(2)*x), 2) - 3*(
I*x^4 + 3*I*x^2 + 2*I)*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2) + 2*sqrt(x^4
 + 3*x^2 + 2)*(x^3 + 5*x))/(x^4 + 3*x^2 + 2)
```

3.311.6 Sympy [F]

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{5x^2+7}{((x^2+1)(x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

3.311.7 Maxima [F]

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.311.8 Giac [F]

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.312 $\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx$

3.312.1 Optimal result

Integrand size = 14, antiderivative size = 149

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = -\frac{3x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(5+3x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}$$

output
```
-3/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(3*x^2+5)/(x^4+3*x^2+2)^(1/2)+3/2
*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^
(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-(x^2+1)^(3/2)*(1/(x^2+1)
)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(
1/2)/(x^4+3*x^2+2)^(1/2)
```

3.312.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{5x+3x^3+3i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+i\sqrt{1+x^2}\sqrt{2+x^2}\operatorname{EllipticF}}{2\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(2 + 3*x^2 + x^4)^(-3/2),x]
```

output
```
(5*x + 3*x^3 + (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqr
t[2]], 2] + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]],
2])/(2*Sqrt[2 + 3*x^2 + x^4])
```

3.312.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1405, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4+3x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x(3x^2+5)}{2\sqrt{x^4+3x^2+2}}-\frac{1}{2}\int \frac{3x^2+4}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{2}\left(-4\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-3\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{x(3x^2+5)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{ 1412}$$

$$\frac{1}{2}\left(-3\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx-\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{x(3x^2+5)}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{ 1455}$$

$$\frac{1}{2}\left(-\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}-3\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)\right)+\frac{x(3x^2+5)}{2\sqrt{x^4+3x^2+2}}$$

input
```
Int[(2 + 3*x^2 + x^4)^(-3/2),x]
```

output
```
(x*(5 + 3*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (-3*((x*(2 + x^2))/Sqrt[2 + 3*
x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan
[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) - (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4])/2
```

3.312.3.1 Defintions of rubi rules used

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.312.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{x(3x^2+5)}{2\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$	128
default	$-\frac{2\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$	129
elliptic	$-\frac{2\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}$	129

input
```
int(1/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(3*x^2+5)/(x^4+3*x^2+2)^(1/2)+I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2
)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-3/4*I*2^(1/2)*(2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,
2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.312.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.56

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{3\,(i\,x^4+3i\,x^2+2i)E(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2)+11\,(-i\,x^4-3i\,x^2-2i)F(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2)-2\,\sqrt{x^4+3\,x^2}}{4\,(x^4+3\,x^2+2)}$$

input
```
integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(3*(I*x^4 + 3*I*x^2 + 2*I)*elliptic_e(arcsin(1/2*I*sqrt(2)*x), 2) + 1
1*(-I*x^4 - 3*I*x^2 - 2*I)*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2) - 2*sqrt
(x^4 + 3*x^2 + 2)*(3*x^3 + 5*x))/(x^4 + 3*x^2 + 2)
```

3.312.6 Sympy [F]

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral((x**4 + 3*x**2 + 2)**(-3/2), x)
```

3.312.7 Maxima [F]

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

3.312.8 Giac [F]

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/(3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int(1/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.313 $\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}} \, dx$

3.313.1 Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}} \, dx = \frac{x}{6\sqrt{2+3x^2+x^4}} + \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}} - \frac{9(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{4\sqrt{2+3x^2+x^4}} + \frac{125(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{84\sqrt{2}\sqrt{2+3x^2+x^4}}$$

output
```
1/6*x/(x^4+3*x^2+2)^(1/2)+125/168*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Elliptic
Pi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3
*x^2+2)^(1/2)+1/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2
),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-9/4*(x^
2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+
2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

3.313.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.80

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx = \frac{35x+14x^3+14i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-7i\sqrt{1+x^2}\sqrt{2}}{}$$

input
```
Integrate[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

output
```
(35*x + 14*x^3 + (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/
Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqr
t[2]], 2] + (25*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[
x/Sqrt[2]], 2])/(42*Sqrt[2 + 3*x^2 + x^4])
```

3.313.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.51, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1545, 25, 1492, 27, 1503, 1412, 1455, 1538, 27, 1412, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)(x^4+3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1545}$$

$$-\frac{1}{6}\int -\frac{5x^2+8}{(x^4+3x^2+2)^{3/2}}dx-\frac{25}{6}\int \frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\int \frac{5x^2+8}{(x^4+3x^2+2)^{3/2}}dx-\frac{25}{6}\int \frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{6}\left(\frac{x(2x^2+5)}{\sqrt{x^4+3x^2+2}}-\frac{1}{2}\int \frac{2(2x^2+1)}{\sqrt{x^4+3x^2+2}}dx\right)-\frac{25}{6}\int \frac{1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{x\left(2x^2+5\right)}{\sqrt{x^4+3x^2+2}}-\int \frac{2x^2+1}{\sqrt{x^4+3x^2+2}}dx\right)-\frac{25}{6}\int \frac{1}{\left(5x^2+7\right)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{6}\left(-\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-2\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{x\left(2x^2+5\right)}{\sqrt{x^4+3x^2+2}}\right)-\frac{25}{6}\int \frac{1}{\left(5x^2+7\right)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 1412}$$

$$\frac{1}{6}\left(-2\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx-\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{x\left(2x^2+5\right)}{\sqrt{x^4+3x^2+2}}\right)-\frac{25}{6}\int \frac{1}{\left(5x^2+7\right)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 1455}$$

$$\frac{1}{6}\left(-\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-2\left(\frac{x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{}{\sqrt{}}\right.$$
$$\frac{25}{6}\int \frac{1}{\left(5x^2+7\right)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 1538}$$

$$\frac{1}{6}\left(-\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-2\left(\frac{x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{}{\sqrt{}}\right.$$
$$\frac{25}{6}\left(\frac{1}{2}\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{4}\int \frac{2\left(x^2+1\right)}{\left(5x^2+7\right)\sqrt{x^4+3x^2+2}}dx\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{6}\left(-\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-2\left(\frac{x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{}{\sqrt{}}\right.$$
$$\frac{25}{6}\left(\frac{1}{2}\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{5}{2}\int \frac{x^2+1}{\left(5x^2+7\right)\sqrt{x^4+3x^2+2}}dx\right)$$

$$\Big\downarrow \text{ 1412}$$

3.313. $\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx$

$$\frac{1}{6}\left(-\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-2\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{}{\sqrt{}}\right.$$
$$\left.\frac{25}{6}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5}{2}\int\frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx\right)\right.$$

$$\Big\downarrow \; 1786$$

$$\frac{1}{6}\left(-\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-2\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{}{\sqrt{}}\right.$$
$$\left.\frac{25}{6}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{x^2+1}\sqrt{x^2+2}\int\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(5x^2+7)}dx}{2\sqrt{x^4+3x^2+2}}\right)\right.$$

$$\Big\downarrow \; 414$$

$$\frac{1}{6}\left(-\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-2\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{}{\sqrt{}}\right.$$
$$\left.\frac{25}{6}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}\right)\right)$$

input
```
Int[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

output
```
((x*(5 + 2*x^2))/Sqrt[2 + 3*x^2 + x^4] - 2*((x*(2 + x^2))/Sqrt[2 + 3*x^2 +
 x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
1/2])/Sqrt[2 + 3*x^2 + x^4]) - ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellipt
icF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]))/6 - (25*(((1 + x^2)*
Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*Sqrt[2 + 3
*x^2 + x^4]) - (5*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(14*Sqrt[2]*S
qrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])))/6
```

3.313. $\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx$

3.313.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.313. $\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}} \, dx$

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1538
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/(2*c*d - e*(b - q)))   I
nt[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[e/(2*c*d - e*(b - q))   Int[(b
- q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !LtQ[c, 0]
```

rule 1545
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x^2)*(a + b*x^
2 + c*x^4)^p, x], x] + Simp[e^2/(c*d^2 - b*d*e + a*e^2)   Int[(a + b*x^2 +
c*x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0] && (EqQ[c*
d^2 - a*e^2, 0] || NiceSqrtQ[b^2 - 4*a*c])
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

3.313.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.93

method	result
default	$-\frac{2\left(-\frac{1}{6}x^{3}-\frac{5}{12}x\right)}{\sqrt{x^{4}+3x^{2}+2}}-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{12\sqrt{x^{4}+3x^{2}+2}}+\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^{4}+3x^{2}+2}}+\frac{25i\sqrt{2}\,\sqrt{1+\frac{x^{2}}{2}}\,\sqrt{x^{2}+1}\,\Pi\left(\frac{i\sqrt{}}{2}\right.}{42\sqrt{x^{4}+3x^{2}+2}}$
elliptic	$-\frac{2\left(-\frac{1}{6}x^{3}-\frac{5}{12}x\right)}{\sqrt{x^{4}+3x^{2}+2}}-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{12\sqrt{x^{4}+3x^{2}+2}}+\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^{4}+3x^{2}+2}}+\frac{25i\sqrt{2}\,\sqrt{1+\frac{x^{2}}{2}}\,\sqrt{x^{2}+1}\,\Pi\left(\frac{i\sqrt{}}{2}\right.}{42\sqrt{x^{4}+3x^{2}+2}}$
risch	$\frac{x\left(2x^{2}+5\right)}{6\sqrt{x^{4}+3x^{2}+2}}+\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{12\sqrt{x^{4}+3x^{2}+2}}-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{6\sqrt{x^{4}+3x^{2}+2}}+\frac{25i\sqrt{2}\,\sqrt{1+\frac{x^{2}}{2}}}{42}$

input
```
int(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/6*x^3-5/12*x)/(x^4+3*x^2+2)^(1/2)-1/12*I*2^(1/2)*(2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/6*I*2
^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2
^(1/2)*x,2^(1/2))+25/42*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x
^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

3.313.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx=\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^10 + 37*x^8 + 107*x^6 + 151*x^4 + 104*
x^2 + 28), x)
```

3.313.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx=\int \frac{1}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\cdot\left(5x^2+7\right)}\,dx$$

input
```
integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)), x)
```

3.313.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

3.313.8 Giac [F]

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^2+7)\,(x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2)),x)
```

output
```
int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2)), x)
```

3.314 $\int \frac{1}{(7+5x^2)^2(2+3x^2+x^4)^{3/2}}\,dx$

3.314.1 Optimal result

Integrand size = 24, antiderivative size = 235

$$\int \frac{1}{(7+5x^2)^2(2+3x^2+x^4)^{3/2}}\,dx = -\frac{31x(2+x^2)}{56\sqrt{2+3x^2+x^4}}+\frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}}+\frac{625x\sqrt{2+3x^2+x^4}}{504(7+5x^2)}+\frac{31(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{28\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{463(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{336\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{375(2+x^2)\operatorname{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{784\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

output
```
-31/56*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/36*x*(11*x^2+20)/(x^4+3*x^2+2)^(1/2
)+375/1568*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1
/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+3
1/56*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2)
)*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-463/672*(x^2+1)^(3/2
)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2
)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/504*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+
7)
```

3.314.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.32 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.89

$$\int \frac{1}{(7+5x^2)^2 (2+3x^2+x^4)^{3/2}}\, dx = \frac{7490x + 10157x^3 + 3255x^5 + 651i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)\, E\Big(i\text{arcsi}$$

input
```
Integrate[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```

output
```
(7490*x + 10157*x^3 + 3255*x^5 + (651*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 +
5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (182*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (1575*I)*Sqrt[1 + x
^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (1125*I)*x^2
*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(1
176*(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])
```

3.314.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^2 (x^4+3x^2+2)^{3/2}}\, dx$$

$$\Big\downarrow \ 1556$$

$$\int \left(\frac{5x^2+14}{36(x^4+3x^2+2)^{3/2}} - \frac{25}{36(5x^2+7)\sqrt{x^4+3x^2+2}} - \frac{25}{6(5x^2+7)^2\sqrt{x^4+3x^2+2}}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{463(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{336\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{31(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{28\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{375(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{784\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}+\frac{625\sqrt{x^4+3x^2+2}x}{504(5x^2+7)}-\frac{31(x^2+2)x}{56\sqrt{x^4+3x^2+2}}+\frac{(11x^2+20)x}{36\sqrt{x^4+3x^2+2}}$$

input
```
Int[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```

output
```
(-31*x*(2 + x^2))/(56*Sqrt[2 + 3*x^2 + x^4]) + (x*(20 + 11*x^2))/(36*Sqrt[
2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(504*(7 + 5*x^2)) + (31*
(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(28*Sqrt[2]
*Sqrt[2 + 3*x^2 + x^4]) - (463*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellipti
cF[ArcTan[x], 1/2])/(336*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (375*(2 + x^2)*E
llipticPi[2/7, ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqr
t[2 + 3*x^2 + x^4])
```

3.314.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.314.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.79

method	result
default	$-\frac{2\left(-\frac{11}{72}x^3-\frac{5}{18}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{625x\sqrt{x^4+3x^2+2}}{504(5x^2+7)}+\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{168\sqrt{x^4+3x^2+2}}+\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{112\sqrt{x^4+3x^2+2}}+$
elliptic	$-\frac{2\left(-\frac{11}{72}x^3-\frac{5}{18}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{625x\sqrt{x^4+3x^2+2}}{504(5x^2+7)}+\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{168\sqrt{x^4+3x^2+2}}+\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{112\sqrt{x^4+3x^2+2}}+$
risch	$\frac{x(465x^4+1451x^2+1070)}{168(5x^2+7)\sqrt{x^4+3x^2+2}}+\frac{17i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{48\sqrt{x^4+3x^2+2}}-\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{112\sqrt{x^4+3x^2+2}}+$

input
```
int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-11/72*x^3-5/18*x)/(x^4+3*x^2+2)^(1/2)+625/504*x*(x^4+3*x^2+2)^(1/2)/(
5*x^2+7)+13/168*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2
)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+31/112*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+75/392*I*
2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2
*I*2^(1/2)*x,10/7,2^(1/2))
```

3.314.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^2\,(2+3x^2+x^4)^{3/2}}\,dx=\int \frac{1}{(x^4+3\,x^2+2)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^12 + 220*x^10 + 794*x^8 + 1504*x^6 +
1577*x^4 + 868*x^2 + 196), x)
```

3.314.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^2\,(2+3x^2+x^4)^{3/2}}\,dx=\int \frac{1}{((x^2+1)\,(x^2+2))^{\frac{3}{2}}\,(5x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2), x)
```

3.314.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^2 (2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}}(5x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

3.314.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^2 (2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}}(5x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^2 (2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(5x^2+7)^2 (x^4+3x^2+2)^{3/2}}\,dx$$

input
```
int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2)),x)
```

output
```
int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2)), x)
```

3.315 $\int \frac{1}{(7+5x^2)^3(2+3x^2+x^4)^{3/2}}\,dx$

3.315.1 Optimal result

Integrand size = 24, antiderivative size = 263

$$\int \frac{1}{(7+5x^2)^3(2+3x^2+x^4)^{3/2}}\,dx = -\frac{5797x(2+x^2)}{28224\sqrt{2+3x^2+x^4}}+\frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}}+\frac{625x\sqrt{2+3x^2+x^4}}{1008(7+5x^2)^2}+\frac{41875x\sqrt{2+3x^2+x^4}}{84672(7+5x^2)}+\frac{5797(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{14112\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{49907(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{56448\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{192625(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{395136\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

output
```
-5797/28224*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/216*x*(23*x^2+50)/(x^4+3*x^2+2
)^(1/2)+192625/790272*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x
/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2
+2)^(1/2)+5797/28224*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(
1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-4990
7/112896*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(
1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/1008*x*(x^4+
3*x^2+2)^(1/2)/(5*x^2+7)^2+41875/84672*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

3.315.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.60

$$\int \frac{1}{(7+5x^2)^3 (2+3x^2+x^4)^{3/2}}\,dx = \frac{\frac{7x(550550+1089803x^2+698290x^4+144925x^6)}{(7+5x^2)^2} + 40579i\sqrt{1+x^2}\sqrt{2+x^2}E\Big(i\text{arcsi}$$

input
```
Integrate[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

output
```
((7*x*(550550 + 1089803*x^2 + 698290*x^4 + 144925*x^6))/(7 + 5*x^2)^2 + (4
0579*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (
742*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (3
8525*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]],
 2])/(197568*Sqrt[2 + 3*x^2 + x^4])
```

3.315.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^3 (x^4+3x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1556}$$

$$\int \left(-\frac{-35x^2-62}{216(x^4+3x^2+2)^{3/2}} - \frac{175}{216(5x^2+7)\sqrt{x^4+3x^2+2}} - \frac{25}{36(5x^2+7)^2\sqrt{x^4+3x^2+2}} - \frac{25}{6(5x^2+7)^3\sqrt{x^4+}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{49907(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{56448\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{5797(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{14112\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{192625(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{395136\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}+\frac{41875\sqrt{x^4+3x^2+2}\,x}{84672\,(5x^2+7)}+\frac{625\sqrt{x^4+3x^2+2}\,x}{1008\,(5x^2+7)^2}-\frac{5797(x^2+2)\,x}{28224\sqrt{x^4+3x^2+2}}+\frac{(23x^2+50)\,x}{216\sqrt{x^4+3x^2+2}}$$

input
```
Int[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

output
```
(-5797*x*(2 + x^2))/(28224*Sqrt[2 + 3*x^2 + x^4]) + (x*(50 + 23*x^2))/(216
*Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(1008*(7 + 5*x^2)^
2) + (41875*x*Sqrt[2 + 3*x^2 + x^4])/(84672*(7 + 5*x^2)) + (5797*(1 + x^2)
*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(14112*Sqrt[2]*Sqrt[
2 + 3*x^2 + x^4]) - (49907*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[A
rcTan[x], 1/2])/(56448*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (192625*(2 + x^2)*
EllipticPi[2/7, ArcTan[x], 1/2])/(395136*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]
*Sqrt[2 + 3*x^2 + x^4])
```

3.315.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.315.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.51 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.73

method	result
risch	$\frac{x(144925x^6+698290x^4+1089803x^2+550550)}{28224(5x^2+7)^2\sqrt{x^4+3x^2+2}}+\frac{271i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}F\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)}{2688\sqrt{x^4+3x^2+2}}-\frac{5797i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{i\sqrt{2}x}{2},\sqrt{\right.\right.}}{56448\sqrt{x^4+3x^2+}}$
default	$-\frac{2\left(-\frac{23}{432}x^3-\frac{25}{216}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{625x\sqrt{x^4+3x^2+2}}{1008(5x^2+7)^2}+\frac{41875x\sqrt{x^4+3x^2+2}}{84672(5x^2+7)}-\frac{53i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}F\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)}{28224\sqrt{x^4+3x^2+2}}+\frac{5797i\sqrt{2}\sqrt{2x^2+}}{56448}$
elliptic	$-\frac{2\left(-\frac{23}{432}x^3-\frac{25}{216}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{625x\sqrt{x^4+3x^2+2}}{1008(5x^2+7)^2}+\frac{41875x\sqrt{x^4+3x^2+2}}{84672(5x^2+7)}-\frac{53i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}F\left(\frac{i\sqrt{2}x}{2},\sqrt{2}\right)}{28224\sqrt{x^4+3x^2+2}}+\frac{5797i\sqrt{2}\sqrt{2x^2+}}{56448}$

input
```
int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/28224*x*(144925*x^6+698290*x^4+1089803*x^2+550550)/(5*x^2+7)^2/(x^4+3*x^
2+2)^(1/2)+271/2688*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^
(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-5797/56448*I*2^(1/2)*(2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-
EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))+38525/197568*I*2^(1/2)*(1+1/2*x^2)^(1/
2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/
2))
```

3.315.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^3\left(2+3x^2+x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(x^4+3\,x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^14 + 1275*x^12 + 5510*x^10 + 13078*x
^8 + 18413*x^6 + 15379*x^4 + 7056*x^2 + 1372), x)
```

3.315.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^3 (2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{((x^2+1)(x^2+2))^{\frac{3}{2}} (5x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3), x)
```

3.315.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^3 (2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}} (5x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

3.315.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^3 (2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}} (5x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^3\,(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^2+7)^3\,(x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2)),x)
```

output
```
int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2)), x)
```

3.316 $\int (7+5x^2)^4 \sqrt{2+x^2-x^4}\,dx$

3.316.1 Optimal result

Integrand size = 24, antiderivative size = 116

$$\begin{aligned}\int (7+5x^2)^4 \sqrt{2+x^2-x^4}\,dx &= \frac{1}{231}x(177953+717372x^2)\sqrt{2+x^2-x^4}-\frac{116100}{77}x(2+x^2-x^4)^{3/2}\\ &\quad-\frac{14500}{33}x^3(2+x^2-x^4)^{3/2}-\frac{625}{11}x^5(2+x^2-x^4)^{3/2}\\ &\quad+\frac{3764813}{231}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{539419}{77}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\end{aligned}$$

output
```
-116100/77*x*(-x^4+x^2+2)^(3/2)-14500/33*x^3*(-x^4+x^2+2)^(3/2)-625/11*x^5
*(-x^4+x^2+2)^(3/2)+3764813/231*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-539419/
77*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/231*x*(717372*x^2+177953)*(-x^4+x^
2+2)^(1/2)
```

3.316.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int (7+5x^2)^4 \sqrt{2+x^2-x^4}\,dx = \frac{-1037294x-186503x^3+1125819x^5+231228x^7-105925x^9-75250x^{11}-13125x^{13}+3764813i\sqrt{4+2}}{231\sqrt{2+x^2-x^4}}$$

input
```
Integrate[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```

output
```
(-1037294*x - 186503*x^3 + 1125819*x^5 + 231228*x^7 - 105925*x^9 - 75250*x
^11 - 13125*x^13 + (3764813*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh
[x], -1/2] - (4838091*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -
1/2])/(231*Sqrt[2 + x^2 - x^4])
```

3.316.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {1518, 25, 2207, 27, 2207, 25, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+7\right)^4 \sqrt{-x^4+x^2+2}\,dx$$

$$\Big\downarrow \text{1518}$$

$$-\frac{1}{11}\int -\sqrt{-x^4+x^2+2}\left(43500x^6+87100x^4+75460x^2+26411\right)dx-\frac{625}{11}\left(-x^4+x^2+2\right)^{3/2}x^5$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{11}\int \sqrt{-x^4+x^2+2}\left(43500x^6+87100x^4+75460x^2+26411\right)dx-\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{11}\left(-\frac{1}{9}\int -9\sqrt{-x^4+x^2+2}\left(116100x^4+104460x^2+26411\right)dx-\frac{14500}{3}\left(-x^4+x^2+2\right)^{3/2}x^3\right)-\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\left(\int \sqrt{-x^4+x^2+2}\left(116100x^4+104460x^2+26411\right)dx-\frac{14500}{3}x^3\left(-x^4+x^2+2\right)^{3/2}\right)-\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{11}\left(-\frac{1}{7}\int -\left(\left(1195620x^2+417077\right)\sqrt{-x^4+x^2+2}\right)dx-\frac{116100}{7}\left(-x^4+x^2+2\right)^{3/2}x-\frac{14500}{3}\left(-x^4+x^2+2\right)^{3}\right.$$

$$\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{11}\left(\frac{1}{7}\int \left(1195620x^2+417077\right)\sqrt{-x^4+x^2+2}dx-\frac{116100}{7}\left(-x^4+x^2+2\right)^{3/2}x-\frac{14500}{3}\left(-x^4+x^2+2\right)^{3/2}x^3\right)$$

$$\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \ 1490$$

$$\frac{1}{11}\left(\frac{1}{7}\left(\frac{1}{3}x\left(717372x^2+177953\right)\sqrt{-x^4+x^2+2}-\frac{1}{15}\int -\frac{5\left(3764813x^2+2146556\right)}{\sqrt{-x^4+x^2+2}}dx\right)-\frac{116100}{7}\left(-x^4+x^2+\right.\right.$$

$$\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{11}\left(\frac{1}{7}\left(\frac{1}{3}\int \frac{3764813x^2+2146556}{\sqrt{-x^4+x^2+2}}dx+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(717372x^2+177953\right)\right)-\frac{116100}{7}\left(-x^4+x^2+2\right)^{3/2}x\right.$$

$$\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \ 1494$$

$$\frac{1}{11}\left(\frac{1}{7}\left(\frac{2}{3}\int \frac{3764813x^2+2146556}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(717372x^2+177953\right)\right)-\frac{116100}{7}\left(-x^4+x^2+2\right)^{3/2}x\right.$$

$$\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{11}\left(\frac{1}{7}\left(\frac{1}{3}\int \frac{3764813x^2+2146556}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(717372x^2+177953\right)\right)-\frac{116100}{7}\left(-x^4+x^2+2\right)^{3/2}x\right.$$

$$\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \ 399$$

$$\frac{1}{11}\left(\frac{1}{7}\left(\frac{1}{3}\left(3764813\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-1618257\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(717372x^2+177953\right)\right.\right.$$

$$\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

3.316. $\int \left(7+5x^2\right)^4\sqrt{2+x^2-x^4}\,dx$

$$\Big\downarrow \; 321$$

$$\frac{1}{11}\left(\frac{1}{7}\left(\frac{1}{3}\left(3764813\int\frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-1618257\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{3}x\sqrt{-x^4+x^2+2}(717372x^2+\right.\right.$$
$$\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{11}\left(\frac{1}{7}\left(\frac{1}{3}\left(3764813E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-1618257\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{3}x\sqrt{-x^4+x^2+2}(717\right.\right.$$
$$\frac{625}{11}x^5\left(-x^4+x^2+2\right)^{3/2}$$

input
```
Int[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```

output
```
(-625*x^5*(2 + x^2 - x^4)^(3/2))/11 + ((-116100*x*(2 + x^2 - x^4)^(3/2))/7
 - (14500*x^3*(2 + x^2 - x^4)^(3/2))/3 + ((x*(177953 + 717372*x^2)*Sqrt[2
+ x^2 - x^4])/3 + (3764813*EllipticE[ArcSin[x/Sqrt[2]], -2] - 1618257*Elli
pticF[ArcSin[x/Sqrt[2]], -2])/3)/7)/11
```

3.316.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.316. $\int (7+5x^2)^4\sqrt{2+x^2-x^4}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.316. $\int (7+5x^2)^4 \sqrt{2+x^2-x^4}\,dx$

3.316.4 Maple [A] (verified)

Time = 7.91 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{x(13125x^8+88375x^6+220550x^4+166072x^2-518647)(x^4-x^2-2)}{231\sqrt{-x^4+x^2+2}}+\frac{1073278\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{231\sqrt{-x^4+x^2+2}}-\frac{3764813\sqrt{}}{}$
default	$-\frac{518647x\sqrt{-x^4+x^2+2}}{231}+\frac{1073278\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{231\sqrt{-x^4+x^2+2}}-\frac{3764813\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{}}{2}\right.\right.}{462\sqrt{-x^4+x^2+2}}$
elliptic	$-\frac{518647x\sqrt{-x^4+x^2+2}}{231}+\frac{1073278\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{231\sqrt{-x^4+x^2+2}}-\frac{3764813\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{}}{2}\right.\right.}{462\sqrt{-x^4+x^2+2}}$

input
```
int((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/231*x*(13125*x^8+88375*x^6+220550*x^4+166072*x^2-518647)*(x^4-x^2-2)/(-
x^4+x^2+2)^(1/2)+1073278/231*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+
x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-3764813/462*2^(1/2)*(-2*x^
2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(
1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int (7+5x^2)^4\sqrt{2+x^2-x^4}\,dx$$
$$=\frac{-7529626i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2})+8602904i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2})+(13125x^{10}+88375x^8+220}{231x}$$

input
```
integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/231*(-7529626*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 8602904*
I*sqrt(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) + (13125*x^10 + 88375*x^8
+ 220550*x^6 + 166072*x^4 - 518647*x^2 - 3764813)*sqrt(-x^4 + x^2 + 2))/x
```

3.316.6 Sympy [F]

$$\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\, dx = \int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^4\, dx$$

input
```
integrate((5*x**2+7)**4*(-x**4+x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**4, x)
```

3.316.7 Maxima [F]

$$\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\, dx = \int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^4 dx$$

input
```
integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)
```

3.316.8 Giac [F]

$$\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\, dx = \int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^4 dx$$

input
```
integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\, dx = \int \left(5\,x^2+7\right)^4 \sqrt{-x^4+x^2+2}\, dx$$

input
```
int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(1/2), x)
```

3.317 $\int (7+5x^2)^3 \sqrt{2+x^2-x^4}\,dx$

3.317.1 Optimal result

Integrand size = 24, antiderivative size = 95

$$\int (7+5x^2)^3 \sqrt{2+x^2-x^4}\,dx = \frac{1}{63}x(5956+14691x^2)\sqrt{2+x^2-x^4}-\frac{1825}{21}x(2+x^2-x^4)^{3/2} - \frac{125}{9}x^3(2+x^2-x^4)^{3/2}+\frac{79411}{63}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{8735}{21}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
-1825/21*x*(-x^4+x^2+2)^(3/2)-125/9*x^3*(-x^4+x^2+2)^(3/2)+79411/63*Ellipt
icE(1/2*x*2^(1/2),I*2^(1/2))-8735/21*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/
63*x*(14691*x^2+5956)*(-x^4+x^2+2)^(1/2)
```

3.317.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int (7+5x^2)^3 \sqrt{2+x^2-x^4}\,dx$$
$$= \frac{-9988x+9938x^3+21660x^5-1116x^7-3725x^9-875x^{11}+79411i\sqrt{4+2x^2-2x^4}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right)}{63\sqrt{2+x^2-x^4}}$$

input
```
Integrate[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

output
```
(-9988*x + 9938*x^3 + 21660*x^5 - 1116*x^7 - 3725*x^9 - 875*x^11 + (79411*
I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (106014*I)*Sqrt
[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(63*Sqrt[2 + x^2 - x^4]
)
```

3.317.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1518, 27, 2207, 25, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+7\right)^3 \sqrt{-x^4+x^2+2}\,dx$$

$$\Big\downarrow \text{1518}$$

$$-\frac{1}{9}\int -3\sqrt{-x^4+x^2+2}\left(1825x^4+2455x^2+1029\right)dx-\frac{125}{9}\left(-x^4+x^2+2\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \sqrt{-x^4+x^2+2}\left(1825x^4+2455x^2+1029\right)dx-\frac{125}{9}x^3\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{3}\left(-\frac{1}{7}\int -\left(\left(24485x^2+10853\right)\sqrt{-x^4+x^2+2}\right)dx-\frac{1825}{7}x\left(-x^4+x^2+2\right)^{3/2}\right)-\frac{125}{9}x^3\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{7}\int \left(24485x^2+10853\right)\sqrt{-x^4+x^2+2}dx-\frac{1825}{7}x\left(-x^4+x^2+2\right)^{3/2}\right)-\frac{125}{9}x^3\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{3}x\left(14691x^2+5956\right)\sqrt{-x^4+x^2+2}-\frac{1}{15}\int -\frac{5\left(79411x^2+53206\right)}{\sqrt{-x^4+x^2+2}}dx\right)-\frac{1825}{7}x\left(-x^4+x^2+2\right)^{3/2}\right)-\frac{125}{9}x^3\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{3}\int \frac{79411x^2+53206}{\sqrt{-x^4+x^2+2}}dx+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(14691x^2+5956\right)\right)-\frac{1825}{7}x\left(-x^4+x^2+2\right)^{3/2}\right)-\frac{125}{9}x^3\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \; 1494$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2}{3}\int \frac{79411x^2+53206}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(14691x^2+5956\right)\right)-\frac{1825}{7}x\left(-x^4+x^2+2\right)^{3/2}\right)-\frac{125}{9}x^3\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{3}\int \frac{79411x^2+53206}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(14691x^2+5956\right)\right)-\frac{1825}{7}x\left(-x^4+x^2+2\right)^{3/2}\right)-\frac{125}{9}x^3\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \; 399$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{3}\left(79411\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-26205\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(14691x^2+5956\right)\right)-\frac{1825}{7}\right.$$
$$\frac{125}{9}x^3\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \; 321$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{3}\left(79411\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-26205\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(14691x^2+5956\right)\right)\right.$$
$$\frac{125}{9}x^3\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{3}\left(79411E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-26205\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(14691x^2\right.\right.\right.$$
$$\frac{125}{9}x^3\left(-x^4+x^2+2\right)^{3/2}$$

3.317. $\int \left(7+5x^2\right)^3\sqrt{2+x^2-x^4}\,dx$

input
```
Int[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

output
```
(-125*x^3*(2 + x^2 - x^4)^(3/2))/9 + ((-1825*x*(2 + x^2 - x^4)^(3/2))/7 +
((x*(5956 + 14691*x^2)*Sqrt[2 + x^2 - x^4])/3 + (79411*EllipticE[ArcSin[x/
Sqrt[2]], -2] - 26205*EllipticF[ArcSin[x/Sqrt[2]], -2])/3)/7)/3
```

3.317.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.317.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{x\left(875x^{6}+4600x^{4}+7466x^{2}-4994\right)\left(x^{4}-x^{2}-2\right)}{63\sqrt{-x^{4}+x^{2}+2}}+\frac{26603\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{63\sqrt{-x^{4}+x^{2}+2}}-\frac{79411\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\right.\right.}{126\sqrt{-x^{4}\cdot}}$
default	$-\frac{4994x\sqrt{-x^{4}+x^{2}+2}}{63}+\frac{26603\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{63\sqrt{-x^{4}+x^{2}+2}}-\frac{79411\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right.}{126\sqrt{-x^{4}+x^{2}+2}}$
elliptic	$-\frac{4994x\sqrt{-x^{4}+x^{2}+2}}{63}+\frac{26603\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{63\sqrt{-x^{4}+x^{2}+2}}-\frac{79411\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right.}{126\sqrt{-x^{4}+x^{2}+2}}$

input
```
int((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/63*x*(875*x^6+4600*x^4+7466*x^2-4994)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+26
603/63*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF
(1/2*x*2^(1/2),I*2^(1/2))-79411/126*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)
/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^
(1/2),I*2^(1/2)))
```

3.317.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int (7+5x^2)^3 \sqrt{2+x^2-x^4}\,dx$$
$$= \frac{-158822i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right) \mid -\frac{1}{2}) + 185425i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right) \mid -\frac{1}{2}) + (875\,x^8 + 4600\,x^6 + 7466\,x^4 -}{63\,x}$$

input
```
integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/63*(-158822*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 185425*I*s
qrt(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) + (875*x^8 + 4600*x^6 + 7466*
x^4 - 4994*x^2 - 79411)*sqrt(-x^4 + x^2 + 2))/x
```

3.317.6 Sympy [F]

$$\int (7+5x^2)^3 \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^3\,dx$$

input
```
integrate((5*x**2+7)**3*(-x**4+x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3, x)
```

3.317.7 Maxima [F]

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^3 dx$$

input
```
integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

3.317.8 Giac [F]

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^3 dx$$

input
```
integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\,dx = \int \left(5\,x^2+7\right)^3 \sqrt{-x^4+x^2+2}\,dx$$

input
```
int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2), x)
```

3.318 $\int (7+5x^2)^2 \sqrt{2+x^2-x^4}\,dx$

3.318.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx = \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4} - \frac{25}{7}x\left(2+x^2-x^4\right)^{3/2} + \frac{2045}{21}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{79}{7}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
-25/7*x*(-x^4+x^2+2)^(3/2)+2045/21*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-79/7
*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/21*x*(354*x^2+275)*(-x^4+x^2+2)^(1/2
)
```

3.318.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx = \frac{250x+683x^3+304x^5-204x^7-75x^9+2045i\sqrt{4+2x^2-2x^4}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right)-2949i\sqrt{4+2x^2-2}}{21\sqrt{2+x^2-x^4}}$$

input
```
Integrate[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

output
```
(250*x + 683*x^3 + 304*x^5 - 204*x^7 - 75*x^9 + (2045*I)*Sqrt[4 + 2*x^2 -
2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2949*I)*Sqrt[4 + 2*x^2 - 2*x^4]*El
lipticF[I*ArcSinh[x], -1/2])/(21*Sqrt[2 + x^2 - x^4])
```

3.318.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1518, 25, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+7\right)^2 \sqrt{-x^4+x^2+2}\,dx$$

$$\Big\downarrow \text{1518}$$

$$-\frac{1}{7}\int -\left(\left(590x^2+393\right)\sqrt{-x^4+x^2+2}\right)dx-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{7}\int \left(590x^2+393\right)\sqrt{-x^4+x^2+2}dx-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{7}\left(\frac{1}{3}x\left(354x^2+275\right)\sqrt{-x^4+x^2+2}-\frac{1}{15}\int -\frac{5\left(2045x^2+1808\right)}{\sqrt{-x^4+x^2+2}}dx\right)-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{1}{3}\int \frac{2045x^2+1808}{\sqrt{-x^4+x^2+2}}dx+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(354x^2+275\right)\right)-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{7}\left(\frac{2}{3}\int \frac{2045x^2+1808}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(354x^2+275\right)\right)-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{1}{3}\int \frac{2045x^2+1808}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{3}x\sqrt{-x^4+x^2+2}\left(354x^2+275\right)\right)-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{7}\left(\frac{1}{3}\left(2045\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-237\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)+\frac{1}{3}x\sqrt{-x^4+x^2+2}(354x^2+275)\right)-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{7}\left(\frac{1}{3}\left(2045\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-237\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{3}x\sqrt{-x^4+x^2+2}(354x^2+275)\right)-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{7}\left(\frac{1}{3}\left(2045E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-237\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{3}x\sqrt{-x^4+x^2+2}(354x^2+275)\right)-\frac{25}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

input
```
Int[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

output
```
(-25*x*(2 + x^2 - x^4)^(3/2))/7 + ((x*(275 + 354*x^2)*Sqrt[2 + x^2 - x^4])
/3 + (2045*EllipticE[ArcSin[x/Sqrt[2]], -2] - 237*EllipticF[ArcSin[x/Sqrt[
2]], -2])/3)/7
```

3.318.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.318. $\int (7+5x^2)^2 \sqrt{2+x^2-x^4}\, dx$

3.318.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(68) = 136$.

Time = 2.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99

method	result
risch	$-\frac{x(75x^4+279x^2+125)(x^4-x^2-2)}{21\sqrt{-x^4+x^2+2}}+\frac{904\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{21\sqrt{-x^4+x^2+2}}-\frac{2045\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\right.\right.}{42\sqrt{-x^4+x^2+2}}$
default	$\frac{125x\sqrt{-x^4+x^2+2}}{21}+\frac{904\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{21\sqrt{-x^4+x^2+2}}-\frac{2045\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{42\sqrt{-x^4+x^2+2}}+$
elliptic	$\frac{125x\sqrt{-x^4+x^2+2}}{21}+\frac{904\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{21\sqrt{-x^4+x^2+2}}-\frac{2045\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{42\sqrt{-x^4+x^2+2}}+$

input
```
int((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*x*(75*x^4+279*x^2+125)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+904/21*2^(1/2)
*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2)
,I*2^(1/2))-2045/42*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1
/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2))
)
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int (7+5x^2)^2\sqrt{2+x^2-x^4}\,dx$$
$$=\frac{-4090i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})+4994i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})+(75\,x^6+279\,x^4+125\,x^2-2045)\sqrt{}}{21\,x}$$

input
```
integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/21*(-4090*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 4994*I*sqrt(
2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) + (75*x^6 + 279*x^4 + 125*x^2 - 2
045)*sqrt(-x^4 + x^2 + 2))/x
```

3.318.6 Sympy [F]

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^2\,dx$$

input
```
integrate((5*x**2+7)**2*(-x**4+x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2, x)
```

3.318.7 Maxima [F]

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2\,dx$$

input
```
integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

3.318.8 Giac [F]

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\left(5\,x^2+7\right)^2\,dx$$

input
```
integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}\,dx = \int \left(5\,x^2+7\right)^2 \sqrt{-x^4+x^2+2}\,dx$$

input
```
int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2), x)
```

3.319 $\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx$

3.319.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx = x(2+x^2)\sqrt{2+x^2-x^4}+7E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + 3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
7*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+3*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+
x*(x^2+2)*(-x^4+x^2+2)^(1/2)
```

3.319.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.95 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx = \frac{4x+4x^3-x^5-x^7+7i\sqrt{4+2x^2-2x^4}\,E\left(i\text{arcsinh}(x)\middle|-\frac{1}{2}\right)-12i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left(i\text{arcsinh}(x)\right.}{\sqrt{2+x^2-x^4}}$$

input
```
Integrate[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
```

output
```
(4*x + 4*x^3 - x^5 - x^7 + (7*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSi
nh[x], -1/2] - (12*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2
])/Sqrt[2 + x^2 - x^4]
```

3.319.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)\sqrt{-x^4+x^2+2}\,dx$$

$$\Big\downarrow \text{1490}$$

$$x(x^2+2)\sqrt{-x^4+x^2+2}-\frac{1}{15}\int -\frac{15(7x^2+10)}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{7x^2+10}{\sqrt{-x^4+x^2+2}}dx+x\sqrt{-x^4+x^2+2}(x^2+2)$$

$$\Big\downarrow \text{1494}$$

$$2\int \frac{7x^2+10}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+x\sqrt{-x^4+x^2+2}(x^2+2)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{7x^2+10}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+x\sqrt{-x^4+x^2+2}(x^2+2)$$

$$\Big\downarrow \text{399}$$

$$3\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+7\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+x\sqrt{-x^4+x^2+2}(x^2+2)$$

$$\Big\downarrow \text{321}$$

$$7\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+x\sqrt{-x^4+x^2+2}(x^2+2)$$

$$\Big\downarrow \text{327}$$

$$3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+7E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\Big|-2\right)+x\sqrt{-x^4+x^2+2}(x^2+2)$$

input
```
Int[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
```

output
```
x*(2 + x^2)*Sqrt[2 + x^2 - x^4] + 7*EllipticE[ArcSin[x/Sqrt[2]], -2] + 3*E
llipticF[ArcSin[x/Sqrt[2]], -2]
```

3.319.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.319. $\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx$

3.319.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(50) = 100.

Time = 1.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.04

method	result
risch	$-\frac{x(x^2+2)(x^4-x^2-2)}{\sqrt{-x^4+x^2+2}}+\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{\sqrt{-x^4+x^2+2}}-\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^4+x^2+2}}$
default	$2x\sqrt{-x^4+x^2+2}+\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{\sqrt{-x^4+x^2+2}}-\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^4+x^2+2}}+x$
elliptic	$2x\sqrt{-x^4+x^2+2}+\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{\sqrt{-x^4+x^2+2}}-\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^4+x^2+2}}+x$

input
```
int((5*x^2+7)*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x*(x^2+2)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+5*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-7/2*2^(1/2)
*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2
),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx$$
$$=\frac{-14i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})+19i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})+(x^4+2x^2-7)\sqrt{-x^4+x^2+2}}{x}$$

input
```
integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
(-14*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 19*I*sqrt(2)*x*elli
ptic_f(arcsin(sqrt(2)/x), -1/2) + (x^4 + 2*x^2 - 7)*sqrt(-x^4 + x^2 + 2))/
x
```

3.319.6 Sympy [F]

$$\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx = \int \sqrt{-(x^2-2)(x^2+1)}(5x^2+7)\,dx$$

input
```
integrate((5*x**2+7)*(-x**4+x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7), x)
```

3.319.7 Maxima [F]

$$\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}(5\,x^2+7)\,dx$$

input
```
integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

3.319.8 Giac [F]

$$\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}(5\,x^2+7)\,dx$$

input
```
integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)\sqrt{2+x^2-x^4}\,dx=\int \left(5\,x^2+7\right)\,\sqrt{-x^4+x^2+2}\,dx$$

input
```
int((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2), x)
```

3.320 $\int \sqrt{2+x^2-x^4}\,dx$

3.320.1 Optimal result

Integrand size = 14, antiderivative size = 44

$$\int \sqrt{2+x^2-x^4}\,dx=\frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
1/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+EllipticF(1/2*x*2^(1/2),I*2^(1/2))+
1/3*x*(-x^4+x^2+2)^(1/2)
```

3.320.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.93 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.05

$$\int \sqrt{2+x^2-x^4}\,dx=\frac{2x+x^3-x^5+i\sqrt{4+2x^2-2x^4}E\left(i\text{arcsinh}(x)\middle|-\frac{1}{2}\right)-3i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)}{3\sqrt{2+x^2-x^4}}$$

input
```
Integrate[Sqrt[2 + x^2 - x^4],x]
```

output
```
(2*x + x^3 - x^5 + I*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2]
 - (3*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(3*Sqrt[2
+ x^2 - x^4])
```

3.320.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1404, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-x^4+x^2+2}\,dx$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{x^2+4}{\sqrt{-x^4+x^2+2}}dx+\frac{1}{3}\sqrt{-x^4+x^2+2}x$$

$$\Big\downarrow \text{1494}$$

$$\frac{2}{3}\int \frac{x^2+4}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{3}\sqrt{-x^4+x^2+2}x$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{x^2+4}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{3}\sqrt{-x^4+x^2+2}x$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{3}\left(3\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{1}{3}\sqrt{-x^4+x^2+2}x$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{3}\left(\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{3}\sqrt{-x^4+x^2+2}x$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{3}\left(3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\Big|-2\right)\right)+\frac{1}{3}\sqrt{-x^4+x^2+2}x$$

input
```
Int[Sqrt[2 + x^2 - x^4],x]
```

output
```
(x*Sqrt[2 + x^2 - x^4])/3 + (EllipticE[ArcSin[x/Sqrt[2]], -2] + 3*Elliptic
F[ArcSin[x/Sqrt[2]], -2])/3
```

3.320.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.320. $\int \sqrt{2+x^2-x^4}\,dx$

3.320.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(44) = 88$.

Time = 0.64 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.84

method	result	size
default	$\frac{x\sqrt{-x^4+x^2+2}}{3}+\frac{2\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{3\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{6\sqrt{-x^4+x^2+2}}$	125
elliptic	$\frac{x\sqrt{-x^4+x^2+2}}{3}+\frac{2\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{3\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{6\sqrt{-x^4+x^2+2}}$	125
risch	$-\frac{x(x^4-x^2-2)}{3\sqrt{-x^4+x^2+2}}+\frac{2\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{3\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{6\sqrt{-x^4+x^2+2}}$	135

input
```
int((-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(-x^4+x^2+2)^(1/2)+2/3*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+
x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-1/6*2^(1/2)*(-2*x^2+4)^(1/
2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-El
lipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.320.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \sqrt{2+x^2-x^4}\,dx$$
$$=\frac{-2i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})+4i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid-\frac{1}{2})+\sqrt{-x^4+x^2+2}(x^2-1)}{3\,x}$$

input
```
integrate((-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-2*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 4*I*sqrt(2)*x*el
liptic_f(arcsin(sqrt(2)/x), -1/2) + sqrt(-x^4 + x^2 + 2)*(x^2 - 1))/x
```

3.320.6 Sympy [F]

$$\int \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\,dx$$

input
```
integrate((-x**4+x**2+2)**(1/2),x)
```

output
```
Integral(sqrt(-x**4 + x**2 + 2), x)
```

3.320.7 Maxima [F]

$$\int \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\,dx$$

input
```
integrate((-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2), x)
```

3.320.8 Giac [F]

$$\int \sqrt{2+x^2-x^4}\,dx = \int \sqrt{-x^4+x^2+2}\,dx$$

input
```
integrate((-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2), x)
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{2 + x^2 - x^4}\, dx = \int \sqrt{-x^4 + x^2 + 2}\, dx$$

input
```
int((x^2 - x^4 + 2)^(1/2),x)
```

output
```
int((x^2 - x^4 + 2)^(1/2), x)
```

3.321 $\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx$

3.321.1 Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx = -\frac{1}{5}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{17}{25}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{34}{175}\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
-1/5*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+17/25*EllipticF(1/2*x*2^(1/2),I*2^
(1/2))-34/175*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.321.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.93 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx = -\frac{1}{175}i\sqrt{2}\left(35E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right)+7\operatorname{EllipticF}\left(i\operatorname{arcsinh}(x),-\frac{1}{2}\right)-17\operatorname{EllipticPi}\left(\frac{5}{7},i\operatorname{arcsinh}(x),-\frac{1}{2}\right)\right)$$

input
```
Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
```

output
```
(-1/175*I)*Sqrt[2]*(35*EllipticE[I*ArcSinh[x], -1/2] + 7*EllipticF[I*ArcSi
nh[x], -1/2] - 17*EllipticPi[5/7, I*ArcSinh[x], -1/2])
```

3.321.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1525, 25, 1494, 27, 399, 321, 327, 1536, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-x^4+x^2+2}}{5x^2+7}dx$$

$$\Big\downarrow \text{ 1525}$$

$$-\frac{1}{25}\int -\frac{12-5x^2}{\sqrt{-x^4+x^2+2}}dx-\frac{34}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{25}\int \frac{12-5x^2}{\sqrt{-x^4+x^2+2}}dx-\frac{34}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{ 1494}$$

$$\frac{2}{25}\int \frac{12-5x^2}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx-\frac{34}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{25}\int \frac{12-5x^2}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-\frac{34}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{ 399}$$

$$\frac{1}{25}\left(17\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-5\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)-\frac{34}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\frac{1}{25}\left(17\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-5\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)-\frac{34}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$\frac{1}{25}\left(17\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)-\frac{34}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{ 1536}$$

$$\frac{1}{25}\left(17\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)-\frac{68}{25}\int\frac{1}{2\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{25}\left(17\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)-\frac{34}{25}\int\frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx$$

$$\Big\downarrow \; 412$$

$$\frac{1}{25}\left(17\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)-\frac{34}{175}\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

input
```
Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
```

output
```
(-5*EllipticE[ArcSin[x/Sqrt[2]], -2] + 17*EllipticF[ArcSin[x/Sqrt[2]], -2]
)/25 - (34*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/175
```

3.321.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1525
```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(c*d^2 - b*d*e + a*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + b*x^2
+ c*x^4]), x], x] - Simp[1/e^2   Int[(c*d - b*e - c*e*x^2)/Sqrt[a + b*x^2 +
 c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

rule 1536
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/((d + e*x^
2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[
c, 0]
```

3.321.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(47) = 94$.

Time = 0.76 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.07

3.321. $\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx$

method	result	siz
default	$\frac{17\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{50\sqrt{-x^{4}+x^{2}+2}}-\frac{\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{10\sqrt{-x^{4}+x^{2}+2}}-\frac{34\sqrt{2}\,\sqrt{1-\frac{x^{2}}{2}}\,\sqrt{x^{2}+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\frac{10}{7},i\sqrt{2}\right)}{175\sqrt{-x^{4}+x^{2}+2}}$	14
elliptic	$\frac{17\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{50\sqrt{-x^{4}+x^{2}+2}}-\frac{\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{10\sqrt{-x^{4}+x^{2}+2}}-\frac{34\sqrt{2}\,\sqrt{1-\frac{x^{2}}{2}}\,\sqrt{x^{2}+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\frac{10}{7},i\sqrt{2}\right)}{175\sqrt{-x^{4}+x^{2}+2}}$	14

input
```
int((-x^4+x^2+2)^(1/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

output
```
17/50*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(
1/2*x*2^(1/2),I*2^(1/2))-1/10*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4
+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-34/175*2^(1/2)*(1-1/2*x^2
)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*
2^(1/2))
```

3.321.5 Fricas [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx=\int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

input
```
integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

3.321.6 Sympy [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx=\int \frac{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}{5x^2+7}\,dx$$

input
```
integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7),x)
```

output
```
Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7), x)
```

3.321.7 Maxima [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

input
```
integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

3.321.8 Giac [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

input
```
integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

input
```
int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7),x)
```

output
```
int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7), x)
```

3.322 $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx$

3.322.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx = \frac{x\sqrt{2+x^2-x^4}}{14(7+5x^2)} + \frac{1}{70}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{6}{175}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) + \frac{99\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{2450}$$

output
```
1/70*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-6/175*EllipticF(1/2*x*2^(1/2),I*2^
(1/2))+99/2450*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/14*x*(-x^4+x^2+
2)^(1/2)/(5*x^2+7)
```

3.322.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.65

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx = \frac{700x+350x^3-350x^5+70i\sqrt{2}(7+5x^2)\sqrt{2+x^2-x^4}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right)-21i\sqrt{2}(7+5x^2)\sqrt{2+x^2-}$$

input
```
Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

output
```
(700*x + 350*x^3 - 350*x^5 + (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4
]*EllipticE[I*ArcSinh[x], -1/2] - (21*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2
- x^4]*EllipticF[I*ArcSinh[x], -1/2] - (693*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]
*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (495*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 -
x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(4900*(7 + 5*x^2)*Sqrt[2 + x^2 -
 x^4])
```

3.322.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1554, 25, 2234, 1494, 27, 399, 321, 327, 1536, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-x^4+x^2+2}}{(5x^2+7)^2}dx$$

$$\Big\downarrow \text{1554}$$

$$\frac{x\sqrt{-x^4+x^2+2}}{14(5x^2+7)} - \frac{1}{14}\int -\frac{x^4+2}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{14}\int \frac{x^4+2}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx + \frac{\sqrt{-x^4+x^2+2}x}{14(5x^2+7)}$$

$$\Big\downarrow \text{2234}$$

$$\frac{1}{14}\left(\frac{99}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx - \frac{1}{25}\int \frac{7-5x^2}{\sqrt{-x^4+x^2+2}}dx\right) + \frac{\sqrt{-x^4+x^2+2}x}{14(5x^2+7)}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{14}\left(\frac{99}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx - \frac{2}{25}\int \frac{7-5x^2}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx\right) + \frac{\sqrt{-x^4+x^2+2}x}{14(5x^2+7)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{14}\left(\frac{99}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx - \frac{1}{25}\int \frac{7-5x^2}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right) + \frac{\sqrt{-x^4+x^2+2}x}{14(5x^2+7)}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{14}\left(\frac{1}{25}\left(5\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-12\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)+\frac{99}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx\right)+\frac{\sqrt{-x^4+x^2+2}x}{14\,(5x^2+7)}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{14}\left(\frac{1}{25}\left(5\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-12\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{99}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx\right)+\frac{\sqrt{-x^4+x^2+2}x}{14\,(5x^2+7)}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{14}\left(\frac{99}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx+\frac{1}{25}\left(5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-12\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)\right)+\frac{\sqrt{-x^4+x^2+2}x}{14\,(5x^2+7)}$$

$$\Big\downarrow \text{1536}$$

$$\frac{1}{14}\left(\frac{198}{25}\int \frac{1}{2\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx+\frac{1}{25}\left(5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-12\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)\right)+\frac{\sqrt{-x^4+x^2+2}x}{14\,(5x^2+7)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{14}\left(\frac{99}{25}\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx+\frac{1}{25}\left(5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-12\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)\right)+\frac{\sqrt{-x^4+x^2+2}x}{14\,(5x^2+7)}$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{14}\left(\frac{99}{175}\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{1}{25}\left(5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-12\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)\right)+\frac{\sqrt{-x^4+x^2+2}x}{14\,(5x^2+7)}$$

input
```
Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

3.322. $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx$

output
```
(x*Sqrt[2 + x^2 - x^4])/(14*(7 + 5*x^2)) + ((5*EllipticE[ArcSin[x/Sqrt[2]]
, -2] - 12*EllipticF[ArcSin[x/Sqrt[2]], -2])/25 + (99*EllipticPi[-10/7, Ar
cSin[x/Sqrt[2]], -2])/175)/14
```

3.322.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

3.322. $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2} \, dx$

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1536
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/((d + e*x^
2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[
c, 0]
```

rule 1554
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(q +
 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*((a*(2*q + 3) +
2*b*(q + 2)*x^2 + c*(2*q + 5)*x^4)/Sqrt[a + b*x^2 + c*x^4]), x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && ILtQ[q, -1]
```

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.322.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(71) = 142$.

Time = 3.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.23

method	result
default	$\frac{x\sqrt{-x^4+x^2+2}}{70x^2+98}-\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{175\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{140\sqrt{-x^4+x^2+2}}+\frac{99\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2}\right.}{2450\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{x\sqrt{-x^4+x^2+2}}{70x^2+98}-\frac{3\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{175\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{140\sqrt{-x^4+x^2+2}}+\frac{99\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2}\right.}{2450\sqrt{-x^4+x^2+2}}$
risch	$-\frac{(x^4-x^2-2)x}{14(5x^2+7)\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{100\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{140\sqrt{-x^4+x^2+2}}+$

input
```
int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

output
```
1/14*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-3/175*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/140*2^(1/2)
*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2)
,I*2^(1/2))+99/2450*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(
1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.322.5 Fricas [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx=\int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

3.322.6 Sympy [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx=\int \frac{\sqrt{-(x^2-2)(x^2+1)}}{(5x^2+7)^2}\,dx$$

input
```
integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

output
```
Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**2, x)
```

3.322.7 Maxima [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

3.322.8 Giac [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^2}\,dx$$

input
```
int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^2,x)
```

output
```
int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^2, x)
```

3.323 $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3} dx$

3.323.1 Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3} dx = \frac{x\sqrt{2+x^2-x^4}}{28(7+5x^2)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328(7+5x^2)} - \frac{31E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640} - \frac{269\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{166600} + \frac{16601\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{2332400}$$

output
```
-31/66640*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-269/166600*EllipticF(1/2*x*2^
(1/2),I*2^(1/2))+16601/2332400*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1
/28*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-31/13328*x*(-x^4+x^2+2)^(1/2)/(5*x^2+
7)
```

3.323.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.35 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3} dx = \frac{181300x - 17850x^3 - 144900x^5 + 54250x^7 - 2170i\sqrt{2}(7+5x^2)^2\sqrt{2+x^2-x^4}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right) + 702\cdots}{\cdots}$$

input
```
Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

output
```
(181300*x - 17850*x^3 - 144900*x^5 + 54250*x^7 - (2170*I)*Sqrt[2]*(7 + 5*x
^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (7021*I)*Sqrt[2]
*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (813449
*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (116
2070*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2
] - (415025*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x
], -1/2])/(4664800*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

3.323.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1554, 25, 2210, 2234, 1494, 27, 399, 321, 327, 1536, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-x^4+x^2+2}}{(5x^2+7)^3}dx$$

$$\Big\downarrow \text{1554}$$

$$\frac{x\sqrt{-x^4+x^2+2}}{28(5x^2+7)^2}-\frac{1}{28}\int -\frac{-x^4+2x^2+6}{(5x^2+7)^2\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{28}\int \frac{-x^4+2x^2+6}{(5x^2+7)^2\sqrt{-x^4+x^2+2}}dx+\frac{\sqrt{-x^4+x^2+2}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{2210}$$

$$\frac{1}{28}\left(\frac{1}{476}\int \frac{-31x^4-182x^2+470}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-\frac{31x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}\right)+\frac{\sqrt{-x^4+x^2+2}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{2234}$$

$$\frac{1}{28}\left(\frac{1}{476}\left(\frac{16601}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-\frac{1}{25}\int \frac{155x^2+693}{\sqrt{-x^4+x^2+2}}dx\right)-\frac{31x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}\right)+\frac{\sqrt{-x^4+x^2+2}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{28}\left(\frac{1}{476}\left(\frac{16601}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-\frac{2}{25}\int \frac{155x^2+693}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)-\frac{31x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}\right)+\frac{\sqrt{-x^4+x^2+2}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\left(\frac{1}{476}\left(\frac{16601}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-\frac{1}{25}\int \frac{155x^2+693}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)-\frac{31x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}\right)+\frac{\sqrt{-x^4+x^2+2}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{28}\left(\frac{1}{476}\left(\frac{1}{25}\left(-538\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-155\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{16601}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx\right)-\frac{3}{}\right.$$
$$\frac{\sqrt{-x^4+x^2+2}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{28}\left(\frac{1}{476}\left(\frac{1}{25}\left(-155\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-538\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{16601}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}\right.\right.$$
$$\frac{\sqrt{-x^4+x^2+2}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{28}\left(\frac{1}{476}\left(\frac{16601}{25}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx+\frac{1}{25}\left(-538\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-155E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right.\right.\right.\right.\right.$$
$$\frac{\sqrt{-x^4+x^2+2}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{1536}$$

$$\frac{1}{28}\left(\frac{1}{476}\left(\frac{33202}{25}\int \frac{1}{2\sqrt{2-x^2}\sqrt{x^2+1}(5x^2+7)}dx+\frac{1}{25}\left(-538\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-155E\left(\arcsin\left(\right.\right.\right.\right.\right.$$
$$\frac{\sqrt{-x^4+x^2+2}x}{28(5x^2+7)^2}$$

3.323. $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx$

$$\downarrow 27$$

$$\frac{1}{28}\left(\frac{1}{476}\left(\frac{16601}{25}\int\frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx+\frac{1}{25}\left(-538\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-155E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right.\right.\right.\right.\right.$$
$$\frac{\sqrt{-x^4+x^2+2}x}{28\,(5x^2+7)^2}$$

$$\downarrow 412$$

$$\frac{1}{28}\left(\frac{1}{476}\left(\frac{16601}{175}\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+\frac{1}{25}\left(-538\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-155E\left(\text{arcsi}\right.\right.\right.\right.$$
$$\frac{\sqrt{-x^4+x^2+2}x}{28\,(5x^2+7)^2}$$

input
```
Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

output
```
(x*Sqrt[2 + x^2 - x^4])/(28*(7 + 5*x^2)^2) + ((-31*x*Sqrt[2 + x^2 - x^4])/
(476*(7 + 5*x^2)) + ((-155*EllipticE[ArcSin[x/Sqrt[2]], -2] - 538*Elliptic
F[ArcSin[x/Sqrt[2]], -2])/25 + (16601*EllipticPi[-10/7, ArcSin[x/Sqrt[2]],
 -2])/175)/476)/28
```

3.323.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.323. $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1536
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/((d + e*x^
2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[
c, 0]
```

rule 1554
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(q +
 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*((a*(2*q + 3) +
2*b*(q + 2)*x^2 + c*(2*q + 5)*x^4)/Sqrt[a + b*x^2 + c*x^4]), x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && ILtQ[q, -1]
```

3.323. $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx$

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.323.4 Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.85

method	result
default	$\frac{x\sqrt{-x^4+x^2+2}}{28(5x^2+7)^2}-\frac{31x\sqrt{-x^4+x^2+2}}{13328(5x^2+7)}-\frac{269\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{333200\sqrt{-x^4+x^2+2}}-\frac{31\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{133280\sqrt{-x^4+x^2+2}}+\frac{166}{}$
elliptic	$\frac{x\sqrt{-x^4+x^2+2}}{28(5x^2+7)^2}-\frac{31x\sqrt{-x^4+x^2+2}}{13328(5x^2+7)}-\frac{269\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{333200\sqrt{-x^4+x^2+2}}-\frac{31\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{133280\sqrt{-x^4+x^2+2}}+\frac{166}{}$
risch	$\frac{(x^4-x^2-2)x(155x^2-259)}{13328(5x^2+7)^2\sqrt{-x^4+x^2+2}}-\frac{99\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{95200\sqrt{-x^4+x^2+2}}+\frac{31\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right.}{133280\sqrt{-x^4+x^2+2}}$

input
```
int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

output
```
1/28*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-31/13328*x*(-x^4+x^2+2)^(1/2)/(5*x^2
+7)-269/333200*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*E
llipticF(1/2*x*2^(1/2),I*2^(1/2))-31/133280*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+16601/23324
00*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1
/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.323.5 Fricas [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

3.323.6 Sympy [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{-(x^2-2)(x^2+1)}}{(5x^2+7)^3}\,dx$$

input
```
integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

output
```
Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**3, x)
```

3.323.7 Maxima [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx=\int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

3.323.8 Giac [F]

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx = \int \frac{\sqrt{-x^4+x^2+2}}{(5\,x^2+7)^3}\,dx$$

input
```
int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^3,x)
```

output
```
int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^3, x)
```

3.324 $\int (7+5x^2)^4 (2+x^2-x^4)^{3/2} \, dx$

3.324.1 Optimal result

Integrand size = 24, antiderivative size = 142

$$\int (7+5x^2)^4 (2+x^2-x^4)^{3/2} \, dx = \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005} - \frac{x(69817-1581440x^2)(2+x^2-x^4)^{3/2}}{1001} - \frac{132300}{143}x(2+x^2-x^4)^{5/2} - \frac{11750}{39}x^3(2+x^2-x^4)^{5/2} - \frac{125}{3}x^5(2+x^2-x^4)^{5/2} + \frac{124141422E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle| -2\right)}{5005} - \frac{50794416 \operatorname{EllipticF}\left(\right.}{5005}$$

output

```
-1/1001*x*(-1581440*x^2+69817)*(-x^4+x^2+2)^(3/2)-132300/143*x*(-x^4+x^2+2
)^(5/2)-11750/39*x^3*(-x^4+x^2+2)^(5/2)-125/3*x^5*(-x^4+x^2+2)^(5/2)+12414
1422/5005*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-50794416/5005*EllipticF(1/2*x
*2^(1/2),I*2^(1/2))+3/5005*x*(7837383*x^2+2193559)*(-x^4+x^2+2)^(1/2)
```

3.324.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.86

$$\int (7+5x^2)^4 (2+x^2-x^4)^{3/2} \, dx = \frac{-75836958x+48624305x^3+172881581x^5+32834763x^7-36649955x^9-24642275x^{11}-15}{}$$

input
```
Integrate[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```

output
```
(-75836958*x + 48624305*x^3 + 172881581*x^5 + 32834763*x^7 - 36649955*x^9
- 24642275*x^11 - 1556625*x^13 + 2646875*x^15 + 625625*x^17 + (372424266*I
)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (482444775*I)*Sq
rt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(15015*Sqrt[2 + x^2 -
 x^4])
```

3.324.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1518, 27, 2207, 27, 2207, 25, 1490, 27, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+7\right)^4 \left(-x^4+x^2+2\right)^{3/2} dx$$

$$\Big\downarrow \text{1518}$$

$$-\frac{1}{15}\int -5\left(-x^4+x^2+2\right)^{3/2} \left(11750x^6+23300x^4+20580x^2+7203\right)dx-\frac{125}{3}\left(-x^4+x^2+2\right)^{5/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \left(-x^4+x^2+2\right)^{3/2} \left(11750x^6+23300x^4+20580x^2+7203\right)dx-\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{3}\left(-\frac{1}{13}\int -3\left(-x^4+x^2+2\right)^{3/2} \left(132300x^4+112680x^2+31213\right)dx-\frac{11750}{13}\left(-x^4+x^2+2\right)^{5/2}x^3\right)-\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3}{13}\int \left(-x^4+x^2+2\right)^{3/2} \left(132300x^4+112680x^2+31213\right)dx-\frac{11750}{13}x^3\left(-x^4+x^2+2\right)^{5/2}\right)-\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{3}\left(\frac{3}{13}\left(-\frac{1}{11}\int -\left(\left(2033280x^2+607943\right)\left(-x^4+x^2+2\right)^{3/2}\right)dx-\frac{132300}{11}x\left(-x^4+x^2+2\right)^{5/2}\right)-\frac{11750}{13}x^3\left(-x^4\right.$$
$$\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$\big\downarrow$ 25

$$\frac{1}{3}\left(\frac{3}{13}\left(\frac{1}{11}\int\left(2033280x^2+607943\right)\left(-x^4+x^2+2\right)^{3/2}dx-\frac{132300}{11}x\left(-x^4+x^2+2\right)^{5/2}\right)-\frac{11750}{13}x^3\left(-x^4+x^2\right.$$
$$\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$\big\downarrow$ 1490

$$\frac{1}{3}\left(\frac{3}{13}\left(\frac{1}{11}\left(-\frac{1}{21}\int -27\left(2612461x^2+961204\right)\sqrt{-x^4+x^2+2}dx-\frac{1}{7}x\left(69817-1581440x^2\right)\left(-x^4+x^2+2\right)^{3/2}\right)\right.\right.$$
$$\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(\frac{3}{13}\left(\frac{1}{11}\left(\frac{9}{7}\int\left(2612461x^2+961204\right)\sqrt{-x^4+x^2+2}dx-\frac{1}{7}x\left(69817-1581440x^2\right)\left(-x^4+x^2+2\right)^{3/2}\right)-\frac{132}{1}\right.\right.$$
$$\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$\big\downarrow$ 1490

$$\frac{1}{3}\left(\frac{3}{13}\left(\frac{1}{11}\left(\frac{9}{7}\left(\frac{1}{15}x\left(7837383x^2+2193559\right)\sqrt{-x^4+x^2+2}-\frac{1}{15}\int -\frac{2\left(20690237x^2+12224501\right)}{\sqrt{-x^4+x^2+2}}dx\right)-\frac{1}{7}x(69\right.\right.\right.$$
$$\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(\frac{3}{13}\left(\frac{1}{11}\left(\frac{9}{7}\left(\frac{2}{15}\int\frac{20690237x^2+12224501}{\sqrt{-x^4+x^2+2}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(7837383x^2+2193559\right)\right)-\frac{1}{7}x\left(69817-\right.\right.\right.\right.$$
$$\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$\big\downarrow$ 1494

$$\frac{1}{3}\left(\frac{3}{13}\left(\frac{1}{11}\left(\frac{9}{7}\left(\frac{4}{15}\int\frac{20690237x^2+12224501}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(7837383x^2+2193559\right)\right)-\frac{1}{7}x\left(69817-\right.\right.\right.\right.$$
$$\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

3.324. $\int\left(7+5x^2\right)^4\left(2+x^2-x^4\right)^{3/2}\,dx$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(\frac{3}{13}\left(\frac{1}{11}\left(\frac{9}{7}\left(\frac{2}{15}\int \frac{20690237x^2+12224501}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(7837383x^2+2193559\right)\right)-\frac{1}{7}x\left(69817-\right.\right.\right.\right.\right.$$
$$\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow 399$$

$$\frac{1}{3}\left(\frac{3}{13}\left(\frac{1}{11}\left(\frac{9}{7}\left(\frac{2}{15}\left(20690237\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-8465736\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(7837383\right.\right.\right.\right.\right.$$
$$\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow 321$$

$$\frac{1}{3}\left(\frac{3}{13}\left(\frac{1}{11}\left(\frac{9}{7}\left(\frac{2}{15}\left(20690237\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-8465736\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(\right.\right.\right.\right.\right.$$
$$\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow 327$$

$$\frac{1}{3}\left(\frac{3}{13}\left(\frac{1}{11}\left(\frac{9}{7}\left(\frac{2}{15}\left(20690237E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-8465736\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{15}x\sqrt{-x^4+\right.\right.\right.\right.\right.$$
$$\frac{125}{3}x^5\left(-x^4+x^2+2\right)^{5/2}$$

input
```
Int[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```

output
```
(-125*x^5*(2 + x^2 - x^4)^(5/2))/3 + ((-11750*x^3*(2 + x^2 - x^4)^(5/2))/1
3 + (3*((-132300*x*(2 + x^2 - x^4)^(5/2))/11 + (-1/7*(x*(69817 - 1581440*x
^2)*(2 + x^2 - x^4)^(3/2)) + (9*((x*(2193559 + 7837383*x^2)*Sqrt[2 + x^2 -
 x^4])/15 + (2*(20690237*EllipticE[ArcSin[x/Sqrt[2]], -2] - 8465736*Ellipt
icF[ArcSin[x/Sqrt[2]], -2]))/15))/7)/11))/13)/3
```

3.324. $\int \left(7+5x^2\right)^4\left(2+x^2-x^4\right)^{3/2}\,dx$

3.324.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.324. $\int (7+5x^2)^4 (2+x^2-x^4)^{3/2} \, dx$

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.324.4 Maple [A] (verified)

Time = 7.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18

method	result
risch	$\frac{x(625625x^{12}+3272500x^{10}+2967125x^8-15130150x^6-45845855x^4-43271392x^2+37918479)(x^4-x^2-2)}{15015\sqrt{-x^4+x^2+2}}+\frac{36673503\sqrt{2}\,\sqrt{-2x^2+}}{5005\sqrt{-}}$
default	$\frac{833561x^5\sqrt{-x^4+x^2+2}}{273}+\frac{43271392x^3\sqrt{-x^4+x^2+2}}{15015}-\frac{12639493x\sqrt{-x^4+x^2+2}}{5005}+\frac{36673503\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{5005\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{833561x^5\sqrt{-x^4+x^2+2}}{273}+\frac{43271392x^3\sqrt{-x^4+x^2+2}}{15015}-\frac{12639493x\sqrt{-x^4+x^2+2}}{5005}+\frac{36673503\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{5005\sqrt{-x^4+x^2+2}}$

input
```
int((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15015*x*(625625*x^12+3272500*x^10+2967125*x^8-15130150*x^6-45845855*x^4-
43271392*x^2+37918479)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+36673503/5005*2^(1/2
)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2
),I*2^(1/2))-62070711/5005*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^
2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2
^(1/2)))
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int (7+5x^2)^4 (2+x^2-x^4)^{3/2}\,dx = \frac{-744848532i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}) + 854869041i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}) - (625625\,x^{14}}{}$$

input
```
integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/15015*(-744848532*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 8548
69041*I*sqrt(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - (625625*x^14 + 327
2500*x^12 + 2967125*x^10 - 15130150*x^8 - 45845855*x^6 - 43271392*x^4 + 37
918479*x^2 + 372424266)*sqrt(-x^4 + x^2 + 2))/x
```

3.324.6 Sympy [F]

$$\int (7+5x^2)^4 (2+x^2-x^4)^{3/2}\,dx = \int \left(-(x^2-2)\,(x^2+1)\right)^{\frac{3}{2}} (5x^2+7)^4\,dx$$

input
```
integrate((5*x**2+7)**4*(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**4, x)
```

3.324.7 Maxima [F]

$$\int (7+5x^2)^4 (2+x^2-x^4)^{3/2}\,dx = \int (-x^4+x^2+2)^{\frac{3}{2}} (5\,x^2+7)^4\,dx$$

input
```
integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

3.324.8 Giac [F]

$$\int (7+5x^2)^4 (2+x^2-x^4)^{3/2}\, dx = \int (-x^4+x^2+2)^{\frac{3}{2}} (5\,x^2+7)^4\, dx$$

input
```
integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int (7+5x^2)^4 (2+x^2-x^4)^{3/2}\, dx = \int (5\,x^2+7)^4 \left(-x^4+x^2+2\right)^{3/2} dx$$

input
```
int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(3/2), x)
```

3.325 $\int (7+5x^2)^3 (2+x^2-x^4)^{3/2}\,dx$

3.325.1 Optimal result

Integrand size = 24, antiderivative size = 121

$$\int (7+5x^2)^3 (2+x^2-x^4)^{3/2}\,dx = \frac{x(2512273+5712051x^2)\sqrt{2+x^2-x^4}}{15015} + \frac{x(33792+374045x^2)(2+x^2-x^4)^{3/2}}{3003} - \frac{7825}{143}x(2+x^2-x^4)^{5/2} - \frac{125}{13}x^3(2+x^2-x^4)^{5/2} + \frac{31072528E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{15015} - \frac{3199778\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{5005}$$

output
```
1/3003*x*(374045*x^2+33792)*(-x^4+x^2+2)^(3/2)-7825/143*x*(-x^4+x^2+2)^(5/
2)-125/13*x^3*(-x^4+x^2+2)^(5/2)+31072528/15015*EllipticE(1/2*x*2^(1/2),I*
2^(1/2))-3199778/5005*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/15015*x*(571205
1*x^2+2512273)*(-x^4+x^2+2)^(1/2)
```

3.325.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97

$$\int (7+5x^2)^3 (2+x^2-x^4)^{3/2}\,dx = \frac{-872614x+11078615x^3+13371048x^5-1756521x^7-4448240x^9-1027775x^{11}+388500x^{1}}{}$$

input
```
Integrate[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

output
```
(-872614*x + 11078615*x^3 + 13371048*x^5 - 1756521*x^7 - 4448240*x^9 - 102
7775*x^11 + 388500*x^13 + 144375*x^15 + (31072528*I)*Sqrt[4 + 2*x^2 - 2*x^
4]*EllipticE[I*ArcSinh[x], -1/2] - (41809125*I)*Sqrt[4 + 2*x^2 - 2*x^4]*El
lipticF[I*ArcSinh[x], -1/2])/(15015*Sqrt[2 + x^2 - x^4])
```

3.325.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {1518, 25, 2207, 25, 1490, 27, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+7\right)^3 \left(-x^4+x^2+2\right)^{3/2} dx$$

$$\Big\downarrow \text{1518}$$

$$-\frac{1}{13}\int -\left(-x^4+x^2+2\right)^{3/2} \left(7825x^4+10305x^2+4459\right) dx-\frac{125}{13}\left(-x^4+x^2+2\right)^{5/2} x^3$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{13}\int \left(-x^4+x^2+2\right)^{3/2} \left(7825x^4+10305x^2+4459\right) dx-\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{13}\left(-\frac{1}{11}\int -\left(\left(160305x^2+64699\right)\left(-x^4+x^2+2\right)^{3/2}\right) dx-\frac{7825}{11}x\left(-x^4+x^2+2\right)^{5/2}\right)-\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{13}\left(\frac{1}{11}\int \left(160305x^2+64699\right)\left(-x^4+x^2+2\right)^{3/2} dx-\frac{7825}{11}x\left(-x^4+x^2+2\right)^{5/2}\right)-\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{21}x\left(374045x^2+33792\right)\left(-x^4+x^2+2\right)^{3/2}-\frac{1}{21}\int -3\left(1904017x^2+883258\right)\sqrt{-x^4+x^2+2}dx\right)-\frac{782}{11}$$
$$\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

$\downarrow$ 27

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\int\left(1904017x^2+883258\right)\sqrt{-x^4+x^2+2}dx+\frac{1}{21}x\left(374045x^2+33792\right)\left(-x^4+x^2+2\right)^{3/2}\right)-\frac{7825}{11}x($$
$$\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

$\downarrow$ 1490

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{1}{15}x\left(5712051x^2+2512273\right)\sqrt{-x^4+x^2+2}-\frac{1}{15}\int -\frac{2\left(15536264x^2+10736597\right)}{\sqrt{-x^4+x^2+2}}dx\right)+\frac{1}{21}x(3740$$
$$\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

$\downarrow$ 27

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{2}{15}\int\frac{15536264x^2+10736597}{\sqrt{-x^4+x^2+2}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(5712051x^2+2512273\right)\right)+\frac{1}{21}x\left(374045x^2+\right.$$
$$\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

$\downarrow$ 1494

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{4}{15}\int\frac{15536264x^2+10736597}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(5712051x^2+2512273\right)\right)+\frac{1}{21}x\left(374045x^2+\right.$$
$$\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

$\downarrow$ 27

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{2}{15}\int\frac{15536264x^2+10736597}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(5712051x^2+2512273\right)\right)+\frac{1}{21}x\left(374045x^2+\right.$$
$$\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

$\downarrow$ 399

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{2}{15}\left(15536264\int\frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-4799667\int\frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(5712051x^2-\right.$$
$$\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

3.325. $\int\left(7+5x^2\right)^3\left(2+x^2-x^4\right)^{3/2}\,dx$

$\downarrow$ 321

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{2}{15}\left(15536264\int\frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-4799667\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}(571\right.\right.\right.$$
$$\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

$\downarrow$ 327

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{7}\left(\frac{2}{15}\left(15536264E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-4799667\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{15}x\sqrt{-x^4+x^2}\right.\right.\right.$$
$$\frac{125}{13}x^3\left(-x^4+x^2+2\right)^{5/2}$$

input
```
Int[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

output
```
(-125*x^3*(2 + x^2 - x^4)^(5/2))/13 + ((-7825*x*(2 + x^2 - x^4)^(5/2))/11
+ ((x*(33792 + 374045*x^2)*(2 + x^2 - x^4)^(3/2))/21 + ((x*(2512273 + 5712
051*x^2)*Sqrt[2 + x^2 - x^4])/15 + (2*(15536264*EllipticE[ArcSin[x/Sqrt[2]
], -2] - 4799667*EllipticF[ArcSin[x/Sqrt[2]], -2]))/15)/7)/11)/13
```

3.325.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.325. $\int (7+5x^2)^3 (2+x^2-x^4)^{3/2}\, dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.325. $\int (7+5x^2)^3 (2+x^2-x^4)^{3/2} \, dx$

3.325.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

method	result
risch	$\frac{x(144375x^{10}+532875x^8-206150x^6-3588640x^4-5757461x^2+436307)(x^4-x^2-2)}{15015\sqrt{-x^4+x^2+2}}+\frac{10736597\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{15015\sqrt{-x^4+x^2+2}}$
default	$\frac{65248x^5\sqrt{-x^4+x^2+2}}{273}+\frac{5757461x^3\sqrt{-x^4+x^2+2}}{15015}-\frac{436307x\sqrt{-x^4+x^2+2}}{15015}+\frac{10736597\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{15015\sqrt{-x^4+x^2+2}}-$
elliptic	$\frac{65248x^5\sqrt{-x^4+x^2+2}}{273}+\frac{5757461x^3\sqrt{-x^4+x^2+2}}{15015}-\frac{436307x\sqrt{-x^4+x^2+2}}{15015}+\frac{10736597\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{15015\sqrt{-x^4+x^2+2}}-$

input
```
int((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15015*x*(144375*x^10+532875*x^8-206150*x^6-3588640*x^4-5757461*x^2+43630
7)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+10736597/15015*2^(1/2)*(-2*x^2+4)^(1/2)*
(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-155362
64/15015*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(Ellipt
icF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69

$$\int (7+5x^2)^3 (2+x^2-x^4)^{3/2} \,dx = \frac{-62145056i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2})+72881653i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2})-(144375x^{12}+}{150}$$

input
```
integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/15015*(-62145056*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 72881
653*I*sqrt(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - (144375*x^12 + 53287
5*x^10 - 206150*x^8 - 3588640*x^6 - 5757461*x^4 + 436307*x^2 + 31072528)*s
qrt(-x^4 + x^2 + 2))/x
```

3.325.6 Sympy [F]

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}} \left(5x^2+7\right)^3 \, dx$$

input
```
integrate((5*x**2+7)**3*(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3, x)
```

3.325.7 Maxima [F]

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^3 \, dx$$

input
```
integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

3.325.8 Giac [F]

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^3 \, dx$$

input
```
integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(5\,x^2+7\right)^3 \left(-x^4+x^2+2\right)^{3/2} dx$$

input
```
int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2), x)
```

3.326 $\int (7+5x^2)^2 (2+x^2-x^4)^{3/2} \, dx$

3.326.1 Optimal result

Integrand size = 24, antiderivative size = 100

$$\int (7+5x^2)^2 (2+x^2-x^4)^{3/2} \, dx = \frac{1}{495}x(11497+14889x^2)\sqrt{2+x^2-x^4} + \frac{1}{99}x(363+920x^2)(2+x^2-x^4)^{3/2} - \frac{25}{11}x(2+x^2-x^4)^{5/2} + \frac{85942}{495}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{3392}{165}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
1/99*x*(920*x^2+363)*(-x^4+x^2+2)^(3/2)-25/11*x*(-x^4+x^2+2)^(5/2)+85942/4
95*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-3392/165*EllipticF(1/2*x*2^(1/2),I*2
^(1/2))+1/495*x*(14889*x^2+11497)*(-x^4+x^2+2)^(1/2)
```

3.326.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.01 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

$$\int (7+5x^2)^2 (2+x^2-x^4)^{3/2} \, dx = \frac{21254x + 53435x^3 + 23097x^5 - 19944x^7 - 10760x^9 + 1225x^{11} + 1125x^{13} + 85942i\sqrt{4+2x^2}}{495\sqrt{2+x^2-}}$$

input
```
Integrate[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```

output
```
(21254*x + 53435*x^3 + 23097*x^5 - 19944*x^7 - 10760*x^9 + 1225*x^11 + 112
5*x^13 + (85942*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] -
 (123825*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(495*Sq
rt[2 + x^2 - x^4])
```

3.326.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1518, 25, 1490, 27, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+7\right)^2 \left(-x^4+x^2+2\right)^{3/2} dx$$

$$\Big\downarrow \text{1518}$$

$$-\frac{1}{11} \int -\left(\left(920x^2+589\right) \left(-x^4+x^2+2\right)^{3/2}\right) dx - \frac{25}{11} x\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{11} \int \left(920x^2+589\right) \left(-x^4+x^2+2\right)^{3/2} dx - \frac{25}{11} x\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{11} \left(\frac{1}{9} x\left(920x^2+363\right) \left(-x^4+x^2+2\right)^{3/2} - \frac{1}{21} \int -7\left(4963x^2+3292\right) \sqrt{-x^4+x^2+2}dx\right) - \frac{25}{11} x\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11} \left(\frac{1}{3} \int \left(4963x^2+3292\right) \sqrt{-x^4+x^2+2}dx + \frac{1}{9} x\left(920x^2+363\right) \left(-x^4+x^2+2\right)^{3/2}\right) - \frac{25}{11} x\left(-x^4+x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{11}\left(\frac{1}{3}\left(\frac{1}{15}x(14889x^2+11497)\sqrt{-x^4+x^2+2}-\frac{1}{15}\int -\frac{2(42971x^2+37883)}{\sqrt{-x^4+x^2+2}}dx\right)+\frac{1}{9}x(920x^2+363)(-x^4+x$$
$$\frac{25}{11}x(-x^4+x^2+2)^{5/2}$$

$\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{3}\left(\frac{2}{15}\int \frac{42971x^2+37883}{\sqrt{-x^4+x^2+2}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}(14889x^2+11497)\right)+\frac{1}{9}x(920x^2+363)(-x^4+x^2+2$$
$$\frac{25}{11}x(-x^4+x^2+2)^{5/2}$$

$\downarrow$ 1494

$$\frac{1}{11}\left(\frac{1}{3}\left(\frac{4}{15}\int \frac{42971x^2+37883}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}(14889x^2+11497)\right)+\frac{1}{9}x(920x^2+363)(-x^4+x^2+2$$
$$\frac{25}{11}x(-x^4+x^2+2)^{5/2}$$

$\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{3}\left(\frac{2}{15}\int \frac{42971x^2+37883}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}(14889x^2+11497)\right)+\frac{1}{9}x(920x^2+363)(-x^4+x^2+2$$
$$\frac{25}{11}x(-x^4+x^2+2)^{5/2}$$

$\downarrow$ 399

$$\frac{1}{11}\left(\frac{1}{3}\left(\frac{2}{15}\left(42971\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-5088\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}(14889x^2+11497)\right)+\frac{1}{9}$$
$$\frac{25}{11}x(-x^4+x^2+2)^{5/2}$$

$\downarrow$ 321

$$\frac{1}{11}\left(\frac{1}{3}\left(\frac{2}{15}\left(42971\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-5088\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}(14889x^2+1149$$
$$\frac{25}{11}x(-x^4+x^2+2)^{5/2}$$

$\downarrow$ 327

$$\frac{1}{11}\left(\frac{1}{3}\left(\frac{2}{15}\left(42971E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-5088\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}(14889x$$
$$\frac{25}{11}x(-x^4+x^2+2)^{5/2}$$

3.326. $\int (7+5x^2)^2(2+x^2-x^4)^{3/2}\,dx$

input
```
Int[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```

output
```
(-25*x*(2 + x^2 - x^4)^(5/2))/11 + ((x*(363 + 920*x^2)*(2 + x^2 - x^4)^(3/
2))/9 + ((x*(11497 + 14889*x^2)*Sqrt[2 + x^2 - x^4])/15 + (2*(42971*Ellipt
icE[ArcSin[x/Sqrt[2]], -2] - 5088*EllipticF[ArcSin[x/Sqrt[2]], -2]))/15)/3
)/11
```

3.326.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.326.4 Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.57

method	result
risch	$\frac{x(1125x^8+2350x^6-6160x^4-21404x^2-10627)(x^4-x^2-2)}{495\sqrt{-x^4+x^2+2}}+\frac{37883\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{495\sqrt{-x^4+x^2+2}}-\frac{42971\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{}}{}$
default	$\frac{112x^5\sqrt{-x^4+x^2+2}}{9}+\frac{21404x^3\sqrt{-x^4+x^2+2}}{495}+\frac{10627x\sqrt{-x^4+x^2+2}}{495}+\frac{37883\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{495\sqrt{-x^4+x^2+2}}-\frac{42971\sqrt{2}}{}$
elliptic	$\frac{112x^5\sqrt{-x^4+x^2+2}}{9}+\frac{21404x^3\sqrt{-x^4+x^2+2}}{495}+\frac{10627x\sqrt{-x^4+x^2+2}}{495}+\frac{37883\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{495\sqrt{-x^4+x^2+2}}-\frac{42971\sqrt{2}}{}$

input
```
int((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/495*x*(1125*x^8+2350*x^6-6160*x^4-21404*x^2-10627)*(x^4-x^2-2)/(-x^4+x^2
+2)^(1/2)+37883/495*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1
/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-42971/495*2^(1/2)*(-2*x^2+4)^(1/2)*
(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-Ellip
ticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int (7+5x^2)^2 (2+x^2-x^4)^{3/2}\,dx = \frac{-171884i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}) + 209767i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}) - (1125\,x^{10} + 2350\,x}{495\,x}$$

input
```
integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/495*(-171884*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 209767*I*
sqrt(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - (1125*x^10 + 2350*x^8 - 61
60*x^6 - 21404*x^4 - 10627*x^2 + 85942)*sqrt(-x^4 + x^2 + 2))/x
```

3.326.6 Sympy [F]

$$\int (7+5x^2)^2 (2+x^2-x^4)^{3/2}\,dx = \int \left(-(x^2-2)(x^2+1)\right)^{\frac{3}{2}} (5x^2+7)^2\,dx$$

input
```
integrate((5*x**2+7)**2*(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2, x)
```

3.326.7 Maxima [F]

$$\int \left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^2 \, dx$$

input
```
integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

3.326.8 Giac [F]

$$\int \left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^2 \, dx$$

input
```
integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^2 \left(2+x^2-x^4\right)^{3/2} \, dx = \int \left(5\,x^2+7\right)^2 \left(-x^4+x^2+2\right)^{3/2} \, dx$$

input
```
int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2), x)
```

3.327 $\int (7+5x^2)(2+x^2-x^4)^{3/2}\,dx$

3.327.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int (7+5x^2)(2+x^2-x^4)^{3/2}\,dx = \frac{1}{315}x(1087+669x^2)\sqrt{2+x^2-x^4} + \frac{1}{63}x(48+35x^2)(2+x^2-x^4)^{3/2} + \frac{4432}{315}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{418}{105}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
1/63*x*(35*x^2+48)*(-x^4+x^2+2)^(3/2)+4432/315*EllipticE(1/2*x*2^(1/2),I*2
^(1/2))+418/105*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/315*x*(669*x^2+1087)*
(-x^4+x^2+2)^(1/2)
```

3.327.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.32

$$\int (7+5x^2)(2+x^2-x^4)^{3/2}\,dx = \frac{3134x+4085x^3-438x^5-1674x^7-110x^9+175x^{11}+4432i\sqrt{4+2x^2-2x^4}E(i\operatorname{arcsinh}(x)|}{315\sqrt{2+x^2-x^4}}$$

input
```
Integrate[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

output
```
(3134*x + 4085*x^3 - 438*x^5 - 1674*x^7 - 110*x^9 + 175*x^11 + (4432*I)*Sq
rt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (7275*I)*Sqrt[4 + 2*
x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(315*Sqrt[2 + x^2 - x^4])
```

3.327.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1490, 25, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+7\right)\left(-x^4+x^2+2\right)^{3/2}dx$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}-\frac{1}{21}\int -\left(\left(223x^2+262\right)\sqrt{-x^4+x^2+2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{21}\int \left(223x^2+262\right)\sqrt{-x^4+x^2+2}dx+\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{21}\left(\frac{1}{15}x\left(669x^2+1087\right)\sqrt{-x^4+x^2+2}-\frac{1}{15}\int -\frac{2\left(2216x^2+2843\right)}{\sqrt{-x^4+x^2+2}}dx\right)+\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\left(\frac{2}{15}\int \frac{2216x^2+2843}{\sqrt{-x^4+x^2+2}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(669x^2+1087\right)\right)+\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{21}\left(\frac{4}{15}\int \frac{2216x^2+2843}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(669x^2+1087\right)\right)+\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{21}\left(\frac{2}{15}\int \frac{2216x^2+2843}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(669x^2+1087\right)\right)+\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \ 399$$

$$\frac{1}{21}\left(\frac{2}{15}\left(627\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+2216\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(669x^2+1087\right)\right)+\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \ 321$$

$$\frac{1}{21}\left(\frac{2}{15}\left(2216\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+627\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(669x^2+1087\right)\right)+\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \ 327$$

$$\frac{1}{21}\left(\frac{2}{15}\left(627\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+2216E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(669x^2+108\right.\right.$$
$$\frac{1}{63}x\left(35x^2+48\right)\left(-x^4+x^2+2\right)^{3/2}$$

input
```
Int[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

output
```
(x*(48 + 35*x^2)*(2 + x^2 - x^4)^(3/2))/63 + ((x*(1087 + 669*x^2)*Sqrt[2 +
 x^2 - x^4])/15 + (2*(2216*EllipticE[ArcSin[x/Sqrt[2]], -2] + 627*Elliptic
F[ArcSin[x/Sqrt[2]], -2]))/15)/21
```

3.327.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.327.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(75) = 150.

Time = 1.71 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.88

method	result
risch	$\frac{x(175x^6+65x^4-1259x^2-1567)(x^4-x^2-2)}{315\sqrt{-x^4+x^2+2}}+\frac{2843\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{315\sqrt{-x^4+x^2+2}}-\frac{2216\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\right.\right.}{315\sqrt{-x^4+x^2+2}}$
default	$-\frac{13x^5\sqrt{-x^4+x^2+2}}{63}+\frac{1259x^3\sqrt{-x^4+x^2+2}}{315}+\frac{1567x\sqrt{-x^4+x^2+2}}{315}+\frac{2843\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{315\sqrt{-x^4+x^2+2}}-\frac{2216\sqrt{2}\,\sqrt{-}}{}$
elliptic	$-\frac{13x^5\sqrt{-x^4+x^2+2}}{63}+\frac{1259x^3\sqrt{-x^4+x^2+2}}{315}+\frac{1567x\sqrt{-x^4+x^2+2}}{315}+\frac{2843\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{315\sqrt{-x^4+x^2+2}}-\frac{2216\sqrt{2}\,\sqrt{-}}{}$

input
```
int((5*x^2+7)*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*x*(175*x^6+65*x^4-1259*x^2-1567)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+2843
/315*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1
/2*x*2^(1/2),I*2^(1/2))-2216/315*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-
x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/
2),I*2^(1/2)))
```

3.327.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int (7+5x^2)\left(2+x^2-x^4\right)^{3/2}\,dx=\frac{-8864i\,\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2})+11707i\,\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2})-(175\,x^8+65\,x^6-125}{315\,x}$$

input
```
integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(-8864*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 11707*I*sqr
t(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - (175*x^8 + 65*x^6 - 1259*x^4
- 1567*x^2 + 4432)*sqrt(-x^4 + x^2 + 2))/x
```

3.327.6 Sympy [F]

$$\int (7+5x^2)\left(2+x^2-x^4\right)^{3/2}\,dx=\int \left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\cdot\left(5x^2+7\right)\,dx$$

input
```
integrate((5*x**2+7)*(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7), x)
```

3.327.7 Maxima [F]

$$\int (7+5x^2)\left(2+x^2-x^4\right)^{3/2}\,dx=\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)dx$$

input
```
integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

3.327.8 Giac [F]

$$\int (7+5x^2)\left(2+x^2-x^4\right)^{3/2}\,dx=\int \left(-x^4+x^2+2\right)^{\frac{3}{2}}\left(5\,x^2+7\right)dx$$

input
```
integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int (7+5x^2)\left(2+x^2-x^4\right)^{3/2}\,dx=\int \left(5\,x^2+7\right)\,\left(-x^4+x^2+2\right)^{3/2}\,dx$$

input
```
int((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2), x)
```

3.328 $\int (2+x^2-x^4)^{3/2}\,dx$

3.328.1 Optimal result

Integrand size = 14, antiderivative size = 74

$$\int (2+x^2-x^4)^{3/2}\,dx=\frac{1}{35}x(19+3x^2)\sqrt{2+x^2-x^4}+\frac{1}{7}x(2+x^2-x^4)^{3/2}+\frac{34}{35}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{48}{35}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
1/7*x*(-x^4+x^2+2)^(3/2)+34/35*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+48/35*El
lipticF(1/2*x*2^(1/2),I*2^(1/2))+1/35*x*(3*x^2+19)*(-x^4+x^2+2)^(1/2)
```

3.328.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int (2+x^2-x^4)^{3/2}\,dx=\frac{58x+45x^3-31x^5-13x^7+5x^9+34i\sqrt{4+2x^2-2x^4}E\left(i\text{arcsinh}(x)\middle|-\frac{1}{2}\right)-75i\sqrt{4+2x^2-}}{35\sqrt{2+x^2-x^4}}$$

input
```
Integrate[(2 + x^2 - x^4)^(3/2),x]
```

output
```
(58*x + 45*x^3 - 31*x^5 - 13*x^7 + 5*x^9 + (34*I)*Sqrt[4 + 2*x^2 - 2*x^4]*
EllipticE[I*ArcSinh[x], -1/2] - (75*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I
*ArcSinh[x], -1/2])/(35*Sqrt[2 + x^2 - x^4])
```

3.328.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1404, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^4+x^2+2\right)^{3/2} dx$$

$$\Big\downarrow \text{1404}$$

$$\frac{3}{7}\int \left(x^2+4\right)\sqrt{-x^4+x^2+2}dx+\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{3}{7}\left(\frac{1}{15}x\left(3x^2+19\right)\sqrt{-x^4+x^2+2}-\frac{1}{15}\int -\frac{2\left(17x^2+41\right)}{\sqrt{-x^4+x^2+2}}dx\right)+\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}\left(\frac{2}{15}\int \frac{17x^2+41}{\sqrt{-x^4+x^2+2}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(3x^2+19\right)\right)+\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{1494}$$

$$\frac{3}{7}\left(\frac{4}{15}\int \frac{17x^2+41}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(3x^2+19\right)\right)+\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}\left(\frac{2}{15}\int \frac{17x^2+41}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(3x^2+19\right)\right)+\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{399}$$

$$\frac{3}{7}\left(\frac{2}{15}\left(24\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+17\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}\left(3x^2+19\right)\right)+\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{321}$$

$$\frac{3}{7}\left(\frac{2}{15}\left(17\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+24\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}(3x^2+19)\right)+\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{327}$$

$$\frac{3}{7}\left(\frac{2}{15}\left(24\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+17E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)+\frac{1}{15}x\sqrt{-x^4+x^2+2}(3x^2+19)\right)+\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}$$

input
```
Int[(2 + x^2 - x^4)^(3/2),x]
```

output
```
(x*(2 + x^2 - x^4)^(3/2))/7 + (3*((x*(19 + 3*x^2)*Sqrt[2 + x^2 - x^4])/15
+ (2*(17*EllipticE[ArcSin[x/Sqrt[2]], -2] + 24*EllipticF[ArcSin[x/Sqrt[2]]
, -2]))/15))/7
```

3.328.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.328. $\int (2+x^2-x^4)^{3/2}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.328.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(68) = 136$.

Time = 0.68 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99

method	result
risch	$\frac{x\left(5x^4-8x^2-29\right)\left(x^4-x^2-2\right)}{35\sqrt{-x^4+x^2+2}}+\frac{41\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{35\sqrt{-x^4+x^2+2}}-\frac{17\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right.}{35\sqrt{-x^4+x^2+2}}$
default	$-\frac{x^5\sqrt{-x^4+x^2+2}}{7}+\frac{8x^3\sqrt{-x^4+x^2+2}}{35}+\frac{29x\sqrt{-x^4+x^2+2}}{35}+\frac{41\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{35\sqrt{-x^4+x^2+2}}-\frac{17\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2}}{3}$
elliptic	$-\frac{x^5\sqrt{-x^4+x^2+2}}{7}+\frac{8x^3\sqrt{-x^4+x^2+2}}{35}+\frac{29x\sqrt{-x^4+x^2+2}}{35}+\frac{41\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{35\sqrt{-x^4+x^2+2}}-\frac{17\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2}}{3}$

input
```
int((-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x*(5*x^4-8*x^2-29)*(x^4-x^2-2)/(-x^4+x^2+2)^(1/2)+41/35*2^(1/2)*(-2*x
^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))-17/35*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(Ell
ipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int (2+x^2-x^4)^{3/2}\,dx = \frac{-68i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}) + 109i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2}) - (5x^6-8x^4-29x^2+34)}{35x}$$

input
```
integrate((-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/35*(-68*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 109*I*sqrt(2)*
x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - (5*x^6 - 8*x^4 - 29*x^2 + 34)*sqrt
(-x^4 + x^2 + 2))/x
```

3.328.6 Sympy [F]

$$\int (2+x^2-x^4)^{3/2}\,dx = \int (-x^4+x^2+2)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x**4+x**2+2)**(3/2),x)
```

output
```
Integral((-x**4 + x**2 + 2)**(3/2), x)
```

3.328.7 Maxima [F]

$$\int \left(2+x^2-x^4\right)^{3/2} dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} dx$$

input
```
integrate((-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2), x)
```

3.328.8 Giac [F]

$$\int \left(2+x^2-x^4\right)^{3/2} dx = \int \left(-x^4+x^2+2\right)^{\frac{3}{2}} dx$$

input
```
integrate((-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2), x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \left(2+x^2-x^4\right)^{3/2} dx = \int \left(-x^4+x^2+2\right)^{3/2} dx$$

input
```
int((x^2 - x^4 + 2)^(3/2),x)
```

output
```
int((x^2 - x^4 + 2)^(3/2), x)
```

3.329 $\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2} dx$

3.329.1 Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2} dx = \frac{1}{75}x(13-3x^2)\sqrt{2+x^2-x^4} + \frac{92}{375}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{178}{625}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) + \frac{1156\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

output
```
92/375*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-178/625*EllipticF(1/2*x*2^(1/2),
I*2^(1/2))+1156/4375*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/75*x*(-3*
x^2+13)*(-x^4+x^2+2)^(1/2)
```

3.329.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.81

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2} dx = \frac{4550x+1225x^3-2800x^5+525x^7+3220i\sqrt{4+2x^2-2x^4}E\left(i\text{arcsinh}(x)\middle|-\frac{1}{2}\right)-}{}$$

input
```
Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

output
```
(4550*x + 1225*x^3 - 2800*x^5 + 525*x^7 + (3220*I)*Sqrt[4 + 2*x^2 - 2*x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (2961*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Elliptic
F[I*ArcSinh[x], -1/2] - (1734*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticPi[5/7, I
*ArcSinh[x], -1/2])/(13125*Sqrt[2 + x^2 - x^4])
```

3.329.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {1532, 1536, 27, 412, 2207, 27, 2207, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-x^4+x^2+2\right)^{3/2}}{5x^2+7}dx$$

$$\Big\downarrow \text{1532}$$

$$\frac{1156}{625}\int \frac{1}{\left(5x^2+7\right)\sqrt{-x^4+x^2+2}}dx+\frac{1}{625}\int \frac{125x^6-425x^4+220x^2+192}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{1536}$$

$$\frac{2312}{625}\int \frac{1}{2\sqrt{2-x^2}\sqrt{x^2+1}\left(5x^2+7\right)}dx+\frac{1}{625}\int \frac{125x^6-425x^4+220x^2+192}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1156}{625}\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}\left(5x^2+7\right)}dx+\frac{1}{625}\int \frac{125x^6-425x^4+220x^2+192}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{625}\int \frac{125x^6-425x^4+220x^2+192}{\sqrt{-x^4+x^2+2}}dx+\frac{1156\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{625}\left(-\frac{1}{5}\int -\frac{5\left(-325x^4+370x^2+192\right)}{\sqrt{-x^4+x^2+2}}dx-25\sqrt{-x^4+x^2+2}x^3\right)+\frac{1156\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{625}\left(\int \frac{-325x^4+370x^2+192}{\sqrt{-x^4+x^2+2}}dx-25x^3\sqrt{-x^4+x^2+2}\right)+\frac{1156\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{1}{625}\left(-\frac{1}{3}\int \frac{2(37-230x^2)}{\sqrt{-x^4+x^2+2}}dx+\frac{325}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3\right)+\frac{1156\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{625}\left(-\frac{2}{3}\int \frac{37-230x^2}{\sqrt{-x^4+x^2+2}}dx+\frac{325}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3\right)+\frac{1156\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

$$\Big\downarrow \text{ 1494}$$

$$\frac{1}{625}\left(-\frac{4}{3}\int \frac{37-230x^2}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{325}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3\right)+\frac{1156\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{625}\left(-\frac{2}{3}\int \frac{37-230x^2}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{325}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3\right)+\frac{1156\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{1}{625}\left(-\frac{2}{3}\left(267\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-230\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{325}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3\right)+\frac{1156\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

$$\Big\downarrow \text{ 321}$$

3.329. $\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx$

$$\frac{1}{625}\left(-\frac{2}{3}\left(267\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-230\int\frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{325}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}\right.$$
$$\frac{1156\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

$\downarrow$ 327

$$\frac{1}{625}\left(-\frac{2}{3}\left(267\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-230E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)+\frac{325}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4}\right.$$
$$\frac{1156\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{4375}$$

input
```
Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

output
```
((325*x*Sqrt[2 + x^2 - x^4])/3 - 25*x^3*Sqrt[2 + x^2 - x^4] - (2*(-230*Ell
ipticE[ArcSin[x/Sqrt[2]], -2] + 267*EllipticF[ArcSin[x/Sqrt[2]], -2]))/3)/
625 + (1156*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/4375
```

3.329.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.329. $\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1532
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/e^(2*p + 1)   Int[1/((d + e*x
^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[1/e^(2*p + 1)   Int[(1/Sqrt[a +
 b*x^2 + c*x^4])*ExpandToSum[(e^(2*p + 1)*(a + b*x^2 + c*x^4)^(p + 1/2) - (
c*d^2 - b*d*e + a*e^2)^(p + 1/2))/(d + e*x^2), x], x], x] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ
[p - 1/2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

rule 1536
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/((d + e*x^
2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[
c, 0]
```

3.329. $\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2} \, dx$

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.329.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(69) = 138.

Time = 2.00 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.40

method	result
default	$-\frac{x^{3}\sqrt{-x^{4}+x^{2}+2}}{25}+\frac{13x\sqrt{-x^{4}+x^{2}+2}}{75}-\frac{89\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{625\sqrt{-x^{4}+x^{2}+2}}+\frac{46\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{375\sqrt{-x^{4}+x^{2}+2}}+$
elliptic	$-\frac{x^{3}\sqrt{-x^{4}+x^{2}+2}}{25}+\frac{13x\sqrt{-x^{4}+x^{2}+2}}{75}-\frac{89\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{625\sqrt{-x^{4}+x^{2}+2}}+\frac{46\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{375\sqrt{-x^{4}+x^{2}+2}}+$
risch	$\frac{x(3x^{2}-13)(x^{4}-x^{2}-2)}{75\sqrt{-x^{4}+x^{2}+2}}-\frac{37\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{1875\sqrt{-x^{4}+x^{2}+2}}-\frac{46\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{375\sqrt{-x^{4}+x^{2}+2}}+$

input
```
int((-x^4+x^2+2)^(3/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

output
```
-1/25*x^3*(-x^4+x^2+2)^(1/2)+13/75*x*(-x^4+x^2+2)^(1/2)-89/625*2^(1/2)*(-2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2
^(1/2))+46/375*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*E
llipticE(1/2*x*2^(1/2),I*2^(1/2))+1156/4375*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2
+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.329.5 Fricas [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

input
```
integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

output
```
integral((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.329.6 Sympy [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(-(x^2-2)\,(x^2+1))^{\frac{3}{2}}}{5x^2+7}\,dx$$

input
```
integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7),x)
```

output
```
Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7), x)
```

3.329.7 Maxima [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

input
```
integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.329.8 Giac [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx=\int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

input
```
integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx=\int \frac{(-x^4+x^2+2)^{3/2}}{5\,x^2+7}\,dx$$

input
```
int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7),x)
```

output
```
int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.330 $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx$

3.330.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx = -\frac{1}{75}x\sqrt{2+x^2-x^4}-\frac{17x\sqrt{2+x^2-x^4}}{175\,(7+5x^2)} - \frac{97}{525}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{458}{875}\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) - \frac{1241\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{6125}$$

output
```
-97/525*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+458/875*EllipticF(1/2*x*2^(1/2)
,I*2^(1/2))-1241/6125*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-1/75*x*(-x
^4+x^2+2)^(1/2)-17/175*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

3.330.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.32 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.16

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx = \frac{-14000x-11900x^3+4550x^5+2450x^7-6790i\sqrt{2}(7+5x^2)\sqrt{2+x^2-x^4}E(i\text{arc}}{}$$

input
```
Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

output
```
(-14000*x - 11900*x^3 + 4550*x^5 + 2450*x^7 - (6790*I)*Sqrt[2]*(7 + 5*x^2)
*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (567*I)*Sqrt[2]*(7 +
5*x^2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (26061*I)*Sqrt[
2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (18615*I)*Sqr
t[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(36750*(
7 + 5*x^2)*Sqrt[2 + x^2 - x^4])
```

3.330.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-x^4+x^2+2)^{3/2}}{(5x^2+7)^2}\,dx$$

$$\Big\downarrow \text{1556}$$

$$\int \left(\frac{x^4}{25\sqrt{-x^4+x^2+2}}-\frac{24x^2}{125\sqrt{-x^4+x^2+2}}-\frac{1292}{625\,(5x^2+7)\sqrt{-x^4+x^2+2}}+\frac{1156}{625\,(5x^2+7)^2\sqrt{-x^4+x^2+2}}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{458}{875}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-\frac{97}{525}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1241\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{6125}-\frac{17\sqrt{-x^4+x^2+2}\,x}{175\,(5x^2+7)}-\frac{1}{75}\sqrt{-x^4+x^2+2}\,x$$

input
```
Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

output
```
-1/75*(x*Sqrt[2 + x^2 - x^4]) - (17*x*Sqrt[2 + x^2 - x^4])/(175*(7 + 5*x^2
)) - (97*EllipticE[ArcSin[x/Sqrt[2]], -2])/525 + (458*EllipticF[ArcSin[x/S
qrt[2]], -2])/875 - (1241*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/6125
```

3.330.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.330.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(86) = 172.

Time = 2.89 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.94

method	result
default	$-\frac{17x\sqrt{-x^4+x^2+2}}{175(5x^2+7)}-\frac{x\sqrt{-x^4+x^2+2}}{75}+\frac{229\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{875\sqrt{-x^4+x^2+2}}-\frac{97\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{1050\sqrt{-x^4+x^2+2}}-$
elliptic	$-\frac{17x\sqrt{-x^4+x^2+2}}{175(5x^2+7)}-\frac{x\sqrt{-x^4+x^2+2}}{75}+\frac{229\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{875\sqrt{-x^4+x^2+2}}-\frac{97\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{1050\sqrt{-x^4+x^2+2}}-$
risch	$\frac{(x^4-x^2-2)x(7x^2+20)}{105(5x^2+7)\sqrt{-x^4+x^2+2}}+\frac{127\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{750\sqrt{-x^4+x^2+2}}+\frac{97\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{1050\sqrt{-x^4+x^2+2}}$

input
```
int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

output
```
-17/175*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/75*x*(-x^4+x^2+2)^(1/2)+229/875*2
^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2
^(1/2),I*2^(1/2))-97/1050*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2
+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1241/6125*2^(1/2)*(1-1/2*x^2)
^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2
^(1/2))
```

3.330.5 Fricas [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

output
```
integral((-x^4 + x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

3.330.6 Sympy [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{(-(x^2-2)\,(x^2+1))^{\frac{3}{2}}}{(5x^2+7)^2}\,dx$$

input
```
integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**2,x)
```

output
```
Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**2, x)
```

3.330.7 Maxima [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.330.8 Giac [F]

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}{\left(5\,x^2+7\right)^2}\,dx$$

input
```
integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx=\int \frac{\left(-x^4+x^2+2\right)^{3/2}}{\left(5\,x^2+7\right)^2}\,dx$$

input
```
int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^2,x)
```

output
```
int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.331 $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx$

3.331.1 Optimal result

Integrand size = 24, antiderivative size = 102

$$\begin{aligned}\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = &-\frac{17x\sqrt{2+x^2-x^4}}{350\,(7+5x^2)^2}+\frac{563x\sqrt{2+x^2-x^4}}{9800\,(7+5x^2)} \\ &+\frac{191E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}-\frac{1251\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{24500} \\ &+\frac{9879\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{343000}\end{aligned}$$

output
```
191/9800*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1251/24500*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))+9879/343000*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-17/3
50*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+563/9800*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7
)
```

3.331.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.39 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.39

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \frac{485100x+636650x^3-45500x^5-197050x^7+13370i\sqrt{2}(7+5x^2)^2\sqrt{2+x^2-x^4}E}{}$$

input
```
Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

output
```
(485100*x + 636650*x^3 - 45500*x^5 - 197050*x^7 + (13370*I)*Sqrt[2]*(7 + 5
*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2541*I)*Sqrt[
2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (4840
71*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (6
91530*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/
2] - (246975*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[
x], -1/2])/(686000*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

3.331.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-x^4+x^2+2\right)^{3/2}}{\left(5x^2+7\right)^3}dx$$

$$\Big\downarrow \text{1556}$$

$$\int \left(\frac{x^2}{125\sqrt{-x^4+x^2+2}}+\frac{429}{625\left(5x^2+7\right)\sqrt{-x^4+x^2+2}}-\frac{1292}{625\left(5x^2+7\right)^2\sqrt{-x^4+x^2+2}}+\frac{1156}{625\left(5x^2+7\right)^3\sqrt{-x^4\right.}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1251\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{24500}+\frac{191E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}+\frac{9879\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{343000}+\frac{563\sqrt{-x^4+x^2+2}x}{9800\left(5x^2+7\right)}-\frac{17\sqrt{-x^4+x^2+2}x}{350\left(5x^2+7\right)^2}$$

input
```
Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

output
```
(-17*x*Sqrt[2 + x^2 - x^4])/(350*(7 + 5*x^2)^2) + (563*x*Sqrt[2 + x^2 - x^
4])/(9800*(7 + 5*x^2)) + (191*EllipticE[ArcSin[x/Sqrt[2]], -2])/9800 - (12
51*EllipticF[ArcSin[x/Sqrt[2]], -2])/24500 + (9879*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/343000
```

3.331. $\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5x^2\right)^3}dx$

3.331.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.331.4 Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.85

method	result
default	$-\frac{17x\sqrt{-x^4+x^2+2}}{350(5x^2+7)^2}+\frac{563x\sqrt{-x^4+x^2+2}}{9800(5x^2+7)}-\frac{1251\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{49000\sqrt{-x^4+x^2+2}}+\frac{191\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{19600\sqrt{-x^4+x^2+2}}$
elliptic	$-\frac{17x\sqrt{-x^4+x^2+2}}{350(5x^2+7)^2}+\frac{563x\sqrt{-x^4+x^2+2}}{9800(5x^2+7)}-\frac{1251\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{49000\sqrt{-x^4+x^2+2}}+\frac{191\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{19600\sqrt{-x^4+x^2+2}}$
risch	$-\frac{(x^4-x^2-2)x(563x^2+693)}{1960(5x^2+7)^2\sqrt{-x^4+x^2+2}}-\frac{221\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{14000\sqrt{-x^4+x^2+2}}-\frac{191\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\right.\right.}{19600\sqrt{-x^4+x^2+2}}$

input
```
int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

output
```
-17/350*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+563/9800*x*(-x^4+x^2+2)^(1/2)/(5*
x^2+7)-1251/49000*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2
)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+191/19600*2^(1/2)*(-2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+9879/343
000*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(
1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.331.5 Fricas [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

output
```
integral((-x^4 + x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

3.331.6 Sympy [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(-(x^2-2)\,(x^2+1))^{\frac{3}{2}}}{(5x^2+7)^3}\,dx$$

input
```
integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**3,x)
```

output
```
Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**3, x)
```

3.331.7 Maxima [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.331.8 Giac [F]

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(-x^4+x^2+2)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
```

output
```
integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(-x^4+x^2+2)^{3/2}}{(5\,x^2+7)^3}\,dx$$

input
```
int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^3,x)
```

output
```
int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.332 $\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx$

3.332.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4} +\frac{3905}{3}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-542\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
3905/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-542*EllipticF(1/2*x*2^(1/2),I*2^
(1/2))-625/3*x*(-x^4+x^2+2)^(1/2)-25*x^3*(-x^4+x^2+2)^(1/2)
```

3.332.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = \frac{-2500x-1550x^3+1100x^5+150x^7+7810i\sqrt{4+2x^2-2x^4}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right)-10089i\sqrt{4+2x^2-2x}}{6\sqrt{2+x^2-x^4}}$$

input
```
Integrate[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```

output
```
(-2500*x - 1550*x^3 + 1100*x^5 + 150*x^7 + (7810*I)*Sqrt[4 + 2*x^2 - 2*x^4
]*EllipticE[I*ArcSinh[x], -1/2] - (10089*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Ellipt
icF[I*ArcSinh[x], -1/2])/(6*Sqrt[2 + x^2 - x^4])
```

3.332.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1518, 27, 2207, 25, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^3}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{1518}$$

$$-\frac{1}{5}\int -\frac{5(625x^4+885x^2+343)}{\sqrt{-x^4+x^2+2}}dx-25\sqrt{-x^4+x^2+2}x^3$$

$$\Big\downarrow \text{27}$$

$$\int \frac{625x^4+885x^2+343}{\sqrt{-x^4+x^2+2}}dx-25x^3\sqrt{-x^4+x^2+2}$$

$$\Big\downarrow \text{2207}$$

$$-\frac{1}{3}\int -\frac{3905x^2+2279}{\sqrt{-x^4+x^2+2}}dx-\frac{625}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{3905x^2+2279}{\sqrt{-x^4+x^2+2}}dx-\frac{625}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3$$

$$\Big\downarrow \text{1494}$$

$$\frac{2}{3}\int \frac{3905x^2+2279}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx-\frac{625}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3905x^2+2279}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-\frac{625}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{3}\left(3905\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-1626\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)-\frac{625}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{3}\left(3905\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-1626\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)-\frac{625}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{3}\left(3905E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-1626\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)-\frac{625}{3}\sqrt{-x^4+x^2+2}x-25\sqrt{-x^4+x^2+2}x^3$$

input
```
Int[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```

output
```
(-625*x*Sqrt[2 + x^2 - x^4])/3 - 25*x^3*Sqrt[2 + x^2 - x^4] + (3905*Ellipt
icE[ArcSin[x/Sqrt[2]], -2] - 1626*EllipticF[ArcSin[x/Sqrt[2]], -2])/3
```

3.332.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.332.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(63) = 126$.

Time = 5.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.18

method	result
default	$\frac{2279\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}} - 25x^3\sqrt{-x^4+x^2+2} - \frac{625x\sqrt{-x^4+x^2+2}}{3} - \frac{3905\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\right.\right.}{6\sqrt{-x^4+}}$
risch	$\frac{25x(3x^2+25)(x^4-x^2-2)}{3\sqrt{-x^4+x^2+2}} + \frac{2279\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}} - \frac{3905\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right.\right.}{6\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{2279\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}} - 25x^3\sqrt{-x^4+x^2+2} - \frac{625x\sqrt{-x^4+x^2+2}}{3} - \frac{3905\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\right.\right.}{6\sqrt{-x^4+}}$

input
```
int((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2279/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF
(1/2*x*2^(1/2),I*2^(1/2))-25*x^3*(-x^4+x^2+2)^(1/2)-625/3*x*(-x^4+x^2+2)^(
1/2)-3905/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(Ell
ipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = \frac{-15620i\,\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2}) + 17899i\,\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2}) - 10\,(15\,x^4+125\,x^2+781)\sqrt{-x^4}}{6\,x}$$

input
```
integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(-15620*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 17899*I*sqrt
(2)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - 10*(15*x^4 + 125*x^2 + 781)*sq
rt(-x^4 + x^2 + 2))/x
```

3.332.6 Sympy [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5x^2+7)^3}{\sqrt{-(x^2-2)(x^2+1)}}\,dx$$

input
```
integrate((5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
```

output
```
Integral((5*x**2 + 7)**3/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

3.332.7 Maxima [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

3.332.8 Giac [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(1/2), x)
```

3.333 $\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx$

3.333.1 Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = -\frac{25}{3}x\sqrt{2+x^2-x^4}+\frac{260}{3}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - 21\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
260/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-21*EllipticF(1/2*x*2^(1/2),I*2^(1
/2))-25/3*x*(-x^4+x^2+2)^(1/2)
```

3.333.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = \frac{-100x-50x^3+50x^5+520i\sqrt{4+2x^2-2x^4}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right)-717i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left(i\text{arcsi}\right.}{6\sqrt{2+x^2-x^4}}$$

input
```
Integrate[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```

output
```
(-100*x - 50*x^3 + 50*x^5 + (520*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*Ar
cSinh[x], -1/2] - (717*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x],
-1/2])/(6*Sqrt[2 + x^2 - x^4])
```

3.333.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1518, 25, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^2}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{1518}$$

$$-\frac{1}{3}\int -\frac{260x^2+197}{\sqrt{-x^4+x^2+2}}dx-\frac{25}{3}\sqrt{-x^4+x^2+2}x$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{260x^2+197}{\sqrt{-x^4+x^2+2}}dx-\frac{25}{3}x\sqrt{-x^4+x^2+2}$$

$$\Big\downarrow \text{1494}$$

$$\frac{2}{3}\int \frac{260x^2+197}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx-\frac{25}{3}x\sqrt{-x^4+x^2+2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{260x^2+197}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-\frac{25}{3}x\sqrt{-x^4+x^2+2}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{3}\left(260\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-63\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)-\frac{25}{3}x\sqrt{-x^4+x^2+2}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{3}\left(260\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx-63\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)-\frac{25}{3}x\sqrt{-x^4+x^2+2}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{3}\left(260E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-63\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)-\frac{25}{3}x\sqrt{-x^4+x^2+2}$$

input
```
Int[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```

output
```
(-25*x*Sqrt[2 + x^2 - x^4])/3 + (260*EllipticE[ArcSin[x/Sqrt[2]], -2] - 63
*EllipticF[ArcSin[x/Sqrt[2]], -2])/3
```

3.333.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.333.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(46) = 92.

Time = 1.95 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.72

method	result	size
default	$\frac{197\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{25x\sqrt{-x^4+x^2+2}}{3}-\frac{130\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{3\sqrt{-x^4+x^2+2}}$	125
elliptic	$\frac{197\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{25x\sqrt{-x^4+x^2+2}}{3}-\frac{130\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{3\sqrt{-x^4+x^2+2}}$	125
risch	$\frac{25x\left(x^4-x^2-2\right)}{3\sqrt{-x^4+x^2+2}}+\frac{197\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{130\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{3\sqrt{-x^4+x^2+2}}$	135

input
```
int((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
197/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(
1/2*x*2^(1/2),I*2^(1/2))-25/3*x*(-x^4+x^2+2)^(1/2)-130/3*2^(1/2)*(-2*x^2+4
)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2
))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = \frac{-1040i\sqrt{2}xE(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid -\frac{1}{2}) + 1237i\sqrt{2}xF(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid -\frac{1}{2}) - 10\sqrt{-x^4+x^2+2}(5x^2+52)}{6x}$$

input
```
integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(-1040*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 1237*I*sqrt(2
)*x*elliptic_f(arcsin(sqrt(2)/x), -1/2) - 10*sqrt(-x^4 + x^2 + 2)*(5*x^2 +
 52))/x
```

3.333.6 Sympy [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5x^2+7)^2}{\sqrt{-(x^2-2)(x^2+1)}}\,dx$$

input
```
integrate((5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

output
```
Integral((5*x**2 + 7)**2/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

3.333.7 Maxima [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

3.333.8 Giac [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(1/2), x)
```

3.334 $\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx$

3.334.1 Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = 5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+2\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
5*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+2*EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

3.334.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = \frac{i\left(10E\left(i\text{arcsinh}(x)\middle|-\frac{1}{2}\right)-17\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)\right)}{\sqrt{2}}$$

input
```
Integrate[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
```

output
```
(I*(10*EllipticE[I*ArcSinh[x], -1/2] - 17*EllipticF[I*ArcSinh[x], -1/2]))/
Sqrt[2]
```

3.334.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+7}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{ 1494}$$

$$2\int \frac{5x^2+7}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{5x^2+7}{\sqrt{2-x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 399}$$

$$2\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+5\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx$$

$$\Big\downarrow \text{ 321}$$

$$5\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+2\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

$$\Big\downarrow \text{ 327}$$

$$2\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

input
```
Int[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
```

output
```
5*EllipticE[ArcSin[x/Sqrt[2]], -2] + 2*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

3.334.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.334.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(31) = 62$.

Time = 0.74 (sec) , antiderivative size = 110, normalized size of antiderivative = 4.40

method	result	size
default	$\frac{7\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{2\sqrt{-x^{4}+x^{2}+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^{4}+x^{2}+2}}$	110
elliptic	$\frac{7\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{2\sqrt{-x^{4}+x^{2}+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^{4}+x^{2}+2}}$	110

input
```
int((5*x^2+7)/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
7/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/
2*x*2^(1/2),I*2^(1/2))-5/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^
2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2
^(1/2)))
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.08

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = \frac{-20i\sqrt{2}xE\left(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid -\frac{1}{2}\right)+27i\sqrt{2}xF\left(\arcsin\left(\frac{\sqrt{2}}{x}\right)\mid -\frac{1}{2}\right)-10\sqrt{-x^4+x^2+2}}{2x}$$

input
```
integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-20*I*sqrt(2)*x*elliptic_e(arcsin(sqrt(2)/x), -1/2) + 27*I*sqrt(2)*x*
elliptic_f(arcsin(sqrt(2)/x), -1/2) - 10*sqrt(-x^4 + x^2 + 2))/x
```

3.334.6 Sympy [F]

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{5x^2+7}{\sqrt{-(x^2-2)(x^2+1)}}\,dx$$

input
```
integrate((5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
```

output
```
Integral((5*x**2 + 7)/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

3.334.7 Maxima [F]

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

3.334.8 Giac [F]

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
int((5*x^2 + 7)/(x^2 - x^4 + 2)^(1/2),x)
```

output
```
int((5*x^2 + 7)/(x^2 - x^4 + 2)^(1/2), x)
```

3.335 $\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx$

3.335.1 Optimal result

Integrand size = 14, antiderivative size = 10

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = \text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

3.335.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = -\frac{i\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/Sqrt[2 + x^2 - x^4],x]
```

output
```
((-I)*EllipticF[I*ArcSinh[x], -1/2])/Sqrt[2]
```

3.335.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^4+x^2+2}}dx$$
$$\Big\downarrow \text{1408}$$
$$2\int \frac{1}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx$$
$$\Big\downarrow \text{321}$$
$$\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

input
```
Int[1/Sqrt[2 + x^2 - x^4],x]
```

output
```
EllipticF[ArcSin[x/Sqrt[2]], -2]
```

3.335.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

3.335.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(13) = 26$.

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 4.70

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{2\sqrt{-x^4+x^2+2}}$	47
elliptic	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{2\sqrt{-x^4+x^2+2}}$	47

input
```
int(1/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/
2*x*2^(1/2),I*2^(1/2))
```

3.335.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = F(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\mid -2)$$

input
```
integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
elliptic_f(arcsin(1/2*sqrt(2)*x), -2)
```

3.335.6 Sympy [F]

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
integrate(1/(-x**4+x**2+2)**(1/2),x)
```

output
```
Integral(1/sqrt(-x**4 + x**2 + 2), x)
```

3.335.7 Maxima [F]

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

3.335.8 Giac [F]

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx=\int \frac{1}{\sqrt{-x^4+x^2+2}}\,dx$$

input
```
int(1/(x^2 - x^4 + 2)^(1/2),x)
```

output
```
int(1/(x^2 - x^4 + 2)^(1/2), x)
```

3.336 $\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx$

3.336.1 Optimal result

Integrand size = 24, antiderivative size = 17

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx = \frac{1}{7}\text{EllipticPi}\left(-\frac{10}{7}, \arcsin\left(\frac{x}{\sqrt{2}}\right), -2\right)$$

output
```
1/7*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.336.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}} dx = -\frac{i\,\text{EllipticPi}\left(\frac{5}{7}, i\text{arcsinh}(x), -\frac{1}{2}\right)}{7\sqrt{2}}$$

input
```
Integrate[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

output
```
((-1/7*I)*EllipticPi[5/7, I*ArcSinh[x], -1/2])/Sqrt[2]
```

3.336.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1536, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{ 1536}$$

$$2\int \frac{1}{2\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx$$

$$\Big\downarrow \text{ 412}$$

$$\frac{1}{7}\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

input
```
Int[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

output
```
EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2]/7
```

3.336.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1536
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/((d + e*x^
2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[
c, 0]
```

3.336.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(16) = 32$.

Time = 0.91 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\frac{10}{7},i\sqrt{2}\right)}{7\sqrt{-x^4+x^2+2}}$	48
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2},-\frac{10}{7},i\sqrt{2}\right)}{7\sqrt{-x^4+x^2+2}}$	48

input
```
int(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(
1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.336.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + x^2 + 2)/(5*x^6 + 2*x^4 - 17*x^2 - 14), x)
```

3.336.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-(x^2-2)(x^2+1)}(5x^2+7)}\,dx$$

input
```
integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)), x)
```

3.336.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

3.336.8 Giac [F]

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{(5\,x^2+7)\,\sqrt{-x^4+x^2+2}}\,dx$$

input
```
int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2)),x)
```

output
```
int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2)), x)
```

3.337 $\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$

3.337.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx = -\frac{25x\sqrt{2+x^2-x^4}}{476(7+5x^2)} - \frac{5}{476}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{1}{238}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) + \frac{167\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{3332}$$

output
```
-5/476*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1/238*EllipticF(1/2*x*2^(1/2),I*
2^(1/2))+167/3332*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-25/476*x*(-x^4
+x^2+2)^(1/2)/(5*x^2+7)
```

3.337.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.32 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.65

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{-700x-350x^3+350x^5-70i\sqrt{2}(7+5x^2)\sqrt{2+x^2-x^4}E\left(i\text{arcsinh}(x)\middle|-\frac{1}{2}\right)+119i\sqrt{2}(7+5x^2)\sqrt{2+x^{\cdots}}}{}$$

input
```
Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

output
```
(-700*x - 350*x^3 + 350*x^5 - (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^
4]*EllipticE[I*ArcSinh[x], -1/2] + (119*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^
2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (1169*I)*Sqrt[2]*Sqrt[2 + x^2 - x
^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (835*I)*Sqrt[2]*x^2*Sqrt[2 + x^2
 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(6664*(7 + 5*x^2)*Sqrt[2 + x^
2 - x^4])
```

3.337.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1551, 2234, 27, 1494, 27, 399, 321, 327, 1536, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^2\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{1551}$$

$$\frac{1}{476}\int \frac{-25x^4-70x^2+118}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-\frac{25x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}$$

$$\Big\downarrow \text{2234}$$

$$\frac{1}{476}\left(167\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-\frac{1}{25}\int \frac{25(5x^2+7)}{\sqrt{-x^4+x^2+2}}dx\right)-\frac{25x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{476}\left(167\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-\int \frac{5x^2+7}{\sqrt{-x^4+x^2+2}}dx\right)-\frac{25x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{476}\left(167\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-2\int \frac{5x^2+7}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)-\frac{25x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{476}\left(167\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-\int \frac{5x^2+7}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)-\frac{25x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{476}\left(-2\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-5\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+167\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx\right)-\frac{25x\sqrt{-x^4+x^2+2}}{476\,(5x^2+7)}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{476}\left(-5\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+167\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-2\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)-\frac{25x\sqrt{-x^4+x^2+2}}{476\,(5x^2+7)}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{476}\left(167\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-2\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)-\frac{25x\sqrt{-x^4+x^2+2}}{476\,(5x^2+7)}$$

$$\Big\downarrow \text{1536}$$

$$\frac{1}{476}\left(334\int \frac{1}{2\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx-2\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)-\frac{25x\sqrt{-x^4+x^2+2}}{476\,(5x^2+7)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{476}\left(167\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx-2\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)-\frac{25x\sqrt{-x^4+x^2+2}}{476\,(5x^2+7)}$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{476}\left(-2\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-5E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{167}{7}\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)-\frac{25x\sqrt{-x^4+x^2+2}}{476\,(5x^2+7)}$$

input
```
Int[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

3.337. $\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$

output
```
(-25*x*Sqrt[2 + x^2 - x^4])/(476*(7 + 5*x^2)) + (-5*EllipticE[ArcSin[x/Sqr
t[2]], -2] - 2*EllipticF[ArcSin[x/Sqrt[2]], -2] + (167*EllipticPi[-10/7, A
rcSin[x/Sqrt[2]], -2])/7)/476
```

3.337.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.337. $\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$

rule 1536
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/((d + e*x^
2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[
c, 0]
```

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.337.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(71) = 142$.

Time = 2.95 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.23

method	result
default	$-\frac{25x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{476\sqrt{-x^4+x^2+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{952\sqrt{-x^4+x^2+2}}+\frac{167\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi}{3332\sqrt{-x^4+}}$
elliptic	$-\frac{25x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{476\sqrt{-x^4+x^2+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{952\sqrt{-x^4+x^2+2}}+\frac{167\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi}{3332\sqrt{-x^4+}}$
risch	$\frac{25(x^4-x^2-2)x}{476(5x^2+7)\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{136\sqrt{-x^4+x^2+2}}+\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{952\sqrt{-x^4+x^2+2}}+$

input
```
int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-25/476*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/476*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-5/952*2^(1
/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1
/2),I*2^(1/2))+167/3332*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+
2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.337.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + x^2 + 2)/(25*x^8 + 45*x^6 - 71*x^4 - 189*x^2 - 98),
x)
```

3.337.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-(x^2-2)(x^2+1)}(5x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2), x)
```

3.337.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

3.337.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^2 \sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^2 \sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{(5\,x^2+7)^2\,\sqrt{-x^4+x^2+2}}\,dx$$

input
```
int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2)),x)
```

output
```
int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2)), x)
```

3.338 $\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx$

3.338.1 Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx = -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2} - \frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)} - \frac{2505E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152} - \frac{263\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{226576} + \frac{58915\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{3172064}$$

output
```
-2505/453152*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-263/226576*EllipticF(1/2*x
*2^(1/2),I*2^(1/2))+58915/3172064*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2)
)-25/952*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-12525/453152*x*(-x^4+x^2+2)^(1/2
)/(5*x^2+7)
```

3.338.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{\frac{350x(-7966-8993x^2+1478x^4+2505x^6)}{(7+5x^2)^2\sqrt{2+x^2-x^4}} - 35070i\sqrt{2}E\left(i\operatorname{arcsinh}(x)\left|-\frac{1}{2}\right.\right) + 56287i\sqrt{2}\operatorname{EllipticF}\left(i\operatorname{arcsinh}(x),-\frac{1}{2}\right) -}{6344128}$$

input
```
Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]
```

output
```
((350*x*(-7966 - 8993*x^2 + 1478*x^4 + 2505*x^6))/((7 + 5*x^2)^2*Sqrt[2 +
x^2 - x^4]) - (35070*I)*Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] + (56287*I)*
Sqrt[2]*EllipticF[I*ArcSinh[x], -1/2] - (58915*I)*Sqrt[2]*EllipticPi[5/7,
I*ArcSinh[x], -1/2])/6344128
```

3.338.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1551, 2210, 2234, 27, 1494, 27, 399, 321, 327, 1536, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^3\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{1551}$$

$$\frac{1}{952}\int \frac{25x^4-190x^2+186}{(5x^2+7)^2\sqrt{-x^4+x^2+2}}dx - \frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}$$

$$\Big\downarrow \text{2210}$$

$$\frac{1}{952}\left(\frac{1}{476}\int \frac{-12525x^4-32690x^2+37698}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx - \frac{12525x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}\right) - \frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}$$

$$\Big\downarrow \text{2234}$$

$$\frac{1}{952}\left(\frac{1}{476}\left(58915\int\frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-\frac{1}{25}\int\frac{25(2505x^2+3031)}{\sqrt{-x^4+x^2+2}}dx\right)-\frac{12525x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}\right)-\frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}$$

$$\downarrow 27$$

$$\frac{1}{952}\left(\frac{1}{476}\left(58915\int\frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-\int\frac{2505x^2+3031}{\sqrt{-x^4+x^2+2}}dx\right)-\frac{12525x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}\right)-\frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}$$

$$\downarrow 1494$$

$$\frac{1}{952}\left(\frac{1}{476}\left(58915\int\frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-2\int\frac{2505x^2+3031}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)-\frac{12525x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}\right)-\frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}$$

$$\downarrow 27$$

$$\frac{1}{952}\left(\frac{1}{476}\left(58915\int\frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-\int\frac{2505x^2+3031}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)-\frac{12525x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}\right)-\frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}$$

$$\downarrow 399$$

$$\frac{1}{952}\left(\frac{1}{476}\left(-526\int\frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-2505\int\frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+58915\int\frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx\right)-\frac{12525}{4}\right.$$
$$\frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}$$

$$\downarrow 321$$

$$\frac{1}{952}\left(\frac{1}{476}\left(-2505\int\frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+58915\int\frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx-526\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)\right.$$
$$\frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}$$

$$\downarrow 327$$

3.338. $\int\frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx$

$$\frac{1}{952}\left(\frac{1}{476}\left(58915\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx - 526\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) - 2505E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\right| \right.\right.$$
$$\frac{25x\sqrt{-x^4+x^2+2}}{952\,(5x^2+7)^2}$$

$\downarrow$ 1536

$$\frac{1}{952}\left(\frac{1}{476}\left(117830\int \frac{1}{2\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx - 526\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) - 2505E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right.\right.\right.\right.$$
$$\frac{25x\sqrt{-x^4+x^2+2}}{952\,(5x^2+7)^2}$$

$\downarrow$ 27

$$\frac{1}{952}\left(\frac{1}{476}\left(58915\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx - 526\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) - 2505E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\right| \right.\right.$$
$$\frac{25x\sqrt{-x^4+x^2+2}}{952\,(5x^2+7)^2}$$

$\downarrow$ 412

$$\frac{1}{952}\left(\frac{1}{476}\left(-526\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) - 2505E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\right|-2\right) + \frac{58915}{7}\text{EllipticPi}\left(-\frac{10}{7},\arcsin\right.\right.\right.$$
$$\frac{25x\sqrt{-x^4+x^2+2}}{952\,(5x^2+7)^2}$$

input
```
Int[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]
```

output
```
(-25*x*Sqrt[2 + x^2 - x^4])/(952*(7 + 5*x^2)^2) + ((-12525*x*Sqrt[2 + x^2
- x^4])/(476*(7 + 5*x^2)) + (-2505*EllipticE[ArcSin[x/Sqrt[2]], -2] - 526*
EllipticF[ArcSin[x/Sqrt[2]], -2] + (58915*EllipticPi[-10/7, ArcSin[x/Sqrt[
2]], -2])/7)/476)/952
```

3.338. $\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx$

3.338.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1536
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/((d + e*x^
2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[
c, 0]
```

3.338. $\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx$

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.338.4 Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.85

method	result
default	$-\frac{25 x \sqrt{-x^{4}+x^{2}+2}}{952 \left(5 x^{2}+7\right)^{2}}-\frac{12525 x \sqrt{-x^{4}+x^{2}+2}}{453152 \left(5 x^{2}+7\right)}-\frac{263 \sqrt{2}\, \sqrt{-2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{x \sqrt{2}}{2}, i \sqrt{2}\right)}{453152 \sqrt{-x^{4}+x^{2}+2}}-\frac{2505 \sqrt{2}\, \sqrt{-2 x^{2}+4}\, \sqrt{x^{2}+1}\, E\left(\frac{x \sqrt{2}}{2}, i \sqrt{\ }\right.}{906304 \sqrt{-x^{4}+x^{2}+2}}$
elliptic	$-\frac{25 x \sqrt{-x^{4}+x^{2}+2}}{952 \left(5 x^{2}+7\right)^{2}}-\frac{12525 x \sqrt{-x^{4}+x^{2}+2}}{453152 \left(5 x^{2}+7\right)}-\frac{263 \sqrt{2}\, \sqrt{-2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{x \sqrt{2}}{2}, i \sqrt{2}\right)}{453152 \sqrt{-x^{4}+x^{2}+2}}-\frac{2505 \sqrt{2}\, \sqrt{-2 x^{2}+4}\, \sqrt{x^{2}+1}\, E\left(\frac{x \sqrt{2}}{2}, i \sqrt{\ }\right.}{906304 \sqrt{-x^{4}+x^{2}+2}}$
risch	$\frac{25 \left(x^{4}-x^{2}-2\right) x \left(2505 x^{2}+3983\right)}{453152 \left(5 x^{2}+7\right)^{2} \sqrt{-x^{4}+x^{2}+2}}-\frac{433 \sqrt{2}\, \sqrt{-2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{x \sqrt{2}}{2}, i \sqrt{2}\right)}{129472 \sqrt{-x^{4}+x^{2}+2}}+\frac{2505 \sqrt{2}\, \sqrt{-2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{x \sqrt{2}}{2}, i \sqrt{2}\right)-E\left(\frac{x \sqrt{2}}{2}\right.\right.}{906304 \sqrt{-x^{4}+x^{2}+2}}$

input
```
int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-25/952*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-12525/453152*x*(-x^4+x^2+2)^(1/2)
/(5*x^2+7)-263/453152*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^
(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-2505/906304*2^(1/2)*(-2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+58
915/3172064*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ell
ipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.338.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + x^2 + 2)/(125*x^10 + 400*x^8 - 40*x^6 - 1442*x^4 - 1
813*x^2 - 686), x)
```

3.338.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-(x^2-2)(x^2+1)}(5x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3), x)
```

3.338.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^3 \sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)
```

3.338.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^3 \sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+x^2+2}(5\,x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^3 \sqrt{2+x^2-x^4}}\,dx = \int \frac{1}{(5\,x^2+7)^3\,\sqrt{-x^4+x^2+2}}\,dx$$

input
```
int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2)),x)
```

output
```
int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2)), x)
```

3.339 $\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx$

3.339.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx = \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} + \frac{27500}{3}x\sqrt{2+x^2-x^4} + 625x^3\sqrt{2+x^2-x^4} - \frac{3482293}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{627857}{6}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
-3482293/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+627857/6*EllipticF(1/2*x*2^
(1/2),I*2^(1/2))+1/18*x*(1419793*x^2+1419985)/(-x^4+x^2+2)^(1/2)+27500/3*x
*(-x^4+x^2+2)^(1/2)+625*x^3*(-x^4+x^2+2)^(1/2)
```

3.339.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx = \frac{1749985x+1607293x^3-153750x^5-11250x^7-3482293i\sqrt{4+2x^2-2x^4}E(i\text{arcs}}{18\sqrt{2+x^2-x}}$$

input
```
Integrate[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

output
```
(1749985*x + 1607293*x^3 - 153750*x^5 - 11250*x^7 - (3482293*I)*Sqrt[4 + 2
*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] + (4281654*I)*Sqrt[4 + 2*x^2 -
 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(18*Sqrt[2 + x^2 - x^4])
```

3.339.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1517, 2207, 27, 2207, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^5}{(-x^4+x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1517}$$

$$\frac{x(1419793x^2+1419985)}{18\sqrt{-x^4+x^2+2}}-\frac{1}{18}\int \frac{56250x^6+450000x^4+3084793x^2+1268722}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{18}\left(\frac{1}{5}\int -\frac{5(495000x^4+3152293x^2+1268722)}{\sqrt{-x^4+x^2+2}}dx+11250\sqrt{-x^4+x^2+2}x^3\right)+\frac{x(1419793x^2+1419985)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{18}\left(11250x^3\sqrt{-x^4+x^2+2}-\int \frac{495000x^4+3152293x^2+1268722}{\sqrt{-x^4+x^2+2}}dx\right)+\frac{x(1419793x^2+1419985)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{18}\left(\frac{1}{3}\int -\frac{3(3482293x^2+1598722)}{\sqrt{-x^4+x^2+2}}dx+165000\sqrt{-x^4+x^2+2}x+11250\sqrt{-x^4+x^2+2}x^3\right)+\frac{x(1419793x^2+1419985)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{18}\left(-\int \frac{3482293x^2+1598722}{\sqrt{-x^4+x^2+2}}dx+165000\sqrt{-x^4+x^2+2}x+11250\sqrt{-x^4+x^2+2}x^3\right)+\frac{x(1419793x^2+1419985)}{18\sqrt{-x^4+x^2+2}}$$

$\Big\downarrow$ 1494

$$\frac{1}{18}\left(-2\int \frac{3482293x^2+1598722}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+165000\sqrt{-x^4+x^2+2}x+11250\sqrt{-x^4+x^2+2}x^3\right)+\frac{x(1419793x^2+1419985)}{18\sqrt{-x^4+x^2+2}}$$

$\Big\downarrow$ 27

$$\frac{1}{18}\left(-\int \frac{3482293x^2+1598722}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+165000\sqrt{-x^4+x^2+2}x+11250\sqrt{-x^4+x^2+2}x^3\right)+\frac{x(1419793x^2+1419985)}{18\sqrt{-x^4+x^2+2}}$$

$\Big\downarrow$ 399

$$\frac{1}{18}\left(1883571\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-3482293\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+165000\sqrt{-x^4+x^2+2}x+11250\sqrt{-x^4+x^2+2x}\right.$$
$$\frac{x(1419793x^2+1419985)}{18\sqrt{-x^4+x^2+2}}$$

$\Big\downarrow$ 321

$$\frac{1}{18}\left(-3482293\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+1883571\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+165000\sqrt{-x^4+x^2+2}x+11250\sqrt{-x^4}\right.$$
$$\frac{x(1419793x^2+1419985)}{18\sqrt{-x^4+x^2+2}}$$

$\Big\downarrow$ 327

$$\frac{1}{18}\left(1883571\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-3482293E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+165000\sqrt{-x^4+x^2+2}x+1125\right.$$
$$\frac{x(1419793x^2+1419985)}{18\sqrt{-x^4+x^2+2}}$$

input
```
Int[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

3.339. $\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx$

output
```
(x*(1419985 + 1419793*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (165000*x*Sqrt[2 +
x^2 - x^4] + 11250*x^3*Sqrt[2 + x^2 - x^4] - 3482293*EllipticE[ArcSin[x/Sq
rt[2]], -2] + 1883571*EllipticF[ArcSin[x/Sqrt[2]], -2])/18
```

3.339.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.339. $\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx$

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.339.4 Maple [A] (verified)

Time = 8.85 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.53

method	result
risch	$-\frac{x\left(11250x^6+153750x^4-1607293x^2-1749985\right)}{18\sqrt{-x^4+x^2+2}}-\frac{799361\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{3482293\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,(}{36\sqrt{-x}}$
elliptic	$\frac{\frac{1419793}{18}x^3+\frac{1419985}{18}x}{\sqrt{-x^4+x^2+2}}+625x^3\sqrt{-x^4+x^2+2}+\frac{27500x\sqrt{-x^4+x^2+2}}{3}-\frac{799361\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+$
default	$\frac{\frac{84035}{18}x-\frac{16807}{18}x^3}{\sqrt{-x^4+x^2+2}}-\frac{799361\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{3482293\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$

input
```
int((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*x*(11250*x^6+153750*x^4-1607293*x^2-1749985)/(-x^4+x^2+2)^(1/2)-7993
61/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(
1/2*x*2^(1/2),I*2^(1/2))+3482293/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)
/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^
(1/2),I*2^(1/2)))
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx = -\frac{6964586\,\sqrt{2}(-i\,x^5+i\,x^3+2i\,x)E(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2})+7763947\,\sqrt{2}(i\,x^5-i\,x^3-2i\,x)F(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2})}{18\,(x^5-x^3-2\,x)}$$

input
```
integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/18*(6964586*sqrt(2)*(-I*x^5 + I*x^3 + 2*I*x)*elliptic_e(arcsin(sqrt(2)/
x), -1/2) + 7763947*sqrt(2)*(I*x^5 - I*x^3 - 2*I*x)*elliptic_f(arcsin(sqrt
(2)/x), -1/2) - 2*(5625*x^8 + 76875*x^6 + 937500*x^4 - 2616139*x^2 - 34822
93)*sqrt(-x^4 + x^2 + 2))/(x^5 - x^3 - 2*x)
```

3.339.6 Sympy [F]

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^5}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**5/(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**5/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

3.339.7 Maxima [F]

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

3.339.8 Giac [F]

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(-x^4+x^2+2)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^5/(x^2 - x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^5/(x^2 - x^4 + 2)^(3/2), x)
```

3.340 $\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx$

3.340.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} - \frac{165239}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{31921}{6}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
-165239/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+31921/6*EllipticF(1/2*x*2^(1
/2),I*2^(1/2))+1/18*x*(83489*x^2+83585)/(-x^4+x^2+2)^(1/2)+625/3*x*(-x^4+x
^2+2)^(1/2)
```

3.340.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \frac{91085x + 87239x^3 - 3750x^5 - 165239i\sqrt{4+2x^2-2x^4}E\left(i\text{arcsinh}(x)\middle|-\frac{1}{2}\right) + 1999}{18\sqrt{2+x^2-x^4}}$$

input
```
Integrate[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```

output
```
(91085*x + 87239*x^3 - 3750*x^5 - (165239*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Ellip
ticE[I*ArcSinh[x], -1/2] + (199977*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*
ArcSinh[x], -1/2])/(18*Sqrt[2 + x^2 - x^4])
```

3.340.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1517, 2207, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^4}{(-x^4+x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1517}$$

$$\frac{x(83489x^2+83585)}{18\sqrt{-x^4+x^2+2}}-\frac{1}{18}\int \frac{11250x^4+157739x^2+61976}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{18}\left(\frac{1}{3}\int -\frac{3(165239x^2+69476)}{\sqrt{-x^4+x^2+2}}dx+3750\sqrt{-x^4+x^2+2}x\right)+\frac{x(83489x^2+83585)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{18}\left(3750x\sqrt{-x^4+x^2+2}-\int \frac{165239x^2+69476}{\sqrt{-x^4+x^2+2}}dx\right)+\frac{x(83489x^2+83585)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{18}\left(3750x\sqrt{-x^4+x^2+2}-2\int \frac{165239x^2+69476}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)+\frac{x(83489x^2+83585)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{18}\left(3750x\sqrt{-x^4+x^2+2}-\int \frac{165239x^2+69476}{\sqrt{2-x^2}\sqrt{x^2+1}}dx\right)+\frac{x(83489x^2+83585)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{18}\left(95763\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-165239\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+3750\sqrt{-x^4+x^2+2}x\right)+\frac{x(83489x^2+83585)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \color{blue}{321}$$

$$\frac{1}{18}\left(-165239\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+95763\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+3750\sqrt{-x^4+x^2+2}x\right)+\frac{x\left(83489x^2+83585\right)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \color{blue}{327}$$

$$\frac{1}{18}\left(95763\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-165239E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+3750\sqrt{-x^4+x^2+2}x\right)+\frac{x\left(83489x^2+83585\right)}{18\sqrt{-x^4+x^2+2}}$$

input
```
Int[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```

output
```
(x*(83585 + 83489*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (3750*x*Sqrt[2 + x^2 -
x^4] - 165239*EllipticE[ArcSin[x/Sqrt[2]], -2] + 95763*EllipticF[ArcSin[x/
Sqrt[2]], -2])/18
```

3.340.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.340. $\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.340. $\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx$

3.340.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.85

method	result
risch	$-\frac{x(3750x^4-87239x^2-91085)}{18\sqrt{-x^4+x^2+2}}-\frac{17369\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{165239\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\right.\right.}{36\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{\frac{83489}{18}x^3+\frac{83585}{18}x}{\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{3}-\frac{17369\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{165239\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\right.\right.}{36\sqrt{-x^4+x^2+2}}$
default	$\frac{\frac{12005}{18}x-\frac{2401}{18}x^3}{\sqrt{-x^4+x^2+2}}-\frac{17369\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{165239\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}+$

input
```
int((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*x*(3750*x^4-87239*x^2-91085)/(-x^4+x^2+2)^(1/2)-17369/9*2^(1/2)*(-2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^
(1/2))+165239/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)
*(EllipticF(1/2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx =$$
$$-\frac{165239\,\sqrt{2}(-i\,x^5+i\,x^3+2i\,x)E(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2})+182608\,\sqrt{2}(i\,x^5-i\,x^3-2i\,x)F(\arcsin\left(\frac{\sqrt{2}}{x}\right)\,|-\frac{1}{2})}{9\,(x^5-x^3-2\,x)}$$

input
```
integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/9*(165239*sqrt(2)*(-I*x^5 + I*x^3 + 2*I*x)*elliptic_e(arcsin(sqrt(2)/x)
, -1/2) + 182608*sqrt(2)*(I*x^5 - I*x^3 - 2*I*x)*elliptic_f(arcsin(sqrt(2)
/x), -1/2) - (1875*x^6 + 39000*x^4 - 128162*x^2 - 165239)*sqrt(-x^4 + x^2
+ 2))/(x^5 - x^3 - 2*x)
```

3.340.6 Sympy [F]

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^4}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**4/(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**4/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

3.340.7 Maxima [F]

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

3.340.8 Giac [F]

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(-x^4+x^2+2)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^4/(x^2 - x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^4/(x^2 - x^4 + 2)^(3/2), x)
```

3.341 $\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx$

3.341.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{7147}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1763}{6}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
-7147/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1763/6*EllipticF(1/2*x*2^(1/2)
,I*2^(1/2))+1/18*x*(4897*x^2+4945)/(-x^4+x^2+2)^(1/2)
```

3.341.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \frac{1}{18}\left(\frac{4945x}{\sqrt{2+x^2-x^4}} + \frac{4897x^3}{\sqrt{2+x^2-x^4}} - 7147i\sqrt{2}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right) + 8076i\sqrt{2}\operatorname{EllipticF}\left(i\operatorname{arcsinh}(x),-\frac{1}{2}\right)\right)$$

input
```
Integrate[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

output
```
((4945*x)/Sqrt[2 + x^2 - x^4] + (4897*x^3)/Sqrt[2 + x^2 - x^4] - (7147*I)*
Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] + (8076*I)*Sqrt[2]*EllipticF[I*ArcSi
nh[x], -1/2])/18
```

3.341.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1517, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^3}{(-x^4+x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1517}$$

$$\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}-\frac{1}{18}\int \frac{7147x^2+1858}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{1494}$$

$$\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}-\frac{1}{9}\int \frac{7147x^2+1858}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}-\frac{1}{18}\int \frac{7147x^2+1858}{\sqrt{2-x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{18}\left(5289\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-7147\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{18}\left(5289\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-7147\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{18}\left(5289\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-7147E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)+\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}$$

input
```
Int[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

output
```
(x*(4945 + 4897*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (-7147*EllipticE[ArcSin[x
/Sqrt[2]], -2] + 5289*EllipticF[ArcSin[x/Sqrt[2]], -2])/18
```

3.341.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.341. $\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx$

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.341.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(53) = 106$.

Time = 2.52 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.40

method	result
risch	$\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}-\frac{929\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{7147\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{\frac{4897}{18}x^3+\frac{4945}{18}x}{\sqrt{-x^4+x^2+2}}-\frac{929\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{7147\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
default	$\frac{\frac{1715}{18}x-\frac{343}{18}x^3}{\sqrt{-x^4+x^2+2}}-\frac{929\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{7147\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}+\frac{\frac{625}{9}x}{\sqrt{-x}}$

input
```
int((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*x*(4897*x^2+4945)/(-x^4+x^2+2)^(1/2)-929/18*2^(1/2)*(-2*x^2+4)^(1/2)*
(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+7147/3
6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2
*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.84

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx =$$
$$-\frac{14294\sqrt{2}(-i\,x^5+i\,x^3+2i\,x)E(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2})+15223\sqrt{2}(i\,x^5-i\,x^3-2i\,x)F(\arcsin\left(\frac{\sqrt{2}}{x}\right)|-\frac{1}{2})-}{18\,(x^5-x^3-2\,x)}$$

input
```
integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/18*(14294*sqrt(2)*(-I*x^5 + I*x^3 + 2*I*x)*elliptic_e(arcsin(sqrt(2)/x)
, -1/2) + 15223*sqrt(2)*(I*x^5 - I*x^3 - 2*I*x)*elliptic_f(arcsin(sqrt(2)/
x), -1/2) - 2*(1125*x^4 - 6046*x^2 - 7147)*sqrt(-x^4 + x^2 + 2))/(x^5 - x^
3 - 2*x)
```

3.341.6 Sympy [F]

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^3}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**3/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

3.341.7 Maxima [F]

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

3.341.8 Giac [F]

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(-x^4+x^2+2)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(3/2), x)
```

3.342 $\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx$

3.342.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{281}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{139}{6}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
-281/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+139/6*EllipticF(1/2*x*2^(1/2),I
*2^(1/2))+1/18*x*(281*x^2+305)/(-x^4+x^2+2)^(1/2)
```

3.342.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = \frac{1}{18}\left(\frac{305x}{\sqrt{2+x^2-x^4}} + \frac{281x^3}{\sqrt{2+x^2-x^4}} - 281i\sqrt{2}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right) + 213i\sqrt{2}\operatorname{EllipticF}\left(i\operatorname{arcsinh}(x),-\frac{1}{2}\right)\right)$$

input
```
Integrate[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

output
```
((305*x)/Sqrt[2 + x^2 - x^4] + (281*x^3)/Sqrt[2 + x^2 - x^4] - (281*I)*Sqr
t[2]*EllipticE[I*ArcSinh[x], -1/2] + (213*I)*Sqrt[2]*EllipticF[I*ArcSinh[x
], -1/2])/18
```

3.342.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1517, 25, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^2}{(-x^4+x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1517}$$

$$\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}-\frac{1}{18}\int -\frac{136-281x^2}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{18}\int \frac{136-281x^2}{\sqrt{-x^4+x^2+2}}dx+\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{9}\int \frac{136-281x^2}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{18}\int \frac{136-281x^2}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{18}\left(417\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-281\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{18}\left(417\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-281\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{18}\left(417\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-281E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)+\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}$$

input
```
Int[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

output
```
(x*(305 + 281*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (-281*EllipticE[ArcSin[x/Sq
rt[2]], -2] + 417*EllipticF[ArcSin[x/Sqrt[2]], -2])/18
```

3.342.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.342.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(53) = 106$.

Time = 2.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.40

method	result
risch	$\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}+\frac{34\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{281\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{\frac{281}{18}x^3+\frac{305}{18}x}{\sqrt{-x^4+x^2+2}}+\frac{34\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{281\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
default	$\frac{\frac{245}{18}x-\frac{49}{18}x^3}{\sqrt{-x^4+x^2+2}}+\frac{34\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{281\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}+\frac{\frac{25}{9}x^3+}{\sqrt{-x^4+}}$

input
```
int((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*x*(281*x^2+305)/(-x^4+x^2+2)^(1/2)+34/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+281/36*2^(
1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^
(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = -\frac{281\,(x^4-x^2-2)E\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\,|-2\right)-553\,(x^4-x^2-2)F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\,|-2\right)+2\sqrt{-x^4+x^2+2}(}{36\,(x^4-x^2-2)}$$

input
```
integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/36*(281*(x^4 - x^2 - 2)*elliptic_e(arcsin(1/2*sqrt(2)*x), -2) - 553*(x^
4 - x^2 - 2)*elliptic_f(arcsin(1/2*sqrt(2)*x), -2) + 2*sqrt(-x^4 + x^2 + 2
)*(281*x^3 + 305*x))/(x^4 - x^2 - 2)
```

3.342.6 Sympy [F]

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^2}{(-(x^2-2)\,(x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**2/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

3.342.7 Maxima [F]

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

3.342.8 Giac [F]

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(-x^4+x^2+2)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(3/2), x)
```

3.343 $\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}}\,dx$

3.343.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}}\,dx = \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}} - \frac{13}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{17}{6}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
-13/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+17/6*EllipticF(1/2*x*2^(1/2),I*2
^(1/2))+1/18*x*(13*x^2+25)/(-x^4+x^2+2)^(1/2)
```

3.343.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}}\,dx = \frac{1}{18}\left(\frac{25x}{\sqrt{2+x^2-x^4}} + \frac{13x^3}{\sqrt{2+x^2-x^4}} - 13i\sqrt{2}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right) - 6i\sqrt{2}\operatorname{EllipticF}\left(i\operatorname{arcsinh}(x),-\frac{1}{2}\right)\right)$$

input
```
Integrate[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

output
```
((25*x)/Sqrt[2 + x^2 - x^4] + (13*x^3)/Sqrt[2 + x^2 - x^4] - (13*I)*Sqrt[2
]*EllipticE[I*ArcSinh[x], -1/2] - (6*I)*Sqrt[2]*EllipticF[I*ArcSinh[x], -1
/2])/18
```

3.343.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1492, 25, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+7}{(-x^4+x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1492}$$

$$\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}-\frac{1}{18}\int -\frac{38-13x^2}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{18}\int \frac{38-13x^2}{\sqrt{-x^4+x^2+2}}dx+\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{9}\int \frac{38-13x^2}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{18}\int \frac{38-13x^2}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{18}\left(51\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx-13\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{18}\left(51\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-13\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{18}\left(51\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)-13E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)+\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}$$

input
```
Int[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

output
```
(x*(25 + 13*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (-13*EllipticE[ArcSin[x/Sqrt[
2]], -2] + 51*EllipticF[ArcSin[x/Sqrt[2]], -2])/18
```

3.343.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.343.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(53) = 106$.

Time = 1.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.40

method	result
risch	$\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}+\frac{19\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{13\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{\frac{13}{18}x^3+\frac{25}{18}x}{\sqrt{-x^4+x^2+2}}+\frac{19\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{13\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$
default	$\frac{\frac{35}{18}x-\frac{7}{18}x^3}{\sqrt{-x^4+x^2+2}}+\frac{19\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{13\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}+\frac{\frac{10}{9}x^3-}{\sqrt{-x^4+x}}$

input
```
int((5*x^2+7)/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*x*(13*x^2+25)/(-x^4+x^2+2)^(1/2)+19/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+13/36*2^(1/
2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1
/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.343.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}}\,dx =$$
$$-\frac{13\,(x^4-x^2-2)E\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\,|-2\right)-89\,(x^4-x^2-2)F\left(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\,|-2\right)+2\sqrt{-x^4+x^2+2}(13}{36\,(x^4-x^2-2)}$$

input
```
integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/36*(13*(x^4 - x^2 - 2)*elliptic_e(arcsin(1/2*sqrt(2)*x), -2) - 89*(x^4
- x^2 - 2)*elliptic_f(arcsin(1/2*sqrt(2)*x), -2) + 2*sqrt(-x^4 + x^2 + 2)*
(13*x^3 + 25*x))/(x^4 - x^2 - 2)
```

3.343.6 Sympy [F]

$$\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{5x^2+7}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

3.343.7 Maxima [F]

$$\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

3.343.8 Giac [F]

$$\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(-x^4+x^2+2)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)/(x^2 - x^4 + 2)^(3/2),x)
```

output
```
int((5*x^2 + 7)/(x^2 - x^4 + 2)^(3/2), x)
```

3.344 $\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx$

3.344.1 Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx=\frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}}+\frac{1}{18}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{6}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
1/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1/6*EllipticF(1/2*x*2^(1/2),I*2^(1
/2))+1/18*x*(-x^2+5)/(-x^4+x^2+2)^(1/2)
```

3.344.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx=\frac{1}{18}\left(\frac{5x}{\sqrt{2+x^2-x^4}}-\frac{x^3}{\sqrt{2+x^2-x^4}}+i\sqrt{2}E\left(i\text{arcsinh}(x)\middle|-\frac{1}{2}\right)-3i\sqrt{2}\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)\right)$$

input
```
Integrate[(2 + x^2 - x^4)^(-3/2),x]
```

output
```
((5*x)/Sqrt[2 + x^2 - x^4] - x^3/Sqrt[2 + x^2 - x^4] + I*Sqrt[2]*EllipticE
[I*ArcSinh[x], -1/2] - (3*I)*Sqrt[2]*EllipticF[I*ArcSinh[x], -1/2])/18
```

3.344.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1405, 25, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{x\left(5-x^2\right)}{18\sqrt{-x^4+x^2+2}}-\frac{1}{18}\int -\frac{x^2+4}{\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{18}\int \frac{x^2+4}{\sqrt{-x^4+x^2+2}}dx+\frac{x\left(5-x^2\right)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{ 1494}$$

$$\frac{1}{9}\int \frac{x^2+4}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{x\left(5-x^2\right)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{18}\int \frac{x^2+4}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{x\left(5-x^2\right)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{1}{18}\left(3\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{x\left(5-x^2\right)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{1}{18}\left(\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{x\left(5-x^2\right)}{18\sqrt{-x^4+x^2+2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{1}{18}\left(3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\right)+\frac{x\left(5-x^2\right)}{18\sqrt{-x^4+x^2+2}}$$

input
```
Int[(2 + x^2 - x^4)^(-3/2),x]
```

output
```
(x*(5 - x^2))/(18*Sqrt[2 + x^2 - x^4]) + (EllipticE[ArcSin[x/Sqrt[2]], -2]
 + 3*EllipticF[ArcSin[x/Sqrt[2]], -2])/18
```

3.344.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

3.344.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(53) = 106$.

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.36

method	result	size
risch	$-\frac{x(x^2-5)}{18\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$	130
default	$\frac{\frac{5}{18}x-\frac{1}{18}x^3}{\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$	133
elliptic	$\frac{\frac{5}{18}x-\frac{1}{18}x^3}{\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$	133

input
```
int(1/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*x*(x^2-5)/(-x^4+x^2+2)^(1/2)+1/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1
/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-1/36*2^(1/2)*(-2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*x*2^(1/2),I*
2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))
```

3.344.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx = \frac{(x^4-x^2-2)E(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\mid -2)+7\,(x^4-x^2-2)F(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\mid -2)+2}{36\,(x^4-x^2-2)}$$

input
```
integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/36*((x^4 - x^2 - 2)*elliptic_e(arcsin(1/2*sqrt(2)*x), -2) + 7*(x^4 - x^2
 - 2)*elliptic_f(arcsin(1/2*sqrt(2)*x), -2) + 2*sqrt(-x^4 + x^2 + 2)*(x^3
- 5*x))/(x^4 - x^2 - 2)
```

3.344.6 Sympy [F]

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral((-x**4 + x**2 + 2)**(-3/2), x)
```

3.344.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.344.8 Giac [F]

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + x^2 + 2)^(-3/2), x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{3/2}}\,dx$$

input
```
int(1/(x^2 - x^4 + 2)^(3/2),x)
```

output
```
int(1/(x^2 - x^4 + 2)^(3/2), x)
```

3.345 $\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx$

3.345.1 Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx = \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} + \frac{8}{153}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1}{102}\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right) - \frac{25}{238}\operatorname{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

output
```
8/153*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1/102*EllipticF(1/2*x*2^(1/2),I*2
^(1/2))-25/238*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/306*x*(-16*x^2+
35)/(-x^4+x^2+2)^(1/2)
```

3.345.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx = \frac{\frac{490x}{\sqrt{2+x^2-x^4}} - \frac{224x^3}{\sqrt{2+x^2-x^4}} + 224i\sqrt{2}E\left(i\operatorname{arcsinh}(x)\middle|-\frac{1}{2}\right) - 357i\sqrt{2}\operatorname{EllipticF}(}{4284}$$

input
```
Integrate[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

output
```
((490*x)/Sqrt[2 + x^2 - x^4] - (224*x^3)/Sqrt[2 + x^2 - x^4] + (224*I)*Sqr
t[2]*EllipticE[I*ArcSinh[x], -1/2] - (357*I)*Sqrt[2]*EllipticF[I*ArcSinh[x
], -1/2] + (225*I)*Sqrt[2]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/4284
```

3.345.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1545, 25, 1492, 27, 1494, 27, 399, 321, 327, 1536, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)(-x^4+x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1545}$$

$$-\frac{1}{34}\int -\frac{12-5x^2}{(-x^4+x^2+2)^{3/2}}dx-\frac{25}{34}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{34}\int \frac{12-5x^2}{(-x^4+x^2+2)^{3/2}}dx-\frac{25}{34}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{34}\left(\frac{x(35-16x^2)}{9\sqrt{-x^4+x^2+2}}-\frac{1}{18}\int -\frac{2(16x^2+19)}{\sqrt{-x^4+x^2+2}}dx\right)-\frac{25}{34}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{34}\left(\frac{1}{9}\int \frac{16x^2+19}{\sqrt{-x^4+x^2+2}}dx+\frac{x(35-16x^2)}{9\sqrt{-x^4+x^2+2}}\right)-\frac{25}{34}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{34}\left(\frac{2}{9}\int \frac{16x^2+19}{2\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{x(35-16x^2)}{9\sqrt{-x^4+x^2+2}}\right)-\frac{25}{34}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{34}\left(\frac{1}{9}\int \frac{16x^2+19}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+\frac{x(35-16x^2)}{9\sqrt{-x^4+x^2+2}}\right)-\frac{25}{34}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \textbf{399}$$

$$\frac{1}{34}\left(\frac{1}{9}\left(3\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}dx+16\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx\right)+\frac{x(35-16x^2)}{9\sqrt{-x^4+x^2+2}}\right)-\frac{25}{34}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \textbf{321}$$

$$\frac{1}{34}\left(\frac{1}{9}\left(16\int \frac{\sqrt{x^2+1}}{\sqrt{2-x^2}}dx+3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)\right)+\frac{x(35-16x^2)}{9\sqrt{-x^4+x^2+2}}\right)-\frac{25}{34}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \textbf{327}$$

$$\frac{1}{34}\left(\frac{1}{9}\left(3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+16E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\Big|-2\right)\right)+\frac{x(35-16x^2)}{9\sqrt{-x^4+x^2+2}}\right)-\frac{25}{34}\int \frac{1}{(5x^2+7)\sqrt{-x^4+x^2+2}}dx$$

$$\Big\downarrow \textbf{1536}$$

$$\frac{1}{34}\left(\frac{1}{9}\left(3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+16E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\Big|-2\right)\right)+\frac{x(35-16x^2)}{9\sqrt{-x^4+x^2+2}}\right)-\frac{25}{17}\int \frac{1}{2\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{34}\left(\frac{1}{9}\left(3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+16E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\Big|-2\right)\right)+\frac{x(35-16x^2)}{9\sqrt{-x^4+x^2+2}}\right)-\frac{25}{34}\int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}\,(5x^2+7)}dx$$

$$\Big\downarrow \textbf{412}$$

$$\frac{1}{34}\left(\frac{1}{9}\left(3\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)+16E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\Big|-2\right)\right)+\frac{x(35-16x^2)}{9\sqrt{-x^4+x^2+2}}\right)-\frac{25}{238}\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

input
```
Int[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

3.345. $\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx$

output
```
((x*(35 - 16*x^2))/(9*Sqrt[2 + x^2 - x^4]) + (16*EllipticE[ArcSin[x/Sqrt[2
]], -2] + 3*EllipticF[ArcSin[x/Sqrt[2]], -2])/9)/34 - (25*EllipticPi[-10/7
, ArcSin[x/Sqrt[2]], -2])/238
```

3.345.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

3.345. $\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx$

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 1536
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/((d + e*x^
2)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[
c, 0]
```

rule 1545
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x^2)*(a + b*x^
2 + c*x^4)^p, x], x] + Simp[e^2/(c*d^2 - b*d*e + a*e^2)   Int[(a + b*x^2 +
c*x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0] && (EqQ[c*
d^2 - a*e^2, 0] || NiceSqrtQ[b^2 - 4*a*c])
```

3.345.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(69) = 138$.

Time = 1.82 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.28

method	result
default	$\frac{-\frac{8}{153}x^3+\frac{35}{306}x}{\sqrt{-x^4+x^2+2}} + \frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{204\sqrt{-x^4+x^2+2}} + \frac{4\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{153\sqrt{-x^4+x^2+2}} - \frac{25\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2}\right.}{238\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{-\frac{8}{153}x^3+\frac{35}{306}x}{\sqrt{-x^4+x^2+2}} + \frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{204\sqrt{-x^4+x^2+2}} + \frac{4\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{153\sqrt{-x^4+x^2+2}} - \frac{25\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{x\sqrt{2}}{2}\right.}{238\sqrt{-x^4+x^2+2}}$
risch	$-\frac{x(16x^2-35)}{306\sqrt{-x^4+x^2+2}} + \frac{19\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{612\sqrt{-x^4+x^2+2}} - \frac{4\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)\right)}{153\sqrt{-x^4+x^2+2}} - 25\sqrt{}$

input
```
int(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-4/153*x^3+35/612*x)/(-x^4+x^2+2)^(1/2)+1/204*2^(1/2)*(-2*x^2+4)^(1/2)*
(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+4/153*
2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*
2^(1/2),I*2^(1/2))-25/238*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^
2+2)^(1/2)*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.345.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)\left(2+x^2-x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-x^4+x^2+2\right)^{\frac{3}{2}}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + x^2 + 2)/(5*x^10 - 3*x^8 - 29*x^6 - x^4 + 48*x^2 + 28
), x)
```

3.345.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)\left(2+x^2-x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\cdot(5x^2+7)}\,dx$$

input
```
integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)), x)
```

3.345.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

3.345.8 Giac [F]

$$\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^2+7)\,(-x^4+x^2+2)^{3/2}}\,dx$$

input
```
int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2)),x)
```

output
```
int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2)), x)
```

3.346 $\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}}\,dx$

3.346.1 Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}}\,dx = \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} + \frac{5143E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656} + \frac{89\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{24276} - \frac{10825\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{113288}$$

output
```
5143/145656*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+89/24276*EllipticF(1/2*x*2^
(1/2),I*2^(1/2))-10825/113288*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/
10404*x*(-287*x^2+580)/(-x^4+x^2+2)^(1/2)+625/16184*x*(-x^4+x^2+2)^(1/2)/(
5*x^2+7)
```

3.346.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.96

$$\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}}\,dx = \frac{953260x+253386x^3-360010x^5+72002i\sqrt{2}(7+5x^2)\sqrt{2+x^2-x^4}E}{}$$

input
```
Integrate[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

output
```
(953260*x + 253386*x^3 - 360010*x^5 + (72002*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2
 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (111741*I)*Sqrt[2]*(7 + 5*x^
2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (681975*I)*Sqrt[2]*
Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (487125*I)*Sqrt[
2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(2039184*(
7 + 5*x^2)*Sqrt[2 + x^2 - x^4])
```

3.346.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^2(-x^4+x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1556}$$

$$\int \left(\frac{194-95x^2}{1156(-x^4+x^2+2)^{3/2}}-\frac{475}{1156(5x^2+7)\sqrt{-x^4+x^2+2}}-\frac{25}{34(5x^2+7)^2\sqrt{-x^4+x^2+2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{89\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{24276}+\frac{5143E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656}-\frac{10825\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{113288}+\frac{625\sqrt{-x^4+x^2+2}x}{16184(5x^2+7)}+\frac{(580-287x^2)x}{10404\sqrt{-x^4+x^2+2}}$$

input
```
Int[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

output
```
(x*(580 - 287*x^2))/(10404*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^
4])/(16184*(7 + 5*x^2)) + (5143*EllipticE[ArcSin[x/Sqrt[2]], -2])/145656 +
 (89*EllipticF[ArcSin[x/Sqrt[2]], -2])/24276 - (10825*EllipticPi[-10/7, Ar
cSin[x/Sqrt[2]], -2])/113288
```

3.346.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.346.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(93) = 186$.

Time = 3.04 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.88

method	result
default	$\frac{-\frac{287}{10404}x^3+\frac{145}{2601}x}{\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{16184(5x^2+7)}+\frac{89\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{48552\sqrt{-x^4+x^2+2}}+\frac{5143\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{291312\sqrt{-x^4+x^2+2}}-$
elliptic	$\frac{-\frac{287}{10404}x^3+\frac{145}{2601}x}{\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{16184(5x^2+7)}+\frac{89\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{48552\sqrt{-x^4+x^2+2}}+\frac{5143\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,E\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{291312\sqrt{-x^4+x^2+2}}-$
risch	$-\frac{x(25715x^4-18099x^2-68090)}{145656(5x^2+7)\sqrt{-x^4+x^2+2}}+\frac{811\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{41616\sqrt{-x^4+x^2+2}}-\frac{5143\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)-E\left(\frac{x\sqrt{}}{2}\right.\right.}{291312\sqrt{-x^4+x^2+2}}$

input
```
int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-287/20808*x^3+145/5202*x)/(-x^4+x^2+2)^(1/2)+625/16184*x*(-x^4+x^2+2)^
(1/2)/(5*x^2+7)+89/48552*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+
2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+5143/291312*2^(1/2)*(-2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*x*2^(1/2),I*2^(1/2))
-10825/113288*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*E
llipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.346.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^2 (2+x^2-x^4)^{3/2}} \, dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}} (5\,x^2+7)^2} \, dx$$

input
```
integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + x^2 + 2)/(25*x^12 + 20*x^10 - 166*x^8 - 208*x^6 + 233
*x^4 + 476*x^2 + 196), x)
```

3.346.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^2 (2+x^2-x^4)^{3/2}} \, dx = \int \frac{1}{(-(x^2-2)(x^2+1))^{\frac{3}{2}} (5x^2+7)^2} \, dx$$

input
```
integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2), x)
```

3.346.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^2 (2+x^2-x^4)^{3/2}} \, dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}} (5\,x^2+7)^2} \, dx$$

input
```
integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

3.346.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^2 (2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^2 (2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^2+7)^2 (-x^4+x^2+2)^{3/2}}\,dx$$

input
```
int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2)),x)
```

output
```
int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2)), x)
```

3.347 $\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}}\, dx$

3.347.1 Optimal result

Integrand size = 24, antiderivative size = 128

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}}\, dx = \frac{x(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\,(7+5x^2)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\,(7+5x^2)} + \frac{3086453E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{138664512} + \frac{60409\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{23110752} - \frac{6898575\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{107850176}$$

output
```
3086453/138664512*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+60409/23110752*Ellipt
icF(1/2*x*2^(1/2),I*2^(1/2))-6898575/107850176*EllipticPi(1/2*x*2^(1/2),-1
0/7,I*2^(1/2))+1/353736*x*(-4909*x^2+9830)/(-x^4+x^2+2)^(1/2)+625/32368*x*
(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+645625/15407168*x*(-x^4+x^2+2)^(1/2)/(5*x^2
+7)
```

3.347.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.44 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.91

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}} \, dx = \frac{3857257460x + 3876617542x^3 - 737347940x^5 - 1080258550x^7 + 43210}{}$$

input
```
Integrate[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

output
```
(3857257460*x + 3876617542*x^3 - 737347940*x^5 - 1080258550*x^7 + (4321034
2*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/
2] - (67352691*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*Ar
cSinh[x], -1/2] + (3042271575*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/
7, I*ArcSinh[x], -1/2] + (4346102250*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*El
lipticPi[5/7, I*ArcSinh[x], -1/2] + (1552179375*I)*Sqrt[2]*x^4*Sqrt[2 + x^
2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(1941303168*(7 + 5*x^2)^2*Sq
rt[2 + x^2 - x^4])
```

3.347.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^3 (-x^4+x^2+2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1556}$$

$$\int \left(-\frac{1635x^2-3278}{39304 (-x^4+x^2+2)^{3/2}} - \frac{8175}{39304 (5x^2+7) \sqrt{-x^4+x^2+2}} - \frac{475}{1156 (5x^2+7)^2 \sqrt{-x^4+x^2+2}} - \frac{}{34 (5x^2+}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{60409\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{23110752}+\frac{3086453E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{138664512}-\frac{6898575\,\text{EllipticPi}\left(-\frac{10}{7},\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{107850176}+\frac{645625\sqrt{-x^4+x^2+2}\,x}{15407168\,(5x^2+7)}+\frac{625\sqrt{-x^4+x^2+2}\,x}{32368\,(5x^2+7)^2}+\frac{(9830-4909x^2)\,x}{353736\sqrt{-x^4+x^2+2}}$$

input
```
Int[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

output
```
(x*(9830 - 4909*x^2))/(353736*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 -
 x^4])/(32368*(7 + 5*x^2)^2) + (645625*x*Sqrt[2 + x^2 - x^4])/(15407168*(7
 + 5*x^2)) + (3086453*EllipticE[ArcSin[x/Sqrt[2]], -2])/138664512 + (60409
*EllipticF[ArcSin[x/Sqrt[2]], -2])/23110752 - (6898575*EllipticPi[-10/7, A
rcSin[x/Sqrt[2]], -2])/107850176
```

3.347.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.347.4 Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{x(77161325x^6+52667710x^4-276901253x^2-275518390)}{138664512(5x^2+7)^2\sqrt{-x^4+x^2+2}}+\frac{492701\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{39618432\sqrt{-x^4+x^2+2}}-\frac{3086453\sqrt{2}\sqrt{-2x^2+}}{2}$
default	$\frac{-\frac{4909}{353736}x^3+\frac{4915}{176868}x}{\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{32368(5x^2+7)^2}+\frac{645625x\sqrt{-x^4+x^2+2}}{15407168(5x^2+7)}+\frac{60409\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{46221504\sqrt{-x^4+x^2+2}}+\frac{3086453\sqrt{}}{}$
elliptic	$\frac{-\frac{4909}{353736}x^3+\frac{4915}{176868}x}{\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{32368(5x^2+7)^2}+\frac{645625x\sqrt{-x^4+x^2+2}}{15407168(5x^2+7)}+\frac{60409\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{2}}{2},i\sqrt{2}\right)}{46221504\sqrt{-x^4+x^2+2}}+\frac{3086453\sqrt{}}{}$

input
```
int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/138664512*x*(77161325*x^6+52667710*x^4-276901253*x^2-275518390)/(5*x^2+
7)^2/(-x^4+x^2+2)^(1/2)+492701/39618432*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*x*2^(1/2),I*2^(1/2))-3086453/2773290
24*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/
2*x*2^(1/2),I*2^(1/2))-EllipticE(1/2*x*2^(1/2),I*2^(1/2)))-6898575/1078501
76*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1
/2*x*2^(1/2),-10/7,I*2^(1/2))
```

3.347.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}} \, dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}} (5\,x^2+7)^3} \, dx$$

input
```
integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + x^2 + 2)/(125*x^14 + 275*x^12 - 690*x^10 - 2202*x^8 -
 291*x^6 + 4011*x^4 + 4312*x^2 + 1372), x)
```

3.347.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}} \, dx = \int \frac{1}{(-(x^2-2)(x^2+1))^{\frac{3}{2}} (5x^2+7)^3} \, dx$$

input
```
integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
```

output
```
Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3), x)
```

3.347.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5\,x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

3.347.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}(5\,x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^2+7)^3\,(-x^4+x^2+2)^{3/2}}\,dx$$

input
```
int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2)),x)
```

output
```
int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2)), x)
```

3.348 $\int (7+5x^2)^4 \sqrt{4+3x^2+x^4}\,dx$

3.348.1 Optimal result

Integrand size = 24, antiderivative size = 242

$$\begin{aligned}\int (7+5x^2)^4 \sqrt{4+3x^2+x^4}\,dx = {} & \frac{51665x\sqrt{4+3x^2+x^4}}{33(2+x^2)} \\ & + \frac{1}{33}x(18727+4516x^2)\sqrt{4+3x^2+x^4} \\ & + \frac{3050}{11}x(4+3x^2+x^4)^{3/2} \\ & + \frac{23500}{99}x^3(4+3x^2+x^4)^{3/2} + \frac{625}{11}x^5(4+3x^2+x^4)^{3/2} \\ & - \frac{51665\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33\sqrt{4+3x^2+x^4}} \\ & + \frac{33159(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{11\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

output
```
3050/11*x*(x^4+3*x^2+4)^(3/2)+23500/99*x^3*(x^4+3*x^2+4)^(3/2)+625/11*x^5*
(x^4+3*x^2+4)^(3/2)+51665/33*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/33*x*(4516*x^
2+18727)*(x^4+3*x^2+4)^(1/2)+33159/22*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/
2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1
/2)-51665/33*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1
/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)
*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.348.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.99 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.46

$$\int (7+5x^2)^4 \sqrt{4+3x^2+x^4}\,dx$$
$$= \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(663924+1257535x^2+1217475x^4+712748x^6+264075x^8+57250x^{10}+5625x^{12})-15499}{}$$

input
```
Integrate[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(663924 + 1257535*x^2 + 1217475*x^4 + 712
748*x^6 + 264075*x^8 + 57250*x^10 + 5625*x^12) - 154995*Sqrt[2]*(3*I + Sqr
t[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt
[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-36253*I + 516
65*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I
+ Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-
3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(396*Sqrt[(-I)/(-3*I
 + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.348.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1518, 2207, 27, 2207, 27, 1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)^4 \sqrt{x^4+3x^2+4}\,dx$$

$$\Big\downarrow \text{1518}$$

$$\frac{1}{11}\int \sqrt{x^4+3x^2+4}(23500x^6+68350x^4+75460x^2+26411)\,dx+\frac{625}{11}(x^4+3x^2+4)^{3/2}x^5$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{11}\left(\frac{1}{9}\int 3\sqrt{x^4+3x^2+4}\left(64050x^4+132380x^2+79233\right)dx+\frac{23500}{9}\left(x^4+3x^2+4\right)^{3/2}x^3\right)+\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\left(\frac{1}{3}\int \sqrt{x^4+3x^2+4}\left(64050x^4+132380x^2+79233\right)dx+\frac{23500}{9}\left(x^4+3x^2+4\right)^{3/2}x^3\right)+\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5$$

$$\Big\downarrow \; 2207$$

$$\frac{1}{11}\left(\frac{1}{3}\left(\frac{1}{7}\int 7\left(22580x^2+42633\right)\sqrt{x^4+3x^2+4}dx+9150x\left(x^4+3x^2+4\right)^{3/2}\right)+\frac{23500}{9}\left(x^4+3x^2+4\right)^{3/2}x^3\right)+\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\left(\frac{1}{3}\left(\int \left(22580x^2+42633\right)\sqrt{x^4+3x^2+4}dx+9150x\left(x^4+3x^2+4\right)^{3/2}\right)+\frac{23500}{9}\left(x^4+3x^2+4\right)^{3/2}x^3\right)+\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5$$

$$\Big\downarrow \; 1490$$

$$\frac{1}{11}\left(\frac{1}{3}\left(\frac{1}{15}\int \frac{15\left(51665x^2+95624\right)}{\sqrt{x^4+3x^2+4}}dx+9150x\left(x^4+3x^2+4\right)^{3/2}+x\left(4516x^2+18727\right)\sqrt{x^4+3x^2+4}\right)+\frac{2350}{9}$$
$$\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\left(\frac{1}{3}\left(\int \frac{51665x^2+95624}{\sqrt{x^4+3x^2+4}}dx+9150x\left(x^4+3x^2+4\right)^{3/2}+x\left(4516x^2+18727\right)\sqrt{x^4+3x^2+4}\right)+\frac{23500}{9}(x^4+3$$
$$\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5$$

$$\Big\downarrow \; 1511$$

$$\frac{1}{11}\left(\frac{1}{3}\left(198954\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-103330\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx+9150x\left(x^4+3x^2+4\right)^{3/2}+x(4516x^2+187\right.\right.$$
$$\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

3.348. $\int \left(7+5x^2\right)^4\sqrt{4+3x^2+x^4}\,dx$

$$\frac{1}{11}\left(\frac{1}{3}\left(198954\int\frac{1}{\sqrt{x^4+3x^2+4}}dx-51665\int\frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+9150x\left(x^4+3x^2+4\right)^{3/2}+x\left(4516x^2+18727\right.\right.\right.$$
$$\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5$$

$$\Big\downarrow \ 1416$$

$$\frac{1}{11}\left(\frac{1}{3}\left(-51665\int\frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+\frac{99477\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}+9150x\left(x^4+3x\right.\right.\right.$$
$$\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5$$

$$\Big\downarrow \ 1509$$

$$\frac{1}{11}\left(\frac{1}{3}\left(\frac{99477\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-51665\left(\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt{x^4+3x^2+4}}\right.\right.\right.$$
$$\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5$$

input
```
Int[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(625*x^5*(4 + 3*x^2 + x^4)^(3/2))/11 + ((23500*x^3*(4 + 3*x^2 + x^4)^(3/2)
)/9 + (x*(18727 + 4516*x^2)*Sqrt[4 + 3*x^2 + x^4] + 9150*x*(4 + 3*x^2 + x^
4)^(3/2) - 51665*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x
^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8
])/Sqrt[4 + 3*x^2 + x^4]) + (99477*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x
^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]
))/3)/11
```

3.348. $\int\left(7+5x^2\right)^4\sqrt{4+3x^2+x^4}\,dx$

3.348.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.348. $\int (7+5x^2)^4 \sqrt{4+3x^2+x^4} \, dx$

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.348.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x(5625x^8+40375x^6+120450x^4+189898x^2+165981)\sqrt{x^4+3x^2+4}}{99}+\frac{382496\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i}}{4}\right.}{33\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$
default	$\frac{55327x\sqrt{x^4+3x^2+4}}{33}+\frac{382496\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{33\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}-\frac{1653280\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x}}{}$
elliptic	$\frac{55327x\sqrt{x^4+3x^2+4}}{33}+\frac{382496\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{33\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}-\frac{1653280\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x}}{}$

input
```
int((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/99*x*(5625*x^8+40375*x^6+120450*x^4+189898*x^2+165981)*(x^4+3*x^2+4)^(1/
2)+382496/33/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-
(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*
I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-1653280/33/(-6+2*I*7^(1/2))^(1
/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/
(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),
1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6
*I*7^(1/2))^(1/2)))
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.57

$$\int (7+5x^2)^4 \sqrt{4+3x^2+x^4}\,dx$$
$$= \frac{154995\sqrt{2}\left(\sqrt{-7}x-3x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2x}\right)\mid \frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-9\sqrt{2}\left(9253\sqrt{-7}x-75571x\right)}{}$$

input
```
integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
1/396*(154995*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arc
sin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 9*sqrt(2)*(92
53*sqrt(-7)*x - 75571*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*
sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(5625*x^10 + 40375*x^8 + 12
0450*x^6 + 189898*x^4 + 165981*x^2 + 154995)*sqrt(x^4 + 3*x^2 + 4))/x
```

3.348.6 Sympy [F]

$$\int (7+5x^2)^4 \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{(x^2-x+2)(x^2+x+2)}\left(5x^2+7\right)^4\,dx$$

input
```
integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**4, x)
```

3.348.7 Maxima [F]

$$\int (7+5x^2)^4 \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^4 dx$$

input
```
integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

3.348.8 Giac [F]

$$\int (7+5x^2)^4 \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^4 dx$$

input
```
integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int (7+5x^2)^4 \sqrt{4+3x^2+x^4}\,dx = \int \left(5\,x^2+7\right)^4 \sqrt{x^4+3\,x^2+4}\,dx$$

input
```
int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(1/2),x)
```

output
```
int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.349 $\int (7+5x^2)^3 \sqrt{4+3x^2+x^4}\,dx$

3.349.1 Optimal result

Integrand size = 24, antiderivative size = 221

$$\int (7+5x^2)^3 \sqrt{4+3x^2+x^4}\,dx = \frac{4717x\sqrt{4+3x^2+x^4}}{21(2+x^2)} + \frac{1}{21}x(1708+407x^2)\sqrt{4+3x^2+x^4} + \frac{275}{7}x(4+3x^2+x^4)^{3/2} + \frac{125}{9}x^3(4+3x^2+x^4)^{3/2} - \frac{4717\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)\mid\frac{1}{8}\right)}{21\sqrt{4+3x^2+x^4}} + \frac{1301(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
275/7*x*(x^4+3*x^2+4)^(3/2)+125/9*x^3*(x^4+3*x^2+4)^(3/2)+4717/21*x*(x^4+3
*x^2+4)^(1/2)/(x^2+2)+1/21*x*(407*x^2+1708)*(x^4+3*x^2+4)^(1/2)+1301/6*(x^
2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*E
llipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^
2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-4717/21*(x^2+2)*(cos(2*arctan(1/2*x*2
^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*
x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^
2+4)^(1/2)
```

3.349.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.62 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.58

$$\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\,dx$$
$$= \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(60096+93656x^2+71862x^4+30946x^6+7725x^8+875x^{10})-14151\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{}}{-3i}}}{}$$

input
```
Integrate[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(60096 + 93656*x^2 + 71862*x^4 + 30946*x^
6 + 7725*x^8 + 875*x^10) - 14151*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt
[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I +
 Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sq
rt[7])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-3409*I + 4717*Sqrt[7])*Sqrt[(-3*I +
Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3
*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I
- Sqrt[7])/(3*I + Sqrt[7])])/(252*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x
^2 + x^4])
```

3.349.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1518, 27, 2207, 1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+7\right)^3 \sqrt{x^4+3x^2+4}\,dx$$
$$\Big\downarrow \text{1518}$$
$$\frac{1}{9}\int 3\sqrt{x^4+3x^2+4}\left(825x^4+1705x^2+1029\right)dx+\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \sqrt{x^4+3x^2+4}\left(825x^4+1705x^2+1029\right)dx+\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{3}\left(\frac{1}{7}\int \left(2035x^2+3903\right)\sqrt{x^4+3x^2+4}dx+\frac{825}{7}x\left(x^4+3x^2+4\right)^{3/2}\right)+\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{1}{15}\int \frac{15\left(4717x^2+8780\right)}{\sqrt{x^4+3x^2+4}}dx+x\sqrt{x^4+3x^2+4}\left(407x^2+1708\right)\right)+\frac{825}{7}x\left(x^4+3x^2+4\right)^{3/2}\right)+\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\int \frac{4717x^2+8780}{\sqrt{x^4+3x^2+4}}dx+x\sqrt{x^4+3x^2+4}\left(407x^2+1708\right)\right)+\frac{825}{7}x\left(x^4+3x^2+4\right)^{3/2}\right)+\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{3}\left(\frac{1}{7}\left(18214\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-9434\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx+x\sqrt{x^4+3x^2+4}\left(407x^2+1708\right)\right)+\frac{825}{7}x(x^4+ \right.$$
$$\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{7}\left(18214\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-4717\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+x\sqrt{x^4+3x^2+4}\left(407x^2+1708\right)\right)+\frac{825}{7}x(x^4+ \right.$$
$$\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{3}\left(\frac{1}{7}\left(-4717\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+\frac{9107\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}+x\sqrt{x^4+3x^2+4}(\right.\right.$$
$$\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

$$\Big\downarrow \text{1509}$$

3.349. $\int \left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}\,dx$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{9107(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-4717\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)\right.}{\sqrt{x^4+3x^2+4}}\right.\right.\right.$$

$$\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

input
```
Int[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(125*x^3*(4 + 3*x^2 + x^4)^(3/2))/9 + ((825*x*(4 + 3*x^2 + x^4)^(3/2))/7 +
 (x*(1708 + 407*x^2)*Sqrt[4 + 3*x^2 + x^4] - 4717*(-((x*Sqrt[4 + 3*x^2 + x
^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*E
llipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) + (9107*(2 + x^
2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8]
)/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]))/7)/3
```

3.349.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.349. $\int (7+5x^2)^3 \sqrt{4+3x^2+x^4}\, dx$

3.349.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.09

method	result
risch	$\frac{x(875x^6+5100x^4+12146x^2+15024)\sqrt{x^4+3x^2+4}}{63}+\frac{35120\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$
default	$\frac{5008x\sqrt{x^4+3x^2+4}}{21}+\frac{35120\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{150944\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{}}{}$
elliptic	$\frac{5008x\sqrt{x^4+3x^2+4}}{21}+\frac{35120\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{150944\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{}}{}$

input
```
int((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/63*x*(875*x^6+5100*x^4+12146*x^2+15024)*(x^4+3*x^2+4)^(1/2)+35120/21/(-6
+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1
/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2)
,1/4*(2+6*I*7^(1/2))^(1/2))-150944/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*
I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/
2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2
))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
))
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.60

$$\int (7+5x^2)^3\sqrt{4+3x^2+x^4}\,dx$$

$$=\frac{14151\sqrt{2}(\sqrt{-7}x-3x)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8})-6\sqrt{2}(1261\sqrt{-7}x-10368x)\sqrt{}}{}$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
1/252*(14151*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcs
in(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 6*sqrt(2)*(126
1*sqrt(-7)*x - 10368*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*s
qrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(875*x^8 + 5100*x^6 + 12146*
x^4 + 15024*x^2 + 14151)*sqrt(x^4 + 3*x^2 + 4))/x
```

3.349.6 Sympy [F]

$$\int (7+5x^2)^3\sqrt{4+3x^2+x^4}\,dx=\int\sqrt{(x^2-x+2)(x^2+x+2)}\left(5x^2+7\right)^3\,dx$$

input
```
integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3, x)
```

3.349.7 Maxima [F]

$$\int (7+5x^2)^3\sqrt{4+3x^2+x^4}\,dx=\int\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^3\,dx$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

3.349.8 Giac [F]

$$\int (7+5x^2)^3\sqrt{4+3x^2+x^4}\,dx=\int\sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^3\,dx$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^3 \sqrt{4+3x^2+x^4}\,dx = \int \left(5\,x^2+7\right)^3 \sqrt{x^4+3\,x^2+4}\,dx$$

input
```
int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2),x)
```

output
```
int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.350 $\int (7+5x^2)^2 \sqrt{4+3x^2+x^4} \, dx$

3.350.1 Optimal result

Integrand size = 24, antiderivative size = 198

$$\begin{aligned}\int (7+5x^2)^2 \sqrt{4+3x^2+x^4} \, dx &= \frac{319x\sqrt{4+3x^2+x^4}}{7(2+x^2)} + \frac{1}{7}x(119+38x^2)\sqrt{4+3x^2+x^4} \\ &\quad + \frac{25}{7}x(4+3x^2+x^4)^{3/2} \\ &\quad - \frac{319\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{7\sqrt{4+3x^2+x^4}} \\ &\quad + \frac{81(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

output
```
25/7*x*(x^4+3*x^2+4)^(3/2)+319/7*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/7*x*(38*x
^2+119)*(x^4+3*x^2+4)^(1/2)+81/2*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^
(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),
1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-3
19/7*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(
1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3
*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.350.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.10 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.73

$$\int (7+5x^2)^2 \sqrt{4+3x^2+x^4}\,dx$$
$$= \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(876+1109x^2+658x^4+188x^6+25x^8)-319\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\Big(i}{28\sqrt{}}$$

input
```
Integrate[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(876 + 1109*x^2 + 658*x^4 + 188*x^6 + 25*
x^8) - 319*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I
 + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I
*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])
] + Sqrt[2]*(-35*I + 319*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I
+ Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*
ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
)/(28*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.350.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1518, 1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)^2 \sqrt{x^4+3x^2+4}\,dx$$
$$\Big\downarrow \text{1518}$$
$$\frac{1}{7}\int (190x^2+243)\sqrt{x^4+3x^2+4}dx+\frac{25}{7}x(x^4+3x^2+4)^{3/2}$$
$$\Big\downarrow \text{1490}$$

$$\frac{1}{7}\left(\frac{1}{15}\int \frac{15\left(319x^2+496\right)}{\sqrt{x^4+3x^2+4}}dx+x\sqrt{x^4+3x^2+4}\left(38x^2+119\right)\right)+\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\int \frac{319x^2+496}{\sqrt{x^4+3x^2+4}}dx+x\sqrt{x^4+3x^2+4}\left(38x^2+119\right)\right)+\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \; 1511$$

$$\frac{1}{7}\left(1134\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-638\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx+x\sqrt{x^4+3x^2+4}\left(38x^2+119\right)\right)+\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(1134\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-319\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+x\sqrt{x^4+3x^2+4}\left(38x^2+119\right)\right)+\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{7}\left(-319\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+\frac{567\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}+x\sqrt{x^4+3x^2+4}\left(38x^2\right.\right.$$
$$\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{7}\left(\frac{567\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-319\left(\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\right.\right.$$
$$\frac{25}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

input
```
Int[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(25*x*(4 + 3*x^2 + x^4)^(3/2))/7 + (x*(119 + 38*x^2)*Sqrt[4 + 3*x^2 + x^4]
 - 319*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4
+ 3*x^2 + x^4]) + (567*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellip
ticF[2*ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]))/7
```

3.350. $\int \left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}\,dx$

3.350.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.350. $\int (7+5x^2)^2 \sqrt{4+3x^2+x^4}\,dx$

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.350.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.19

method	result
risch	$\frac{x(25x^4+113x^2+219)\sqrt{x^4+3x^2+4}}{7}+\frac{1984\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{10208\sqrt{1-\left(-\right.}}{}$
default	$\frac{219x\sqrt{x^4+3x^2+4}}{7}+\frac{1984\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{10208\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-}}{}$
elliptic	$\frac{219x\sqrt{x^4+3x^2+4}}{7}+\frac{1984\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{10208\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-}}{}$

input
```
int((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x*(25*x^4+113*x^2+219)*(x^4+3*x^2+4)^(1/2)+1984/7/(-6+2*I*7^(1/2))^(1/
2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/
2))^(1/2))-10208/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/
2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(E
llipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE
(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.65

$$\int (7+5x^2)^2 \sqrt{4+3x^2+x^4}\,dx$$
$$= \frac{319\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-3\sqrt{2}\left(65\sqrt{-7}x-443\,x\right)\sqrt{\sqrt{-7}}}{28\,x}$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
1/28*(319*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(
1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 3*sqrt(2)*(65*sqr
t(-7)*x - 443*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqr
t(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(25*x^6 + 113*x^4 + 219*x^2 + 319)*
sqrt(x^4 + 3*x^2 + 4))/x
```

3.350.6 Sympy [F]

$$\int (7+5x^2)^2 \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{(x^2-x+2)\,(x^2+x+2)}\left(5x^2+7\right)^2\,dx$$

input
```
integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2, x)
```

3.350.7 Maxima [F]

$$\int (7+5x^2)^2 \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2\,dx$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

3.350.8 Giac [F]

$$\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\left(5\,x^2+7\right)^2\,dx$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^2 \sqrt{4+3x^2+x^4}\,dx = \int \left(5\,x^2+7\right)^2\,\sqrt{x^4+3\,x^2+4}\,dx$$

input
```
int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2),x)
```

output
```
int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.351 $\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx$

3.351.1 Optimal result

Integrand size = 22, antiderivative size = 177

$$\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx = \frac{9x\sqrt{4+3x^2+x^4}}{2+x^2}+\frac{1}{3}x(10+3x^2)\sqrt{4+3x^2+x^4} - \frac{9\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}} + \frac{49(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
9*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/3*x*(3*x^2+10)*(x^4+3*x^2+4)^(1/2)+49/6*
(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2))
)*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+
2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-9*(x^2+2)*(cos(2*arctan(1/2*x*2^(1
/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2
^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4
)^(1/2)
```

3.351.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.66 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.91

$$\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx$$
$$= \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(40+42x^2+19x^4+3x^6)-27\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{}{-3i}}\right.\right.}{12\sqrt{-\frac{i}{-3i+\sqrt{7}}}}$$

input
```
Integrate[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(40 + 42*x^2 + 19*x^4 + 3*x^6) - 27*Sqrt[
2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqr
t[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-
2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*
I + 27*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(
3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I
)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-
3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.351.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)\sqrt{x^4+3x^2+4}\,dx$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{15}\int \frac{5(27x^2+44)}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{3}x\sqrt{x^4+3x^2+4}(3x^2+10)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{27x^2+44}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{3}x\sqrt{x^4+3x^2+4}(3x^2+10)$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{3}\left(98\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-54\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{3}x\sqrt{x^4+3x^2+4}(3x^2+10)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(98\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-27\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{3}x\sqrt{x^4+3x^2+4}(3x^2+10)$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{3}\left(\frac{49(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-27\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+$$
$$\frac{1}{3}x\sqrt{x^4+3x^2+4}(3x^2+10)$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{3}\left(\frac{49(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-27\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x}{\cdot}\right.\right.$$
$$\frac{1}{3}x\sqrt{x^4+3x^2+4}(3x^2+10)$$

input
```
Int[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(x*(10 + 3*x^2)*Sqrt[4 + 3*x^2 + x^4])/3 + (-27*(-((x*Sqrt[4 + 3*x^2 + x^4
])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ell
ipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) + (49*(2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(S
qrt[2]*Sqrt[4 + 3*x^2 + x^4]))/3
```

3.351. $\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx$

3.351.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.351. $\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx$

3.351.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.31

method	result
risch	$\frac{x(3x^2+10)\sqrt{x^4+3x^2+4}}{3}+\frac{176\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{288\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{}$
default	$\frac{10x\sqrt{x^4+3x^2+4}}{3}+\frac{176\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{288\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}\right.}}{}$
elliptic	$\frac{10x\sqrt{x^4+3x^2+4}}{3}+\frac{176\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{288\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}\right.}}{}$

input
```
int((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(3*x^2+10)*(x^4+3*x^2+4)^(1/2)+176/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8
+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4
)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-
288/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8
*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*
(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*
7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.69

$$\int \left(7+5x^2\right)\sqrt{4+3x^2+x^4}\,dx$$

$$=\frac{27\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-2\sqrt{2}\left(8\sqrt{-7}x-57\,x\right)\sqrt{\sqrt{-7}-3}}{12\,x}$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(27*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1
/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 2*sqrt(2)*(8*sqrt(
-7)*x - 57*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-
7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(3*x^4 + 10*x^2 + 27)*sqrt(x^4 + 3*x^2
 + 4))/x
```

3.351.6 Sympy [F]

$$\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx = \int \sqrt{(x^2-x+2)(x^2+x+2)}(5x^2+7)\,dx$$

input
```
integrate((5*x**2+7)*(x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7), x)
```

3.351.7 Maxima [F]

$$\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3x^2+4}(5x^2+7)\,dx$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

3.351.8 Giac [F]

$$\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3x^2+4}(5x^2+7)\,dx$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx = \int \left(5\,x^2+7\right)\,\sqrt{x^4+3\,x^2+4}\,dx$$

input
```
int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2),x)
```

output
```
int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.352 $\int \sqrt{4+3x^2+x^4}\,dx$

3.352.1 Optimal result

Integrand size = 14, antiderivative size = 169

$$\int \sqrt{4+3x^2+x^4}\,dx = \frac{1}{3}x\sqrt{4+3x^2+x^4}+\frac{x\sqrt{4+3x^2+x^4}}{2+x^2} - \frac{\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}} + \frac{7(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
1/3*x*(x^4+3*x^2+4)^(1/2)+x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+7/6*(x^2+2)*(cos(2
*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(si
n(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^
(1/2)/(x^4+3*x^2+4)^(1/2)-(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/c
os(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(
1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.352.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.94 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.96

$$\int \sqrt{4+3x^2+x^4}\,dx$$
$$= \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(4+3x^2+x^4)-3\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{}}{3i+\sqrt{}}\right.}{12\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3}}$$

input
```
Integrate[Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 3*Sqrt[2]*(3*I + Sqrt
[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[
7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])
*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] +
 (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[
7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-3*I + Sqrt[7])]
*Sqrt[4 + 3*x^2 + x^4])
```

3.352.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1404, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^4+3x^2+4}\,dx$$
$$\Big\downarrow \text{ 1404}$$
$$\frac{1}{3}\int \frac{3x^2+8}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{3}\sqrt{x^4+3x^2+4}x$$
$$\Big\downarrow \text{ 1511}$$
$$\frac{1}{3}\left(14\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-6\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{3}\sqrt{x^4+3x^2+4}x$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(14\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-3\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{3}\sqrt{x^4+3x^2+4}x$$

$$\Big\downarrow 1416$$

$$\frac{1}{3}\left(\frac{7(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-3\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{3}\sqrt{x^4+3x^2+4}x$$

$$\Big\downarrow 1509$$

$$\frac{1}{3}\left(\frac{7(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-3\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-x\sqrt{\right.\right.$$

$$\frac{1}{3}\sqrt{x^4+3x^2+4}x$$

input
```
Int[Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(x*Sqrt[4 + 3*x^2 + x^4])/3 + (-3*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2))
+ (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTa
n[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) + (7*(2 + x^2)*Sqrt[(4 + 3*x^2
+ x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 +
 3*x^2 + x^4]))/3
```

3.352.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.352.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.33

method	result
default	$\frac{x\sqrt{x^4+3x^2+4}}{3}+\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$
risch	$\frac{x\sqrt{x^4+3x^2+4}}{3}+\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$
elliptic	$\frac{x\sqrt{x^4+3x^2+4}}{3}+\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-$

input
```
int((x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(x^4+3*x^2+4)^(1/2)+32/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/
2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*Elli
pticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-32/(-6+2*I*7
^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x
^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/
2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2
),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.68

$$\int \sqrt{4+3x^2+x^4}\,dx$$
$$= \frac{3\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-\sqrt{2}\left(\sqrt{-7}x-15\,x\right)\sqrt{\sqrt{-7}-3}F(\mathrm{ar}}{12\,x}$$

input
```
integrate((x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1/
2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(sqrt(-7)*x
 - 15*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) -
3)/x), 3/8*sqrt(-7) + 1/8) + 4*sqrt(x^4 + 3*x^2 + 4)*(x^2 + 3))/x
```

3.352.6 Sympy [F]

$$\int \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3x^2+4}\,dx$$

input
```
integrate((x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral(sqrt(x**4 + 3*x**2 + 4), x)
```

3.352.7 Maxima [F]

$$\int \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

3.352.8 Giac [F]

$$\int \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{4+3x^2+x^4}\,dx = \int \sqrt{x^4+3\,x^2+4}\,dx$$

input
```
int((3*x^2 + x^4 + 4)^(1/2),x)
```

output
```
int((3*x^2 + x^4 + 4)^(1/2), x)
```

3.353 $\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$

3.353.1 Optimal result

Integrand size = 24, antiderivative size = 322

$$\begin{aligned}\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx &= \frac{x\sqrt{4+3x^2+x^4}}{5(2+x^2)}+\frac{1}{5}\sqrt{\frac{11}{35}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)\\
&\quad-\frac{\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{9(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{25\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad-\frac{11\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{75\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{187(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{525\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

output
```
1/175*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/5*x*(x^4+3*
x^2+4)^(1/2)/(x^2+2)+1/30*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/c
os(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(
1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+187/1050
*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2
+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-1/5*(x^2+2)*(cos(2*arctan
(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arc
tan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)
```

3.353.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.63 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = -\frac{\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(35\left(3i+\sqrt{7}\right)E\left(i\text{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\left(7i-35\sqrt{7}\right)\text{EllipticF}\left(i\right.\right.}{350\sqrt{2}\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{}}$$

input
```
Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

output
```
-1/350*(Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I +
 Sqrt[7])]*(35*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqr
t[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I - 35*Sqrt[7])*EllipticF
[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7
])] + (88*I)*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*
I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(Sqrt[2]*Sqrt[(-I)/(-
3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.353.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1523, 27, 1511, 27, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+3x^2+4}}{5x^2+7}dx$$

$$\Big\downarrow \text{1523}$$

$$\frac{88}{15}\int \frac{x^2+2}{2\left(5x^2+7\right)\sqrt{x^4+3x^2+4}}dx-\frac{2}{15}\int \frac{4-3x^2}{2\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{44}{15}\int \frac{x^2+2}{\left(5x^2+7\right)\sqrt{x^4+3x^2+4}}dx-\frac{1}{15}\int \frac{4-3x^2}{\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{15}\left(2\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-6\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)+\frac{44}{15}\int \frac{x^2+2}{\left(5x^2+7\right)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(2\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-3\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{44}{15}\int \frac{x^2+2}{\left(5x^2+7\right)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{15}\left(\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-3\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+$$
$$\frac{44}{15}\int \frac{x^2+2}{\left(5x^2+7\right)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{1509}$$

$$\frac{44}{15}\int \frac{x^2+2}{\left(5x^2+7\right)\sqrt{x^4+3x^2+4}}dx+$$
$$\frac{1}{15}\left(\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-3\left(\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-x\sqrt{\cdots}\right.\right.$$

$$\Big\downarrow \text{2220}$$

$$\frac{44}{15}\left(\frac{3\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{4\sqrt{385}}+\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+4}}\right)+$$

$$\frac{1}{15}\left(\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-3\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{}}{}\right.\right.$$

input
```
Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

output
```
(-3*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 +
 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3
*x^2 + x^4]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*
ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]))/15 + (44*((3*Arc
Tan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(4*Sqrt[385]) + (17*(2 + x^2
)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2
]], 1/8])/(140*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])))/15
```

3.353.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

3.353. $\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1523
```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e - d*q))   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] - Simp[1/(e*(e
 - d*q))   Int[(c*d - b*e + a*e*q - (c*e - a*d*q^3)*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.353.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.20

method	result
default	$\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\cdots}}{4}\cdots\right.}{5\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(3+i\sqrt{7}\right)}$
elliptic	$\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\cdots}}{4}\cdots\right.}{5\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(3+i\sqrt{7}\right)}$

input
```
int((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

output
```
32/25/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^
2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(
1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-32/5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*
x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^
2+4)^(1/2)/(3+I*7^(1/2))*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I
*7^(1/2))^(1/2))+32/5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))
^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2
))*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+44/17
5/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^
2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1
/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/
8*I*7^(1/2))^(1/2))
```

3.353.5 Fricas [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

3.353.6 Sympy [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{(x^2-x+2)\,(x^2+x+2)}}{5x^2+7}\,dx$$

input
```
integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7),x)
```

output
```
Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7), x)
```

3.353.7 Maxima [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

3.353.8 Giac [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

input
```
int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7),x)
```

output
```
int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7), x)
```

3.354 $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx$

3.354.1 Optimal result

Integrand size = 24, antiderivative size = 284

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx = -\frac{x\sqrt{4+3x^2+x^4}}{70(2+x^2)}+\frac{x\sqrt{4+3x^2+x^4}}{14(7+5x^2)}+\frac{51\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{280\sqrt{385}}+\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{4+3x^2+x^4}}-\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{35\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{289(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{9800\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
51/107800*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/70*x*(x
^4+3*x^2+4)^(1/2)/(x^2+2)+1/14*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+1/70*(x^2+2
)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Elli
pticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^
2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-1/70*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)
))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1
/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(
1/2)+289/19600*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan
(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2
))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.354.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.67 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx$$
$$= \frac{700\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(4+3x^2+x^4)+35\left(3i+\sqrt{7}\right)(7+5x^2)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\cdots}}\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

output
```
(700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I + Sqrt[7])*
(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3
*I + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I
 - Sqrt[7])/(3*I + Sqrt[7])] - EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqr
t[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]) - (98*I)*(7 + 5*x^2)*Sqrt[2 -
 ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*Ellip
ticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sq
rt[7])] - (102*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[
1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcS
inh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(9
800*Sqrt[(-I)/(-3*I + Sqrt[7])]*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])
```

3.354.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1554, 25, 2232, 27, 1509, 2224, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+3x^2+4}}{(5x^2+7)^2}dx$$

$$\Big\downarrow \text{1554}$$

$$\frac{x\sqrt{x^4+3x^2+4}}{14(5x^2+7)}-\frac{1}{14}\int -\frac{4-x^4}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{14}\int \frac{4-x^4}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+\frac{\sqrt{x^4+3x^2+4}x}{14(5x^2+7)}$$

$$\Big\downarrow \text{2232}$$

$$\frac{1}{14}\left(\frac{2}{5}\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx+\frac{1}{5}\int \frac{3(2-x^2)}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx\right)+\frac{\sqrt{x^4+3x^2+4}x}{14(5x^2+7)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{14}\left(\frac{1}{5}\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+\frac{3}{5}\int \frac{2-x^2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx\right)+\frac{\sqrt{x^4+3x^2+4}x}{14(5x^2+7)}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{14}\left(\frac{3}{5}\int \frac{2-x^2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+\frac{1}{5}\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2+4}}{x^2+2}\right)\right)+\frac{\sqrt{x^4+3x^2+4}x}{14(5x^2+7)}$$

$$\Big\downarrow \text{2224}$$

$$\frac{1}{14}\left(\frac{3}{5}\left(\frac{17}{3}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx-\frac{4}{3}\int \frac{1}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{5}\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}\right.\right.$$

$$\frac{\sqrt{x^4+3x^2+4}x}{14(5x^2+7)}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{14}\left(\frac{3}{5}\left(\frac{17}{3}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx-\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{x^4+3x^2+4}}\right)+\frac{1}{5}\left(\frac{\sqrt{2}(x\right.\right.$$

$$\frac{\sqrt{x^4+3x^2+4}x}{14(5x^2+7)}$$

$$\Big\downarrow \text{2220}$$

3.354. $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2+4}}{x^2+2}\right)+\frac{3}{5}\left(\frac{17}{3}\left(\frac{3\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{4\sqrt{385}}+\right.\right.\right.$$

$$\frac{\sqrt{x^4+3x^2+4}x}{14\left(5x^2+7\right)}$$

input
```
Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

output
```
(x*Sqrt[4 + 3*x^2 + x^4])/(14*(7 + 5*x^2)) + ((-((x*Sqrt[4 + 3*x^2 + x^4])
/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellip
ticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4])/5 + (3*(-1/3*(Sqrt[
2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt
[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (17*((3*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[
4 + 3*x^2 + x^4]])/(4*Sqrt[385]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(140*Sqrt[2]*Sqrt
[4 + 3*x^2 + x^4])))/3))/5)/14
```

3.354.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1554
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(q +
 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*((a*(2*q + 3) +
2*b*(q + 2)*x^2 + c*(2*q + 5)*x^4)/Sqrt[a + b*x^2 + c*x^4]), x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && ILtQ[q, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2224
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[2*A*(B/(B*d + A*e))   Int[1/Sqrt[a + b*x
^2 + c*x^4], x], x] - Simp[(B*d - A*e)/(B*d + A*e)   Int[(A - B*x^2)/((d +
e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x]
&& NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && NegQ[B/A]
```

rule 2232
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + b*x^2
 + c*x^4], x], x] + Simp[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d -
 a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
&&  !GtQ[b^2 - 4*a*c, 0]
```

3.354. $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx$

3.354.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.19

method	result
risch	$\frac{x\sqrt{x^4+3x^2+4}}{70x^2+98}+\frac{2\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{16\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right.}}{35\sqrt{-}}$ …
default	$\frac{x\sqrt{x^4+3x^2+4}}{70x^2+98}+\frac{2\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{16\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\right.}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,(}$ …
elliptic	$\frac{x\sqrt{x^4+3x^2+4}}{70x^2+98}+\frac{2\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{16\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\right.}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,(}$ …

input
```
int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

output
```
1/14*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+2/25/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+
1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)
^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1
6/35/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/
8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x
*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I
*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+51/2450/(-3/8+1/8*I*7^(1/2))^(
1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/
2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+
1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

3.354.5 Fricas [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx=\int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^4 + 70*x^2 + 49), x)
```

3.354.6 Sympy [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{(x^2-x+2)\,(x^2+x+2)}}{(5x^2+7)^2}\,dx$$

input
```
integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**2,x)
```

output
```
Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**2, x)
```

3.354.7 Maxima [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

3.354.8 Giac [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^2}\,dx$$

input
```
int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^2,x)
```

output
```
int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^2, x)
```

3.355 $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx$

3.355.1 Optimal result

Integrand size = 24, antiderivative size = 312

$$\begin{aligned}\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx &= -\frac{139x\sqrt{4+3x^2+x^4}}{86240\,(2+x^2)}+\frac{x\sqrt{4+3x^2+x^4}}{28\,(7+5x^2)^2}\\
&\quad+\frac{139x\sqrt{4+3x^2+x^4}}{17248\,(7+5x^2)}+\frac{14999\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{344960\sqrt{385}}\\
&\quad+\frac{139(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{43120\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad-\frac{23(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2940\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&\quad+\frac{254983(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{36220800\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

output
```
14999/132809600*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-139
/86240*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/28*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^
2+139/17248*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+139/86240*(x^2+2)*(cos(2*arcta
n(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*ar
ctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/
(x^4+3*x^2+4)^(1/2)-23/5880*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)
/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2
^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+254983
/72441600*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*
x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((
x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.355.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.59 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx$$
$$= \frac{\frac{700x(1589+695x^2)(4+3x^2+x^4)}{(7+5x^2)^2} + i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(4865\left(3-i\sqrt{7}\right)E\left(i\text{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{}}}\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

output
```
((700*x*(1589 + 695*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*
I)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3
*I + Sqrt[7])]*(4865*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I
 + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9597 + (4865*I)*Sqrt
[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])
/(3*I + Sqrt[7])] - 29998*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqr
t[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(1207360
0*Sqrt[4 + 3*x^2 + x^4])
```

3.355.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1554, 25, 2210, 25, 2232, 27, 1509, 2226, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+3x^2+4}}{(5x^2+7)^3}dx$$

$$\Big\downarrow \text{1554}$$

$$\frac{x\sqrt{x^4+3x^2+4}}{28(5x^2+7)^2}-\frac{1}{28}\int -\frac{x^4+6x^2+12}{(5x^2+7)^2\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{28}\int \frac{x^4+6x^2+12}{(5x^2+7)^2\sqrt{x^4+3x^2+4}}dx+\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{2210}$$

$$\frac{1}{28}\left(\frac{139x\sqrt{x^4+3x^2+4}}{616(5x^2+7)}-\frac{1}{616}\int -\frac{-139x^4-266x^2+500}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx\right)+\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{28}\left(\frac{1}{616}\int \frac{-139x^4-266x^2+500}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+\frac{139\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}\right)+\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{2232}$$

$$\frac{1}{28}\left(\frac{1}{616}\left(\frac{278}{5}\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx+\frac{1}{5}\int \frac{554-1747x^2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx\right)+\frac{139\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}\right)+\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\left(\frac{1}{616}\left(\frac{139}{5}\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{5}\int \frac{554-1747x^2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx\right)+\frac{139\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}\right)+\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{28}\left(\frac{1}{616}\left(\frac{1}{5}\int\frac{554-1747x^2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+\frac{139}{5}\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2}}{x^2+2}\right.\right.\right.$$

$$\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}$$

$\big\downarrow$ 2226

$$\frac{1}{28}\left(\frac{1}{616}\left(\frac{1}{5}\left(\frac{29998}{3}\int\frac{x^2+2}{2(5x^2+7)\sqrt{x^4+3x^2+4}}dx-\frac{4048}{3}\int\frac{1}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{139}{5}\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2}{(x^2+2}}}{\sqrt{x^4}}\right.\right.\right.$$

$$\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}$$

$\big\downarrow$ 27

$$\frac{1}{28}\left(\frac{1}{616}\left(\frac{1}{5}\left(\frac{14999}{3}\int\frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx-\frac{4048}{3}\int\frac{1}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{139}{5}\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+}{(x^2+2)^2}}}{\sqrt{x^4+}}\right.\right.\right.$$

$$\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}$$

$\big\downarrow$ 1416

$$\frac{1}{28}\left(\frac{1}{616}\left(\frac{1}{5}\left(\frac{14999}{3}\int\frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx-\frac{1012\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{x^4+3x^2+4}}\right.\right.\right.$$

$$\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}$$

$\big\downarrow$ 2220

$$\frac{1}{28}\left(\frac{1}{616}\left(\frac{139}{5}\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2+4}}{x^2+2}\right)+\frac{1}{5}\left(\frac{14999}{3}\left(\frac{3\arctan\left(\frac{2}{\sqrt{x^4}}\right.}{4\sqrt{385}}\right.\right.\right.\right.$$

$$\frac{\sqrt{x^4+3x^2+4}x}{28(5x^2+7)^2}$$

input
```
Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

3.355. $\int\frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx$

output
```
(x*Sqrt[4 + 3*x^2 + x^4])/(28*(7 + 5*x^2)^2) + ((139*x*Sqrt[4 + 3*x^2 + x^
4])/(616*(7 + 5*x^2)) + ((139*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)) + (S
qrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/
Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]))/5 + ((-1012*Sqrt[2]*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sq
rt[4 + 3*x^2 + x^4]) + (14999*((3*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2
+ x^4]])/(4*Sqrt[385]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(140*Sqrt[2]*Sqrt[4 + 3*x^2
 + x^4])))/3)/5)/616)/28
```

3.355.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1554
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(q +
 1))), x] + Simp[1/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*((a*(2*q + 3) +
2*b*(q + 2)*x^2 + c*(2*q + 5)*x^4)/Sqrt[a + b*x^2 + c*x^4]), x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && ILtQ[q, -1]
```

3.355. $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx$

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2232
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + b*x^2
 + c*x^4], x], x] + Simp[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d -
 a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
&&  !GtQ[b^2 - 4*a*c, 0]
```

3.355. $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3} \, dx$

3.355.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.11

method	result
risch	$\frac{\sqrt{x^4+3x^2+4}\,x(695x^2+1589)}{17248(5x^2+7)^2} - \frac{51\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{15400\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}} + \frac{139\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x}}{}$
default	$\frac{x\sqrt{x^4+3x^2+4}}{28(5x^2+7)^2} + \frac{139x\sqrt{x^4+3x^2+4}}{17248(5x^2+7)} - \frac{51\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{15400\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}} + \frac{139\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{}}{8}}}{2695\sqrt{}}$
elliptic	$\frac{x\sqrt{x^4+3x^2+4}}{28(5x^2+7)^2} + \frac{139x\sqrt{x^4+3x^2+4}}{17248(5x^2+7)} - \frac{51\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{15400\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}} + \frac{139\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{}}{8}}}{2695\sqrt{}}$

input
```
int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

output
```
1/17248*(x^4+3*x^2+4)^(1/2)*x*(695*x^2+1589)/(5*x^2+7)^2-51/15400/(-6+2*I*
7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*
x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*
(2+6*I*7^(1/2))^(1/2))+139/2695/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1
/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+
I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/
2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+149
99/3018400/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*
(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1
/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)
/(-3/8+1/8*I*7^(1/2))^(1/2))
```

3.355.5 Fricas [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

3.355.6 Sympy [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx = \int \frac{\sqrt{(x^2-x+2)\,(x^2+x+2)}}{(5x^2+7)^3}\,dx$$

input
```
integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**3,x)
```

output
```
Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**3, x)
```

3.355.7 Maxima [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

3.355.8 Giac [F]

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

3.355.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx = \int \frac{\sqrt{x^4+3\,x^2+4}}{(5\,x^2+7)^3}\,dx$$

input
```
int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^3,x)
```

output
```
int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^3, x)
```

3.356 $\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2} \, dx$

3.356.1 Optimal result

Integrand size = 24, antiderivative size = 268

$$\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2} \, dx = \frac{12665086x\sqrt{4+3x^2+x^4}}{2145(2+x^2)} + \frac{7x(661429+174989x^2)\sqrt{4+3x^2+x^4}}{2145} + \frac{x(452001+131080x^2)(4+3x^2+x^4)^{3/2}}{1287} + \frac{92150}{429}x(4+3x^2+x^4)^{5/2} + \frac{2250}{13}x^3(4+3x^2+x^4)^{5/2} + \frac{125}{3}x^5(4+3x^2+x^4)^{5/2} - \frac{12665086\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2145\sqrt{4+3x^2+x^4}} +$$

output
```
1/1287*x*(131080*x^2+452001)*(x^4+3*x^2+4)^(3/2)+92150/429*x*(x^4+3*x^2+4)
^(5/2)+2250/13*x^3*(x^4+3*x^2+4)^(5/2)+125/3*x^5*(x^4+3*x^2+4)^(5/2)+12665
086/2145*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+7/2145*x*(174989*x^2+661429)*(x^4+3
*x^2+4)^(1/2)-12665086/2145*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)
/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2
^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+238355
6/429*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^
(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)
/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.356.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.37 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.36

$$\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2} \, dx = \frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(180184116+391419623x^2+472235001x^4+377574349x^6+212188905x^8+83}{}$$

input
```
Integrate[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(180184116 + 391419623*x^2 + 472235001*x^
4 + 377574349*x^6 + 212188905*x^8 + 83076275*x^10 + 21862875*x^12 + 352687
5*x^14 + 268125*x^16) - 18997629*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt
[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I +
 Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sq
rt[7])/(3*I + Sqrt[7])] + 21*Sqrt[2]*(-477617*I + 904649*Sqrt[7])*Sqrt[(-3
*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^
2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x],
(3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12870*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[
4 + 3*x^2 + x^4])
```

3.356.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1518, 27, 2207, 2207, 1490, 27, 1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)^4 (x^4+3x^2+4)^{3/2} \, dx$$

$$\Big\downarrow \textbf{1518}$$

$$\frac{1}{15}\int 5(x^4+3x^2+4)^{3/2}(6750x^6+19550x^4+20580x^2+7203)\,dx+\frac{125}{3}(x^4+3x^2+4)^{5/2}x^5$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{3}\int \left(x^4+3x^2+4\right)^{3/2}\left(6750x^6+19550x^4+20580x^2+7203\right)dx+\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5$$

$\big\downarrow$ 2207

$$\frac{1}{3}\left(\frac{1}{13}\int \left(x^4+3x^2+4\right)^{3/2}\left(92150x^4+186540x^2+93639\right)dx+\frac{6750}{13}\left(x^4+3x^2+4\right)^{5/2}x^3\right)+\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5$$

$\big\downarrow$ 2207

$$\frac{1}{3}\left(\frac{1}{13}\left(\frac{1}{11}\int \left(393240x^2+661429\right)\left(x^4+3x^2+4\right)^{3/2}dx+\frac{92150}{11}x\left(x^4+3x^2+4\right)^{5/2}\right)+\frac{6750}{13}\left(x^4+3x^2+4\right)^{5/2}\right.$$
$$\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5$$

$\big\downarrow$ 1490

$$\frac{1}{3}\left(\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{21}\int 147\left(174989x^2+291864\right)\sqrt{x^4+3x^2+4}dx+\frac{1}{3}x\left(131080x^2+452001\right)\left(x^4+3x^2+4\right)^{3/2}\right)+\right.\right.$$
$$\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{13}\left(\frac{1}{11}\left(7\int \left(174989x^2+291864\right)\sqrt{x^4+3x^2+4}dx+\frac{1}{3}x\left(131080x^2+452001\right)\left(x^4+3x^2+4\right)^{3/2}\right)+\frac{92150}{11}\right.\right.$$
$$\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5$$

$\big\downarrow$ 1490

$$\frac{1}{3}\left(\frac{1}{13}\left(\frac{1}{11}\left(7\left(\frac{1}{15}\int \frac{6\left(904649x^2+1595782\right)}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+4}\left(174989x^2+661429\right)\right)+\frac{1}{3}x\left(131080x^2+4\right.\right.\right.\right.$$
$$\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{13}\left(\frac{1}{11}\left(7\left(\frac{2}{5}\int \frac{904649x^2+1595782}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+4}\left(174989x^2+661429\right)\right)+\frac{1}{3}x\left(131080x^2+45200\right.\right.\right.\right.$$
$$\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5$$

$\big\downarrow$ 1511

3.356. $\int \left(7+5x^2\right)^4\left(4+3x^2+x^4\right)^{3/2}\,dx$

$$\frac{1}{3}\left(\frac{1}{13}\left(\frac{1}{11}\left(7\left(\frac{2}{5}\left(3405080\int\frac{1}{\sqrt{x^4+3x^2+4}}dx-1809298\int\frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{5}x\sqrt{x^4+3x^2+4}(174989x\right.\right.\right.\right.\right.$$
$$\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{13}\left(\frac{1}{11}\left(7\left(\frac{2}{5}\left(3405080\int\frac{1}{\sqrt{x^4+3x^2+4}}dx-904649\int\frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{5}x\sqrt{x^4+3x^2+4}(174989x^2\right.\right.\right.\right.\right.$$
$$\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{3}\left(\frac{1}{13}\left(\frac{1}{11}\left(7\left(\frac{2}{5}\left(\frac{851270\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-904649\int\frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)\right.\right.\right.\right.$$
$$\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{3}\left(\frac{1}{13}\left(\frac{1}{11}\left(7\left(\frac{2}{5}\left(\frac{851270\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-904649\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x}{(x^2+}}}{\sqrt{x^4}}\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{3}\left(x^4+3x^2+4\right)^{5/2}x^5$$

input
```
Int[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(125*x^5*(4 + 3*x^2 + x^4)^(5/2))/3 + ((6750*x^3*(4 + 3*x^2 + x^4)^(5/2))/
13 + ((92150*x*(4 + 3*x^2 + x^4)^(5/2))/11 + ((x*(452001 + 131080*x^2)*(4
+ 3*x^2 + x^4)^(3/2))/3 + 7*((x*(661429 + 174989*x^2)*Sqrt[4 + 3*x^2 + x^4
])/5 + (2*(-904649*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 +
 x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1
/8])/Sqrt[4 + 3*x^2 + x^4]) + (851270*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 +
x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^
4]))/5))/11)/13)/3
```

3.356. $\int (7+5x^2)^4 \left(4+3x^2+x^4\right)^{3/2} \, dx$

3.356.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.356. $\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2} \, dx$

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.356.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.96

method	result
risch	$\frac{x\left(268125x^{12}+2722500x^{10}+12622875x^{8}+34317650x^{6}+58744455x^{4}+64070384x^{2}+45046029\right)\sqrt{x^{4}+3x^{2}+4}}{6435}+\frac{89363792\sqrt{1-\left(-\frac{3}{8}\right.}}{}$
default	$\frac{356027x^{5}\sqrt{x^{4}+3x^{2}+4}}{39}+\frac{64070384x^{3}\sqrt{x^{4}+3x^{2}+4}}{6435}+\frac{15015343x\sqrt{x^{4}+3x^{2}+4}}{2145}+\frac{89363792\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)}}{2145\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^{4}+}}$
elliptic	$\frac{356027x^{5}\sqrt{x^{4}+3x^{2}+4}}{39}+\frac{64070384x^{3}\sqrt{x^{4}+3x^{2}+4}}{6435}+\frac{15015343x\sqrt{x^{4}+3x^{2}+4}}{2145}+\frac{89363792\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^{2}}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)}}{2145\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^{4}+}}$

input
```
int((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6435*x*(268125*x^12+2722500*x^10+12622875*x^8+34317650*x^6+58744455*x^4+
64070384*x^2+45046029)*(x^4+3*x^2+4)^(1/2)+89363792/2145/(-6+2*I*7^(1/2))^
(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2
)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^
(1/2))^(1/2))-405282752/2145/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2)
)*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7
^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))
-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.55

$$\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2}\, dx = \frac{37995258\sqrt{2}(\sqrt{-7}x-3\,x)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8})-21\sqrt{2}(1011407}{}$$

input
```
integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
1/25740*(37995258*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e
(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 21*sqrt(2
)*(1011407*sqrt(-7)*x - 7821567*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/
2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(268125*x^14 + 27
22500*x^12 + 12622875*x^10 + 34317650*x^8 + 58744455*x^6 + 64070384*x^4 +
45046029*x^2 + 37995258)*sqrt(x^4 + 3*x^2 + 4))/x
```

3.356.6 Sympy [F]

$$\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2}\, dx = \int \left((x^2-x+2)(x^2+x+2)\right)^{\frac{3}{2}} (5x^2+7)^4\, dx$$

input
```
integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**4, x)
```

3.356.7 Maxima [F]

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2} \, dx = \int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^4 dx$$

input
```
integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

3.356.8 Giac [F]

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2} \, dx = \int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^4 dx$$

input
```
integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2} \, dx = \int \left(5\,x^2+7\right)^4 \left(x^4+3\,x^2+4\right)^{3/2} dx$$

input
```
int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(3/2),x)
```

output
```
int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.357 $\int (7+5x^2)^3 (4+3x^2+x^4)^{3/2} \, dx$

3.357.1 Optimal result

Integrand size = 24, antiderivative size = 247

$$\int (7+5x^2)^3 (4+3x^2+x^4)^{3/2} \, dx = \frac{4525662x\sqrt{4+3x^2+x^4}}{5005(2+x^2)} + \frac{x(1653701+435441x^2)\sqrt{4+3x^2+x^4}}{5005} + \frac{x(53504+15365x^2)(4+3x^2+x^4)^{3/2}}{1001} + \frac{3825}{143}x(4+3x^2+x^4)^{5/2} + \frac{125}{13}x^3(4+3x^2+x^4)^{5/2} - \frac{4525662\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)\middle|\frac{1}{8}\right)}{5005\sqrt{4+3x^2+x^4}} + \frac{121826\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2}{(2+x}}}{143\sqrt{4}}$$

output
```
1/1001*x*(15365*x^2+53504)*(x^4+3*x^2+4)^(3/2)+3825/143*x*(x^4+3*x^2+4)^(5
/2)+125/13*x^3*(x^4+3*x^2+4)^(5/2)+4525662/5005*x*(x^4+3*x^2+4)^(1/2)/(x^2
+2)+1/5005*x*(435441*x^2+1653701)*(x^4+3*x^2+4)^(1/2)-4525662/5005*(x^2+2)
*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellip
ticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2
+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+121826/143*(x^2+2)*(cos(2*arctan(1/2*x*2^
(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x
*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2
+4)^(1/2)
```

3.357.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.79 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.45

$$\int (7+5x^2)^3 (4+3x^2+x^4)^{3/2} \, dx = \frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(19463124+36710547x^2+37166164x^4+24107711x^6+10713970x^8+3158575}{}$$

input
```
Integrate[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(19463124 + 36710547*x^2 + 37166164*x^4 +
 24107711*x^6 + 10713970*x^8 + 3158575*x^10 + 567000*x^12 + 48125*x^14) -
2262831*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I +
Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*Ar
cSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] +
 Sqrt[2]*(-1215823*I + 2262831*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/
(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*Ellipt
icF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqr
t[7])])/(10010*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.357.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1518, 2207, 1490, 27, 1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)^3 (x^4+3x^2+4)^{3/2} \, dx$$

$$\Big\downarrow \text{1518}$$

$$\frac{1}{13}\int (x^4+3x^2+4)^{3/2} (3825x^4+8055x^2+4459) \, dx + \frac{125}{13}(x^4+3x^2+4)^{5/2} x^3$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{13}\left(\frac{1}{11}\int \left(19755x^2+33749\right)\left(x^4+3x^2+4\right)^{3/2}dx+\frac{3825}{11}x\left(x^4+3x^2+4\right)^{5/2}\right)+\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

$\big\downarrow$ 1490

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{1}{21}\int 9\left(145147x^2+243652\right)\sqrt{x^4+3x^2+4}dx+\frac{1}{7}x\left(15365x^2+53504\right)\left(x^4+3x^2+4\right)^{3/2}\right)+\frac{3825}{11}x\left(x^4\right.\right.$$
$$\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

$\big\downarrow$ 27

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{3}{7}\int \left(145147x^2+243652\right)\sqrt{x^4+3x^2+4}dx+\frac{1}{7}x\left(15365x^2+53504\right)\left(x^4+3x^2+4\right)^{3/2}\right)+\frac{3825}{11}x\left(x^4+\right.\right.$$
$$\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

$\big\downarrow$ 1490

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{3}{7}\left(\frac{1}{15}\int \frac{2\left(2262831x^2+4002158\right)}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{15}x\sqrt{x^4+3x^2+4}\left(435441x^2+1653701\right)\right)+\frac{1}{7}x\left(15365x^2+5\right.\right.\right.$$
$$\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

$\big\downarrow$ 27

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{3}{7}\left(\frac{2}{15}\int \frac{2262831x^2+4002158}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{15}x\sqrt{x^4+3x^2+4}\left(435441x^2+1653701\right)\right)+\frac{1}{7}x\left(15365x^2+5350\right.\right.\right.$$
$$\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

$\big\downarrow$ 1511

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{3}{7}\left(\frac{2}{15}\left(8527820\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-4525662\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{15}x\sqrt{x^4+3x^2+4}\left(435441x\right.\right.\right.\right.$$
$$\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

$\big\downarrow$ 27

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{3}{7}\left(\frac{2}{15}\left(8527820\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-2262831\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{15}x\sqrt{x^4+3x^2+4}\left(435441x^2\right.\right.\right.\right.$$
$$\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

$\big\downarrow$ 1416

3.357. $\int \left(7+5x^2\right)^3\left(4+3x^2+x^4\right)^{3/2}\,dx$

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{3}{7}\left(\frac{2}{15}\left(\frac{2131955\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-2262831\int\frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)\right.\right.\right.$$

$$\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

$$\Big\downarrow \ 1509$$

$$\frac{1}{13}\left(\frac{1}{11}\left(\frac{3}{7}\left(\frac{2}{15}\left(\frac{2131955\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-2262831\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x}{(x^2+}}}{\sqrt{x^4}}\right.\right.\right.\right.$$

$$\frac{125}{13}\left(x^4+3x^2+4\right)^{5/2}x^3$$

input
```
Int[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(125*x^3*(4 + 3*x^2 + x^4)^(5/2))/13 + ((3825*x*(4 + 3*x^2 + x^4)^(5/2))/1
1 + ((x*(53504 + 15365*x^2)*(4 + 3*x^2 + x^4)^(3/2))/7 + (3*((x*(1653701 +
 435441*x^2)*Sqrt[4 + 3*x^2 + x^4])/15 + (2*(-2262831*(-((x*Sqrt[4 + 3*x^2
 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) + (2131955*
Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x
/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]))/15))/7)/11)/13
```

3.357.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.357. $\int (7+5x^2)^3 \left(4+3x^2+x^4\right)^{3/2} \, dx$

3.357.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x(48125x^{10}+422625x^8+1698200x^6+3928870x^4+5528301x^2+4865781)\sqrt{x^4+3x^2+4}}{5005}+\frac{32017264\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\right.}}{5005\sqrt{-6+2i\sqrt{7}}\sqrt{}}$
default	$\frac{71434x^5\sqrt{x^4+3x^2+4}}{91}+\frac{5528301x^3\sqrt{x^4+3x^2+4}}{5005}+\frac{4865781x\sqrt{x^4+3x^2+4}}{5005}+\frac{32017264\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}}{5005\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2}}$
elliptic	$\frac{71434x^5\sqrt{x^4+3x^2+4}}{91}+\frac{5528301x^3\sqrt{x^4+3x^2+4}}{5005}+\frac{4865781x\sqrt{x^4+3x^2+4}}{5005}+\frac{32017264\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}}{5005\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2}}$

input
```
int((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5005*x*(48125*x^10+422625*x^8+1698200*x^6+3928870*x^4+5528301*x^2+486578
1)*(x^4+3*x^2+4)^(1/2)+32017264/5005/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I
*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2
)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-144821
184/5005/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/
8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1
/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6
+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.357.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.58

$$\int (7+5x^2)^3 (4+3x^2+x^4)^{3/2} \, dx = \frac{4525662\sqrt{2}(\sqrt{-7}x-3x)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2x}\right) \mid \frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-\sqrt{2}(2524583\sqrt{-}}{}$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
1/20020*(4525662*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(
arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(2
524583*sqrt(-7)*x - 19580223*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*s
qrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(48125*x^12 + 422625
*x^10 + 1698200*x^8 + 3928870*x^6 + 5528301*x^4 + 4865781*x^2 + 4525662)*s
qrt(x^4 + 3*x^2 + 4))/x
```

3.357.6 Sympy [F]

$$\int (7+5x^2)^3 (4+3x^2+x^4)^{3/2}\,dx = \int \left((x^2-x+2)(x^2+x+2)\right)^{\frac{3}{2}} (5x^2+7)^3\,dx$$

input
```
integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3, x)
```

3.357.7 Maxima [F]

$$\int (7+5x^2)^3 (4+3x^2+x^4)^{3/2}\,dx = \int (x^4+3\,x^2+4)^{\frac{3}{2}} (5\,x^2+7)^3\,dx$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

3.357.8 Giac [F]

$$\int (7+5x^2)^3 (4+3x^2+x^4)^{3/2}\,dx = \int (x^4+3\,x^2+4)^{\frac{3}{2}} (5\,x^2+7)^3\,dx$$

input
```
integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2} \, dx = \int \left(5\,x^2+7\right)^3 \left(x^4+3\,x^2+4\right)^{3/2} dx$$

input
```
int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2),x)
```

output
```
int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.358 $\int (7+5x^2)^2 (4+3x^2+x^4)^{3/2}\, dx$

3.358.1 Optimal result

Integrand size = 24, antiderivative size = 226

$$\begin{aligned}\int (7+5x^2)^2 (4+3x^2+x^4)^{3/2}\, dx &= \frac{175346x\sqrt{4+3x^2+x^4}}{1155(2+x^2)} \\ &+ \frac{x(64533+18253x^2)\sqrt{4+3x^2+x^4}}{1155} + \frac{1}{693}x(6831+2240x^2)(4+3x^2+x^4)^{3/2} \\ &+ \frac{25}{11}x(4+3x^2+x^4)^{5/2} - \frac{175346\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1155\sqrt{4+3x^2+x^4}} \\ &+ \frac{4628\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{33\sqrt{4+3x^2+x^4}}\end{aligned}$$

output
```
1/693*x*(2240*x^2+6831)*(x^4+3*x^2+4)^(3/2)+25/11*x*(x^4+3*x^2+4)^(5/2)+17
5346/1155*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/1155*x*(18253*x^2+64533)*(x^4+3*
x^2+4)^(1/2)-175346/1155*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/co
s(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1
/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+4628/33*(
x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))
*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2
)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.358.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.14 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.57

$$\int (7+5x^2)^2 (4+3x^2+x^4)^{3/2} \, dx = \frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(1824876+2932753x^2+2435811x^4+1229714x^6+408480x^8+82075x^{10}+787}{}$$

input
```
Integrate[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(1824876 + 2932753*x^2 + 2435811*x^4 + 12
29714*x^6 + 408480*x^8 + 82075*x^10 + 7875*x^12) - 263019*Sqrt[2]*(3*I + S
qrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sq
rt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I
+ Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-34209*I + 8
7673*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*
I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/
(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(6930*Sqrt[(-I)/(-
3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.358.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1518, 1490, 1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)^2 (x^4+3x^2+4)^{3/2} \, dx$$

$$\Big\downarrow \text{1518}$$

$$\frac{1}{11}\int (320x^2+439)(x^4+3x^2+4)^{3/2} dx + \frac{25}{11}x(x^4+3x^2+4)^{5/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{11}\left(\frac{1}{21}\int \left(18253x^2+27768\right)\sqrt{x^4+3x^2+4}dx+\frac{1}{63}x\left(2240x^2+6831\right)\left(x^4+3x^2+4\right)^{3/2}\right)+\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}$$

$$\Big\downarrow \; 1490$$

$$\frac{1}{11}\left(\frac{1}{21}\left(\frac{1}{15}\int \frac{6\left(87673x^2+148614\right)}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+4}\left(18253x^2+64533\right)\right)+\frac{1}{63}x\left(2240x^2+6831\right)\left(x^4+3\right.\right.$$
$$\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\left(\frac{1}{21}\left(\frac{2}{5}\int \frac{87673x^2+148614}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+4}\left(18253x^2+64533\right)\right)+\frac{1}{63}x\left(2240x^2+6831\right)\left(x^4+3x^2+\right.\right.$$
$$\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}$$

$$\Big\downarrow \; 1511$$

$$\frac{1}{11}\left(\frac{1}{21}\left(\frac{2}{5}\left(323960\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-175346\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{5}x\sqrt{x^4+3x^2+4}\left(18253x^2+64533\right.\right.\right.$$
$$\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\left(\frac{1}{21}\left(\frac{2}{5}\left(323960\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-87673\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{5}x\sqrt{x^4+3x^2+4}\left(18253x^2+64533\right)\right.\right.$$
$$\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{11}\left(\frac{1}{21}\left(\frac{2}{5}\left(\frac{80990\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-87673\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{5}x\sqrt{\right.\right.$$
$$\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}$$

$$\Big\downarrow \; 1509$$

3.358. $\int \left(7+5x^2\right)^2\left(4+3x^2+x^4\right)^{3/2}\,dx$

$$\frac{1}{11}\left(\frac{1}{21}\left(\frac{2}{5}\left(\frac{80990\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-87673\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\,\text{a}\right.}{\sqrt{x^4+3x^2+4}}\right.\right.\right.\right.$$

$$\frac{25}{11}x\left(x^4+3x^2+4\right)^{5/2}$$

input
```
Int[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(25*x*(4 + 3*x^2 + x^4)^(5/2))/11 + ((x*(6831 + 2240*x^2)*(4 + 3*x^2 + x^4
)^(3/2))/63 + ((x*(64533 + 18253*x^2)*Sqrt[4 + 3*x^2 + x^4])/5 + (2*(-8767
3*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3
*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x
^2 + x^4]) + (80990*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*
EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]))/5)/21)/11
```

3.358.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

3.358. $\int \left(7+5x^2\right)^2\left(4+3x^2+x^4\right)^{3/2}\,dx$

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.358.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.09

method	result
risch	$\frac{x\left(7875x^8+58450x^6+201630x^4+391024x^2+456219\right)\sqrt{x^4+3x^2+4}}{3465}+\frac{396304\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\Bigg(\frac{x\sqrt{-6+2i}}{4}}{385\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
default	$\frac{1222x^5\sqrt{x^4+3x^2+4}}{21}+\frac{391024x^3\sqrt{x^4+3x^2+4}}{3465}+\frac{50691x\sqrt{x^4+3x^2+4}}{385}+\frac{396304\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\Bigg(x\sqrt{}}{385\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{1222x^5\sqrt{x^4+3x^2+4}}{21}+\frac{391024x^3\sqrt{x^4+3x^2+4}}{3465}+\frac{50691x\sqrt{x^4+3x^2+4}}{385}+\frac{396304\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\Bigg(x\sqrt{}}{385\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

input
```
int((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3465*x*(7875*x^8+58450*x^6+201630*x^4+391024*x^2+456219)*(x^4+3*x^2+4)^(
1/2)+396304/385/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*
(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6
+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-5611072/1155/(-6+2*I*7^(1/2
))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(
1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4
*(2+6*I*7^(1/2))^(1/2)))
```

3.358.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.61

$$\int (7+5x^2)^2 (4+3x^2+x^4)^{3/2}\,dx = \frac{526038\sqrt{2}(\sqrt{-7}x-3x)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2x}\right) \mid \frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-3\sqrt{2}(101039\sqrt{-7}}{}$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
1/13860*(526038*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(a
rcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 3*sqrt(2)*(
101039*sqrt(-7)*x - 748959*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqr
t(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(7875*x^10 + 58450*x^8
 + 201630*x^6 + 391024*x^4 + 456219*x^2 + 526038)*sqrt(x^4 + 3*x^2 + 4))/x
```

3.358.6 Sympy [F]

$$\int (7+5x^2)^2 (4+3x^2+x^4)^{3/2}\,dx = \int \left((x^2-x+2)(x^2+x+2)\right)^{\frac{3}{2}} (5x^2+7)^2\,dx$$

input
```
integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2, x)
```

3.358.7 Maxima [F]

$$\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2} \, dx = \int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^2 dx$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

3.358.8 Giac [F]

$$\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2} \, dx = \int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}} \left(5\,x^2+7\right)^2 dx$$

input
```
integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2} \, dx = \int \left(5\,x^2+7\right)^2 \left(x^4+3\,x^2+4\right)^{3/2} dx$$

input
```
int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2),x)
```

output
```
int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.359 $\int (7+5x^2)(4+3x^2+x^4)^{3/2}\,dx$

3.359.1 Optimal result

Integrand size = 22, antiderivative size = 207

$$\int (7+5x^2)(4+3x^2+x^4)^{3/2}\,dx = \frac{2798x\sqrt{4+3x^2+x^4}}{105(2+x^2)} + \frac{1}{105}x(1029+289x^2)\sqrt{4+3x^2+x^4} + \frac{1}{63}x(108+35x^2)(4+3x^2+x^4)^{3/2} - \frac{2798\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{105\sqrt{4+3x^2+x^4}} + \frac{74\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{4+3x^2+x^4}}$$

output
```
1/63*x*(35*x^2+108)*(x^4+3*x^2+4)^(3/2)+2798/105*x*(x^4+3*x^2+4)^(1/2)/(x^
2+2)+1/105*x*(289*x^2+1029)*(x^4+3*x^2+4)^(1/2)-2798/105*(x^2+2)*(cos(2*ar
ctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2
*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/
2)/(x^4+3*x^2+4)^(1/2)+74/3*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)
/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2
^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.359.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.38 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.69

$$\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(20988+28489x^2+19068x^4+7082x^6+1590x^8+175x^{10})-4197\sqrt{2}(3i+\sqrt{7})}{}$$

input
```
Integrate[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(20988 + 28489*x^2 + 19068*x^4 + 7082*x^6
 + 1590*x^8 + 175*x^10) - 4197*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7
] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + S
qrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt
[7])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-567*I + 1399*Sqrt[7])*Sqrt[(-3*I + Sqr
t[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I
+ Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - S
qrt[7])/(3*I + Sqrt[7])])/(630*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2
+ x^4])
```

3.359.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1490, 1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+7)\left(x^4+3x^2+4\right)^{3/2}dx$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{21}\int (289x^2+444)\sqrt{x^4+3x^2+4}dx+\frac{1}{63}x(35x^2+108)\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{21}\left(\frac{1}{15}\int \frac{6(1399x^2+2382)}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+4}(289x^2+1029)\right)+\frac{1}{63}x(35x^2+108)\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{21}\left(\frac{2}{5}\int \frac{1399x^2+2382}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{5}x\sqrt{x^4+3x^2+4}(289x^2+1029)\right)+\frac{1}{63}x(35x^2+108)\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \ 1511$$

$$\frac{1}{21}\left(\frac{2}{5}\left(5180\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-2798\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{5}x\sqrt{x^4+3x^2+4}(289x^2+1029)\right)+\frac{1}{63}x(35x^2+108)\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{21}\left(\frac{2}{5}\left(5180\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-1399\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{5}x\sqrt{x^4+3x^2+4}(289x^2+1029)\right)+\frac{1}{63}x(35x^2+108)\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \ 1416$$

$$\frac{1}{21}\left(\frac{2}{5}\left(\frac{1295\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-1399\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{5}x\sqrt{x^4+3x\right.$$
$$\frac{1}{63}x(35x^2+108)\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \ 1509$$

$$\frac{1}{21}\left(\frac{2}{5}\left(\frac{1295\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-1399\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt{x^4+3x^2+4}}\right.\right.\right.$$
$$\frac{1}{63}x(35x^2+108)\left(x^4+3x^2+4\right)^{3/2}$$

input
```
Int[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

3.359. $\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx$

output
```
(x*(108 + 35*x^2)*(4 + 3*x^2 + x^4)^(3/2))/63 + ((x*(1029 + 289*x^2)*Sqrt[
4 + 3*x^2 + x^4])/5 + (2*(-1399*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)) +
(Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[
x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) + (1295*Sqrt[2]*(2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 +
 3*x^2 + x^4]))/5)/21
```

3.359.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.359. $\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx$

3.359.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.16

method	result
risch	$\frac{x(175x^6+1065x^4+3187x^2+5247)\sqrt{x^4+3x^2+4}}{315}+\frac{6352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{35\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}-\frac{89}{}$
default	$\frac{71x^5\sqrt{x^4+3x^2+4}}{21}+\frac{3187x^3\sqrt{x^4+3x^2+4}}{315}+\frac{583x\sqrt{x^4+3x^2+4}}{35}+\frac{6352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\right.}{35\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{71x^5\sqrt{x^4+3x^2+4}}{21}+\frac{3187x^3\sqrt{x^4+3x^2+4}}{315}+\frac{583x\sqrt{x^4+3x^2+4}}{35}+\frac{6352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\right.}{35\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$

input
```
int((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*x*(175*x^6+1065*x^4+3187*x^2+5247)*(x^4+3*x^2+4)^(1/2)+6352/35/(-6+2
*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2
))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1
/4*(2+6*I*7^(1/2))^(1/2))-89536/105/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*
7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))
^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.64

$$\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx=\frac{8394\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-3\sqrt{2}\left(1607\sqrt{-7}x-\right.}{}$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
1/1260*(8394*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcs
in(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 3*sqrt(2)*(160
7*sqrt(-7)*x - 11967*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*s
qrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(175*x^8 + 1065*x^6 + 3187*x
^4 + 5247*x^2 + 8394)*sqrt(x^4 + 3*x^2 + 4))/x
```

3.359.6 Sympy [F]

$$\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx=\int\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\cdot\left(5x^2+7\right)\,dx$$

input
```
integrate((5*x**2+7)*(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7), x)
```

3.359.7 Maxima [F]

$$\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx=\int\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)\,dx$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

3.359.8 Giac [F]

$$\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx=\int\left(x^4+3\,x^2+4\right)^{\frac{3}{2}}\left(5\,x^2+7\right)\,dx$$

input
```
integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

3.359.9 Mupad [F(-1)]

Timed out.

$$\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx = \int \left(5\,x^2+7\right)\,\left(x^4+3\,x^2+4\right)^{3/2}\,dx$$

input
```
int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2),x)
```

output
```
int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.360 $\int (4+3x^2+x^4)^{3/2} \, dx$

3.360.1 Optimal result

Integrand size = 14, antiderivative size = 198

$$\int (4+3x^2+x^4)^{3/2} \, dx = \frac{138x\sqrt{4+3x^2+x^4}}{35(2+x^2)} + \frac{1}{35}x(49+9x^2)\sqrt{4+3x^2+x^4} + \frac{1}{7}x(4+3x^2+x^4)^{3/2} - \frac{138\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{4+3x^2+x^4}} + \frac{4\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}$$

output
```
1/7*x*(x^4+3*x^2+4)^(3/2)+138/35*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/35*x*(9*x
^2+49)*(x^4+3*x^2+4)^(1/2)-138/35*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)
^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2)))
,1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+
4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2
)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^
2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.360.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.78 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.73

$$\int (4+3x^2+x^4)^{3/2}\,dx = \frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(276+303x^2+161x^4+39x^6+5x^8)-69\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2i}{3i+\sqrt{7}}}}{}$$

input
```
Integrate[(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(276 + 303*x^2 + 161*x^4 + 39*x^6 + 5*x^8
) - 69*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + S
qrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*Arc
Sinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] +
Sqrt[2]*(-77*I + 69*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSi
nh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(70
*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.360.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1404, 1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^4+3x^2+4)^{3/2}\,dx$$

$$\Big\downarrow \text{1404}$$

$$\frac{3}{7}\int (3x^2+8)\sqrt{x^4+3x^2+4}dx+\frac{1}{7}x(x^4+3x^2+4)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{3}{7}\left(\frac{1}{15}\int \frac{2\left(69x^2+142\right)}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{15}x\sqrt{x^4+3x^2+4}\left(9x^2+49\right)\right)+\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}\left(\frac{2}{15}\int \frac{69x^2+142}{\sqrt{x^4+3x^2+4}}dx+\frac{1}{15}x\sqrt{x^4+3x^2+4}\left(9x^2+49\right)\right)+\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \text{1511}$$

$$\frac{3}{7}\left(\frac{2}{15}\left(280\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-138\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{15}x\sqrt{x^4+3x^2+4}\left(9x^2+49\right)\right)+\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}\left(\frac{2}{15}\left(280\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-69\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{15}x\sqrt{x^4+3x^2+4}\left(9x^2+49\right)\right)+\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \text{1416}$$

$$\frac{3}{7}\left(\frac{2}{15}\left(\frac{70\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-69\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{1}{15}x\sqrt{x^4+3x^2+\right.$$

$$\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

$$\Big\downarrow \text{1509}$$

$$\frac{3}{7}\left(\frac{2}{15}\left(\frac{70\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-69\left(\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)\right.}{\sqrt{x^4+3x^2+4}}\right.\right.\right.$$

$$\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}$$

input
```
Int[(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(x*(4 + 3*x^2 + x^4)^(3/2))/7 + (3*((x*(49 + 9*x^2)*Sqrt[4 + 3*x^2 + x^4])
/15 + (2*(-69*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/
Sqrt[4 + 3*x^2 + x^4]) + (70*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
 x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]))/15))
/7
```

3.360. $\int \left(4+3x^2+x^4\right)^{3/2}\,dx$

3.360.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.360. $\int (4+3x^2+x^4)^{3/2}\,dx$

3.360.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.19

method	result
risch	$\frac{x(5x^4+24x^2+69)\sqrt{x^4+3x^2+4}}{35}+\frac{1136\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{35\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}-\frac{4416\sqrt{1-\left(-\frac{3}{8}+\frac{i}{}\right.}}{}$
default	$\frac{x^5\sqrt{x^4+3x^2+4}}{7}+\frac{24x^3\sqrt{x^4+3x^2+4}}{35}+\frac{69x\sqrt{x^4+3x^2+4}}{35}+\frac{1136\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6}}{4}\right.}{35\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{x^5\sqrt{x^4+3x^2+4}}{7}+\frac{24x^3\sqrt{x^4+3x^2+4}}{35}+\frac{69x\sqrt{x^4+3x^2+4}}{35}+\frac{1136\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6}}{4}\right.}{35\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$

input
```
int((x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x*(5*x^4+24*x^2+69)*(x^4+3*x^2+4)^(1/2)+1136/35/(-6+2*I*7^(1/2))^(1/2
)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x
^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2
))^(1/2))-4416/35/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2
)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(El
lipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(
1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.360.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.65

$$\int (4+3x^2+x^4)^{3/2}\,dx = \frac{138\sqrt{2}(\sqrt{-7}x-3\,x)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8})-\sqrt{2}(67\sqrt{-7}x-627\,x}{1}$$

input
```
integrate((x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
1/140*(138*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin
(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(67*sqrt
(-7)*x - 627*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt
(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 4*(5*x^6 + 24*x^4 + 69*x^2 + 138)*sqrt
(x^4 + 3*x^2 + 4))/x
```

3.360.6 Sympy [F]

$$\int \left(4+3x^2+x^4\right)^{3/2} dx = \int \left(x^4+3x^2+4\right)^{\frac{3}{2}} dx$$

input
```
integrate((x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral((x**4 + 3*x**2 + 4)**(3/2), x)
```

3.360.7 Maxima [F]

$$\int \left(4+3x^2+x^4\right)^{3/2} dx = \int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}} dx$$

input
```
integrate((x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

3.360.8 Giac [F]

$$\int \left(4+3x^2+x^4\right)^{3/2} dx = \int \left(x^4+3\,x^2+4\right)^{\frac{3}{2}} dx$$

input
```
integrate((x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

3.360.9 Mupad [F(-1)]

Timed out.

$$\int (4 + 3x^2 + x^4)^{3/2} \, dx = \int (x^4 + 3\,x^2 + 4)^{3/2} \, dx$$

input
```
int((3*x^2 + x^4 + 4)^(3/2),x)
```

output
```
int((3*x^2 + x^4 + 4)^(3/2), x)
```

3.361 $\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2} dx$

3.361.1 Optimal result

Integrand size = 24, antiderivative size = 284

$$\begin{aligned}\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2} dx &= \frac{94x\sqrt{4+3x^2+x^4}}{125(2+x^2)} \\ &+ \frac{1}{75}x(11+3x^2)\sqrt{4+3x^2+x^4} + \frac{44}{125}\sqrt{\frac{11}{35}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) \\ &- \frac{94\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{4+3x^2+x^4}} \\ &+ \frac{54\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{125\sqrt{4+3x^2+x^4}} \\ &+ \frac{4114\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{13125\sqrt{4+3x^2+x^4}}\end{aligned}$$

output
```
44/4375*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+94/125*x*(x
^4+3*x^2+4)^(1/2)/(x^2+2)+1/75*x*(3*x^2+11)*(x^4+3*x^2+4)^(1/2)-94/125*(x^
2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*E
llipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/
(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+54/125*(x^2+2)*(cos(2*arctan(1/2*x*2^
(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x
*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2
+4)^(1/2)+4114/13125*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*
arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*
2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.361.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.56 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.68

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \frac{350\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(44+45x^2+20x^4+3x^6)-4935\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i}{}}$$

input
```
Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

output
```
(350*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(44 + 45*x^2 + 20*x^4 + 3*x^6) - 4935*S
qrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]
*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqr
t[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*Sqrt[2
]*(-241*I + 705*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7]
)]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[S
qrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (5808*
I)*Sqrt[2]*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I +
 Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14,
I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7]
)])/(26250*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.361.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1530, 27, 2207, 27, 2207, 27, 1511, 27, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^4+3x^2+4\right)^{3/2}}{5x^2+7}dx$$

$$\Big\downarrow \text{ 1530}$$

$$\frac{1936\int \frac{17(x^2+2)}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx}{6375}-\frac{\int -\frac{17(75x^6+345x^4+792x^2+304)}{\sqrt{x^4+3x^2+4}}dx}{6375}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1936}{375}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+\frac{1}{375}\int \frac{75x^6+345x^4+792x^2+304}{\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{1936}{375}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+\frac{1}{375}\left(\frac{1}{5}\int \frac{5\left(165x^4+612x^2+304\right)}{\sqrt{x^4+3x^2+4}}dx+15\sqrt{x^4+3x^2+4}x^3\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1936}{375}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+\frac{1}{375}\left(\int \frac{165x^4+612x^2+304}{\sqrt{x^4+3x^2+4}}dx+15\sqrt{x^4+3x^2+4}x^3\right)$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{1936}{375}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+\frac{1}{375}\left(\frac{1}{3}\int \frac{18\left(47x^2+14\right)}{\sqrt{x^4+3x^2+4}}dx+55\sqrt{x^4+3x^2+4}x+15\sqrt{x^4+3x^2+4}x^3\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1936}{375}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+\frac{1}{375}\left(6\int \frac{47x^2+14}{\sqrt{x^4+3x^2+4}}dx+55\sqrt{x^4+3x^2+4}x+15\sqrt{x^4+3x^2+4}x^3\right)$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{1936}{375}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+$$
$$\frac{1}{375}\left(6\left(108\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-94\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)+55\sqrt{x^4+3x^2+4}x+15\sqrt{x^4+3x^2+4}x^3\right)$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1936}{375}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+$$
$$\frac{1}{375}\left(6\left(108\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-47\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+55\sqrt{x^4+3x^2+4}x+15\sqrt{x^4+3x^2+4}x^3\right)$$

$$\Big\downarrow \textbf{1416}$$

$$\frac{1}{375}\left(6\left(\frac{27\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-47\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+55\sqrt{x^4+3x^2+4}\right.$$
$$\frac{1936}{375}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \textbf{1509}$$

$$\frac{1936}{375}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+$$
$$\frac{1}{375}\left(6\left(\frac{27\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-47\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)\right.}{\sqrt{x^4+3x^2+4}}\right.\right.\right.$$

$$\Big\downarrow \textbf{2220}$$

$$\frac{1936}{375}\left(\frac{3\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{4\sqrt{385}}+\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+4}}\right)+$$
$$\frac{1}{375}\left(6\left(\frac{27\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-47\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)\right.}{\sqrt{x^4+3x^2+4}}\right.\right.\right.$$

input
```
Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

3.361. $\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx$

output
```
(55*x*Sqrt[4 + 3*x^2 + x^4] + 15*x^3*Sqrt[4 + 3*x^2 + x^4] + 6*(-47*(-((x*
Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x
^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4
]) + (27*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2
*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]))/375 + (1936*((3*ArcTan[(
2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(4*Sqrt[385]) + (17*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1
/8])/(140*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])))/375
```

3.361.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.361. $\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2} \, dx$

rule 1530
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[-(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(e^(2*p)*(c*d^2 - a*e^2))
Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2)/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x] + Simp[1/(e^(2*p)*(c*d^2 - a*e^2))   Int[(1/Sqrt
[a + b*x^2 + c*x^4])*ExpandToSum[(e^(2*p)*(c*d^2 - a*e^2)*(a + b*x^2 + c*x^
4)^(p + 1/2) + (c*d^2 - b*d*e + a*e^2)^(p + 1/2)*(a*d*Rt[c/a, 2] + a*e + (c
*d + a*e*Rt[c/a, 2])*x^2))/(d + e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p - 1/
2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.361.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.19

method	result
risch	$\frac{x(3x^2+11)\sqrt{x^4+3x^2+4}}{75}+\frac{9424\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{1875\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{3008\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}}{}$
default	$\frac{x^3\sqrt{x^4+3x^2+4}}{25}+\frac{11x\sqrt{x^4+3x^2+4}}{75}+\frac{9424\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{1875\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{3008\sqrt{1+\frac{3x^2}{8}-}}{125}$
elliptic	$\frac{x^3\sqrt{x^4+3x^2+4}}{25}+\frac{11x\sqrt{x^4+3x^2+4}}{75}+\frac{9424\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{1875\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{3008\sqrt{1+\frac{3x^2}{8}-}}{125}$

input
```
int((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

output
```
1/75*x*(3*x^2+11)*(x^4+3*x^2+4)^(1/2)+9424/1875/(-6+2*I*7^(1/2))^(1/2)*(1-
(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*
x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1
/2))-3008/125/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1
-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(Ellipt
icF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*
x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+1936/4375/(-3/8+1/8*I
*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7
^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,
-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^
(1/2))
```

3.361.5 Fricas [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx=\int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

output
```
integral((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

3.361.6 Sympy [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}{5x^2+7}\,dx$$

input
```
integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7),x)
```

output
```
Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7), x)
```

3.361.7 Maxima [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

3.361.8 Giac [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = \int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{5\,x^2+7}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

3.361.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx=\int \frac{(x^4+3\,x^2+4)^{3/2}}{5\,x^2+7}\,dx$$

input
```
int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7),x)
```

output
```
int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7), x)
```

3.362 $\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx$

3.362.1 Optimal result

Integrand size = 24, antiderivative size = 305

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \frac{1}{75}x\sqrt{4+3x^2+x^4}+\frac{4x\sqrt{4+3x^2+x^4}}{175(2+x^2)}+\frac{22x\sqrt{4+3x^2+x^4}}{175(7+5x^2)}+\frac{13}{350}\sqrt{\frac{11}{35}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)-\frac{4\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{175\sqrt{4+3x^2+x^4}}+\frac{4\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{175\sqrt{4+3x^2+x^4}}+\frac{2431(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{36750\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
13/12250*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/75*x*(x^
4+3*x^2+4)^(1/2)+4/175*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+22/175*x*(x^4+3*x^2+4
)^(1/2)/(5*x^2+7)+2431/73500*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2
)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/
280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/
2)-4/175*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x
*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x
^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+4/175*(x^2+2)*(cos(2*arct
an(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*a
rctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)
/(x^4+3*x^2+4)^(1/2)
```

3.362.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.48 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.01

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \frac{\frac{175x(23+7x^2)(4+3x^2+x^4)}{7+5x^2} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(105(3-i\sqrt{7})\,E\left(i\right.\right.}{}$$

input
```
Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

output
```
((175*x*(23 + 7*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2) - I*Sqrt[6 + (2*I)*Sqr
t[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + S
qrt[7])]*(105*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt
[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(158 + (15*I)*Sqrt[7])*Elli
pticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + S
qrt[7])] + 429*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-
3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(18375*Sqrt[4 + 3*x
^2 + x^4])
```

3.362.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4+3x^2+4)^{3/2}}{(5x^2+7)^2}\,dx$$

$$\Big\downarrow \text{1556}$$

$$\int \left(\frac{x^4}{25\sqrt{x^4+3x^2+4}} + \frac{16x^2}{125\sqrt{x^4+3x^2+4}} + \frac{88}{625(5x^2+7)\sqrt{x^4+3x^2+4}} + \frac{1936}{625(5x^2+7)^2\sqrt{x^4+3x^2+4}} + \frac{}{62}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{13}{350}\sqrt{\frac{11}{35}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)+\frac{4\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}}-\frac{4\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}}+\frac{187\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}+\frac{6919(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{183750\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{4\sqrt{x^4+3x^2+4}x}{175(x^2+2)}+\frac{22\sqrt{x^4+3x^2+4}x}{175(5x^2+7)}+\frac{1}{75}\sqrt{x^4+3x^2+4}x$$

input
```
Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

output
```
(x*Sqrt[4 + 3*x^2 + x^4])/75 + (4*x*Sqrt[4 + 3*x^2 + x^4])/(175*(2 + x^2))
 + (22*x*Sqrt[4 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) + (13*Sqrt[11/35]*ArcTan
[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/350 - (4*Sqrt[2]*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(175*
Sqrt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) +
 (6919*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*
ArcTan[x/Sqrt[2]], 1/8])/(183750*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (187*Sqr
t[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*Ar
cTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4])
```

3.362.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.362.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.13

method	result
risch	$\frac{\sqrt{x^4+3x^2+4}\,x(7x^2+23)}{525x^2+735}+\frac{232\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{375\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{128\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\cdots}{\cdots}$
default	$\frac{22x\sqrt{x^4+3x^2+4}}{175(5x^2+7)}+\frac{x\sqrt{x^4+3x^2+4}}{75}+\frac{232\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{375\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{128\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{\cdots}}{8}}\cdots}{175\sqrt{\cdots}}$
elliptic	$\frac{22x\sqrt{x^4+3x^2+4}}{175(5x^2+7)}+\frac{x\sqrt{x^4+3x^2+4}}{75}+\frac{232\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{375\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{128\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{\cdots}}{8}}\cdots}{175\sqrt{\cdots}}$

input
```
int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

output
```
1/105*(x^4+3*x^2+4)^(1/2)*x*(7*x^2+23)/(5*x^2+7)+232/375/(-6+2*I*7^(1/2))^
(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2
)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^
(1/2))^(1/2))-128/175/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^
(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))
*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-Ellipt
icE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+286/6125/(-3/
8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*
I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(
1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^
(1/2))^(1/2))
```

3.362.5 Fricas [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx=\int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

output
```
integral((x^4 + 3*x^2 + 4)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

3.362.6 Sympy [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{((x^2-x+2)\,(x^2+x+2))^{\frac{3}{2}}}{(5x^2+7)^2}\,dx$$

input
```
integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**2,x)
```

output
```
Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**2, x)
```

3.362.7 Maxima [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

3.362.8 Giac [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{(5\,x^2+7)^2}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

3.362.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \frac{(x^4+3\,x^2+4)^{3/2}}{(5\,x^2+7)^2}\,dx$$

input
```
int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^2,x)
```

output
```
int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

3.363 $\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3} dx$

3.363.1 Optimal result

Integrand size = 24, antiderivative size = 440

$$\begin{aligned}\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3} dx &= \frac{9x\sqrt{4+3x^2+x^4}}{1960(2+x^2)} + \frac{11x\sqrt{4+3x^2+x^4}}{175(7+5x^2)^2} \\ &+ \frac{167x\sqrt{4+3x^2+x^4}}{9800(7+5x^2)} + \frac{1347\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{7840\sqrt{385}} \\ &+ \frac{111(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{24500\sqrt{2}\sqrt{4+3x^2+x^4}} \\ &- \frac{6\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{875\sqrt{4+3x^2+x^4}} \\ &- \frac{817(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{91875\sqrt{2}\sqrt{4+3x^2+x^4}} \\ &- \frac{22\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{13125\sqrt{4+3x^2+x^4}} \\ &+ \frac{7633(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{274400\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

output
```
1347/3018400*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+9/1960
*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+11/175*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+16
7/9800*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-9/1960*(x^2+2)*(cos(2*arctan(1/2*x*
2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2
*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x
^2+4)^(1/2)-3/490*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arc
tan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((
x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+7633/548800*(x^2
+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*El
lipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(
x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.363.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.55 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.70

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \frac{\frac{140x(357+167x^2)(4+3x^2+x^4)}{(7+5x^2)^2} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\Big(315\big(3-i\sqrt{7}\big)\,E}{}$$

input
```
Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

output
```
((140*x*(357 + 167*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 - I*Sqrt[6 + (2*I
)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*
I + Sqrt[7])]*(315*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
 Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(103 + (45*I)*Sqrt[7])
*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*
I + Sqrt[7])] + 2694*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2
*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(274400*Sqrt[
4 + 3*x^2 + x^4])
```

3.363.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4+3x^2+4)^{3/2}}{(5x^2+7)^3}\,dx$$

$$\Big\downarrow \text{1556}$$

$$\int \left(\frac{x^2}{125\sqrt{x^4+3x^2+4}}+\frac{89}{625\,(5x^2+7)\sqrt{x^4+3x^2+4}}+\frac{88}{625\,(5x^2+7)^2\sqrt{x^4+3x^2+4}}+\frac{1936}{625\,(5x^2+7)^3\sqrt{x^4+3}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1347\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{7840\sqrt{385}}-\frac{22\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}-\frac{817(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{91875\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{6\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{875\sqrt{x^4+3x^2+4}}+\frac{111(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{24500\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{7633(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{274400\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{9\sqrt{x^4+3x^2+4}x}{1960\,(x^2+2)}+\frac{167\sqrt{x^4+3x^2+4}x}{9800\,(5x^2+7)}+\frac{11\sqrt{x^4+3x^2+4}x}{175\,(5x^2+7)^2}$$

input
```
Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

output
```
(9*x*Sqrt[4 + 3*x^2 + x^4])/(1960*(2 + x^2)) + (11*x*Sqrt[4 + 3*x^2 + x^4]
)/(175*(7 + 5*x^2)^2) + (167*x*Sqrt[4 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) +
 (1347*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(7840*Sqrt[385]) +
 (111*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/S
qrt[2]], 1/8])/(24500*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (6*Sqrt[2]*(2 + x^2
)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])
/(875*Sqrt[4 + 3*x^2 + x^4]) - (817*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(91875*Sqrt[2]*Sqrt[4 + 3*x^2
 + x^4]) - (22*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellip
ticF[2*ArcTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4]) + (7633*(2 +
 x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sq
rt[2]], 1/8])/(274400*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

3.363.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.363.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.79

method	result
risch	$\frac{\sqrt{x^4+3x^2+4}\,x(167x^2+357)}{1960(5x^2+7)^2}+\frac{17\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{350\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{36\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}}{}$
default	$\frac{11x\sqrt{x^4+3x^2+4}}{175(5x^2+7)^2}+\frac{167x\sqrt{x^4+3x^2+4}}{9800(5x^2+7)}+\frac{17\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{350\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{36\sqrt{1+\frac{3x^2}{8}-\frac{ix^2}{8}}}{245}$
elliptic	$\frac{11x\sqrt{x^4+3x^2+4}}{175(5x^2+7)^2}+\frac{167x\sqrt{x^4+3x^2+4}}{9800(5x^2+7)}+\frac{17\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{350\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{36\sqrt{1+\frac{3x^2}{8}-\frac{ix^2}{8}}}{245}$

input
```
int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1960*(x^4+3*x^2+4)^(1/2)*x*(167*x^2+357)/(5*x^2+7)^2+17/350/(-6+2*I*7^(1
/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)
^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6
*I*7^(1/2))^(1/2))-36/245/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x
^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1
/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-El
lipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+1347/6860
0/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^
2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1
/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/
8*I*7^(1/2))^(1/2))
```

3.363.5 Fricas [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

output
```
integral((x^4 + 3*x^2 + 4)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

3.363. $\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx$

3.363.6 Sympy [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}{(5x^2+7)^3}\,dx$$

input
```
integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**3,x)
```

output
```
Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**3, x)
```

3.363.7 Maxima [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

3.363.8 Giac [F]

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(x^4+3\,x^2+4)^{\frac{3}{2}}}{(5\,x^2+7)^3}\,dx$$

input
```
integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \frac{(x^4+3\,x^2+4)^{3/2}}{(5\,x^2+7)^3}\,dx$$

input
```
int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^3,x)
```

output
```
int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

3.364 $\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx$

3.364.1 Optimal result

Integrand size = 24, antiderivative size = 187

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}-\frac{15x\sqrt{4+3x^2+x^4}}{2+x^2} + \frac{15\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}} + \frac{13(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
75*x*(x^4+3*x^2+4)^(1/2)+25*x^3*(x^4+3*x^2+4)^(1/2)-15*x*(x^4+3*x^2+4)^(1/
2)/(x^2+2)+13/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arcta
n(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^
4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+15*(x^2+2)*(cos(2*
arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin
(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(
1/2)/(x^4+3*x^2+4)^(1/2)
```

3.364.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.34 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.80

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{100\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(12+13x^2+6x^4+x^6)+15\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\left(i\text{arcsinh}\left(\sqrt{-\cdots}\right.\right.}{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\cdots}$$

input
```
Integrate[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(100*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(12 + 13*x^2 + 6*x^4 + x^6) + 15*Sqrt[2
]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt
[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2
*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(131*
I + 15*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(
3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I
)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(4*Sqrt[(-I)/(-3
*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.364.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1518, 27, 2207, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^3}{\sqrt{x^4+3x^2+4}}\,dx$$
$$\Big\downarrow \ 1518$$
$$\frac{1}{5}\int \frac{5(225x^4+435x^2+343)}{\sqrt{x^4+3x^2+4}}dx+25\sqrt{x^4+3x^2+4}x^3$$
$$\Big\downarrow \ 27$$

$$\int \frac{225x^4+435x^2+343}{\sqrt{x^4+3x^2+4}}dx+25\sqrt{x^4+3x^2+4}x^3$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{3}\int \frac{3\left(43-15x^2\right)}{\sqrt{x^4+3x^2+4}}dx+75\sqrt{x^4+3x^2+4}x+25\sqrt{x^4+3x^2+4}x^3$$

$$\Big\downarrow \text{27}$$

$$\int \frac{43-15x^2}{\sqrt{x^4+3x^2+4}}dx+75\sqrt{x^4+3x^2+4}x+25\sqrt{x^4+3x^2+4}x^3$$

$$\Big\downarrow \text{1511}$$

$$13\int \frac{1}{\sqrt{x^4+3x^2+4}}dx+30\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx+75\sqrt{x^4+3x^2+4}x+25\sqrt{x^4+3x^2+4}x^3$$

$$\Big\downarrow \text{27}$$

$$13\int \frac{1}{\sqrt{x^4+3x^2+4}}dx+15\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+75\sqrt{x^4+3x^2+4}x+25\sqrt{x^4+3x^2+4}x^3$$

$$\Big\downarrow \text{1416}$$

$$15\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+\frac{13\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}+75\sqrt{x^4+3x^2+4}x+25\sqrt{x^4+3x^2+4}x^3$$

$$\Big\downarrow \text{1509}$$

$$\frac{13\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}+15\left(\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2+4}}{x^2+2}\right)+75\sqrt{x^4+3x^2+4}x+25\sqrt{x^4+3x^2+4}x^3$$

input
```
Int[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
75*x*Sqrt[4 + 3*x^2 + x^4] + 25*x^3*Sqrt[4 + 3*x^2 + x^4] + 15*(-((x*Sqrt[
4 + 3*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(
2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) +
(13*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqr
t[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

3.364. $\int \frac{\left(7+5x^2\right)^3}{\sqrt{4+3x^2+x^4}}\,dx$

3.364.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.364. $\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx$

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.364.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.22

method	result
risch	$25x(x^2+3)\sqrt{x^4+3x^2+4}+\frac{172\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{480\sqrt{1-\left(-\frac{3}{8}\right.}}{}$
default	$\frac{172\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+25x^3\sqrt{x^4+3x^2+4}+75x\sqrt{x^4+3x^2+4}$
elliptic	$\frac{172\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+25x^3\sqrt{x^4+3x^2+4}+75x\sqrt{x^4+3x^2+4}$

input
```
int((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
25*x*(x^2+3)*(x^4+3*x^2+4)^(1/2)+172/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I
*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2
)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+480/(-
6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(
1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*
I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2
))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.66

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = -\frac{60\sqrt{2}(\sqrt{-7}x-3x)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2x}\right) \mid \frac{3}{8}\sqrt{-7}+\frac{1}{8}) - \sqrt{2}(103\sqrt{-7}x-51x)\sqrt{\sqrt{-7}-}}{16x}$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*(60*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(
1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(103*sqrt
(-7)*x - 51*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(
-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 80*(5*x^4 + 15*x^2 - 3)*sqrt(x^4 + 3*x^
2 + 4))/x
```

3.364.6 Sympy [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5x^2+7)^3}{\sqrt{(x^2-x+2)(x^2+x+2)}}\,dx$$

input
```
integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral((5*x**2 + 7)**3/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

3.364.7 Maxima [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5x^2+7)^3}{\sqrt{x^4+3x^2+4}}\,dx$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

3.364.8 Giac [F]

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(1/2),x)
```

output
```
int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(1/2), x)
```

3.365 $\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}} \, dx$

3.365.1 Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}} \, dx = \frac{25}{3}x\sqrt{4+3x^2+x^4} + \frac{20x\sqrt{4+3x^2+x^4}}{2+x^2} - \frac{20\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}} + \frac{167(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{6\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
25/3*x*(x^4+3*x^2+4)^(1/2)+20*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+167/12*(x^2+2)
*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellip
ticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(
1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-20*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^
2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2)
)),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2
)
```

3.365.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.31 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.95

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{50\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(4+3x^2+x^4)-30\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-}{3i-}\right.}{6\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3}}$$

input
```
Integrate[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(50*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 30*Sqrt[2]*(3*I + Sq
rt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqr
t[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
 Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(43*I + 30*Sqrt[
7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7
] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(6*Sqrt[(-I)/(-3*I + Sqrt[7]
)]*Sqrt[4 + 3*x^2 + x^4])
```

3.365.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1518, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^2}{\sqrt{x^4+3x^2+4}}\,dx$$
$$\Big\downarrow \ 1518$$
$$\frac{1}{3}\int \frac{60x^2+47}{\sqrt{x^4+3x^2+4}}dx+\frac{25}{3}\sqrt{x^4+3x^2+4}x$$
$$\Big\downarrow \ 1511$$

$$\frac{1}{3}\left(167\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-120\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)+\frac{25}{3}\sqrt{x^4+3x^2+4}x$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(167\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-60\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{25}{3}\sqrt{x^4+3x^2+4}x$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{3}\left(\frac{167(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}-60\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{25}{3}\sqrt{x^4+3x^2+4}x$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{3}\left(\frac{167(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}-60\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\right.\right.$$
$$\frac{25}{3}\sqrt{x^4+3x^2+4}x$$

input
```
Int[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(25*x*Sqrt[4 + 3*x^2 + x^4])/3 + (-60*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^
2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*A
rcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) + (167*(2 + x^2)*Sqrt[(4 +
3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*
Sqrt[4 + 3*x^2 + x^4]))/3
```

3.365.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.365. $\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx$

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.365.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.32

method	result
default	$\frac{188\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{25x\sqrt{x^4+3x^2+4}}{3}-\frac{640\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\cdots\right.}}{\cdots}$
risch	$\frac{188\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{25x\sqrt{x^4+3x^2+4}}{3}-\frac{640\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\cdots\right.}}{\cdots}$
elliptic	$\frac{188\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{25x\sqrt{x^4+3x^2+4}}{3}-\frac{640\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\cdots\right.}}{\cdots}$

input
```
int((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
188/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1
/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/
2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+25/3*x*(x^4+3*x^2+4)^(1/2)-640/(-6+2*
I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2)
)*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^
(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{240\sqrt{2}\left(\sqrt{-7}x-3\,x\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2\,x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-\sqrt{2}\left(193\sqrt{-7}x-861\,x\right)\sqrt{\sqrt{-7}-}}{48\,x}$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
1/48*(240*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(
1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(193*sqrt
(-7)*x - 861*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt
(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 80*sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 12))
/x
```

3.365.6 Sympy [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5x^2+7)^2}{\sqrt{(x^2-x+2)\,(x^2+x+2)}}\,dx$$

input
```
integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral((5*x**2 + 7)**2/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

3.365.7 Maxima [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

3.365.8 Giac [F]

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{(5\,x^2+7)^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(1/2),x)
```

output
```
int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(1/2), x)
```

3.366 $\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx$

3.366.1 Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = \frac{5x\sqrt{4+3x^2+x^4}}{2+x^2} - \frac{5\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}} + \frac{17(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
5*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+17/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))
^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2
))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/
2)-5*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(
1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3
*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.366.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.42

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = \frac{\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(-5\left(3i+\sqrt{7}\right)E\left(i\text{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\left(i+5\sqrt{7}\right)\text{EllipticF}\left(i\text{arcs}\right.\right.}{2\sqrt{2}\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

input
```
Integrate[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
(Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7
])]*(-5*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*
x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (I + 5*Sqrt[7])*EllipticF[I*ArcSinh
[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(2*S
qrt[2]*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.366.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+7}{\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{ 1511}$$

$$17\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-10\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{ 27}$$

$$17\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-5\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}-5\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}-5\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2+4}}{x^2+2}\right)$$

input
```
Int[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
-5*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 +
3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*
x^2 + x^4]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[
2*ArcTan[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

3.366.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.366.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.38

method	result
default	$\frac{28\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{160\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\left(F\left(\frac{x\sqrt{-6}}{4}\right.\right.}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x}}$
elliptic	$\frac{28\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{160\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\left(F\left(\frac{x\sqrt{-6}}{4}\right.\right.}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x}}$

input
```
int((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
28/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*
I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))
^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-160/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*
I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/
2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2
))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
))
```

3.366.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = \frac{20\sqrt{2}(\sqrt{-7}x-3x)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2x}\right) \mid \frac{3}{8}\sqrt{-7}+\frac{1}{8})-\sqrt{2}(13\sqrt{-7}x-81x)\sqrt{\sqrt{-7}-3}I}{16x}$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
1/16*(20*sqrt(2)*(sqrt(-7)*x - 3*x)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1
/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - sqrt(2)*(13*sqrt(-
7)*x - 81*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7
) - 3)/x), 3/8*sqrt(-7) + 1/8) + 80*sqrt(x^4 + 3*x^2 + 4))/x
```

3.366.6 Sympy [F]

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{5x^2+7}{\sqrt{(x^2-x+2)(x^2+x+2)}}\,dx$$

input
```
integrate((5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral((5*x**2 + 7)/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

3.366.7 Maxima [F]

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

3.366.8 Giac [F]

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(1/2),x)
```

output
```
int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(1/2), x)
```

3.367 $\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$

3.367.1 Optimal result

Integrand size = 14, antiderivative size = 64

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx = \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
1/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1
/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(
x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.367.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx = -\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}x\right),\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

input
```
Integrate[1/Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
((-I)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt[7])
]*EllipticF[I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7])]*x], (-3 - I*Sqrt[7])/(-3 +
 I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^(-1)]*Sqrt[4 + 3*x^2 + x^4])
```

3.367.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \; 1416$$

$$\frac{\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}$$

input
```
Int[1/Sqrt[4 + 3*x^2 + x^4],x]
```

output
```
((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2
]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

3.367.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.367.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$	85
elliptic	$\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$	85

input
```
int(1/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I
*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^
(1/2),1/4*(2+6*I*7^(1/2))^(1/2))
```

3.367.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx = -\frac{1}{16}\sqrt{2}(\sqrt{-7}+3)\sqrt{\sqrt{-7}-3}F(\arcsin\left(\frac{1}{4}\sqrt{2}x\sqrt{\sqrt{-7}-3}\right) \mid \frac{3}{8}\sqrt{-7}+\frac{1}{8})$$

input
```
integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*sqrt(2)*(sqrt(-7) + 3)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/4*sqrt
(2)*x*sqrt(sqrt(-7) - 3)), 3/8*sqrt(-7) + 1/8)
```

3.367.6 Sympy [F]

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3x^2+4}}\,dx$$

input
```
integrate(1/(x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral(1/sqrt(x**4 + 3*x**2 + 4), x)
```

3.367.7 Maxima [F]

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

3.367.8 Giac [F]

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx=\int \frac{1}{\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
int(1/(3*x^2 + x^4 + 4)^(1/2),x)
```

output
```
int(1/(3*x^2 + x^4 + 4)^(1/2), x)
```

3.368 $\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$

3.368.1 Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx = \frac{1}{4}\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right) - \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{6\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{17(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{84\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
1/308*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/12*(x^2+2)*
(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellipt
icF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1
/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+17/168*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)
))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(
1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*
x^2+4)^(1/2)
```

3.368.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx = -\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,\text{EllipticPi}\left(-\frac{5}{14}\left(-3-i\sqrt{7}\right),i\text{arcsinh}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}x\right),\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{7\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

input
```
Integrate[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

output
```
((-1/7*I)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt
[7])]*EllipticPi[(-5*(-3 - I*Sqrt[7]))/14, I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[
7])]*x], (-3 - I*Sqrt[7])/(-3 + I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7
])^(-1)]*Sqrt[4 + 3*x^2 + x^4])
```

3.368.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1540, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{1540}$$

$$\frac{10}{3}\int \frac{x^2+2}{2(5x^2+7)\sqrt{x^4+3x^2+4}}dx - \frac{1}{3}\int \frac{1}{\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{3}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx - \frac{1}{3}\int \frac{1}{\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{5}{3}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{2220}$$

$$\frac{5}{3}\left(\frac{3\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{4\sqrt{385}}+\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+4}}\right)-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}$$

input
```
Int[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

output
```
-1/6*((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/S
qrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (5*((3*ArcTan[(2*Sqrt[11/
35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(4*Sqrt[385]) + (17*(2 + x^2)*Sqrt[(4 + 3*x
^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(140*
Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])))/3
```

3.368.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.368.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\sqrt{1+\frac{3x^2}{8}-\frac{i x^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{i x^2\sqrt{7}}{8}}\,\Pi\left(\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,x,-\frac{5}{7\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)},\frac{\sqrt{-\frac{3}{8}-\frac{i\sqrt{7}}{8}}}{\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}}\right)}{7\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,\sqrt{x^4+3x^2+4}}$	107
elliptic	$\frac{\sqrt{1+\frac{3x^2}{8}-\frac{i x^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{i x^2\sqrt{7}}{8}}\,\Pi\left(\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,x,-\frac{5}{7\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)},\frac{\sqrt{-\frac{3}{8}-\frac{i\sqrt{7}}{8}}}{\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}}\right)}{7\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,\sqrt{x^4+3x^2+4}}$	107

input
```
int(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*
x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^
(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+
1/8*I*7^(1/2))^(1/2))
```

3.368.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^6 + 22*x^4 + 41*x^2 + 28), x)
```

3.368.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2-x+2)(x^2+x+2)}\,(5x^2+7)}\,dx$$

input
```
integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)), x)
```

3.368.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

3.368.8 Giac [F]

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{(5\,x^2+7)\,\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2)),x)
```

output
```
int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2)), x)
```

3.369 $\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx$

3.369.1 Optimal result

Integrand size = 24, antiderivative size = 286

$$\begin{aligned}\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx &= -\frac{5x\sqrt{4+3x^2+x^4}}{616(2+x^2)}+\frac{25x\sqrt{4+3x^2+x^4}}{616(7+5x^2)}+\frac{37\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{2464}\\
&+\frac{5(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{308\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&-\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{42\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&+\frac{629(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{51744\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

output
```
37/189728*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-5/616*x*(
x^4+3*x^2+4)^(1/2)/(x^2+2)+25/616*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+5/616*(x
^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*
EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)
/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-1/84*(x^2+2)*(cos(2*arctan(1/2*x*2^(
1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*
2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+
4)^(1/2)+629/103488*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*a
rctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2
^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.369.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.75 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.68

$$\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{700\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(4+3x^2+x^4)+35(3i+\sqrt{7})(7+5x^2)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{2i}{-3i+}}\right.\right.\right.}{}$$

input
```
Integrate[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```

output
```
(700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I + Sqrt[7])*
(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3
*I + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I
 - Sqrt[7])/(3*I + Sqrt[7])] - EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqr
t[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]) + (98*I)*(7 + 5*x^2)*Sqrt[2 -
 ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*Ellip
ticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sq
rt[7])] - (74*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1
 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSi
nh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(17
248*Sqrt[(-I)/(-3*I + Sqrt[7])]*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])
```

3.369.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1551, 2232, 27, 1509, 2226, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^2\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{1551}$$

$$\frac{25x\sqrt{x^4+3x^2+4}}{616(5x^2+7)}-\frac{1}{616}\int \frac{25x^4+70x^2+12}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{2232}$$

$$\frac{1}{616}\left(10\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx-\frac{1}{5}\int \frac{5(85x^2+82)}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx\right)+\frac{25\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{616}\left(5\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx-\int \frac{85x^2+82}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx\right)+\frac{25\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{616}\left(5\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2+4}}{x^2+2}\right)-\int \frac{85x^2+82}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx\right)+$$
$$\frac{25\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}$$

$$\Big\downarrow \text{2226}$$

$$\frac{1}{616}\left(-\frac{88}{3}\int \frac{1}{\sqrt{x^4+3x^2+4}}dx+\frac{370}{3}\int \frac{x^2+2}{2(5x^2+7)\sqrt{x^4+3x^2+4}}dx+5\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt{x^4+3x^2+4}}\right.\right.$$
$$\frac{25\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{616}\left(-\frac{88}{3}\int \frac{1}{\sqrt{x^4+3x^2+4}}dx+\frac{185}{3}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+5\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)\right.}{\sqrt{x^4+3x^2+4}}\right.\right.$$

$$\frac{25\sqrt{x^4+3x^2+4}x}{616\,(5x^2+7)}$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{616}\left(\frac{185}{3}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx-\frac{22\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{x^4+3x^2+4}}+5\left(\frac{\sqrt{2}(x^2\right.\right.$$

$$\frac{25\sqrt{x^4+3x^2+4}x}{616\,(5x^2+7)}$$

$$\Big\downarrow \; 2220$$

$$\frac{1}{616}\left(-\frac{22\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3\sqrt{x^4+3x^2+4}}+5\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right.}{\sqrt{x^4+3x^2+4}}\right.\right.$$

$$\frac{25\sqrt{x^4+3x^2+4}x}{616\,(5x^2+7)}$$

input
```
Int[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```

output
```
(25*x*Sqrt[4 + 3*x^2 + x^4])/(616*(7 + 5*x^2)) + (5*(-((x*Sqrt[4 + 3*x^2 +
 x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) - (22*Sqrt[2]
*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2
]], 1/8])/(3*Sqrt[4 + 3*x^2 + x^4]) + (185*((3*ArcTan[(2*Sqrt[11/35]*x)/Sq
rt[4 + 3*x^2 + x^4]])/(4*Sqrt[385]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)
/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(140*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])))/3)/616
```

3.369. $\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx$

3.369.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.369. $\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx$

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2232
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + b*x^2
 + c*x^4], x], x] + Simp[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d -
 a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
&&  !GtQ[b^2 - 4*a*c, 0]
```

3.369.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.19

method	result
risch	$\frac{25x\sqrt{x^4+3x^2+4}}{616(5x^2+7)} - \frac{\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{22\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} + \frac{20\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{}}{}\right.}}{77\sqrt{}}$
default	$\frac{25x\sqrt{x^4+3x^2+4}}{616(5x^2+7)} - \frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{22\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} + \frac{20\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\right.}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{25x\sqrt{x^4+3x^2+4}}{616(5x^2+7)} - \frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{22\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} + \frac{20\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\right.}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

input
```
int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
25/616*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-1/22/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/
8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+
4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))
+20/77/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-
1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4
*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2
*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+37/4312/(-3/8+1/8*I*7^(1/2))
^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(
1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/
8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

3.369.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^8 + 145*x^6 + 359*x^4 + 427*x^2 + 196
), x)
```

3.369.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2-x+2)\,(x^2+x+2)}\,(5x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2), x)
```

3.369.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

3.369.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

3.369.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{(5\,x^2+7)^2\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2)),x)
```

output
```
int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2)), x)
```

3.370 $\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx$

3.370.1 Optimal result

Integrand size = 24, antiderivative size = 314

$$\begin{aligned}
&\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx \\
&= -\frac{555x\sqrt{4+3x^2+x^4}}{758912\,(2+x^2)}+\frac{25x\sqrt{4+3x^2+x^4}}{1232\,(7+5x^2)^2}+\frac{2775x\sqrt{4+3x^2+x^4}}{758912\,(7+5x^2)} \\
&\quad -\frac{3285\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{3035648}+\frac{555(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad -\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{8624\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&\quad -\frac{18615(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{21249536\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

output
```
-3285/233744896*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-555
/758912*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+25/1232*x*(x^4+3*x^2+4)^(1/2)/(5*x^2
+7)^2+2775/758912*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+555/758912*(x^2+2)*(cos(
2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(s
in(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)
^(1/2)/(x^4+3*x^2+4)^(1/2)-1/17248*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2
)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))
),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
-18615/42499072*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arcta
n(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/
2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.370.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.72 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.98

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{\frac{700x(1393+555x^2)(4+3x^2+x^4)}{(7+5x^2)^2} + i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(3885(3-i\sqrt{7})\,E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\right.\right.\right.}{}$$

input
```
Integrate[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

output
```
((700*x*(1393 + 555*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*
I)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3
*I + Sqrt[7])]*(3885*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I
 + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9401 + (3885*I)*Sqrt
[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])
/(3*I + Sqrt[7])] + 6570*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt
[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(21249536
*Sqrt[4 + 3*x^2 + x^4])
```

3.370.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1551, 25, 2210, 2232, 27, 1509, 2226, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^3\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{1551}$$

$$\frac{25x\sqrt{x^4+3x^2+4}}{1232(5x^2+7)^2}-\frac{\int -\frac{25x^4+10x^2+76}{(5x^2+7)^2\sqrt{x^4+3x^2+4}}dx}{1232}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{25x^4+10x^2+76}{(5x^2+7)^2\sqrt{x^4+3x^2+4}}dx}{1232}+\frac{25\sqrt{x^4+3x^2+4}x}{1232(5x^2+7)^2}$$

$$\Big\downarrow \text{2210}$$

$$\frac{\frac{2775x\sqrt{x^4+3x^2+4}}{616(5x^2+7)}-\frac{1}{616}\int \frac{2775x^4+4690x^2+4412}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx}{1232}+\frac{25\sqrt{x^4+3x^2+4}x}{1232(5x^2+7)^2}$$

$$\Big\downarrow \text{2232}$$

$$\frac{\frac{1}{616}\left(1110\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx-\frac{1}{5}\int \frac{5(6355x^2+12182)}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx\right)+\frac{2775\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}}{1232}+\frac{25\sqrt{x^4+3x^2+4}x}{1232(5x^2+7)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{616}\left(555\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx-\int \frac{6355x^2+12182}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx\right)+\frac{2775\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}}{1232}+\frac{25\sqrt{x^4+3x^2+4}x}{1232(5x^2+7)^2}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\frac{1}{616}\left(555\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2+4}}{x^2+2}\right)-\int \frac{6355x^2+12182}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx\right)+\frac{2775\sqrt{x^4+3x^2+4}x}{616(5x^2+7)}}{1232}+\frac{25\sqrt{x^4+3x^2+4}x}{1232(5x^2+7)^2}$$

$\downarrow$ 2226

$$\frac{\frac{1}{616}\left(-176\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-10950\int \frac{x^2+2}{2(5x^2+7)\sqrt{x^4+3x^2+4}}dx+555\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2}}{x^2+2}\right.\right.}{1232}$$

$$\frac{25\sqrt{x^4+3x^2+4}x}{1232\left(5x^2+7\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{1}{616}\left(-176\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-5475\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+555\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2+4}}{x^2+2}\right.\right.}{1232}$$

$$\frac{25\sqrt{x^4+3x^2+4}x}{1232\left(5x^2+7\right)^2}$$

$\downarrow$ 1416

$$\frac{\frac{1}{616}\left(-5475\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx-\frac{44\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+555\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\right.}{\sqrt{x^4+3x^2+4}}\right.\right.}{1232}$$

$$\frac{25\sqrt{x^4+3x^2+4}x}{1232\left(5x^2+7\right)^2}$$

$\downarrow$ 2220

$$\frac{\frac{1}{616}\left(-\frac{44\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+555\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2+4}}{x^2+2}\right)-\right.}{1232}$$

$$\frac{25\sqrt{x^4+3x^2+4}x}{1232\left(5x^2+7\right)^2}$$

input
```
Int[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

3.370. $\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx$

output
```
(25*x*Sqrt[4 + 3*x^2 + x^4])/(1232*(7 + 5*x^2)^2) + ((2775*x*Sqrt[4 + 3*x^
2 + x^4])/(616*(7 + 5*x^2)) + (555*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2))
 + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcT
an[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) - (44*Sqrt[2]*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4
+ 3*x^2 + x^4] - 5475*((3*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])
/(4*Sqrt[385]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipti
cPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(140*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
))/616)/1232
```

3.370.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2232
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + b*x^2
 + c*x^4], x], x] + Simp[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d -
 a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
&&  !GtQ[b^2 - 4*a*c, 0]
```

3.370. $\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx$

3.370.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.10

method	result
risch	$\frac{25\sqrt{x^4+3x^2+4}\,x(555x^2+1393)}{758912(5x^2+7)^2} - \frac{23\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{27104\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} + \frac{555\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right.}}{}$
default	$\frac{25x\sqrt{x^4+3x^2+4}}{1232(5x^2+7)^2} + \frac{2775x\sqrt{x^4+3x^2+4}}{758912(5x^2+7)} - \frac{23\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{27104\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} + \frac{555\sqrt{1+\frac{3x^2}{8}-}}{237}$
elliptic	$\frac{25x\sqrt{x^4+3x^2+4}}{1232(5x^2+7)^2} + \frac{2775x\sqrt{x^4+3x^2+4}}{758912(5x^2+7)} - \frac{23\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{27104\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} + \frac{555\sqrt{1+\frac{3x^2}{8}-}}{237}$

input
```
int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
25/758912*(x^4+3*x^2+4)^(1/2)*x*(555*x^2+1393)/(5*x^2+7)^2-23/27104/(-6+2*
I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2)
)*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
4*(2+6*I*7^(1/2))^(1/2))+555/23716/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7
^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/
(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^
(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-
3285/5312384/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2
)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8
+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/
2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

3.370.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}\,(5\,x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^10 + 900*x^8 + 2810*x^6 + 4648*x^4 +
 3969*x^2 + 1372), x)
```

3.370.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2-x+2)(x^2+x+2)}\,(5x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

output
```
Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3), x)
```

3.370.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

3.370.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+4}(5\,x^2+7)^3}\,dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx = \int \frac{1}{(5\,x^2+7)^3\,\sqrt{x^4+3\,x^2+4}}\,dx$$

input
```
int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2)),x)
```

output
```
int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2)), x)
```

3.371 $\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx$

3.371.1 Optimal result

Integrand size = 24, antiderivative size = 219

$$\begin{aligned}\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx &= \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{5000}{3}x\sqrt{4+3x^2+x^4}\\ &+625x^3\sqrt{4+3x^2+x^4}-\frac{220779x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}\\ &+\frac{220779(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}\\ &-\frac{130729(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

output
```
1/28*x*(45779*x^2+99493)/(x^4+3*x^2+4)^(1/2)+5000/3*x*(x^4+3*x^2+4)^(1/2)+
625*x^3*(x^4+3*x^2+4)^(1/2)-220779/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+220779
/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1
/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*
x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-130729/24*(x^2+2)*(cos(2*arcta
n(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*ar
ctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/
(x^4+3*x^2+4)^(1/2)
```

3.371.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.55

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(858479+767337x^2+297500x^4+52500x^6)+662337\sqrt{2}(3i+\sqrt{7})}{}$$

input
```
Integrate[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(858479 + 767337*x^2 + 297500*x^4 + 52500
*x^6) + 662337*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(
-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*Ellipti
cE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt
[7])] - Sqrt[2]*(975947*I + 662337*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x
^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*El
lipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I +
 Sqrt[7])])/(336*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.371.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1517, 2207, 27, 2207, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^5}{(x^4+3x^2+4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1517}$$

$$\frac{1}{28}\int \frac{87500x^6+350000x^4+269221x^2+18156}{\sqrt{x^4+3x^2+4}}dx+\frac{x(45779x^2+99493)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{28}\left(\frac{1}{5}\int \frac{5(140000x^4+59221x^2+18156)}{\sqrt{x^4+3x^2+4}}dx+17500\sqrt{x^4+3x^2+4}x^3\right)+\frac{x(45779x^2+99493)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{28}\left(\int \frac{140000x^4+59221x^2+18156}{\sqrt{x^4+3x^2+4}}dx+17500\sqrt{x^4+3x^2+4}x^3\right)+\frac{x(45779x^2+99493)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \ 2207$$

$$\frac{1}{28}\left(\frac{1}{3}\int -\frac{662337x^2+505532}{\sqrt{x^4+3x^2+4}}dx+\frac{140000}{3}\sqrt{x^4+3x^2+4}x+17500\sqrt{x^4+3x^2+4}x^3\right)+\frac{x(45779x^2+99493)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{28}\left(-\frac{1}{3}\int \frac{662337x^2+505532}{\sqrt{x^4+3x^2+4}}dx+\frac{140000}{3}\sqrt{x^4+3x^2+4}x+17500\sqrt{x^4+3x^2+4}x^3\right)+\frac{x(45779x^2+99493)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \ 1511$$

$$\frac{1}{28}\left(\frac{1}{3}\left(1324674\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx-1830206\int \frac{1}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{140000}{3}\sqrt{x^4+3x^2+4}x+17500\sqrt{x^4+\right.$$
$$\frac{x(45779x^2+99493)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{28}\left(\frac{1}{3}\left(662337\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx-1830206\int \frac{1}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{140000}{3}\sqrt{x^4+3x^2+4}x+17500\sqrt{x^4+3\right.$$
$$\frac{x(45779x^2+99493)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \ 1416$$

$$\frac{1}{28}\left(\frac{1}{3}\left(662337\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx-\frac{915103(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}\right)+\frac{140000}{3}\sqrt{x\right.$$
$$\frac{x(45779x^2+99493)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \ 1509$$

3.371. $\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx$

$$\frac{1}{28}\left(\frac{1}{3}\left(662337\left(\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2+4}}{x^2+2}\right)-\frac{915103\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+}{(x^2+2)^2}}}{\sqrt{2}\sqrt{x}}\right.\right.$$

$$\frac{x\left(45779x^2+99493\right)}{28\sqrt{x^4+3x^2+4}}$$

input
```
Int[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(x*(99493 + 45779*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + ((140000*x*Sqrt[4 + 3
*x^2 + x^4])/3 + 17500*x^3*Sqrt[4 + 3*x^2 + x^4] + (662337*(-((x*Sqrt[4 +
3*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) - (915
103*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqr
t[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]))/3)/28
```

3.371.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.371.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.10

method	result
risch	$\frac{x(52500x^6+297500x^4+767337x^2+858479)}{84\sqrt{x^4+3x^2+4}} - \frac{505532\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}} + \frac{176}{}$
elliptic	$-\frac{2\left(-\frac{45779}{56}x^3-\frac{99493}{56}x\right)}{\sqrt{x^4+3x^2+4}} + 625x^3\sqrt{x^4+3x^2+4} + \frac{5000x\sqrt{x^4+3x^2+4}}{3} - \frac{505532\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x}}{21\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3}}$
default	$-\frac{33614\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}} - \frac{505532\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}} + \frac{1766232\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right.}}{}$

input
```
int((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/84*x*(52500*x^6+297500*x^4+767337*x^2+858479)/(x^4+3*x^2+4)^(1/2)-505532
/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8
*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2)
)^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1766232/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3
/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2
+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I
*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2)
)^(1/2)))
```

3.371.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{662337\sqrt{2}\left(3x^5+9x^3-\sqrt{-7}(x^5+3x^3+4x)+12x\right)\sqrt{\sqrt{-7}-3}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{}}{}\right.\right.}{}$$

input
```
integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
1/336*(662337*sqrt(2)*(3*x^5 + 9*x^3 - sqrt(-7)*(x^5 + 3*x^3 + 4*x) + 12*x
)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x),
3/8*sqrt(-7) + 1/8) - 2*sqrt(2)*(1183080*x^5 + 3549240*x^3 - 267977*sqrt(-
7)*(x^5 + 3*x^3 + 4*x) + 4732320*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1
/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) + 16*(13125*x^8 + 74
375*x^6 + 26250*x^4 - 282133*x^2 - 662337)*sqrt(x^4 + 3*x^2 + 4))/(x^5 + 3
*x^3 + 4*x)
```

3.371.6 Sympy [F]

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^5}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**5/(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**5/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

3.371.7 Maxima [F]

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.371.8 Giac [F]

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.371.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^5}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^5/(3*x^2 + x^4 + 4)^(3/2),x)
```

output
```
int((5*x^2 + 7)^5/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.372 $\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx$

3.372.1 Optimal result

Integrand size = 24, antiderivative size = 200

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{625}{3}x\sqrt{4+3x^2+x^4} + \frac{14523x\sqrt{4+3x^2+x^4}}{28(2+x^2)} - \frac{14523(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{4243(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
1/28*x*(-4023*x^2+2719)/(x^4+3*x^2+4)^(1/2)+625/3*x*(x^4+3*x^2+4)^(1/2)+14
523/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-14523/28*(x^2+2)*(cos(2*arctan(1/2*x*
2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2
*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x
^2+4)^(1/2)+4243/24*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*a
rctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*
((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.372.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.31 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.66

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(78157+40431x^2+17500x^4)-43569\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{}$$

input
```
Integrate[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(78157 + 40431*x^2 + 17500*x^4) - 43569*S
qrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]
*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqr
t[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*
(186179*I + 43569*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[
7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh
[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(336*
Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.372.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1517, 2207, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^4}{(x^4+3x^2+4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1517}$$

$$\frac{1}{28}\int \frac{17500x^4+49523x^2+14088}{\sqrt{x^4+3x^2+4}}dx+\frac{x(2719-4023x^2)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{1}{28}\left(\frac{1}{3}\int -\frac{27736-43569x^2}{\sqrt{x^4+3x^2+4}}dx+\frac{17500}{3}\sqrt{x^4+3x^2+4}x\right)+\frac{x(2719-4023x^2)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{28}\left(\frac{17500}{3}x\sqrt{x^4+3x^2+4}-\frac{1}{3}\int \frac{27736-43569x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{x(2719-4023x^2)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \; 1511$$

$$\frac{1}{28}\left(\frac{1}{3}\left(59402\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-87138\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)+\frac{17500}{3}\sqrt{x^4+3x^2+4}x\right)+\frac{x(2719-4023x^2)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{28}\left(\frac{1}{3}\left(59402\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-43569\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{17500}{3}\sqrt{x^4+3x^2+4}x\right)+\frac{x(2719-4023x^2)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{28}\left(\frac{1}{3}\left(\frac{29701(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-43569\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{17500}{3}\sqrt{x^4+\right.$$

$$\frac{x(2719-4023x^2)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{28}\left(\frac{1}{3}\left(\frac{29701(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-43569\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt{x^4+3x^2+4}}\right.\right.\right.$$

$$\frac{x(2719-4023x^2)}{28\sqrt{x^4+3x^2+4}}$$

input
```
Int[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(x*(2719 - 4023*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + ((17500*x*Sqrt[4 + 3*x^
2 + x^4])/3 + (-43569*(-((x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]]
, 1/8])/Sqrt[4 + 3*x^2 + x^4]) + (29701*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(
2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 +
 x^4]))/3)/28
```

3.372. $\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx$

3.372.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.372.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.18

method	result
risch	$\frac{x(17500x^4+40431x^2+78157)}{84\sqrt{x^4+3x^2+4}} - \frac{27736\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} - 116184\sqrt{1-\left(-\frac{3}{8}\right.}$
elliptic	$-\frac{2\left(\frac{4023}{56}x^3-\frac{2719}{56}x\right)}{\sqrt{x^4+3x^2+4}} + \frac{625x\sqrt{x^4+3x^2+4}}{3} - \frac{27736\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} - 1161$
default	$-\frac{4802\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}} - \frac{27736\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{21\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} - 116184\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}$

input
```
int((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/84*x*(17500*x^4+40431*x^2+78157)/(x^4+3*x^2+4)^(1/2)-27736/21/(-6+2*I*7^
(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^
2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2
+6*I*7^(1/2))^(1/2))-116184/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2
))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*
7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
)-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.372.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{43569\sqrt{2}\left(3x^5+9x^3-\sqrt{-7}\,(x^5+3x^3+4x)+12x\right)\sqrt{\sqrt{-7}-3}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-$$

input
```
integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
-1/336*(43569*sqrt(2)*(3*x^5 + 9*x^3 - sqrt(-7)*(x^5 + 3*x^3 + 4*x) + 12*x
)*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x),
3/8*sqrt(-7) + 1/8) - sqrt(2)*(109905*x^5 + 329715*x^3 - 50503*sqrt(-7)*(x
^5 + 3*x^3 + 4*x) + 439620*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqr
t(2)*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 16*(4375*x^6 + 21000*x^4
 + 52216*x^2 + 43569)*sqrt(x^4 + 3*x^2 + 4))/(x^5 + 3*x^3 + 4*x)
```

3.372.6 Sympy [F]

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^4}{\left((x^2-x+2)(x^2+x+2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**4/(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**4/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

3.372.7 Maxima [F]

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^4}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.372.8 Giac [F]

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^4/(3*x^2 + x^4 + 4)^(3/2),x)
```

output
```
int((5*x^2 + 7)^4/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.373 $\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx$

3.373.1 Optimal result

Integrand size = 24, antiderivative size = 181

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = -\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{4449x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)} - \frac{4449(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{973(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
-1/28*x*(949*x^2+2323)/(x^4+3*x^2+4)^(1/2)+4449/28*x*(x^4+3*x^2+4)^(1/2)/(
x^2+2)-4449/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan
(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/
2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+973/8*(x^2+2)*(cos(
2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(s
in(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2
^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.373.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.81

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{-4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(2323+949x^2) - 4449\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E(}{}$$

input
```
Integrate[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(-4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(2323 + 949*x^2) - 4449*Sqrt[2]*(3*I + S
qrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sq
rt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I
+ Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(3899*I + 4449*
Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + S
qrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I
 + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I +
Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.373.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1517, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^3}{(x^4+3x^2+4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1517}$$

$$\frac{1}{28}\int \frac{4449x^2+4724}{\sqrt{x^4+3x^2+4}}dx - \frac{x(949x^2+2323)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{28}\left(13622\int \frac{1}{\sqrt{x^4+3x^2+4}}dx - 8898\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right) - \frac{x(949x^2+2323)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{28}\left(13622\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-4449\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)-\frac{x\left(949x^2+2323\right)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{28}\left(\frac{6811\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-4449\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)-\frac{x\left(949x^2+2323\right)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{28}\left(\frac{6811\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-4449\left(\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right.}{\sqrt{x^4+3x^2+4}}\right.\right.$$
$$\frac{x\left(949x^2+2323\right)}{28\sqrt{x^4+3x^2+4}}$$

input
```
Int[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
-1/28*(x*(2323 + 949*x^2))/Sqrt[4 + 3*x^2 + x^4] + (-4449*(-((x*Sqrt[4 + 3
*x^2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x
^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) + (6811
*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2
]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]))/28
```

3.373.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.373. $\int \frac{\left(7+5x^2\right)^3}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.373.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{x(949x^2+2323)}{28\sqrt{x^4+3x^2+4}}+\frac{4724\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}-\frac{35592\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-}$
elliptic	$-\frac{2\left(\frac{949}{56}x^3+\frac{2323}{56}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{4724\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}-\frac{35592\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{}}{}$
default	$-\frac{686\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}+\frac{4724\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}-\frac{35592\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1}}{}$

3.373. $\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx$

input
```
int((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/28*x*(949*x^2+2323)/(x^4+3*x^2+4)^(1/2)+4724/7/(-6+2*I*7^(1/2))^(1/2)*(
1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+
3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^
(1/2))-35592/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(
1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(Ellip
ticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4
*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.373.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{4449\sqrt{2}(3x^5+9x^3-\sqrt{-7}(x^5+3x^3+4x)+12x)\sqrt{\sqrt{-7}-3}E(\arcsin\left(\frac{\sqrt{2}\sqrt{\sqrt{-7}-3}}{2x}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8})-2}{}$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
-1/112*(4449*sqrt(2)*(3*x^5 + 9*x^3 - sqrt(-7)*(x^5 + 3*x^3 + 4*x) + 12*x)
*sqrt(sqrt(-7) - 3)*elliptic_e(arcsin(1/2*sqrt(2)*sqrt(sqrt(-7) - 3)/x), 3
/8*sqrt(-7) + 1/8) - 2*sqrt(2)*(8445*x^5 + 25335*x^3 - 1634*sqrt(-7)*(x^5
+ 3*x^3 + 4*x) + 33780*x)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/2*sqrt(2)
*sqrt(sqrt(-7) - 3)/x), 3/8*sqrt(-7) + 1/8) - 16*(875*x^4 + 2756*x^2 + 444
9)*sqrt(x^4 + 3*x^2 + 4))/(x^5 + 3*x^3 + 4*x)
```

3.373.6 Sympy [F]

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^3}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**3/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

3.373.7 Maxima [F]

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.373.8 Giac [F]

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(3/2),x)
```

output
```
int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.374 $\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx$

3.374.1 Optimal result

Integrand size = 24, antiderivative size = 181

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx = -\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}} - \frac{113x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)} + \frac{113(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{9(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
-1/28*x*(-113*x^2+9)/(x^4+3*x^2+4)^(1/2)-113/28*x*(x^4+3*x^2+4)^(1/2)/(x^2
+2)+113/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2
*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*(
(x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+9/8*(x^2+2)*(cos(2*arct
an(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*a
rctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)
/(x^4+3*x^2+4)^(1/2)
```

3.374.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.82

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(-9+113x^2)+113\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\Big(i\text{arc}}{}$$

input
```
Integrate[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(-9 + 113*x^2) + 113*Sqrt[2]*(3*I + Sqrt[
7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7
] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(1043*I + 113*Sqrt[
7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7
] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[
7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.374.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1517, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+7)^2}{(x^4+3x^2+4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1517}$$

$$\frac{1}{28}\int \frac{352-113x^2}{\sqrt{x^4+3x^2+4}}dx - \frac{x(9-113x^2)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{28}\left(126\int \frac{1}{\sqrt{x^4+3x^2+4}}dx + 226\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right) - \frac{x(9-113x^2)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{28}\left(126\int \frac{1}{\sqrt{x^4+3x^2+4}}dx+113\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)-\frac{x(9-113x^2)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \ 1416$$

$$\frac{1}{28}\left(113\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+\frac{63(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}\right)-\frac{x(9-113x^2)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \ 1509$$

$$\frac{1}{28}\left(\frac{63(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}+113\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\right.\right.$$

$$\frac{x(9-113x^2)}{28\sqrt{x^4+3x^2+4}}$$

input
```
Int[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
-1/28*(x*(9 - 113*x^2))/Sqrt[4 + 3*x^2 + x^4] + (113*(-((x*Sqrt[4 + 3*x^2
+ x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2
]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) + (63*(2 + x
^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8
])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]))/28
```

3.374.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.374. $\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx$

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

3.374.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.28

method	result
risch	$\frac{x(113x^2-9)}{28\sqrt{x^4+3x^2+4}}+\frac{352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{904\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-(-\frac{3}{8}-}}{7}$
elliptic	$-\frac{2\left(-\frac{113}{56}x^3+\frac{9}{56}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{904\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-(}}{}$
default	$-\frac{98\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}+\frac{352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{904\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-(-}}{}$

input
```
int((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/28*x*(113*x^2-9)/(x^4+3*x^2+4)^(1/2)+352/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3
/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2
+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
)+904/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8
-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/
4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+
2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.374.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.88

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx =$$

$$-\frac{113\sqrt{2}\left(3\,x^4+9\,x^2-\sqrt{-7}(x^4+3\,x^2+4)+12\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{1}{4}\sqrt{2}x\sqrt{\sqrt{-7}-3}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)+}{}$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
-1/448*(113*sqrt(2)*(3*x^4 + 9*x^2 - sqrt(-7)*(x^4 + 3*x^2 + 4) + 12)*sqrt
(sqrt(-7) - 3)*elliptic_e(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7) - 3)), 3/8*sq
rt(-7) + 1/8) + 3*sqrt(2)*(239*x^4 + 717*x^2 + 155*sqrt(-7)*(x^4 + 3*x^2 +
 4) + 956)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7
) - 3)), 3/8*sqrt(-7) + 1/8) - 16*sqrt(x^4 + 3*x^2 + 4)*(113*x^3 - 9*x))/(
x^4 + 3*x^2 + 4)
```

3.374.6 Sympy [F]

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5x^2+7)^2}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)**2/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

3.374.7 Maxima [F]

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.374.8 Giac [F]

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

input
```
int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(3/2),x)
```

output
```
int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.375 $\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}} dx$

3.375.1 Optimal result

Integrand size = 22, antiderivative size = 181

$$\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}} dx = \frac{x(53+19x^2)}{28\sqrt{4+3x^2+x^4}} - \frac{19x\sqrt{4+3x^2+x^4}}{28(2+x^2)} + \frac{19(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} - \frac{3(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
1/28*x*(19*x^2+53)/(x^4+3*x^2+4)^(1/2)-19/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)
+19/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2
^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4
+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-3/8*(x^2+2)*(cos(2*arctan(1
/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arcta
n(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^
4+3*x^2+4)^(1/2)
```

3.375.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.82

$$\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(53+19x^2)+19\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\left(i\operatorname{arcsin}\right.}{}$$

input
```
Integrate[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(53 + 19*x^2) + 19*Sqrt[2]*(3*I + Sqrt[7]
)*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
+ (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt
[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(49*I + 19*Sqrt[7])*S
qrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (
2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*
Sqrt[4 + 3*x^2 + x^4])
```

3.375.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1492, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+7}{(x^4+3x^2+4)^{3/2}}\,dx$$

$$\Big\downarrow \; 1492$$

$$\frac{1}{28}\int -\frac{19x^2+4}{\sqrt{x^4+3x^2+4}}dx+\frac{x(19x^2+53)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \; 25$$

$$\frac{x(19x^2+53)}{28\sqrt{x^4+3x^2+4}}-\frac{1}{28}\int \frac{19x^2+4}{\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{28}\left(38\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx-42\int \frac{1}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{x\left(19x^2+53\right)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\left(19\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx-42\int \frac{1}{\sqrt{x^4+3x^2+4}}dx\right)+\frac{x\left(19x^2+53\right)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{28}\left(19\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx-\frac{21\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}\right)+\frac{x\left(19x^2+53\right)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{28}\left(19\left(\frac{\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{x^4+3x^2+4}}{x^2+2}\right)-\frac{21\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\text{EllipticF}\left(2\right.}{\sqrt{2}\sqrt{x^4+3x^2+}}\right.$$

$$\frac{x\left(19x^2+53\right)}{28\sqrt{x^4+3x^2+4}}$$

input
```
Int[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(x*(53 + 19*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (19*(-((x*Sqrt[4 + 3*x^2 +
x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*
EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) - (21*(2 + x^2
)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])
/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]))/28
```

3.375.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.375.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.28

method	result
risch	$\frac{x(19x^2+53)}{28\sqrt{x^4+3x^2+4}}-\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{152\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i}{}\right.}}{7\sqrt{}}$
elliptic	$-\frac{2\left(-\frac{19}{56}x^3-\frac{53}{56}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{152\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{}\right.}}{}$
default	$-\frac{14\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}-\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}+\frac{152\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{}\right.}}{}$

input
```
int((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/28*x*(19*x^2+53)/(x^4+3*x^2+4)^(1/2)-4/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8
+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4
)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+
152/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1
/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*
x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*
I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.375.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.88

$$\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{19\sqrt{2}\left(3x^4+9x^2-\sqrt{-7}(x^4+3x^2+4)+12\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{1}{4}\sqrt{2}x\sqrt{\sqrt{-7}-3}\right)\mid\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)-}{}$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
-1/448*(19*sqrt(2)*(3*x^4 + 9*x^2 - sqrt(-7)*(x^4 + 3*x^2 + 4) + 12)*sqrt(
sqrt(-7) - 3)*elliptic_e(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7) - 3)), 3/8*sqr
t(-7) + 1/8) - 3*sqrt(2)*(23*x^4 + 69*x^2 - 5*sqrt(-7)*(x^4 + 3*x^2 + 4) +
 92)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7) - 3)
), 3/8*sqrt(-7) + 1/8) - 16*sqrt(x^4 + 3*x^2 + 4)*(19*x^3 + 53*x))/(x^4 +
3*x^2 + 4)
```

3.375.6 Sympy [F]

$$\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{5x^2+7}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral((5*x**2 + 7)/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

3.375.7 Maxima [F]

$$\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.375.8 Giac [F]

$$\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{5\,x^2+7}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx=\int \frac{5\,x^2+7}{{\left(x^4+3\,x^2+4\right)}^{3/2}} \,d x$$

input
```
int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(3/2),x)
```

output
```
int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.376 $\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx$

3.376.1 Optimal result

Integrand size = 14, antiderivative size = 181

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = -\frac{x(1+3x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{3x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)} - \frac{3(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}} + \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{4\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
-1/28*x*(3*x^2+1)/(x^4+3*x^2+4)^(1/2)+3/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-3
/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1
/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*
x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+1/8*(x^2+2)*(cos(2*arctan(1/2*
x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1
/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3
*x^2+4)^(1/2)
```

3.376.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.20 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.81

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{-4\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(1+3x^2)-3\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\Big(i\text{arcsinh}}{}$$

input
```
Integrate[(4 + 3*x^2 + x^4)^(-3/2),x]
```

output
```
(-4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(1 + 3*x^2) - 3*Sqrt[2]*(3*I + Sqrt[7])*
Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] +
(2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7
])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqrt
[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*
x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqr
t[4 + 3*x^2 + x^4])
```

3.376.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1405, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4+3x^2+4)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{1}{28}\int \frac{3x^2+8}{\sqrt{x^4+3x^2+4}}dx-\frac{x(3x^2+1)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{28}\left(14\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-6\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)-\frac{x(3x^2+1)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\left(14\int \frac{1}{\sqrt{x^4+3x^2+4}}dx-3\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)-\frac{x(3x^2+1)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{28}\left(\frac{7(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-3\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)-\frac{x(3x^2+1)}{28\sqrt{x^4+3x^2+4}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{28}\left(\frac{7(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-3\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{x\sqrt{}}{}\right.\right.$$

$$\frac{x(3x^2+1)}{28\sqrt{x^4+3x^2+4}}$$

input
```
Int[(4 + 3*x^2 + x^4)^(-3/2),x]
```

output
```
-1/28*(x*(1 + 3*x^2))/Sqrt[4 + 3*x^2 + x^4] + (-3*(-((x*Sqrt[4 + 3*x^2 + x
^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*E
llipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) + (7*(2 + x^2)*
Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(
Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]))/28
```

3.376.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

3.376. $\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx$

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.376.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{x(3x^2+1)}{28\sqrt{x^4+3x^2+4}}+\frac{8\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{24\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\right.}}{7}$
default	$-\frac{2\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}+\frac{8\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{24\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\right.}}{7}$
elliptic	$-\frac{2\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}+\frac{8\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{24\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\right.}}{7}$

input
```
int(1/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/28*x*(3*x^2+1)/(x^4+3*x^2+4)^(1/2)+8/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+
1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)
^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-2
4/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8
*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*
(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*
7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

3.376.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.87

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{3\sqrt{2}\left(3x^4+9x^2-\sqrt{-7}(x^4+3x^2+4)+12\right)\sqrt{\sqrt{-7}-3}E\left(\arcsin\left(\frac{1}{4}\sqrt{2}x\sqrt{\sqrt{-}\right.\right.}{}$$

input
```
integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
1/448*(3*sqrt(2)*(3*x^4 + 9*x^2 - sqrt(-7)*(x^4 + 3*x^2 + 4) + 12)*sqrt(sq
rt(-7) - 3)*elliptic_e(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7) - 3)), 3/8*sqrt(
-7) + 1/8) - sqrt(2)*(33*x^4 + 99*x^2 + 5*sqrt(-7)*(x^4 + 3*x^2 + 4) + 132
)*sqrt(sqrt(-7) - 3)*elliptic_f(arcsin(1/4*sqrt(2)*x*sqrt(sqrt(-7) - 3)),
3/8*sqrt(-7) + 1/8) - 16*sqrt(x^4 + 3*x^2 + 4)*(3*x^3 + x))/(x^4 + 3*x^2 +
 4)
```

3.376.6 Sympy [F]

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral((x**4 + 3*x**2 + 4)**(-3/2), x)
```

3.376.7 Maxima [F]

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

3.376.8 Giac [F]

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

input
```
int(1/(3*x^2 + x^4 + 4)^(3/2),x)
```

output
```
int(1/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.377 $\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx$

3.377.1 Optimal result

Integrand size = 24, antiderivative size = 284

$$\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx = -\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}}+\frac{x\sqrt{4+3x^2+x^4}}{77(2+x^2)} +\frac{25}{176}\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)-\frac{\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{77\sqrt{4+3x^2+x^4}} -\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{12\sqrt{2}\sqrt{4+3x^2+x^4}} +\frac{425(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{3696\sqrt{2}\sqrt{4+3x^2+x^4}}$$

output
```
25/13552*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/308*x*(4
*x^2+13)/(x^4+3*x^2+4)^(1/2)+1/77*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-1/24*(x^2+
2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ell
ipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)
^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+425/7392*(x^2+2)*(cos(2*arctan(1/2*x*2^
(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*
x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x
^4+3*x^2+4)^(1/2)-1/77*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(
2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2
))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.377.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.43 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.70

$$\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx = \frac{-26\sqrt{-\frac{i}{-3i+\sqrt{7}}}x - 8\sqrt{-\frac{i}{-3i+\sqrt{7}}}x^3 - 2\sqrt{2}(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+}{3}}}{}$$

input
```
Integrate[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```

output
```
(-26*Sqrt[(-I)/(-3*I + Sqrt[7])]*x - 8*Sqrt[(-I)/(-3*I + Sqrt[7])]*x^3 - 2
*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7]
)]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[S
qrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2
]*(7*I + 2*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sq
rt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (25*I)*Sqrt
[2]*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7
] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSi
nh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(61
6*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

3.377.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1547, 27, 2206, 27, 1511, 27, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)(x^4+3x^2+4)^{3/2}}dx$$

$$\Big\downarrow \text{1547}$$

$$\frac{125}{66}\int \frac{x^2+2}{2(5x^2+7)\sqrt{x^4+3x^2+4}}dx - \frac{1}{66}\int \frac{25x^4+90x^2+124}{2(x^4+3x^2+4)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{125}{132}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx-\frac{1}{132}\int \frac{25x^4+90x^2+124}{(x^4+3x^2+4)^{3/2}}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{132}\left(-\frac{1}{28}\int \frac{8(89-6x^2)}{\sqrt{x^4+3x^2+4}}dx-\frac{3x(4x^2+13)}{7\sqrt{x^4+3x^2+4}}\right)+\frac{125}{132}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{132}\left(-\frac{2}{7}\int \frac{89-6x^2}{\sqrt{x^4+3x^2+4}}dx-\frac{3x(4x^2+13)}{7\sqrt{x^4+3x^2+4}}\right)+\frac{125}{132}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{132}\left(-\frac{2}{7}\left(77\int \frac{1}{\sqrt{x^4+3x^2+4}}dx+12\int \frac{2-x^2}{2\sqrt{x^4+3x^2+4}}dx\right)-\frac{3x(4x^2+13)}{7\sqrt{x^4+3x^2+4}}\right)+ \frac{125}{132}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{132}\left(-\frac{2}{7}\left(77\int \frac{1}{\sqrt{x^4+3x^2+4}}dx+6\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx\right)-\frac{3x(4x^2+13)}{7\sqrt{x^4+3x^2+4}}\right)+ \frac{125}{132}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{132}\left(-\frac{2}{7}\left(6\int \frac{2-x^2}{\sqrt{x^4+3x^2+4}}dx+\frac{77(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}\right)-\frac{3x(4x^2+13)}{7\sqrt{x^4+3x^2+4}}\right) \frac{125}{132}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx$$

$$\Big\downarrow \text{1509}$$

$$\frac{125}{132}\int \frac{x^2+2}{(5x^2+7)\sqrt{x^4+3x^2+4}}dx+ \frac{1}{132}\left(-\frac{2}{7}\left(\frac{77(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}+6\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\right.}{\sqrt{x^4+3x^2+4}}\right.\right.\right.$$

$$\Big\downarrow \text{2220}$$

3.377. $\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx$

$$\frac{125}{132}\left(\frac{3\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{4\sqrt{385}}+\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+4}}\right)+$$

$$\frac{1}{132}\left(-\frac{2}{7}\left(\frac{77(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}+6\left(\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\right.}{\sqrt{x^4+3x^2+4}}\right.\right.\right.$$

input
```
Int[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```

output
```
((-3*x*(13 + 4*x^2))/(7*Sqrt[4 + 3*x^2 + x^4]) - (2*(6*(-((x*Sqrt[4 + 3*x^
2 + x^4])/(2 + x^2)) + (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)
^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]) + (77*(2 +
 x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1
/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])))/7)/132 + (125*((3*ArcTan[(2*Sqrt[
11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(4*Sqrt[385]) + (17*(2 + x^2)*Sqrt[(4 +
3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(1
40*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])))/132
```

3.377.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

3.377. $\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx$

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1547
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_.) + (e_.)*(x_)^2), x_Sym
bol] :> Simp[-(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(e^(2*p)*(Rt[c/a, 2]*d - e)
)   Int[(1 + Rt[c/a, 2]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]
+ Simp[(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(Rt[c/a, 2]*d - e)   Int[(a + b*x^
2 + c*x^4)^p*ExpandToSum[((Rt[c/a, 2]*d - e)*(c*d^2 - b*d*e + a*e^2)^(-p -
1/2) + ((1 + Rt[c/a, 2]*x^2)*(a + b*x^2 + c*x^4)^(-p - 1/2))/e^(2*p))/(d +
e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0] && NeQ[c*d^2 - a*e^2, 0] &
& PosQ[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.377. $\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx$

3.377.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{x(4x^2+13)}{308\sqrt{x^4+3x^2+4}}-\frac{\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\right.}{77}$
default	$-\frac{2\left(\frac{1}{154}x^3+\frac{13}{616}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2-}}$
elliptic	$-\frac{2\left(\frac{1}{154}x^3+\frac{13}{616}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2-}}$

input
```
int(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/308*x*(4*x^2+13)/(x^4+3*x^2+4)^(1/2)-1/77/(-6+2*I*7^(1/2))^(1/2)*(1-(-3
/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2
+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
)-32/77/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8
-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/
4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+
2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+25/308/(-3/8+1/8*I*7^(1/2))
^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(
1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/
8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

3.377.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx=\int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^10 + 37*x^8 + 127*x^6 + 239*x^4 + 248*
x^2 + 112), x)
```

3.377.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{\left((x^2-x+2)(x^2+x+2)\right)^{\frac{3}{2}}\cdot(5x^2+7)}\,dx$$

input
```
integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)), x)
```

3.377.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

3.377.8 Giac [F]

$$\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)}\,dx$$

input
```
integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)\left(4+3x^2+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(5\,x^2+7\right)\,\left(x^4+3\,x^2+4\right)^{3/2}}\,dx$$

input
```
int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2)),x)
```

output
```
int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2)), x)
```

3.378 $\int \frac{1}{(7+5x^2)^2(4+3x^2+x^4)^{3/2}}\,dx$

3.378.1 Optimal result

Integrand size = 24, antiderivative size = 312

$$\begin{aligned}\int \frac{1}{(7+5x^2)^2(4+3x^2+x^4)^{3/2}}\,dx &= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}}-\frac{199x\sqrt{4+3x^2+x^4}}{27104(2+x^2)}\\
&+\frac{625x\sqrt{4+3x^2+x^4}}{27104(7+5x^2)}+\frac{575\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{108416}\\
&+\frac{199(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13552\sqrt{2}\sqrt{4+3x^2+x^4}}\\
&-\frac{2\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{231\sqrt{4+3x^2+x^4}}\\
&+\frac{9775(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2276736\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

output
```
575/8348032*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/13552
*x*(37*x^2+24)/(x^4+3*x^2+4)^(1/2)-199/27104*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)
+625/27104*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+199/27104*(x^2+2)*(cos(2*arctan
(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arc
tan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)+9775/4553472*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(
1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),
-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^
(1/2)-2/231*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/
2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*
x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.378.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00

$$\int \frac{1}{(7+5x^2)^2\,(4+3x^2+x^4)^{3/2}}\,dx = \frac{28x(2836+2633x^2+995x^4)+i\sqrt{6+2i\sqrt{7}}(7+5x^2)\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{\cdots}}{\cdots}$$

input
```
Integrate[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```

output
```
(28*x*(2836 + 2633*x^2 + 995*x^4) + I*Sqrt[6 + (2*I)*Sqrt[7]]*(7 + 5*x^2)*
Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7]
)]*(1393*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]
*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(101 + (199*I)*Sqrt[7])*Elliptic
F[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[
7])] - 1150*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I
 + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(758912*(7 + 5*x^2)*Sq
rt[4 + 3*x^2 + x^4])
```

3.378.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^2 (x^4+3x^2+4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1556}$$

$$\int \left(\frac{5x^2-36}{1936 (x^4+3x^2+4)^{3/2}}-\frac{25}{1936 (5x^2+7) \sqrt{x^4+3x^2+4}}+\frac{25}{44 (5x^2+7)^2 \sqrt{x^4+3x^2+4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{575\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{108416}-\frac{2\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{231\sqrt{x^4+3x^2+4}}+\frac{199(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13552\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{9775(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{2276736\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{199\sqrt{x^4+3x^2+4}x}{27104(x^2+2)}+\frac{625\sqrt{x^4+3x^2+4}x}{27104(5x^2+7)}+\frac{(37x^2+24)x}{13552\sqrt{x^4+3x^2+4}}$$

input

```
Int[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```

output

```
(x*(24 + 37*x^2))/(13552*Sqrt[4 + 3*x^2 + x^4]) - (199*x*Sqrt[4 + 3*x^2 +
x^4])/(27104*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(27104*(7 + 5*x^2)
) + (575*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/10841
6 + (199*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[
x/Sqrt[2]], 1/8])/(13552*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (2*Sqrt[2]*(2 +
x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/
8])/(231*Sqrt[4 + 3*x^2 + x^4]) + (9775*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(
2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(2276736*Sqrt[2]
*Sqrt[4 + 3*x^2 + x^4])
```

3.378.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.378.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.12

method	result
risch	$\frac{x(995x^4+2633x^2+2836)}{27104(5x^2+7)\sqrt{x^4+3x^2+4}} - \frac{349\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{6776\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} + \frac{199\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}}{}$
default	$\frac{625x\sqrt{x^4+3x^2+4}}{27104(5x^2+7)} - \frac{2\left(-\frac{37}{27104}x^3-\frac{3}{3388}x\right)}{\sqrt{x^4+3x^2+4}} - \frac{349\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{6776\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} + \frac{199\sqrt{1+\frac{3x}{8}}}{}$
elliptic	$\frac{625x\sqrt{x^4+3x^2+4}}{27104(5x^2+7)} - \frac{2\left(-\frac{37}{27104}x^3-\frac{3}{3388}x\right)}{\sqrt{x^4+3x^2+4}} - \frac{349\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{6776\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}} + \frac{199\sqrt{1+\frac{3x}{8}}}{}$

input
```
int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/27104*x*(995*x^4+2633*x^2+2836)/(5*x^2+7)/(x^4+3*x^2+4)^(1/2)-349/6776/(
-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^
(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/
2),1/4*(2+6*I*7^(1/2))^(1/2))+199/847/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*
I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/
2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2
))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
))+575/189728/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/
2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/
8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1
/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

3.378.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^2 (4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^12 + 220*x^10 + 894*x^8 + 2084*x^6 +
2913*x^4 + 2296*x^2 + 784), x)
```

3.378.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^2 (4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{((x^2-x+2)\,(x^2+x+2))^{\frac{3}{2}}\,(5x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2), x)
```

3.378.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^2 (4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

3.378.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^2 (4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+3\,x^2+4)^{\frac{3}{2}}(5\,x^2+7)^2}\,dx$$

input
```
integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^2 (4+3x^2+x^4)^{3/2}}\,dx = \int \frac{1}{(5\,x^2+7)^2\,(x^4+3\,x^2+4)^{3/2}}\,dx$$

input
```
int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2)),x)
```

output
```
int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2)), x)
```

3.379 $\int \frac{1}{(7+5x^2)^3(4+3x^2+x^4)^{3/2}}\,dx$

3.379.1 Optimal result

Integrand size = 24, antiderivative size = 340

$$\begin{aligned}\int \frac{1}{(7+5x^2)^3(4+3x^2+x^4)^{3/2}}\,dx &= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} \\ &- \frac{18159x\sqrt{4+3x^2+x^4}}{33392128(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208(7+5x^2)^2} + \frac{51875x\sqrt{4+3x^2+x^4}}{33392128(7+5x^2)} \\ &- \frac{529425\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{133568512} + \frac{18159(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{16696064\sqrt{2}\sqrt{4+3x^2+x^4}} \\ &+ \frac{843(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{4+3x^2+x^4}} \\ &- \frac{3000075(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\operatorname{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{934979584\sqrt{2}\sqrt{4+3x^2+x^4}}\end{aligned}$$

output
```
-529425/10284775424*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)
+1/596288*x*(139*x^2+548)/(x^4+3*x^2+4)^(1/2)-18159/33392128*x*(x^4+3*x^2+
4)^(1/2)/(x^2+2)+625/54208*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+51875/3339212
8*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+18159/33392128*(x^2+2)*(cos(2*arctan(1/2
*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(
1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+
3*x^2+4)^(1/2)+843/758912*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/c
os(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(
1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-3000075/
1869959168*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2
*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*(
(x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

3.379.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.68 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.94

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}}\, dx = \frac{28x(4496212+5811451x^2+2838330x^4+453975x^6)+3i\sqrt{6+2i\sqrt{7}}(}{}$$

input
```
Integrate[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```

output
```
(28*x*(4496212 + 5811451*x^2 + 2838330*x^4 + 453975*x^6) + (3*I)*Sqrt[6 +
(2*I)*Sqrt[7]]*(7 + 5*x^2)^2*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1
 + ((2*I)*x^2)/(3*I + Sqrt[7])]*(42371*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh
[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I
)*(23633*I + 6053*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 352950*EllipticPi[(5*(3 + I*Sqrt
[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I
 + Sqrt[7])]))/(934979584*(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4])
```

3.379.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5x^2+7)^3 (x^4+3x^2+4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1556}$$

$$\int \left(\frac{215x^2+388}{85184 (x^4+3x^2+4)^{3/2}} - \frac{1075}{85184 (5x^2+7) \sqrt{x^4+3x^2+4}} - \frac{25}{1936 (5x^2+7)^2 \sqrt{x^4+3x^2+4}} + \frac{}{44 (5x^2+7)^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{529425\sqrt{\frac{5}{77}}\arctan\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{133568512} + \frac{843(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{x^4+3x^2+4}} + \frac{18159(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\arctan\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{16696064\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{3000075(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\text{EllipticPi}\left(-\frac{9}{280},2\arctan\left(\frac{x}{\sqrt{2}}\right),\frac{1}{8}\right)}{934979584\sqrt{2}\sqrt{x^4+3x^2+4}} - \frac{18159\sqrt{x^4+3x^2+4}x}{33392128(x^2+2)} + \frac{51875\sqrt{x^4+3x^2+4}x}{33392128(5x^2+7)} + \frac{625\sqrt{x^4+3x^2+4}x}{54208(5x^2+7)^2} + \frac{(139x^2+548)x}{596288\sqrt{x^4+3x^2+4}}$$

input
```
Int[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```

output
```
(x*(548 + 139*x^2))/(596288*Sqrt[4 + 3*x^2 + x^4]) - (18159*x*Sqrt[4 + 3*x
^2 + x^4])/(33392128*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(54208*(7
+ 5*x^2)^2) + (51875*x*Sqrt[4 + 3*x^2 + x^4])/(33392128*(7 + 5*x^2)) - (52
9425*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/133568512
 + (18159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan
[x/Sqrt[2]], 1/8])/(16696064*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (843*(2 + x^
2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8]
)/(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3000075*(2 + x^2)*Sqrt[(4 + 3*
x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(934
979584*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

3.379.3.1 Defintions of rubi rules used

rule 1556
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.379.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.05

method	result
risch	$\frac{x(453975x^6+2838330x^4+5811451x^2+4496212)}{33392128(5x^2+7)^2\sqrt{x^4+3x^2+4}}+\frac{1173\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{1192576\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}+$
default	$\frac{625x\sqrt{x^4+3x^2+4}}{54208(5x^2+7)^2}+\frac{51875x\sqrt{x^4+3x^2+4}}{33392128(5x^2+7)}-\frac{2\left(-\frac{139}{1192576}x^3-\frac{137}{298144}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{1173\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\right.}{1192576\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{625x\sqrt{x^4+3x^2+4}}{54208(5x^2+7)^2}+\frac{51875x\sqrt{x^4+3x^2+4}}{33392128(5x^2+7)}-\frac{2\left(-\frac{139}{1192576}x^3-\frac{137}{298144}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{1173\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,F\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\right.}{1192576\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$

input
```
int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/33392128*x*(453975*x^6+2838330*x^4+5811451*x^2+4496212)/(5*x^2+7)^2/(x^4
+3*x^2+4)^(1/2)+1173/1192576/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2)
)*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipt
icF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+18159/1043504/
(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7
^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+
2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1
/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-529425/233744896/(-3/8+1/8*I*7^(1/2
))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))
^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-
3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

3.379.5 Fricas [F]

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}} \, dx = \int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}} (5x^2+7)^3} \, dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^14 + 1275*x^12 + 6010*x^10 + 16678*x
^8 + 29153*x^6 + 31871*x^4 + 19992*x^2 + 5488), x)
```

3.379.6 Sympy [F]

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}} \, dx = \int \frac{1}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}} (5x^2+7)^3} \, dx$$

input
```
integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
```

output
```
Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3), x)
```

3.379.7 Maxima [F]

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}} \, dx = \int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}} (5x^2+7)^3} \, dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

3.379.8 Giac [F]

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}} \, dx = \int \frac{1}{(x^4+3x^2+4)^{\frac{3}{2}} (5x^2+7)^3} \, dx$$

input
```
integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}} \, dx = \int \frac{1}{(5x^2+7)^3 (x^4+3x^2+4)^{3/2}} \, dx$$

input
```
int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2)),x)
```

output
```
int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2)), x)
```

3.380 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}} dx$

3.380.1 Optimal result

Integrand size = 26, antiderivative size = 467

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))\,x\sqrt{a+bx^2+cx^4}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}(15c^2d^3-15acde^2+4abe^3)}{\sqrt{a}}+e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{Ellipti}}{30c^{11/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/15*e^2*(-4*b*e+15*c*d)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/5*e^3*x^3*(c*x^4+b*
x^2+a)^(1/2)/c+1/15*e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d))*x*(c*x^4
+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+x^2*c^(1/2))-1/15*a^(1/4)*e*(45*c^2*d^2+8
*b^2*e^2-3*c*e*(3*a*e+10*b*d))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)+1/30*a^(1/4)
*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)
))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1
/2))*(a^(1/2)+x^2*c^(1/2))*(e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d))+
(4*a*b*e^3-15*a*c*d*e^2+15*c^2*d^3)*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

3.380.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.96 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}e^2x(a+bx^2+cx^4)\left(-4be+3c(5d+ex^2)\right)+i\left(-b+\sqrt{b^2-4ac}\right)e(45c^2d^2+8b^2e^2-3ce(10b}{}$$

input
```
Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4)*(-4*b*e + 3
*c*(5*d + e*x^2)) + I*(-b + Sqrt[b^2 - 4*a*c])*e*(45*c^2*d^2 + 8*b^2*e^2 -
 3*c*e*(10*b*d + 3*a*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(30*c^3*d^3 + 8*b^2*(-
b + Sqrt[b^2 - 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*Sqrt[b^2 - 4*a*c]*d -
2*a*e) + c*e^2*(30*b^2*d - 30*b*Sqrt[b^2 - 4*a*c]*d + 17*a*b*e - 9*a*Sqrt[
b^2 - 4*a*c]*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

3.380.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1518, 2207, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \ 1518$$

$$\frac{\int \frac{e^2(15cd-4be)x^4+3e(5cd^2-ae^2)x^2+5cd^3}{\sqrt{cx^4+bx^2+a}}dx}{5c}+\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$\downarrow$ 2207

$$\frac{\frac{\int \frac{15c^2d^3-15ace^2d+4abe^3+e\left(45c^2d^2+8b^2e^2-3ce(10bd+3ae)\right)x^2}{\sqrt{cx^4+bx^2+a}}dx}{3c}+\frac{e^2x\sqrt{a+bx^2+cx^4}(15cd-4be)}{3c}}{5c}+\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$\downarrow$ 1511

$$\frac{\frac{\left(\frac{\sqrt{a}e\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)}{\sqrt{c}}+4abe^3-15acde^2+15c^2d^3\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}e\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}+}{5c}$$
$$\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$\downarrow$ 27

$$\frac{\frac{\left(\frac{\sqrt{a}e\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)}{\sqrt{c}}+4abe^3-15acde^2+15c^2d^3\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{e\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}+e^2x\sqrt{}}{5c}$$
$$\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$\downarrow$ 1416

$$\frac{\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)}{\sqrt{c}}+4abe^3-15acde^2+15c^2d^3\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{e\left(-3ce(3ae+\right.}{}}{3c}}{5c}$$
$$\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$\downarrow$ 1509

$$\frac{\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)}{\sqrt{c}}+4abe^3-15acde^2+15c^2d^3\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{e\left(-3ce(3ae+\right.}{}}{3c}}{5c}$$
$$\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}$$

3.380. $\int \frac{\left(d+ex^2\right)^3}{\sqrt{a+bx^2+cx^4}}\,dx$

input
```
Int[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(e^3*x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) + ((e^2*(15*c*d - 4*b*e)*x*Sqrt[a
+ b*x^2 + c*x^4])/(3*c) + (-((e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d +
3*a*e))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))))/Sqrt[c]) + ((15*c^2*d^3 - 15*a*c*d*e^2
 + 4*a*b*e^3 + (Sqrt[a]*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)
))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqr
t[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/(3*c))/(5*c)
```

3.380.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.380.4 Maple [A] (verified)

Time = 7.42 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.04

method	result
elliptic	$\frac{e^3x^3\sqrt{c\,x^4+b\,x^2+a}}{5c}+\frac{\left(3d\,e^2-\frac{4e^3b}{5c}\right)x\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\left(d^3-\frac{\left(3d\,e^2-\frac{4e^3b}{5c}\right)a}{3c}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b\right.}}{a}}}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4}}$
risch	$-\frac{e^2x\left(-3c\,x^2e+4be-15cd\right)\sqrt{c\,x^4+b\,x^2+a}}{15c^2}+\frac{15c^2d^3\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}$
default	Expression too large to display

input
```
int((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*e^3*x^3*(c*x^4+b*x^2+a)^(1/2)/c+1/3*(3*d*e^2-4/5*e^3/c*b)/c*x*(c*x^4+b
*x^2+a)^(1/2)+1/4*(d^3-1/3*(3*d*e^2-4/5*e^3/c*b)/c*a)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2))-1/2*(3*d^2*e-3/5*e^3/c*a-2/3*(3*d*e^2-4/5*e^3/c*b)/c*b)*a*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*
a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2)))
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\sqrt{\frac{1}{2}}\Big((45\,ac^3d^2e - 30\,abc^2de^2 + (8\,ab^2c - 9\,a^2c^2)e^3)x\sqrt{\frac{b^2-4\,ac}{c^2}} - (45\,abc^2d^2e - 30\,ab^2cde^2 + (8\,ab^3 - 9\,a^2b}{}$$

input
```
integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(sqrt(1/2)*((45*a*c^3*d^2*e - 30*a*b*c^2*d*e^2 + (8*a*b^2*c - 9*a^2*c
^2)*e^3)*x*sqrt((b^2 - 4*a*c)/c^2) - (45*a*b*c^2*d^2*e - 30*a*b^2*c*d*e^2
+ (8*a*b^3 - 9*a^2*b*c)*e^3)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) -
b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/
x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((1
5*c^4*d^3 - 45*a*c^3*d^2*e + 15*(2*a*b*c^2 - a*c^3)*d*e^2 - (8*a*b^2*c - (
9*a^2 + 4*a*b)*c^2)*e^3)*x*sqrt((b^2 - 4*a*c)/c^2) + (15*b*c^3*d^3 + 45*a*
b*c^2*d^2*e - 15*(2*a*b^2*c + a*b*c^2)*d*e^2 + (8*a*b^3 - (9*a^2*b - 4*a*b
^2)*c)*e^3)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(
arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt
((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(3*a*c^3*e^3*x^4 + 45*a*c^3*
d^2*e - 30*a*b*c^2*d*e^2 + (8*a*b^2*c - 9*a^2*c^2)*e^3 + (15*a*c^3*d*e^2 -
 4*a*b*c^2*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*c^4*x)
```

3.380.6 Sympy [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x**2)**3/sqrt(a + b*x**2 + c*x**4), x)
```

3.380.7 Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

3.380.8 Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(e\,x^2+d)^3}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.381 $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$

3.381.1 Optimal result

Integrand size = 26, antiderivative size = 356

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}+\frac{2e(3cd-be)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{2\sqrt[4]{a}e(3cd-be)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(2e(3cd-be)+\frac{\sqrt{c}(3cd^2-ae^2)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/3*e^2*x*(c*x^4+b*x^2+a)^(1/2)/c+2/3*e*(-b*e+3*c*d)*x*(c*x^4+b*x^2+a)^(1/
2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-2/3*a^(1/4)*e*(-b*e+3*c*d)*(cos(2*arctan(
c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x
^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4
+b*x^2+a)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*e*(-b*e+3*c*d)+(-
a*e^2+3*c*d^2)*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^
(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

3.381.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.10 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}e^2x(a+bx^2+cx^4) - i\left(-b+\sqrt{b^2-4ac}\right)e(-3cd+be)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E}{}$$

input
```
Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4) - I*(-b + S
qrt[b^2 - 4*a*c])*e*(-3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - S
qrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*
c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-3*c^2*d^2
+ b*(-b + Sqrt[b^2 - 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[b^2 - 4*a*c]*d + a*
e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(
2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])])/(6*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a
+ b*x^2 + c*x^4])
```

3.381.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1518, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\downarrow 1518$$

$$\frac{\int \frac{3cd^2-ae^2+2e(3cd-be)x^2}{\sqrt{cx^4+bx^2+a}}dx}{3c}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \text{1511}$$

$$\frac{\left(\frac{2\sqrt{a}e(3cd-be)}{\sqrt{c}}-ae^2+3cd^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}e(3cd-be)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{2\sqrt{a}e(3cd-be)}{\sqrt{c}}-ae^2+3cd^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2e(3cd-be)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{2\sqrt{a}e(3cd-be)}{\sqrt{c}}-ae^2+3cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2e(3cd-be)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{2\sqrt{a}e(3cd-be)}{\sqrt{c}}-ae^2+3cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2e(3cd-be)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2\cdots}{(\sqrt{a}+\sqrt{\cdots}}}}{\cdots}\right.}{\cdots}}{3c}$$

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}$$

input
```
Int[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(e^2*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + ((-2*e*(3*c*d - b*e)*(-((x*Sqrt[a
+ b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x
^2 + c*x^4])))/Sqrt[c] + ((3*c*d^2 - a*e^2 + (2*Sqrt[a]*e*(3*c*d - b*e))/S
qrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]
))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/(3*c)
```

3.381. $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$

3.381.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.381.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{e^2 x\sqrt{c x^4+b x^2+a}}{3c}+\frac{\left(d^2-\frac{a e^2}{3c}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4\cdots\right.}}{ac}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4+b x^2+a}}$
risch	$\frac{e^2 x\sqrt{c x^4+b x^2+a}}{3c}-\frac{\frac{a e^2\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4+b x^2+a}}-\cdots}{\cdots}$
default	$\frac{d^2\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4+b x^2+a}}+e^2\left(\frac{x\sqrt{c x^4+b x^{\cdots}}}{3c}\cdots\right.$

input
```
int((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*e^2*x*(c*x^4+b*x^2+a)^(1/2)/c+1/4*(d^2-1/3*a/c*e^2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2))-1/2*(2*e*d-2/3*b/c*e^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(
1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

3.381.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{2\sqrt{\frac{1}{2}}\left((3\,ac^2de-abce^2)x\sqrt{\frac{b^2-4\,ac}{c^2}}-(3\,abcde-ab^2e^2)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid \frac{b}{}\right.}{}$$

input
```
integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(1/2)*((3*a*c^2*d*e - a*b*c*e^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (3
*a*b*c*d*e - a*b^2*e^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)
*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1
/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((3*c^3*
d^2 - 6*a*c^2*d*e + (2*a*b*c - a*c^2)*e^2)*x*sqrt((b^2 - 4*a*c)/c^2) + (3*
b*c^2*d^2 + 6*a*b*c*d*e - (2*a*b^2 + a*b*c)*e^2)*x)*sqrt(c)*sqrt((c*sqrt((
b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 -
4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a
*c)) + 2*(a*c^2*e^2*x^2 + 6*a*c^2*d*e - 2*a*b*c*e^2)*sqrt(c*x^4 + b*x^2 +
a))/(a*c^3*x)
```

3.381.6 Sympy [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x**2)**2/sqrt(a + b*x**2 + c*x**4), x)
```

3.381.7 Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

3.381.8 Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(e\,x^2+d)^2}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.382 $\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$

3.382.1 Optimal result

Integrand size = 24, antiderivative size = 283

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
e*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*a
rctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipt
icE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(
1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/
(c*x^4+b*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/
4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a
^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x
^2+a)^(1/2)
```

3.382.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\left(-b+\sqrt{b^2-4ac}\right)eE\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(-2\right.}{2\sqrt{2}c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*Ellip
ticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*E
llipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

3.382.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \; 1511$$
$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$
$$\Big\downarrow \; 27$$

$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

input
```
Int[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
-((e*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^
(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + ((d + (Sqrt[a]*e)/Sqrt[c])*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a
^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

3.382. $\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$

3.382.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.382.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.28

method	result
default	$\frac{d\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}-\frac{ea\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{\cdots\right.}\right.}{}}}{}$
elliptic	$\frac{d\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}-\frac{ea\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{\cdots\right.}\right.}{}}}{}$

input
```
int((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*e*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ellipti
cF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

3.382.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.05

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{cx^4+bx^2+a}ace + \sqrt{\frac{1}{2}}\left(acex\sqrt{\frac{b^2-4ac}{c^2}} - abex\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}}{x}\right) \mid \frac{bc\sqrt{\frac{b^2-4}{c^2}}}{}\right.}{}$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(c*x^4 + b*x^2 + a)*a*c*e + sqrt(1/2)*(a*c*e*x*sqrt((b^2 - 4*a*
c)/c^2) - a*b*e*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*ellipti
c_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*
sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((c^2*d - a*c*e)
*x*sqrt((b^2 - 4*a*c)/c^2) + (b*c*d + a*b*e)*x)*sqrt(c)*sqrt((c*sqrt((b^2
- 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*
c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c))
)/(a*c^2*x)
```

3.382.6 Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

3.382.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.382.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x^2)/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.383 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

3.383.1 Optimal result

Integrand size = 26, antiderivative size = 401

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{e}\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{cd^2-bde+ae^2}} + \frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}} - \frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}d\,(cd^2-ae^2)\sqrt{a+bx^2+cx^4}}$$

output
```
1/2*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/
2))*e^(1/2)/d^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2
*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(-e*a^(1
/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)-1/4*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan
(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^
2*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)
/(c*x^4+b*x^2+a)^(1/2)
```

3.383.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.53

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticPi}\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd}, i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right), \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}d\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 - 4*a*c]
)*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*d*Sqrt[a + b*x^2 + c*x^4])
```

3.383.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1540, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1540}$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e} - \frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e} - \frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \ 1416$$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{e\int\frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \ 2220$$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{e\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{x\sqrt{ae^2-bde}}{\sqrt{d}\sqrt{e}\sqrt{a+bx}}\right.}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}\right.}{\sqrt{c}d-\sqrt{a}e}$$

input
```
Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (e*(
-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt
[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e
 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[c]
*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + b*x^2 + c*x^4])))/(S
qrt[c]*d - Sqrt[a]*e)
```

3.383. $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

3.383.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

3.383.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^{2}}{2a}-\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{1+\frac{b\,x^{2}}{2a}+\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\Pi\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},-\frac{2ae}{\left(-b+\sqrt{-4ac+b^{2}}\right)d},\frac{\sqrt{-\frac{b+\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{2}}{\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}\right)}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$	200
elliptic	$\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^{2}}{2a}-\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{1+\frac{b\,x^{2}}{2a}+\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\Pi\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},-\frac{2ae}{\left(-b+\sqrt{-4ac+b^{2}}\right)d},\frac{\sqrt{-\frac{b+\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{2}}{\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}\right)}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$	200

3.383. $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

input
```
int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(
-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2
)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

3.383.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.383.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.383.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.383.8 Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.384 $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx$

3.384.1 Optimal result

Integrand size = 26, antiderivative size = 718

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx &= -\frac{\sqrt{c}ex\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&+\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}+\frac{\sqrt{e}(3cd^2-e(2bd-ae))\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}\left(cd^2-bde+ae^2\right)^{3/2}} \\
&+\frac{\sqrt[4]{a}\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} \\
&+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}} \\
&-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(3cd^2-e(2bd-ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

output
```
1/4*(3*c*d^2-e*(-a*e+2*b*d))*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^
(1/2)/(c*x^4+b*x^2+a)^(1/2))*e^(1/2)/d^(3/2)/(a*e^2-b*d*e+c*d^2)^(3/2)+1/2
*e^2*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1/2*e*x*c^(1/
2)*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(a^(1/2)+x^2*c^(1/2))+1/2*a
^(1/4)*c^(1/4)*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c
^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(
1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c
^(1/2))^2)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))
*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2
))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a
^(1/4)/d/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)-1/8*(3*c*d^2-e*(-a*e
+2*b*d))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x
/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*
c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(e*a^(1/2)
+d*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2
)^(1/2)/a^(1/4)/c^(1/4)/d^2/(a*e^2-b*d*e+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*
x^4+b*x^2+a)^(1/2)
```

3.384.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.51 (sec) , antiderivative size = 1069, normalized size of antiderivative = 1.49

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de^2x(a+bx^2+cx^4)+i\sqrt{2}\left(b-\sqrt{b^2-4ac}\right)de\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}(d+ex^2)\Big(E}{}$$

input
```
Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e^2*x*(a + b*x^2 + c*x^4) + I*Sqrt[2]
*(b - Sqrt[b^2 - 4*a*c])*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)
*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqr
t[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])]) + (2*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^
2)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sq
rt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (6*I)*Sqrt[2]*c*d^2*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)
/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (4*I)*Sqrt[2]*b*d*e*Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2
*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (2*I)*Sqrt[2]*a*e^2*Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2...
```

3.384.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 694, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1551, 25, 2232, 27, 1509, 2226, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1551}$$

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d(d+ex^2)(ae^2-bde+cd^2)}-\frac{\int -\frac{-ce^2x^4-2cdex^2+2cd^2-e(2bd-ae)}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{2d(ae^2-bde+cd^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-ce^2x^4-2cdex^2+2cd^2-e(2bd-ae)}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{2d(ae^2-bde+cd^2)}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d(d+ex^2)(ae^2-bde+cd^2)}$$

$\Big\downarrow$ 2232

$$\frac{\frac{\int \frac{ce\left(2cd^2-\sqrt{a}\sqrt{c}ed-\sqrt{ce}(\sqrt{c}d+\sqrt{a}e)x^2-e(2bd-ae)\right)}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx}{ce}+\sqrt{a}\sqrt{ce}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{2d\left(ae^2-bde+cd^2\right)}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{2cd^2-\sqrt{a}\sqrt{c}ed-\sqrt{ce}(\sqrt{c}d+\sqrt{a}e)x^2-e(2bd-ae)}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx+\sqrt{ce}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{2d\left(ae^2-bde+cd^2\right)}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 1509

$$\frac{\int \frac{2cd^2-\sqrt{a}\sqrt{c}ed-\sqrt{ce}(\sqrt{c}d+\sqrt{a}e)x^2-e(2bd-ae)}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx+\sqrt{ce}\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a}}{\sqrt{}}\cdots\right.}{2d\left(ae^2-bde+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 2226

$$\frac{\frac{2\sqrt{c}(ae^2-bde+cd^2)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\sqrt{a}e\left(3cd^2-e(2bd-ae)\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\sqrt{ce}\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\cdots\right.}{\sqrt[4]{c}\sqrt{a+bx^2}\cdots}\right.}{2d\left(ae^2-bde+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\sqrt{c}(ae^2-bde+cd^2)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{e\left(3cd^2-e(2bd-ae)\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\sqrt{ce}\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\cdots\right.}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.}{2d\left(ae^2-bde+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 1416

3.384. $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx$

$$\frac{-\frac{e(3cd^2-e(2bd-ae))\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(ae^2-bde+cd^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{a}\sqrt{a+bx^2+cx^4}(\sqrt{c}d-\sqrt{a}e)}}{2d\,(ae^2-bde+cd^2)}$$

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\,(d+ex^2)\,(ae^2-bde+cd^2)}$$

$$\Big\downarrow \text{2220}$$

$$\frac{\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(ae^2-bde+cd^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{a}\sqrt{a+bx^2+cx^4}(\sqrt{c}d-\sqrt{a}e)}-\frac{e(3cd^2-e(2bd-ae))\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)}}}{}\right.}{}}{}$$

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\,(d+ex^2)\,(ae^2-bde+cd^2)}$$

input
```
Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(e^2*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2))
+ (Sqrt[c]*e*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^
(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])) + (c^(1/4)*(c*d^2 - b*d*e + a*e^2)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (e*(3*c*d^2 -
e*(2*b*d - a*e))*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 - b*d*e
 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*
Sqrt[c*d^2 - b*d*e + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*
(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + b*
x^2 + c*x^4])))/(Sqrt[c]*d - Sqrt[a]*e))/(2*d*(c*d^2 - b*d*e + a*e^2))
```

3.384. $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx$

3.384.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2232
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + b*x^2
 + c*x^4], x], x] + Simp[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d -
 a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
&&  !GtQ[b^2 - 4*a*c, 0]
```

3.384.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 1279, normalized size of antiderivative = 1.78

method	result	size
default	Expression too large to display	1279
elliptic	Expression too large to display	1279

input
```
int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*e^2*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1/8*c/(a*e
^2-b*d*e+c*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2
*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^
(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/d/(a*e^
2-b*d*e+c*d^2)*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-
2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))
^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2))-1/4*c*e/d/(a*e^2-b*d*e+c*d^2)*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1
/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x
^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1/2)/(
-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(
1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^
2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(
-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))*a-1/(a*e^2-b*d*e+c*d^2)*e/d*2^(1/2)/(-
b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)...
```

3.384.5 Fricas [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)/(c*e^2*x^8 + (2*c*d*e + b*e^2)*x^6 + (c*d
^2 + 2*b*d*e + a*e^2)*x^4 + a*d^2 + (b*d^2 + 2*a*d*e)*x^2), x)
```

3.384.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 + c*x**4)), x)
```

3.384.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

3.384.8 Giac [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(e\,x^2+d)^2\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.385 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$

3.385.1 Optimal result

Integrand size = 27, antiderivative size = 553

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e(45c^2d^2+8b^2e^2+3ce(10bd+3ae))\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}} + \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\left(\frac{2c(15c^2d^3+15acde^2+4abe^3)}{b-\sqrt{b^2+4ac}}+e(45c^2d^2+8b^2e^2+3ce(10bd+3ae))\right)\sqrt{1-}}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}$$

output
```
-1/15*e^2*(4*b*e+15*c*d)*x*(-c*x^4+b*x^2+a)^(1/2)/c^2-1/5*e^3*x^3*(-c*x^4+
b*x^2+a)^(1/2)/c-1/60*e*(45*c^2*d^2+8*b^2*e^2+3*c*e*(3*a*e+10*b*d))*Ellipt
icE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(
b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^
2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/
2)))^(1/2)/c^(7/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/60*EllipticF(x*2^(1/2)
*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)
^(1/2)))^(1/2))*(e*(45*c^2*d^2+8*b^2*e^2+3*c*e*(3*a*e+10*b*d))+2*c*(4*a*b*
e^3+15*a*c*d*e^2+15*c^2*d^3)/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))*
(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c
*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(7/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

3.385.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.72 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$$
$$= \frac{-4c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}e^2x(a+bx^2-cx^4)\left(4be+3c(5d+ex^2)\right)-i\sqrt{2}\left(-b+\sqrt{b^2+4ac}\right)e(45c^2d^2+8b^2e^2+3c}{}$$

input
```
Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
(-4*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(a + b*x^2 - c*x^4)*(4*b*e
+ 3*c*(5*d + e*x^2)) - I*Sqrt[2]*(-b + Sqrt[b^2 + 4*a*c])*e*(45*c^2*d^2 +
8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)
/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sq
rt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a
*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-
30*c^3*d^3 + 8*b^2*(-b + Sqrt[b^2 + 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*S
qrt[b^2 + 4*a*c]*d - 2*a*e) + c*e^2*(-30*b^2*d + 30*b*Sqrt[b^2 + 4*a*c]*d
- 17*a*b*e + 9*a*Sqrt[b^2 + 4*a*c]*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x
^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b +
 Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 +
4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(60*c^3*Sq
rt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

3.385.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1518, 25, 2207, 25, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$$
$$\Big\downarrow \; 1518$$

$$-\frac{\int -\frac{e^2(15cd+4be)x^4+3e(5cd^2+ae^2)x^2+5cd^3}{\sqrt{-cx^4+bx^2+a}}dx}{5c}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

$\downarrow$ 25

$$\frac{\int \frac{e^2(15cd+4be)x^4+3e(5cd^2+ae^2)x^2+5cd^3}{\sqrt{-cx^4+bx^2+a}}dx}{5c}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

$\downarrow$ 2207

$$-\frac{-\frac{\int -\frac{15c^2d^3+15ace^2d+4abe^3+e\left(45c^2d^2+8b^2e^2+3ce(10bd+3ae)\right)x^2}{\sqrt{-cx^4+bx^2+a}}dx}{3c}-\frac{e^2x\sqrt{a+bx^2-cx^4}(4be+15cd)}{3c}}{5c}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{15c^2d^3+15ace^2d+4abe^3+e\left(45c^2d^2+8b^2e^2+3ce(10bd+3ae)\right)x^2}{\sqrt{-cx^4+bx^2+a}}dx}{3c}-\frac{e^2x\sqrt{a+bx^2-cx^4}(4be+15cd)}{3c}}{5c}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

$\downarrow$ 1514

$$\frac{\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{15c^2d^3+15ace^2d+4abe^3+e\left(45c^2d^2+8b^2e^2+3ce(10bd+3ae)\right)x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{3c\sqrt{a+bx^2-cx^4}}-\frac{e^2x\sqrt{a+bx^2-cx^4}(4be+15cd)}{3c}}{5c}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

$\downarrow$ 399

$$\frac{\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(b-\sqrt{4ac+b^2}\right)\left(\frac{2c\left(4abe^3+15acde^2+15c^2d^3\right)}{b-\sqrt{4ac+b^2}}+e\left(3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\ldots}{2c}\ldots\right)}{3c\sqrt{a+bx^2-cx^4}}\ldots}{5c}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

$\downarrow$ 321

3.385. $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2c\left(4abe^3+15acde^2+15c^2d^3\right)}{b-\sqrt{4ac+b^2}}+e\left(3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{}{\sqrt{b}}\right.\right.}{2\sqrt{2}c^{3/2}}\right.}{3c\sqrt{a+bx^2-cx^4}} \quad 5c$$

$$\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

$\downarrow$ 327

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2c\left(4abe^3+15acde^2+15c^2d^3\right)}{b-\sqrt{4ac+b^2}}+e\left(3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{}{\sqrt{b}}\right.\right.}{2\sqrt{2}c^{3/2}}\right.}{3c\sqrt{a+bx^2-cx^4}}$$

$$\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

input
```
Int[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
-1/5*(e^3*x^3*Sqrt[a + b*x^2 - c*x^4])/c + (-1/3*(e^2*(15*c*d + 4*b*e)*x*S
qrt[a + b*x^2 - c*x^4])/c + (Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*S
qrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(-1/2*((b - Sqrt[b^2 + 4*a*c])*
Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*
a*e))*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (
b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2)) + ((b -
 Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*((2*c*(15*c^2*d^3 + 15*a*c
*d*e^2 + 4*a*b*e^3))/(b - Sqrt[b^2 + 4*a*c]) + e*(45*c^2*d^2 + 8*b^2*e^2 +
 3*c*e*(10*b*d + 3*a*e)))*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sq
rt[2]*c^(3/2))))/(3*c*Sqrt[a + b*x^2 - c*x^4]))/(5*c)
```

3.385. $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$

3.385.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.385. $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}} dx$

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.385.4 Maple [A] (verified)

Time = 6.41 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.88

method	result
elliptic	$-\frac{e^{3} x^{3} \sqrt{-c\,x^{4}+b\,x^{2}+a}}{5 c}-\frac{\left(3 d\,e^{2}+\frac{4 e^{3} b}{5 c}\right) x \sqrt{-c\,x^{4}+b\,x^{2}+a}}{3 c}+\frac{\left(d^{3}+\frac{a \left(3 d\,e^{2}+\frac{4 e^{3} b}{5 c}\right)}{3 c}\right) \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{4a \cdots}\right.}{a}}\cdots}{4 \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}\, \sqrt{-\cdots}}$
risch	$-\frac{e^{2} x \left(3 c\,x^{2} e+4 b e+15 c d\right) \sqrt{-c\,x^{4}+b\,x^{2}+a}}{15 c^{2}}+\frac{\frac{15 c^{2} d^{3} \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}}{2}, \sqrt{-4\cdots}\right.}{4 \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}\, \sqrt{-c\,x^{4}+b\,x^{2}+a}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*e^3*x^3*(-c*x^4+b*x^2+a)^(1/2)/c-1/3*(3*d*e^2+4/5*e^3/c*b)/c*x*(-c*x^
4+b*x^2+a)^(1/2)+1/4*(d^3+1/3*a/c*(3*d*e^2+4/5*e^3/c*b))*2^(1/2)/((-b+(4*a
*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1
/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c
)^(1/2))-1/2*(3*d^2*e+3/5*e^3/c*a+2/3*b/c*(3*d*e^2+4/5*e^3/c*b))*a*2^(1/2)
/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+
b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

3.385.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left((45\,ac^3d^2e+30\,abc^2de^2+(8\,ab^2c+9\,a^2c^2)e^3)\sqrt{-c}x\sqrt{\frac{b^2+4\,ac}{c^2}}+(45\,abc^2d^2e+30\,ab^2cde^2+(8\,ab^3\right.}{}$$

input
```
integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(sqrt(1/2)*((45*a*c^3*d^2*e + 30*a*b*c^2*d*e^2 + (8*a*b^2*c + 9*a^2*
c^2)*e^3)*sqrt(-c)*x*sqrt((b^2 + 4*a*c)/c^2) + (45*a*b*c^2*d^2*e + 30*a*b^
2*c*d*e^2 + (8*a*b^3 + 9*a^2*b*c)*e^3)*sqrt(-c)*x)*sqrt((c*sqrt((b^2 + 4*a
*c)/c^2) + b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^
2) + b)/c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^2 - 2*a*c)/(a*c)) - sq
rt(1/2)*((15*c^4*d^3 + 45*a*c^3*d^2*e + 15*(2*a*b*c^2 + a*c^3)*d*e^2 + (8*
a*b^2*c + (9*a^2 + 4*a*b)*c^2)*e^3)*sqrt(-c)*x*sqrt((b^2 + 4*a*c)/c^2) - (
15*b*c^3*d^3 - 45*a*b*c^2*d^2*e - 15*(2*a*b^2*c - a*b*c^2)*d*e^2 - (8*a*b^
3 + (9*a^2*b - 4*a*b^2)*c)*e^3)*sqrt(-c)*x)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2
) + b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)
/c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^2 - 2*a*c)/(a*c)) + 2*(3*a*c^
3*e^3*x^4 + 45*a*c^3*d^2*e + 30*a*b*c^2*d*e^2 + (8*a*b^2*c + 9*a^2*c^2)*e^
3 + (15*a*c^3*d*e^2 + 4*a*b*c^2*e^3)*x^2)*sqrt(-c*x^4 + b*x^2 + a))/(a*c^4
*x)
```

3.385.6 Sympy [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate((e*x**2+d)**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x**2)**3/sqrt(a + b*x**2 - c*x**4), x)
```

3.385.7 Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{(ex^2+d)^3}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.385.8 Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{(ex^2+d)^3}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{(e\,x^2+d)^3}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x^2)^3/(a + b*x^2 - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^3/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.386 $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx$

3.386.1 Optimal result

Integrand size = 27, antiderivative size = 454

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e(3cd+be)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\middle|\frac{b\cdots}{b\cdots}\right.}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\left(3c^2d^2+b\left(b-\sqrt{b^2+4ac}\right)e^2+ce\left(3bd-3\sqrt{b^2+4ac}d+ae\right)\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\cdots}}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

output
```
-1/3*e^2*x*(-c*x^4+b*x^2+a)^(1/2)/c-1/6*e*(b*e+3*c*d)*EllipticE(x*2^(1/2)*
c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^
(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/
2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(
5/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/6*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*
a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))
*(3*c^2*d^2+b*e^2*(b-(4*a*c+b^2)^(1/2))+c*e*(3*b*d+a*e-3*d*(4*a*c+b^2)^(1/
2)))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(
1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(5/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1
/2)
```

3.386.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.91 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx$$
$$= \frac{2c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}e^2x(-a-bx^2+cx^4)-i\sqrt{2}\left(-b+\sqrt{b^2+4ac}\right)e(3cd+be)\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2}{-b+\sqrt{b^2+4ac}}}}{}$$

input
```
Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
(2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(-a - b*x^2 + c*x^4) - I*Sqr
t[2]*(-b + Sqrt[b^2 + 4*a*c])*e*(3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 + 4*a*c]
- 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2
)/(-b + Sqrt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt
[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*
Sqrt[2]*(-3*c^2*d^2 + b*(-b + Sqrt[b^2 + 4*a*c])*e^2 - c*e*(3*b*d - 3*Sqrt
[b^2 + 4*a*c]*d + a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b
^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a
*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (
b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*c^2*Sqrt[-(c/(b + Sqrt
[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

3.386.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1518, 25, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx$$
$$\Big\downarrow \text{1518}$$

$$-\frac{\int -\frac{3cd^2+ae^2+2e(3cd+be)x^2}{\sqrt{-cx^4+bx^2+a}}dx}{3c}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{3cd^2+ae^2+2e(3cd+be)x^2}{\sqrt{-cx^4+bx^2+a}}dx}{3c}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$

$\Big\downarrow$ 1514

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{3cd^2+ae^2+2e(3cd+be)x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{3c\sqrt{a+bx^2-cx^4}}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$

$\Big\downarrow$ 399

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)+3c^2d^2\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{c}-\cdots\right.}{3c\sqrt{a+bx^2-cx^4}}$$

$$\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)+3c^2d^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{b+\sqrt{b^2\cdots}}}\right.\right.}{\sqrt{2}c^{3/2}}\right.}{3c\sqrt{a+bx^2-cx^4}}$$

$$\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)+3c^2d^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{b+\sqrt{b^2\cdots}}}\right.\right.}{\sqrt{2}c^{3/2}}\right.}{3c\sqrt{a+bx^2-cx^4}}$$

$$\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$

3.386. $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx$

input
```
Int[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
-1/3*(e^2*x*Sqrt[a + b*x^2 - c*x^4])/c + (Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2
 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(-(((b - Sqrt[b^2
+ 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*(3*c*d + b*e)*EllipticE[ArcSin[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b
- Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2))) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(
3*c^2*d^2 + b*(b - Sqrt[b^2 + 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[b^2 + 4*a*
c]*d + a*e))*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*
c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2)))
)/(3*c*Sqrt[a + b*x^2 - c*x^4])
```

3.386.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.386.4 Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.91

method	result
elliptic	$-\frac{e^{2} x \sqrt{-c x^{4}+b x^{2}+a}}{3 c}+\frac{\left(d^{2}+\frac{a e^{2}}{3 c}\right) \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4-\frac{2 b \left(b+\sqrt{4 a c\cdots}\right)}{a c}}}{2}\cdots\right.}{4 \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}\, \sqrt{-c x^{4}+b x^{2}+a}\cdots}$
risch	$-\frac{e^{2} x \sqrt{-c x^{4}+b x^{2}+a}}{3 c}+\frac{\frac{a e^{2} \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4-\frac{2 b \left(b+\sqrt{4 a c+b^{2}}\right)}{a c}}}{2}\right)}{4 \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}\, \sqrt{-c x^{4}+b x^{2}+a}}+\cdots}{\cdots}$
default	$\frac{d^{2} \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{4 a c+b^{2}}\right) x^{2}}{a}}\, F\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4-\frac{2 b \left(b+\sqrt{4 a c+b^{2}}\right)}{a c}}}{2}\right)}{4 \sqrt{\frac{-b+\sqrt{4 a c+b^{2}}}{a}}\, \sqrt{-c x^{4}+b x^{2}+a}}+e^{2} \left(-\frac{x \sqrt{-c x^{4}+b x^{2}+\cdots}}{3 c}\cdots\right.$

input
```
int((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*e^2*x*(-c*x^4+b*x^2+a)^(1/2)/c+1/4*(d^2+1/3*a/c*e^2)*2^(1/2)/((-b+(4*
a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+
(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(
1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/
c)^(1/2))-1/2*(2*e*d+2/3*b/c*e^2)*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^
(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a
/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

3.386.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{2\sqrt{\frac{1}{2}}\left((3\,ac^2de+abce^2)\sqrt{-c}x\sqrt{\frac{b^2+4\,ac}{c^2}}+(3\,abcde+ab^2e^2)\sqrt{-c}x\right)\sqrt{\frac{c\sqrt{\frac{b^2+4\,ac}{c^2}}+b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2+4}{c^2}}}{c}}}{x}\right.}{}$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(1/2)*((3*a*c^2*d*e + a*b*c*e^2)*sqrt(-c)*x*sqrt((b^2 + 4*a*c)
/c^2) + (3*a*b*c*d*e + a*b^2*e^2)*sqrt(-c)*x)*sqrt((c*sqrt((b^2 + 4*a*c)/c
^2) + b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) +
b)/c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^2 - 2*a*c)/(a*c)) - sqrt(1/
2)*((3*c^3*d^2 + 6*a*c^2*d*e + (2*a*b*c + a*c^2)*e^2)*sqrt(-c)*x*sqrt((b^2
 + 4*a*c)/c^2) - (3*b*c^2*d^2 - 6*a*b*c*d*e - (2*a*b^2 - a*b*c)*e^2)*sqrt(
-c)*x)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)*elliptic_f(arcsin(sqrt(1/2)
*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c
^2) - b^2 - 2*a*c)/(a*c)) + 2*(a*c^2*e^2*x^2 + 6*a*c^2*d*e + 2*a*b*c*e^2)*
sqrt(-c*x^4 + b*x^2 + a))/(a*c^3*x)
```

3.386.6 Sympy [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate((e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x**2)**2/sqrt(a + b*x**2 - c*x**4), x)
```

3.386.7 Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.386.8 Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.386.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*x^2 - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)^2/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.387 $\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx$

3.387.1 Optimal result

Integrand size = 25, antiderivative size = 385

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+}}}\right.\right.}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

output
```
1/4*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2
)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d+e*(b-(4*a*c+b^2)^(1/2)))*(1-
2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^
2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)-1/4*
e*EllipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^
(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(
4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+
b^2)^(1/2)))^(1/2)/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

3.387.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.76

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\left(\left(-b+\sqrt{b^2+4ac}\right)eE\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right) -}{2\sqrt{2}c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+b}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
((-1/2*I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/
(b + Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*EllipticE[I*ArcSinh[S
qrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b -
 Sqrt[b^2 + 4*a*c])] + (2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*Arc
Sinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c]
)/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]
*Sqrt[a + b*x^2 - c*x^4])
```

3.387.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

$$\Big\downarrow \text{1514}$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{ex^2+d}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{\sqrt{a+bx^2-cx^4}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{2c}-\frac{e\left(b-\sqrt{4ac+b^2}\right)\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}}{2c}\right.}{\sqrt{a+bx^2-cx^4}}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}}-\frac{e\left(b-\sqrt{4a\right.}}{}\right.}{\sqrt{a+bx^2-cx^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}}-\frac{e\left(b-\sqrt{4a\right.}}{}\right.}{\sqrt{a+bx^2-cx^4}}$$

input
```
Int[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
(Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*(-1/2*((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*
e*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b +
Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2)) + (Sqrt[b +
 Sqrt[b^2 + 4*a*c]]*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[ArcSin[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(
b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2))))/Sqrt[a + b*x^2 - c*x^4]
```

3.387. $\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx$

3.387.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

3.387.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.95

method	result
default	$\frac{d\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}}-\frac{ea\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)\cdots}{a}}\cdots}{\cdots}$
elliptic	$\frac{d\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}}-\frac{ea\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)\cdots}{a}}\cdots}{\cdots}$

input
```
int((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*d*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b
+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*
x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2
))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

3.387.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.81

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{2\sqrt{-cx^4+bx^2+a}\,ace+\sqrt{\frac{1}{2}}\left(a\sqrt{-c}cex\sqrt{\frac{b^2+4ac}{c^2}}+ab\sqrt{-c}ex\right)\sqrt{\frac{c\sqrt{\frac{b^2+4ac}{c^2}}+b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2+4ac}{c^2}}+b}{c}}}{x}\right)\right)}{}$$

input
```
integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(-c*x^4 + b*x^2 + a)*a*c*e + sqrt(1/2)*(a*sqrt(-c)*c*e*x*sqrt(
(b^2 + 4*a*c)/c^2) + a*b*sqrt(-c)*e*x)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b
)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)/x
), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((c^
2*d + a*c*e)*sqrt(-c)*x*sqrt((b^2 + 4*a*c)/c^2) - (b*c*d - a*b*e)*sqrt(-c)
*x)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)*elliptic_f(arcsin(sqrt(1/2)*sq
rt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2)
 - b^2 - 2*a*c)/(a*c)))/(a*c^2*x)
```

3.387.6 Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate((e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x**2)/sqrt(a + b*x**2 - c*x**4), x)
```

3.387.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.387.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x^2)/(a + b*x^2 - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.388 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx$

3.388.1 Optimal result

Integrand size = 27, antiderivative size = 197

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticPi}\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4}}{b-\sqrt{b^2+4}}\right.}{\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}}$$

output
```
1/2*EllipticPi(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),-1/2*e*(b+(4*
a*c+b^2)^(1/2))/c/d,((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(
1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*
x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

3.388.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticPi}\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},i\,\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right),-\frac{b+\sqrt{b^2+4}}{-b+\sqrt{b^2+}}\right.}{\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}d\sqrt{a+bx^2-cx^4}}$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```

output
```
((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
 Sqrt[b^2 + 4*a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2 + 4*a*c])*e)/(c*d), I*
ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4
*a*c])/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c
]))]*d*Sqrt[a + b*x^2 - c*x^4])
```

3.388.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1544, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx$$

$$\Big\downarrow \; 1544$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(ex^2+d)}dx}{\sqrt{a+bx^2-cx^4}}$$

$$\Big\downarrow \; 412$$

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\text{EllipticPi}\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}}$$

input
```
Int[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```

output
```
(Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*S
qrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2
+ 4*a*c])*e)/(c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]
], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sq
rt[a + b*x^2 - c*x^4])
```

3.388.3.1 Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1544
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/((d + e*x^2)*S
qrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& NegQ[c/a]
```

3.388.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{2}\sqrt{1+\frac{b x^2}{2a}-\frac{x^2\sqrt{4ac+b^2}}{2a}}\sqrt{1+\frac{b x^2}{2a}+\frac{x^2\sqrt{4ac+b^2}}{2a}}\,\Pi\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},-\frac{2ae}{\left(-b+\sqrt{4ac+b^2}\right)d},\frac{\sqrt{-\frac{b+\sqrt{4ac+b^2}}{2a}}\sqrt{2}}{\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}\right)}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{4ac+b^2}}{a}}\sqrt{-c x^4+b x^2+a}}$	201
elliptic	$\frac{\sqrt{2}\sqrt{1+\frac{b x^2}{2a}-\frac{x^2\sqrt{4ac+b^2}}{2a}}\sqrt{1+\frac{b x^2}{2a}+\frac{x^2\sqrt{4ac+b^2}}{2a}}\,\Pi\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},-\frac{2ae}{\left(-b+\sqrt{4ac+b^2}\right)d},\frac{\sqrt{-\frac{b+\sqrt{4ac+b^2}}{2a}}\sqrt{2}}{\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}\right)}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{4ac+b^2}}{a}}\sqrt{-c x^4+b x^2+a}}$	201

input
```
int(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(4
*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)/(
-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^
(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)
*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))
```

3.388.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.388.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(1/(e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x**2)*sqrt(a + b*x**2 - c*x**4)), x)
```

3.388.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.388.8 Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{(e\,x^2+d)\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/((d + e*x^2)*(a + b*x^2 - c*x^4)^(1/2)),x)
```

output
```
int(1/((d + e*x^2)*(a + b*x^2 - c*x^4)^(1/2)), x)
```

3.389 $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx$

3.389.1 Optimal result

Integrand size = 27, antiderivative size = 718

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d(cd^2+bde-ae^2)(d+ex^2)} + \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{4\sqrt{2}\sqrt{c}d\left(cd^2+e(bd-ae)\right)\sqrt{a+bx^2-cx^4}} - \frac{\sqrt{b+\sqrt{b^2+4ac}}\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2+}}}\right.\right.}{4\sqrt{2}\sqrt{c}d\left(cd^2+e(bd-ae)\right)\sqrt{a+bx^2-cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\left(3cd^2+e(2bd-ae)\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticPi}\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\text{arcsin}\right.}{2\sqrt{2}\sqrt{c}d^2\left(cd^2+e(bd-ae)\right)\sqrt{a+bx^2-cx^4}}$$

output
```
-1/2*e^2*x*(-c*x^4+b*x^2+a)^(1/2)/d/(-a*e^2+b*d*e+c*d^2)/(e*x^2+d)+1/4*(3*
c*d^2+e*(-a*e+2*b*d))*EllipticPi(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(
1/2),-1/2*e*(b+(4*a*c+b^2)^(1/2))/c/d,((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2
)^(1/2)))^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1
/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d^2/(c*d^2+e*(-a*e+b*d)
)*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)-1/8*EllipticF(x*2^(1/2)*c^(1/2)/(
b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(
1/2))*(2*c*d+e*(b-(4*a*c+b^2)^(1/2)))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1
/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d/
(c*d^2+e*(-a*e+b*d))*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/8*e*Elliptic
E(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-
(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)
^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)
))^(1/2)/d/(c*d^2+e*(-a*e+b*d))*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

3.389.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.81 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.65

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{\sqrt{a+bx^2-cx^4}\left(4de^2x+\frac{i\sqrt{2+\frac{4cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d+ex^2)\left(\left(-b+\sqrt{b^2+4ac}\right)deE\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

output
```
-1/8*(Sqrt[a + b*x^2 - c*x^4]*(4*d*e^2*x + (I*Sqrt[2 + (4*c*x^2)/(-b + Sqr
t[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(d + e*x^2)*(
(-b + Sqrt[b^2 + 4*a*c])*d*e*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqr
t[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + d
*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/
(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*
c])] + 2*(-3*c*d^2 + e*(-2*b*d + a*e))*EllipticPi[-1/2*((b + Sqrt[b^2 + 4*
a*c])*e)/(c*d), I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (
b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[-(c/(b + Sqrt[b^2
+ 4*a*c]))]*(-a - b*x^2 + c*x^4))))/(d^2*(c*d^2 + e*(b*d - a*e))*(d + e*x^
2))
```

3.389.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 610, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1551, 2234, 27, 1514, 399, 321, 327, 1544, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}dx$$

$$\Big\downarrow \ 1551$$

$$\frac{\int \frac{-ce^2x^4-2cdex^2+2cd^2+e(2bd-ae)}{(ex^2+d)\sqrt{-cx^4+bx^2+a}}dx}{2d\left(e(bd-ae)+cd^2\right)}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(d+ex^2\right)\left(e(bd-ae)+cd^2\right)}$$

$$\Big\downarrow \ 2234$$

$$\frac{\left(e(2bd-ae)+3cd^2\right)\int \frac{1}{(ex^2+d)\sqrt{-cx^4+bx^2+a}}dx-\frac{\int \frac{ce^2\left(ex^2+d\right)}{\sqrt{-cx^4+bx^2+a}}dx}{e^2}}{2d\left(e(bd-ae)+cd^2\right)}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(d+ex^2\right)\left(e(bd-ae)+cd^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(e(2bd-ae)+3cd^2\right)\int \frac{1}{(ex^2+d)\sqrt{-cx^4+bx^2+a}}dx-c\int \frac{ex^2+d}{\sqrt{-cx^4+bx^2+a}}dx}{2d\left(e(bd-ae)+cd^2\right)}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(d+ex^2\right)\left(e(bd-ae)+cd^2\right)}$$

$$\Big\downarrow \ 1514$$

$$\frac{\left(e(2bd-ae)+3cd^2\right)\int\frac{1}{(ex^2+d)\sqrt{-cx^4+bx^2+a}}dx-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int\frac{ex^2+d}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{\sqrt{a+bx^2-cx^4}}}{2d\left(e(bd-ae)+cd^2\right)}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(d+ex^2\right)\left(e(bd-ae)+cd^2\right)}$$

$\big\downarrow$ 399

$$\frac{\left(e(2bd-ae)+3cd^2\right)\int\frac{1}{(ex^2+d)\sqrt{-cx^4+bx^2+a}}dx-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\int\frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\ldots}{2c}\ldots\right)}{\sqrt{a+bx^2-cx^4}}}{2d\left(e(bd-ae)+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(d+ex^2\right)\left(e(bd-ae)+cd^2\right)}$$

$\big\downarrow$ 321

$$\frac{\left(e(2bd-ae)+3cd^2\right)\int\frac{1}{(ex^2+d)\sqrt{-cx^4+bx^2+a}}dx-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(\text{arc}\ldots\right.}{2\sqrt{2}c^{3/2}}\ldots\right)}{\sqrt{a+bx^2-cx^4}}}{2d\left(e(bd-ae)+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(d+ex^2\right)\left(e(bd-ae)+cd^2\right)}$$

$\big\downarrow$ 327

$$\frac{\left(e(2bd-ae)+3cd^2\right)\int\frac{1}{(ex^2+d)\sqrt{-cx^4+bx^2+a}}dx-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)\text{EllipticF}\left(\text{arc}\ldots\right.}{2\sqrt{2}c^{3/2}}\ldots\right)}{\ldots}}{2d\left(e(bd-ae)+cd^2\right)}$$

$$\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(d+ex^2\right)\left(e(bd-ae)+cd^2\right)}$$

$\big\downarrow$ 1544

3.389. $\int\frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx$

$$\frac{\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e(2bd-ae)+3cd^2\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\left(ex^2+d\right)}dx}{\sqrt{a+bx^2-cx^4}} - \frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\sqrt{\sqrt{4\cdots}}\right.}{\cdots}}{2d\left(e(bd-\cdots\right.}$$

$$\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(d+ex^2\right)\left(e(bd-ae)+cd^2\right)}$$

$\downarrow$ 412

$$\frac{\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e(2bd-ae)+3cd^2\right)\text{EllipticPi}\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}} - \frac{c\sqrt{1-\cdots}}{\cdots}}{\cdots}$$

$$\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(d+ex^2\right)\left(e(bd-ae)+cd^2\right)}$$

input
```
Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

output
```
-1/2*(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(d*(c*d^2 + e*(b*d - a*e))*(d + e*x^2
)) + (-((c*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/
(b + Sqrt[b^2 + 4*a*c])]*(-1/2*((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2
+ 4*a*c]]*e*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c
]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2)) +
 (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Elliptic
F[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 +
 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2))))/Sqrt[a + b*x^2 -
c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(3*c*d^2 + e*(2*b*d - a*e))*Sqrt[1
- (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*
a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2 + 4*a*c])*e)/(c*d), ArcSin[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt
[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[a + b*x^2 - c*x^4]))/(2*d*(c*d^2
+ e*(b*d - a*e)))
```

3.389. $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx$

3.389.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

3.389. $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx$

rule 1544
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/((d + e*x^2)*S
qrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& NegQ[c/a]
```

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.389.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. 2(613) = 1226.

Time = 1.62 (sec) , antiderivative size = 1293, normalized size of antiderivative = 1.80

method	result	size
default	Expression too large to display	1293
elliptic	Expression too large to display	1293

input
```
int(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*e^2/(a*e^2-b*d*e-c*d^2)/d*x*(-c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)+1/8*c/(a*
e^2-b*d*e-c*d^2)*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2
*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(4*a*c+b^2)^(1/2))^(1
/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*c*e/(a*e^2-b*d
*e-c*d^2)/d*a*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^
2/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)
/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b
+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
+1/4*c*e/(a*e^2-b*d*e-c*d^2)/d*a*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2
)*(4+2*b*x^2/a-2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(4*a*
c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticE
(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)
^(1/2))/a/c)^(1/2))+1/2/(a*e^2-b*d*e-c*d^2)/d^2*e^2*2^(1/2)/(-b/a+1/a*(4*a
*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+
1/2*b*x^2/a+1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*Elli
pticPi(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(
1/2))*a*e/d,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^
(1/2))/a)^(1/2))*a-1/(a*e^2-b*d*e-c*d^2)*e/d*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)
^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*...
```

3.389.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.389.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 - c*x**4)), x)
```

3.389.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

3.389.8 Giac [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\left(e\,x^2+d\right)^2\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/((d + e*x^2)^2*(a + b*x^2 - c*x^4)^(1/2)),x)
```

output
```
int(1/((d + e*x^2)^2*(a + b*x^2 - c*x^4)^(1/2)), x)
```

3.390 $\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$

3.390.1 Optimal result

Integrand size = 26, antiderivative size = 479

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\left(b-\sqrt{b^2+4ac}\right)ex\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)}{2c\sqrt{-a+bx^2+cx^4}} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}e\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\middle|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}d\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}}$$

output
```
1/2*e*x*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))/c/(c*x^4+b
*x^2-a)^(1/2)+1/2*d*(1/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2
/(b+(4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(
1/2))^(1/2)/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/
(b-(4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b+(4*a*c+
b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*
a*c+b^2)^(1/2)))/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)-1/4*e*(1/(1+2*c*
x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)*
EllipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(b+(4*a
*c+b^2)^(1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/(b-(4*a*c+b^2)^(1/2)))^(1/2))*
(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))*(b+(4*a*c+b^2)^(1/
2))^(1/2)/c^(3/2)*2^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*a*c+b^2)
^(1/2)))/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)
```

3.390.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.63

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx = \frac{i\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\left(\left(-b+\sqrt{b^2+4ac}\right)eE\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+(-2}{2\sqrt{2}c\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{-a+bx^2+}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

output
```
((I/2)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*Ellip
ticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 +
 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + (-2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*E
llipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b
^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 +
4*a*c])]*Sqrt[-a + b*x^2 + c*x^4])
```

3.390.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1514}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\int \frac{ex^2+d}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx}{\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(d\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx+e\int \frac{x^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx\right)}{\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(e\int \frac{x^2}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx+\frac{d\sqrt{\sqrt{4ac+b^2}+b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\,\text{EllipticF}\left(\arctan\left(\ldots\right.\right.}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{4ac+\ldots}}}}\right.}{\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(e\left(\frac{x\left(b-\sqrt{4ac+b^2}\right)\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}}{2c\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}-\frac{\left(b-\sqrt{4ac+b^2}\right)\int \frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}}{\left(\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1\right)^{3/2}}dx}{2c}\right)+\frac{d\sqrt{\sqrt{4ac+\ldots}}}{\ldots}\right.}{\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\left(e\left(\frac{x\left(b-\sqrt{4ac+b^2}\right)\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}}{2c\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}-\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}E\left(\arctan\left(\right.\right.}{2\sqrt{2}c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{\frac{2cx}{\sqrt{4ac+}}}}\right.\right.}{\sqrt{-a+bx^2+cx}}$$

input
```
Int[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

output
```
(Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*(e*(((b - Sqrt[b^2 + 4*a*c])*x*Sqrt[1 + (2*c*x^2)/(b - Sqrt
[b^2 + 4*a*c])])/(2*c*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]) - ((b -
 Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 + 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 +
 4*a*c]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(
3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt
[b^2 + 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])])) + (Sqrt[b +
 Sqrt[b^2 + 4*a*c]]*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Elliptic
F[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b
- Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 + 4*a*c])])))/Sqrt[-a + b*x^2 + c*x^4]
```

3.390.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

3.390.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.74

method	result
default	$\frac{d\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c x^4+b x^2-a}}+\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}}{}$
elliptic	$\frac{d\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c x^4+b x^2-a}}+\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}}{}$

input
```
int((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*Ell
ipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2
)^(1/2))/a/c)^(1/2))+e*a/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4
+b*x^2-a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1
/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))
/a/c)^(1/2)))
```

3.390.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.63

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$$

$$= \frac{2\sqrt{cx^4+bx^2-a}ace+\sqrt{\frac{1}{2}}\left(acex\sqrt{\frac{b^2+4ac}{c^2}}-abex\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2+4ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2+4ac}{c^2}}-b}{c}}}{x}\right)\mid-\frac{bc\sqrt{b^2}}{}\right.}{}$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(c*x^4 + b*x^2 - a)*a*c*e + sqrt(1/2)*(a*c*e*x*sqrt((b^2 + 4*a*
c)/c^2) - a*b*e*x)*sqrt(c)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) - b)/c)*ellipti
c_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) - b)/c)/x), -1/2*(b*c
*sqrt((b^2 + 4*a*c)/c^2) + b^2 + 2*a*c)/(a*c)) - sqrt(1/2)*((c^2*d + a*c*e
)*x*sqrt((b^2 + 4*a*c)/c^2) + (b*c*d - a*b*e)*x)*sqrt(c)*sqrt((c*sqrt((b^2
 + 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a
*c)/c^2) - b)/c)/x), -1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) + b^2 + 2*a*c)/(a*c
)))/(a*c^2*x)
```

3.390.6 Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$$

input
```
integrate((e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral((d + e*x**2)/sqrt(-a + b*x**2 + c*x**4), x)
```

3.390.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2-a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

3.390.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{cx^4+bx^2-a}}\,dx$$

input
```
integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{e\,x^2+d}{\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

input
```
int((d + e*x^2)/(b*x^2 - a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(b*x^2 - a + c*x^4)^(1/2), x)
```

3.391 $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx$

3.391.1 Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\sqrt{-b+\sqrt{b^2+4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2+4ac}}}\right),\frac{b-\sqrt{b^2+}}{b+\sqrt{b^2+}}\right.}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

output
```
1/2*EllipticPi(x*2^(1/2)*c^(1/2)/(-b+(4*a*c+b^2)^(1/2))^(1/2),1/2*e*(b-(4*
a*c+b^2)^(1/2))/c/d,((b-(4*a*c+b^2)^(1/2))/(b+(4*a*c+b^2)^(1/2)))^(1/2))*(
1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(-b+(4*a*c+b^2)^(1/2))^(1/2)*(1+2*c
*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)
```

3.391.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\,\text{EllipticPi}\left(\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},i\,\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}x\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}d\sqrt{-a+bx^2+cx^4}}$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*EllipticPi[((b + Sqrt[b^2 + 4*a*c]
)*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + S
qrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
+ 4*a*c])]*d*Sqrt[-a + b*x^2 + c*x^4])
```

3.391.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1544, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}dx$$

$$\Big\downarrow \; 1544$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}(ex^2+d)}dx}{\sqrt{-a+bx^2+cx^4}}$$

$$\Big\downarrow \; 412$$

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right),\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}d\sqrt{-a+bx^2+cx^4}}$$

input
```
Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[-b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[((b - Sqrt[b^2 + 4*
a*c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]]
, (b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqr
t[-a + b*x^2 + c*x^4])
```

3.391.3.1 Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1544
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/((d + e*x^2)*S
qrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& NegQ[c/a]
```

3.391.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{1-\frac{b\,x^{2}}{2a}+\frac{x^{2}\sqrt{4ac+b^{2}}}{2a}}\,\sqrt{1-\frac{b\,x^{2}}{2a}-\frac{x^{2}\sqrt{4ac+b^{2}}}{2a}}\,\Pi\left(\sqrt{-\frac{-b+\sqrt{4ac+b^{2}}}{2a}}\,x,\frac{2ae}{\left(-b+\sqrt{4ac+b^{2}}\right)d},\frac{\sqrt{2}\,\sqrt{\frac{b+\sqrt{4ac+b^{2}}}{a}}}{2\sqrt{-\frac{-b+\sqrt{4ac+b^{2}}}{2a}}}\right)}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{4ac+b^{2}}}{2a}}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}$	198
elliptic	$\frac{\sqrt{1-\frac{b\,x^{2}}{2a}+\frac{x^{2}\sqrt{4ac+b^{2}}}{2a}}\,\sqrt{1-\frac{b\,x^{2}}{2a}-\frac{x^{2}\sqrt{4ac+b^{2}}}{2a}}\,\Pi\left(\sqrt{-\frac{-b+\sqrt{4ac+b^{2}}}{2a}}\,x,\frac{2ae}{\left(-b+\sqrt{4ac+b^{2}}\right)d},\frac{\sqrt{2}\,\sqrt{\frac{b+\sqrt{4ac+b^{2}}}{a}}}{2\sqrt{-\frac{-b+\sqrt{4ac+b^{2}}}{2a}}}\right)}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{4ac+b^{2}}}{2a}}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}$	198

input
```
int(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(1/2*b/a-1/2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a+1/2*x^2/a*(4*a*
c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a-1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)/(c*x
^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,2/(-b
+(4*a*c+b^2)^(1/2))*a*e/d,1/2*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)^(1/2)/(-1/
2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2))
```

3.391.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.391.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/(e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 + c*x**4)), x)
```

3.391.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

3.391.8 Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{1}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

input
```
int(1/((d + e*x^2)*(b*x^2 - a + c*x^4)^(1/2)),x)
```

output
```
int(1/((d + e*x^2)*(b*x^2 - a + c*x^4)^(1/2)), x)
```

3.392 $\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}} dx$

3.392.1 Optimal result

Integrand size = 27, antiderivative size = 293

$$\begin{aligned}\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx &= -\frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\ &\quad - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{-a+bx^2-cx^4}} \\ &\quad + \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}}\end{aligned}$$

output
```
-e*x*(-c*x^4+b*x^2-a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elli
pticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a
^(1/2)+x^2*c^(1/2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4
)/(-c*x^4+b*x^2-a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2
)/a^(1/2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-c*x^4
+b*x^2-a)^(1/2)
```

3.392.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.01

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx =$$
$$-\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\left(\left(-b+\sqrt{b^2-4ac}\right)eE\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right.}{2\sqrt{2}c\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{-a+b}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

output
```
((-1/2*I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*EllipticE[I*ArcSinh[S
qrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])] + (2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*EllipticF[I*Arc
Sinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]
*Sqrt[-a + b*x^2 - c*x^4])
```

3.392.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}dx$$
$$\Big\downarrow \text{1511}$$
$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{-cx^4+bx^2-a}}dx-\frac{\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{-cx^4+bx^2-a}}dx}{\sqrt{c}}$$
$$\Big\downarrow \text{27}$$

$$\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{-cx^4+bx^2-a}}dx-\frac{e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{-cx^4+bx^2-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{-a+bx^2-cx^4}}-\frac{e\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{-cx^4+bx^2-a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{-a+bx^2-cx^4}}-\frac{e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{\sqrt[4]{c}\sqrt{-a+bx^2-cx^4}}+\frac{x\sqrt{-a+bx^2-cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

input
```
Int[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

output
```
-((e*((x*Sqrt[-a + b*x^2 - c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) + (a^(1/4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1
/4)*Sqrt[-a + b*x^2 - c*x^4])))/Sqrt[c]) + ((d + (Sqrt[a]*e)/Sqrt[c])*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^
(1/4)*c^(1/4)*Sqrt[-a + b*x^2 - c*x^4])
```

3.392. $\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$

3.392.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

3.392.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.22

method	result
default	$\frac{d\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-cx^4+bx^2-a}}+\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{-4ac\right.}}{a}}}{}$
elliptic	$\frac{d\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{2\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-cx^4+bx^2-a}}+\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{-4ac\right.}}{a}}}{}$

input
```
int((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d/(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)
*EllipticF(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))+e*a/(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(
-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(-c*x^4+b*x^2-a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)-EllipticE(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2)))
```

3.392.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.06

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx =$$

$$-\frac{2\sqrt{-cx^4+bx^2-a}\,ace+\sqrt{\frac{1}{2}}\left(a\sqrt{-c}cex\sqrt{\frac{b^2-4ac}{c^2}}+ab\sqrt{-c}ex\right)\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}+b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}+b}{c}}}{x}\right.}{}$$

input
```
integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(-c*x^4 + b*x^2 - a)*a*c*e + sqrt(1/2)*(a*sqrt(-c)*c*e*x*sqrt(
(b^2 - 4*a*c)/c^2) + a*b*sqrt(-c)*e*x)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b
)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c)/x
), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 + 2*a*c)/(a*c)) + sqrt(1/2)*((c
^2*d - a*c*e)*sqrt(-c)*x*sqrt((b^2 - 4*a*c)/c^2) - (b*c*d + a*b*e)*sqrt(-c
)*x)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c)*elliptic_f(arcsin(sqrt(1/2)*s
qrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c)/x), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^
2) - b^2 + 2*a*c)/(a*c)))/(a*c^2*x)
```

3.392.6 Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$$

input
```
integrate((e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral((d + e*x**2)/sqrt(-a + b*x**2 - c*x**4), x)
```

3.392.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-cx^4+bx^2-a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

3.392.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{ex^2+d}{\sqrt{-cx^4+bx^2-a}}\,dx$$

input
```
integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{-c\,x^4+b\,x^2-a}}\,dx$$

input
```
int((d + e*x^2)/(b*x^2 - a - c*x^4)^(1/2),x)
```

output
```
int((d + e*x^2)/(b*x^2 - a - c*x^4)^(1/2), x)
```

3.393 $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx$

3.393.1 Optimal result

Integrand size = 29, antiderivative size = 412

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \frac{\sqrt{e}\arctan\left(\frac{\sqrt{-cd^2-e(bd+ae)}x}{\sqrt{d}\sqrt{e}\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2-e(bd+ae)}} + \frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-a+bx^2-cx^4}} - \frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a-bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}d\,(cd^2-ae^2)\sqrt{-a+bx^2-cx^4}}$$

output
```
1/2*arctan(x*(-a*e^2-b*d*e-c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(-c*x^4+b*x^2-a)^(
1/2))*e^(1/2)/d^(1/2)/(-a*e^2-b*d*e-c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+
x^2*c^(1/2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(-e*a
^(1/2)+d*c^(1/2))/(-c*x^4+b*x^2-a)^(1/2)-1/4*a^(3/4)*(cos(2*arctan(c^(1/4)
*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*ar
ctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2)
,1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/
2))^2*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*
d^2)/(-c*x^4+b*x^2-a)^(1/2)
```

3.393.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.50

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx =$$
$$-\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{EllipticPi}\left(-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd}, i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}x\right), -\frac{b+\sqrt{b^2-4}}{-b+\sqrt{b^2-}}\right.}{\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}d\sqrt{-a+bx^2-cx^4}}$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

output
```
((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2 - 4*a*c])*e)/(c*d), I*
ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], -((b + Sqrt[b^2 - 4
*a*c])/(-b + Sqrt[b^2 - 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c
]))]*d*Sqrt[-a + b*x^2 - c*x^4])
```

3.393.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1540, 27, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx$$

$$\Big\downarrow \text{1540}$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{-cx^4+bx^2-a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\sqrt{a}e\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{-cx^4+bx^2-a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c}\int \frac{1}{\sqrt{-cx^4+bx^2-a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{e\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{-cx^4+bx^2-a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \ 1416$$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{a}\sqrt{-a+bx^2-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{e\int\frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{-cx^4+bx^2-a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \ 2222$$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2\sqrt[4]{a}\sqrt{-a+bx^2-cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-$$

$$\frac{e\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{-a+bx^2-cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{x\sqrt{e(ae}}{\sqrt{d}\sqrt{e}\sqrt{-}}\right.}{2\sqrt{d}\sqrt{e}\sqrt{e(ae+bd)+c}}\right.}{\sqrt{c}d-\sqrt{a}e}$$

input
```
Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

output
```
(c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c
]))/4])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a + b*x^2 - c*x^4]) - (e*
(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTanh[(Sqrt[c*d^2 + e*(b*d + a*e)]*x)/(Sq
rt[d]*Sqrt[e]*Sqrt[-a + b*x^2 - c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 + e*
(b*d + a*e)]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a -
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt
[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 +
b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[-a + b*x^2 - c*x^4]))
)/(Sqrt[c]*d - Sqrt[a]*e)
```

3.393. $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx$

3.393.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

3.393.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\sqrt{1-\frac{b\,x^{2}}{2a}+\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{1-\frac{b\,x^{2}}{2a}-\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\Pi\left(\sqrt{-\frac{-b+\sqrt{-4ac+b^{2}}}{2a}}\,x,\frac{2ae}{\left(-b+\sqrt{-4ac+b^{2}}\right)d},\frac{\sqrt{2}\,\sqrt{\frac{b+\sqrt{-4ac+b^{2}}}{a}}}{2\sqrt{-\frac{-b+\sqrt{-4ac+b^{2}}}{2a}}}\right)}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{-c\,x^{4}+b\,x^{2}-a}}$	199
elliptic	$\frac{\sqrt{1-\frac{b\,x^{2}}{2a}+\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{1-\frac{b\,x^{2}}{2a}-\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\Pi\left(\sqrt{-\frac{-b+\sqrt{-4ac+b^{2}}}{2a}}\,x,\frac{2ae}{\left(-b+\sqrt{-4ac+b^{2}}\right)d},\frac{\sqrt{2}\,\sqrt{\frac{b+\sqrt{-4ac+b^{2}}}{a}}}{2\sqrt{-\frac{-b+\sqrt{-4ac+b^{2}}}{2a}}}\right)}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{-c\,x^{4}+b\,x^{2}-a}}$	199

input
```
int(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(1/2*b/a-1/2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a+1/2*x^2/a*(-4*
a*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(
-c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,
2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)/(-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

3.393.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.393.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx$$

input
```
integrate(1/(e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 - c*x**4)), x)
```

3.393.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

3.393.8 Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2-a}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(e*x^2 + d)), x)
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \int \frac{1}{(e\,x^2+d)\,\sqrt{-c\,x^4+b\,x^2-a}}\,dx$$

input
```
int(1/((d + e*x^2)*(b*x^2 - a - c*x^4)^(1/2)),x)
```

output
```
int(1/((d + e*x^2)*(b*x^2 - a - c*x^4)^(1/2)), x)
```

3.394 $\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$

3.394.1 Optimal result

Integrand size = 24, antiderivative size = 229

$$\begin{aligned}\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx &= \frac{3e(5d^2-10de+6e^2)\,x(2+x^2)}{5\sqrt{2+3x^2+x^4}} \\ &\quad + \frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4} + \frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4} \\ &\quad - \frac{3\sqrt{2}e(5d^2-10de+6e^2)\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} \\ &\quad + \frac{(5d^3-10de^2+8e^3)\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{5\sqrt{2}\sqrt{2+3x^2+x^4}}\end{aligned}$$

output
```
3/5*e*(5*d^2-10*d*e+6*e^2)*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/10*(5*d^3-10*d*
e^2+8*e^3)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2
^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-3/5*e*(5*d^2-1
0*d*e+6*e^2)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2
*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/5*(5*d-4*e
)*e^2*x*(x^4+3*x^2+2)^(1/2)+1/5*e^3*x^3*(x^4+3*x^2+2)^(1/2)
```

3.394.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \frac{e^2x(2+3x^2+x^4)(5d+e(-4+x^2)) - 3ie(5d^2-10de+6e^2)\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 5i}{5\sqrt{2+3x^2+x^4}}$$

input

```
Integrate[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

output

```
(e^2*x*(2 + 3*x^2 + x^4)*(5*d + e*(-4 + x^2)) - (3*I)*e*(5*d^2 - 10*d*e +
6*e^2)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (5
*I)*(d^3 - 3*d^2*e + 4*d*e^2 - 2*e^3)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elliptic
F[I*ArcSinh[x/Sqrt[2]], 2])/(5*Sqrt[2 + 3*x^2 + x^4])
```

3.394.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1518, 2207, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{1518}$$

$$\frac{1}{5}\int \frac{3(5d-4e)e^2x^4+3e(5d^2-2e^2)x^2+5d^3}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{5}e^3\sqrt{x^4+3x^2+2}x^3$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3(5d^3-10e^2d+8e^3+3e(5d^2-10ed+6e^2)x^2)}{\sqrt{x^4+3x^2+2}}dx+e^2x\sqrt{x^4+3x^2+2}(5d-4e)\right)+\frac{1}{5}e^3\sqrt{x^4+3x^2+2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\int \frac{5d^3-10e^2d+8e^3+3e\left(5d^2-10ed+6e^2\right)x^2}{\sqrt{x^4+3x^2+2}}dx+e^2x\sqrt{x^4+3x^2+2}(5d-4e)\right)+\frac{1}{5}e^3\sqrt{x^4+3x^2+2}x^3$$

$$\Big\downarrow \; 1503$$

$$\frac{1}{5}\left(\left(5d^3-10de^2+8e^3\right)\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+3e\left(5d^2-10de+6e^2\right)\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+e^2x\sqrt{x^4+3x^2+2}(5\right.$$
$$\frac{1}{5}e^3\sqrt{x^4+3x^2+2}x^3$$

$$\Big\downarrow \; 1412$$

$$\frac{1}{5}\left(3e\left(5d^2-10de+6e^2\right)\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\left(5d^3-10de^2+8e^3\right)\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+e\right.$$
$$\frac{1}{5}e^3\sqrt{x^4+3x^2+2}x^3$$

$$\Big\downarrow \; 1455$$

$$\frac{1}{5}\left(\frac{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\left(5d^3-10de^2+8e^3\right)\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+3e\left(5d^2-10de+6e^2\right)\left(\frac{x\left(x^2+2\right)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}}{}\right.\right.$$
$$\frac{1}{5}e^3\sqrt{x^4+3x^2+2}x^3$$

input
```
Int[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(e^3*x^3*Sqrt[2 + 3*x^2 + x^4])/5 + ((5*d - 4*e)*e^2*x*Sqrt[2 + 3*x^2 + x^
4] + 3*e*(5*d^2 - 10*d*e + 6*e^2)*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (
Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqr
t[2 + 3*x^2 + x^4]) + ((5*d^3 - 10*d*e^2 + 8*e^3)*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]))/5
```

3.394. $\int \frac{\left(d+ex^2\right)^3}{\sqrt{2+3x^2+x^4}}\,dx$

3.394.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.394. $\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.394.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.79 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.80

method	result
elliptic	$\frac{e^3x^3\sqrt{x^4+3x^2+2}}{5}+\left(d\,e^2-\frac{4}{5}e^3\right)x\sqrt{x^4+3x^2+2}-\frac{i\left(d^3-2d\,e^2+\frac{8}{5}e^3\right)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{i\left(3d^2e+\right.}{}$
risch	$\frac{x\,e^2\left(e\,x^2+5d-4e\right)\sqrt{x^4+3x^2+2}}{5}-\frac{id^3\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{4ie^3\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{5\sqrt{x^4+3x^2+2}}+\frac{id\,e^2\sqrt{2}\,\sqrt{}}{}$
default	$-\frac{id^3\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+e^3\left(\frac{x^3\sqrt{x^4+3x^2+2}}{5}-\frac{4x\sqrt{x^4+3x^2+2}}{5}-\frac{4i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{5\sqrt{x^4+3x^2+2}}+\right.$

input
```
int((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*e^3*x^3*(x^4+3*x^2+2)^(1/2)+(d*e^2-4/5*e^3)*x*(x^4+3*x^2+2)^(1/2)-1/2*
I*(d^3-2*d*e^2+8/5*e^3)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I*(3*d^2*e+18/5*e^3-6*d*e^2
)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2
*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.394.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.54

$$\int \frac{\left(d+ex^2\right)^3}{\sqrt{2+3x^2+x^4}}\,dx$$
$$=\frac{-3i\left(5\,d^2e-10\,de^2+6\,e^3\right)xE\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+i\left(5\,d^3+15\,d^2e-40\,de^2+26\,e^3\right)xF\left(\arcsin\left(\frac{i}{x}\right)\,|\,2\right)+\left(e^3x\right.}{5\,x}$$

input
```
integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*(5*d^2*e - 10*d*e^2 + 6*e^3)*x*elliptic_e(arcsin(I/x), 2) + I*(5
*d^3 + 15*d^2*e - 40*d*e^2 + 26*e^3)*x*elliptic_f(arcsin(I/x), 2) + (e^3*x
^4 + 15*d^2*e - 30*d*e^2 + 18*e^3 + (5*d*e^2 - 4*e^3)*x^2)*sqrt(x^4 + 3*x^
2 + 2))/x
```

3.394.6 Sympy [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(d+ex^2)^3}{\sqrt{(x^2+1)(x^2+2)}}\,dx$$

input
```
integrate((e*x**2+d)**3/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral((d + e*x**2)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.394.7 Maxima [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

3.394.8 Giac [F]

$$\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(ex^2+d)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{\left(e\,x^2+d\right)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int((d + e*x^2)^3/(3*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int((d + e*x^2)^3/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.395 $\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

3.395.1 Optimal result

Integrand size = 24, antiderivative size = 168

$$\begin{aligned}\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx &= \frac{2(d-e)ex(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}e^2x\sqrt{2+3x^2+x^4}\\ &\quad -\frac{2\sqrt{2}(d-e)e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}\\ &\quad +\frac{(3d^2-2e^2)(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+3x^2+x^4}}\end{aligned}$$

output
```
2*(d-e)*e*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/6*(3*d^2-2*e^2)*(x^2+1)^(3/2)*(1
/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(
1/2)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-2*(d-e)*e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)
*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x
^4+3*x^2+2)^(1/2)+1/3*e^2*x*(x^4+3*x^2+2)^(1/2)
```

3.395.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{e^2x(2+3x^2+x^4)-6i(d-e)e\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-i(3d^2-6de+4e^2)\sqrt{1+x^2}\sqrt{2+}}{3\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(e^2*x*(2 + 3*x^2 + x^4) - (6*I)*(d - e)*e*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ell
ipticE[I*ArcSinh[x/Sqrt[2]], 2] - I*(3*d^2 - 6*d*e + 4*e^2)*Sqrt[1 + x^2]*
Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4]
)
```

3.395.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1518, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{\sqrt{x^4+3x^2+2}}\,dx$$
$$\Big\downarrow \text{1518}$$
$$\frac{1}{3}\int \frac{3d^2-2e^2+6(d-e)ex^2}{\sqrt{x^4+3x^2+2}}dx+\frac{1}{3}e^2x\sqrt{x^4+3x^2+2}$$
$$\Big\downarrow \text{1503}$$
$$\frac{1}{3}\left((3d^2-2e^2)\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+6e(d-e)\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx\right)+\frac{1}{3}e^2x\sqrt{x^4+3x^2+2}$$
$$\Big\downarrow \text{1412}$$

$$\frac{1}{3}\left(6e(d-e)\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(3d^2-2e^2)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}\right)+\frac{1}{3}e^2x\sqrt{x^4+3x^2+2}$$

$$\Big\downarrow \text{1455}$$

$$\frac{1}{3}\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}(3d^2-2e^2)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+6e(d-e)\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E(\text{ar}}{\sqrt{x^4+3x^2+}}\right.\right.$$
$$\frac{1}{3}e^2x\sqrt{x^4+3x^2+2}$$

input
```
Int[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
(e^2*x*Sqrt[2 + 3*x^2 + x^4])/3 + (6*(d - e)*e*((x*(2 + x^2))/Sqrt[2 + 3*x
^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[
x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + ((3*d^2 - 2*e^2)*(1 + x^2)*Sqrt[(2 + x^
2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]))/
3
```

3.395.3.1 Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

3.395.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.61 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

method	result
elliptic	$\frac{e^{2} x \sqrt{x^{4}+3 x^{2}+2}}{3}-\frac{i \left(d^{2}-\frac{2 e^{2}}{3}\right) \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}+\frac{i \left(2 e d -2 e^{2}\right) \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)-E\left(\frac{i \sqrt{2}\, x}{2}\right)\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}$
risch	$\frac{e^{2} x \sqrt{x^{4}+3 x^{2}+2}}{3}-\frac{i d^{2} \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}+\frac{i e^{2} \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{3 \sqrt{x^{4}+3 x^{2}+2}}+\frac{i \left(6 e d -6 e^{2}\right) \sqrt{2}\, \sqrt{2 x^{2}+4}}{6}$
default	$-\frac{i d^{2} \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}+e^{2} \left(\frac{x \sqrt{x^{4}+3 x^{2}+2}}{3}+\frac{i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{3 \sqrt{x^{4}+3 x^{2}+2}}-\frac{i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}}{\sqrt{}}\right)$

input
```
int((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*e^2*x*(x^4+3*x^2+2)^(1/2)-1/2*I*(d^2-2/3*e^2)*2^(1/2)*(2*x^2+4)^(1/2)*
(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I
*(2*d*e-2*e^2)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(
EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

3.395.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{-6i\,(de-e^2)xE(\arcsin\left(\frac{i}{x}\right)\,|\,2)+i\,(3\,d^2+6\,de-8\,e^2)xF(\arcsin\left(\frac{i}{x}\right)\,|\,2)+(e^2x^2+6\,de-6\,e^2)\sqrt{x^4+3\,x}}{3\,x}$$

input
```
integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-6*I*(d*e - e^2)*x*elliptic_e(arcsin(I/x), 2) + I*(3*d^2 + 6*d*e - 8*
e^2)*x*elliptic_f(arcsin(I/x), 2) + (e^2*x^2 + 6*d*e - 6*e^2)*sqrt(x^4 + 3
*x^2 + 2))/x
```

3.395.6 Sympy [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(d+ex^2)^2}{\sqrt{(x^2+1)\,(x^2+2)}}\,dx$$

input
```
integrate((e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral((d + e*x**2)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.395.7 Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

3.395.8 Giac [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{(e\,x^2+d)^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int((d + e*x^2)^2/(3*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int((d + e*x^2)^2/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.396 $\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx$

3.396.1 Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{ex(2+x^2)}{\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{d(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

output
```
e*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*d*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Elli
pticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*
x^2+2)^(1/2)-e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1
/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

3.396.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\left(eE\left(i\,\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|\,2\right)+(d-e)\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)\right)}{\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(e*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] +
(d - e)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

3.396.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{1503}$$

$$d\int \frac{1}{\sqrt{x^4+3x^2+2}}dx+e\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{1412}$$

$$e\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{1455}$$

$$\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+e\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)$$

input
```
Int[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

output
```
e*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]) + (d*(1 + x^
2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 +
3*x^2 + x^4])
```

3.396.3.1 Defintions of rubi rules used

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.396.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{id\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{ie\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	108
elliptic	$-\frac{id\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{ie\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	108

input
```
int((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Ellipti
cF(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I*e*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/
(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^
(1/2)*x,2^(1/2)))
```

3.396.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.37

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{-i\,exE(\arcsin\left(\frac{i}{x}\right)\,|2)+i\,(d+e)xF(\arcsin\left(\frac{i}{x}\right)\,|2)+\sqrt{x^4+3\,x^2+2}e}{x}$$

input
```
integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
(-I*e*x*elliptic_e(arcsin(I/x), 2) + I*(d + e)*x*elliptic_f(arcsin(I/x), 2
) + sqrt(x^4 + 3*x^2 + 2)*e)/x
```

3.396.6 Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{d+ex^2}{\sqrt{(x^2+1)\,(x^2+2)}}\,dx$$

input
```
integrate((e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral((d + e*x**2)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

3.396.7 Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{ex^2+d}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

3.396.8 Giac [F]

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{ex^2+d}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx = \int \frac{e\,x^2+d}{\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int((d + e*x^2)/(3*x^2 + x^4 + 2)^(1/2),x)
```

output
```
int((d + e*x^2)/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.397 $\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx$

3.397.1 Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}(d-e)\sqrt{2+3x^2+x^4}} - \frac{e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\operatorname{EllipticPi}\left(1-\frac{e}{d},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}d(d-e)\sqrt{2+3x^2+x^4}}$$

output
```
1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))
*((x^2+2)/(x^2+1))^(1/2)/(d-e)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-1/2*e*(x^2+1)^(
3/2)*(1/(x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-e/d,1/2*2^(1/2))*((x^2
+2)/(x^2+1))^(1/2)/d/(d-e)*2^(1/2)/(x^4+3*x^2+2)^(1/2)
```

3.397.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.48

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\operatorname{EllipticPi}\left(\frac{2e}{d},i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{d\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[(2*e)/d, I*ArcSinh[x/Sqrt[2]]
, 2])/(d*Sqrt[2 + 3*x^2 + x^4])
```

3.397.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1538, 27, 1412, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}\,(d+ex^2)}dx$$

$$\Big\downarrow \ 1538$$

$$\frac{\int \frac{1}{\sqrt{x^4+3x^2+2}}dx}{d-e}-\frac{e\int \frac{2(x^2+1)}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx}{2(d-e)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{1}{\sqrt{x^4+3x^2+2}}dx}{d-e}-\frac{e\int \frac{x^2+1}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx}{d-e}$$

$$\Big\downarrow \ 1412$$

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)}-\frac{e\int \frac{x^2+1}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx}{d-e}$$

$$\Big\downarrow \ 1786$$

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)}-\frac{e\sqrt{x^2+1}\sqrt{x^2+2}\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(ex^2+d)}dx}{\sqrt{x^4+3x^2+2}(d-e)}$$

$$\Big\downarrow \ 414$$

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)}-\frac{e(x^2+2)\,\text{EllipticPi}\left(1-\frac{e}{d},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}d\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}(d-e)}$$

input
```
Int[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*(
d - e)*Sqrt[2 + 3*x^2 + x^4]) - (e*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x]
, 1/2])/(Sqrt[2]*d*(d - e)*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4]
)
```

3.397.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1538
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/(2*c*d - e*(b - q)))   I
nt[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[e/(2*c*d - e*(b - q))   Int[(b
- q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !LtQ[c, 0]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

3.397.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{2e}{d},\sqrt{2}\right)}{d\sqrt{x^4+3x^2+2}}$	55
elliptic	$-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{2e}{d},\sqrt{2}\right)}{d\sqrt{x^4+3x^2+2}}$	55

input
```
int(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I/d*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticP
i(1/2*I*2^(1/2)*x,2*e/d,2^(1/2))
```

3.397.5 Fricas [F]

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 2)/(e*x^6 + (d + 3*e)*x^4 + (3*d + 2*e)*x^2 +
2*d), x)
```

3.397.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2+1)\,(x^2+2)}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)), x)
```

3.397.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.397.8 Giac [F]

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(ex^2+d)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)), x)
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{(e\,x^2+d)\,\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int(1/((d + e*x^2)*(3*x^2 + x^4 + 2)^(1/2)),x)
```

output
```
int(1/((d + e*x^2)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.398 $\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx$

3.398.1 Optimal result

Integrand size = 24, antiderivative size = 316

$$\begin{aligned}\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx &= -\frac{ex(2+x^2)}{2d(d^2-3de+2e^2)\sqrt{2+3x^2+x^4}}+\frac{e^2x\sqrt{2+3x^2+x^4}}{2d(d^2-3de+2e^2)(d+ex^2)}\\ &\quad+\frac{e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{2}d(d-2e)(d-e)\sqrt{2+3x^2+x^4}}\\ &\quad+\frac{(2d-e)(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\operatorname{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2d(d-e)^2\sqrt{2+3x^2+x^4}}\\ &\quad-\frac{e(3d^2-6de+2e^2)(2+x^2)\operatorname{EllipticPi}\left(1-\frac{e}{d},\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}d^2(d-2e)(d-e)^2\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

output
```
-1/2*e*x*(x^2+2)/d/(d^2-3*d*e+2*e^2)/(x^4+3*x^2+2)^(1/2)-1/4*e*(3*d^2-6*d*
e+2*e^2)*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2
),1-e/d,1/2*2^(1/2))/d^2/(d-2*e)/(d-e)^2*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(
x^4+3*x^2+2)^(1/2)+1/2*e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+
1)^(1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)/d/(d-2*e)/(d-e)*2^(1/2)/(x^4
+3*x^2+2)^(1/2)+1/2*(2*d-e)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x
^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/d/(d-e)^2/(x^4+3*x^2+2)
^(1/2)+1/2*e^2*x*(x^4+3*x^2+2)^(1/2)/d/(d^2-3*d*e+2*e^2)/(e*x^2+d)
```

3.398.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.55

$$\int \frac{1}{(d+ex^2)^2 \sqrt{2+3x^2+x^4}}\,dx = \frac{\frac{e^2x(2+3x^2+x^4)}{(d^2-3de+2e^2)(d+ex^2)} + \frac{i\sqrt{1+x^2}\sqrt{2+x^2}\left(deE\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+d(d-e)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)+(-3d^2+6de-2e^2)\operatorname{EllipticPi}\left(\frac{2e}{d}\right.\right.}{d(d-2e)(d-e)}}{2d\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
((e^2*x*(2 + 3*x^2 + x^4))/((d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (I*Sqrt[1
 + x^2]*Sqrt[2 + x^2]*(d*e*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + d*(d - e)*
EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (-3*d^2 + 6*d*e - 2*e^2)*EllipticPi[(
2*e)/d, I*ArcSinh[x/Sqrt[2]], 2]))/(d*(d - 2*e)*(d - e)))/(2*d*Sqrt[2 + 3*
x^2 + x^4])
```

3.398.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1551, 25, 2234, 27, 1503, 1412, 1455, 1538, 27, 1412, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^4+3x^2+2}\,(d+ex^2)^2}dx$$

$$\Big\downarrow \text{ 1551}$$

$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\left(d^2-3de+2e^2\right)(d+ex^2)} - \frac{\int -\frac{-e^2x^4-2dex^2+2(d^2-3ed+e^2)}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx}{2d(d-2e)(d-e)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-e^2x^4-2dex^2+2(d^2-3ed+e^2)}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx}{2d(d-2e)(d-e)} + \frac{e^2x\sqrt{x^4+3x^2+2}}{2d\left(d^2-3de+2e^2\right)(d+ex^2)}$$

$$\Big\downarrow \text{ 2234}$$

$$\frac{\left(3d^2-6de+2e^2\right)\int \frac{1}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx-\frac{\int \frac{e^2\left(ex^2+d\right)}{\sqrt{x^4+3x^2+2}}dx}{e^2}}{2d(d-2e)(d-e)}+\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(3d^2-6de+2e^2\right)\int \frac{1}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx-\int \frac{ex^2+d}{\sqrt{x^4+3x^2+2}}dx}{2d(d-2e)(d-e)}+\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}$$

$$\Big\downarrow \; 1503$$

$$\frac{\left(3d^2-6de+2e^2\right)\int \frac{1}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx-d\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-e\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx}{2d(d-2e)(d-e)}+\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}$$

$$\Big\downarrow \; 1412$$

$$\frac{\left(3d^2-6de+2e^2\right)\int \frac{1}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx-e\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx-\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}}{2d(d-2e)(d-e)}+\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}$$

$$\Big\downarrow \; 1455$$

$$\frac{\left(3d^2-6de+2e^2\right)\int \frac{1}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx-\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-e\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E(\text{arc}}{\sqrt{x^4+3x^2+2}}\right.}{2d(d-2e)(d-e)}$$
$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}$$

$$\Big\downarrow \; 1538$$

$$\frac{\left(3d^2-6de+2e^2\right)\left(\frac{\int \frac{1}{\sqrt{x^4+3x^2+2}}dx}{d-e}-\frac{e\int \frac{2\left(x^2+1\right)}{\left(ex^2+d\right)\sqrt{x^4+3x^2+2}}dx}{2(d-e)}\right)-\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-e\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\right.}{2d(d-2e)(d-e)}$$
$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\left(d^2-3de+2e^2\right)\left(d+ex^2\right)}$$

$$\Big\downarrow \; 27$$

3.398. $\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx$

$$\frac{(3d^2-6de+2e^2)\left(\frac{\int \frac{1}{\sqrt{x^4+3x^2+2}}dx}{d-e}-\frac{e\int \frac{x^2+1}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx}{d-e}\right)-\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-e\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\right.}{2d(d-2e)(d-e)}$$

$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\,(d^2-3de+2e^2)\,(d+ex^2)}$$

$\Big\downarrow$ 1412

$$\frac{(3d^2-6de+2e^2)\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)}-\frac{e\int \frac{x^2+1}{(ex^2+d)\sqrt{x^4+3x^2+2}}dx}{d-e}\right)-\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-}{2d(d-2e)(d-e)}$$

$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\,(d^2-3de+2e^2)\,(d+ex^2)}$$

$\Big\downarrow$ 1786

$$\frac{(3d^2-6de+2e^2)\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)}-\frac{e\sqrt{x^2+1}\sqrt{x^2+2}\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(ex^2+d)}dx}{\sqrt{x^4+3x^2+2}(d-e)}\right)-\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}(\text{arcta}}{\sqrt{2}\sqrt{x^4+3x^2+2}}}{2d(d-2e)(d-e)}$$

$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\,(d^2-3de+2e^2)\,(d+ex^2)}$$

$\Big\downarrow$ 414

$$\frac{(3d^2-6de+2e^2)\left(\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}(d-e)}-\frac{e(x^2+2)\,\text{EllipticPi}\left(1-\frac{e}{d},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}d\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}(d-e)}\right)-\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}(\text{arc}}{\sqrt{2}\sqrt{x^4+3x^2+2}}}{2d(d-2e)(d-e)}$$

$$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d\,(d^2-3de+2e^2)\,(d+ex^2)}$$

input
```
Int[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

3.398. $\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx$

output
```
(e^2*x*Sqrt[2 + 3*x^2 + x^4])/(2*d*(d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (-
(e*((x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2
)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4])) - (d*(1 +
x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2
+ 3*x^2 + x^4]) + (3*d^2 - 6*d*e + 2*e^2)*(((1 + x^2)*Sqrt[(2 + x^2)/(1 +
x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*(d - e)*Sqrt[2 + 3*x^2 + x^4]) -
 (e*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x], 1/2])/(Sqrt[2]*d*(d - e)*Sqrt
[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])))/(2*d*(d - 2*e)*(d - e))
```

3.398.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

3.398. $\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx$

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1538
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/(2*c*d - e*(b - q)))   I
nt[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[e/(2*c*d - e*(b - q))   Int[(b
- q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !LtQ[c, 0]
```

rule 1551
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2
*q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c
*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

3.398. $\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx$

3.398.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.40

method	result
default	$\frac{e^{2} x \sqrt{x^{4}+3 x^{2}+2}}{2 d \left(d^{2}-3 e d+2 e^{2}\right) \left(e \,x^{2}+d \right)}+\frac{i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{4 \left(d^{2}-3 e d+2 e^{2}\right) \sqrt{x^{4}+3 x^{2}+2}}-\frac{i e \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{4 \left(d^{2}-3 e d+2 e^{2}\right) d \sqrt{x^{4}+3 x^{2}+2}}+\frac{i e \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+\cdots}}{4 \left(d^{2}-3 e d+2 e^{2}\right) d\cdots}$
elliptic	$\frac{e^{2} x \sqrt{x^{4}+3 x^{2}+2}}{2 d \left(d^{2}-3 e d+2 e^{2}\right) \left(e \,x^{2}+d \right)}+\frac{i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{4 \left(d^{2}-3 e d+2 e^{2}\right) \sqrt{x^{4}+3 x^{2}+2}}-\frac{i e \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, F\left(\frac{i \sqrt{2}\, x}{2}, \sqrt{2}\right)}{4 \left(d^{2}-3 e d+2 e^{2}\right) d \sqrt{x^{4}+3 x^{2}+2}}+\frac{i e \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+\cdots}}{4 \left(d^{2}-3 e d+2 e^{2}\right) d\cdots}$

input
```
int(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*e^2*x*(x^4+3*x^2+2)^(1/2)/d/(d^2-3*d*e+2*e^2)/(e*x^2+d)+1/4*I/(d^2-3*d
*e+2*e^2)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Ellipt
icF(1/2*I*2^(1/2)*x,2^(1/2))-1/4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2)
)+1/4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3
*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-3/2*I/(d^2-3*d*e+2*e^2)*2
^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*
I*2^(1/2)*x,2*e/d,2^(1/2))+3*I/(d^2-3*d*e+2*e^2)*e/d*2^(1/2)*(1+1/2*x^2)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,2*e/d,2^
(1/2))-I/(d^2-3*d*e+2*e^2)/d^2*e^2*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)
/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,2*e/d,2^(1/2))
```

3.398.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.398.6 Sympy [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{(x^2+1)(x^2+2)}\,(d+ex^2)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)**2), x)
```

3.398.7 Maxima [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)
```

3.398.8 Giac [F]

$$\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+3\,x^2+2}(ex^2+d)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(e*x^2 + d)^2), x)
```

3.398.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx=\int \frac{1}{(e\,x^2+d)^2\,\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int(1/((d + e*x^2)^2*(3*x^2 + x^4 + 2)^(1/2)),x)
```

output
```
int(1/((d + e*x^2)^2*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.399 $\int (c+ex^2)^q (a+cx^2+bx^4)^p \, dx$

3.399.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (c+ex^2)^q (a+cx^2+bx^4)^p \, dx = \text{Int}\left((c+ex^2)^q (a+cx^2+bx^4)^p, x\right)$$

output
```
Unintegrable((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

3.399.2 Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+ex^2)^q (a+cx^2+bx^4)^p \, dx = \int (c+ex^2)^q (a+cx^2+bx^4)^p \, dx$$

input
```
Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]
```

output
```
Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

3.399.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {1570}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+ex^2)^q (a+bx^4+cx^2)^p \, dx$$

$$\Big\downarrow \text{1570}$$

$$\int (c+ex^2)^q (a+bx^4+cx^2)^p dx$$

input
```
Int[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]
```

output
```
$Aborted
```

3.399.3.1 Defintions of rubi rules used

rule 1570
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Unintegrable[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ
[{a, b, c, d, e, p, q}, x]
```

3.399.4 Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(e\,x^{2}+c\right)^{q} \left(b\,x^{4}+c\,x^{2}+a\right)^{p} dx$$

input
```
int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

output
```
int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

3.399.5 Fricas [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx = \int (bx^4+cx^2+a)^p (ex^2+c)^q \, dx$$

input
```
integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

3.399.6 Sympy [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x**2+c)**q*(b*x**4+c*x**2+a)**p,x)
```

output
```
Timed out
```

3.399.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx = \int (bx^4+cx^2+a)^p (ex^2+c)^q \, dx$$

input
```
integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

3.399.8 Giac [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx = \int \left(bx^4+cx^2+a\right)^p \left(ex^2+c\right)^q \, dx$$

input
```
integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + c*x^2 + a)^p*(e*x^2 + c)^q, x)
```

3.399.9 Mupad [N/A]

Not integrable

Time = 8.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(c+ex^2\right)^q \left(a+cx^2+bx^4\right)^p \, dx = \int \left(e\,x^2+c\right)^q \,\left(b\,x^4+c\,x^2+a\right)^p \, dx$$

input
```
int((c + e*x^2)^q*(a + b*x^4 + c*x^2)^p,x)
```

output
```
int((c + e*x^2)^q*(a + b*x^4 + c*x^2)^p, x)
```

3.400 $\int (c+ex^2)^3 (a+cx^2+bx^4)^p \, dx$

3.400.1 Optimal result

Integrand size = 24, antiderivative size = 498

$$\int (c+ex^2)^3 (a+cx^2+bx^4)^p \, dx$$
$$= \frac{ce^2(21b-5e+12bp-2ep)x(a+cx^2+bx^4)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{e^3x^3(a+cx^2+bx^4)^{1+p}}{b(7+4p)}$$
$$+ \frac{c(ae^3(5+2p)-3abe^2(7+4p)+b^2c^2(35+48p+16p^2))\,x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+ }{b^2(5+4p)(7+4p)}$$
$$+ \frac{e(c^2e^2(15+16p+4p^2)+3b^2c^2(35+48p+16p^2)-3be(ae(5+4p)+c^2(21+26p+8p^2)))\,x^3\left(1+\frac{2}{c-\sqrt{-}}\right.}{3b^2(5+4p)}$$

output
```
-c*e^2*(e*(5+2*p)-3*b*(7+4*p))*x*(b*x^4+c*x^2+a)^(p+1)/b^2/(16*p^2+48*p+35
)+e^3*x^3*(b*x^4+c*x^2+a)^(p+1)/b/(7+4*p)+c*(a*e^3*(5+2*p)-3*a*b*e^2*(7+4*
p)+b^2*c^2*(16*p^2+48*p+35))*x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2
*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/b^2/(5+4*p)
/(7+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c^2
)^(1/2)))^p)+1/3*e*(c^2*e^2*(4*p^2+16*p+15)+3*b^2*c^2*(16*p^2+48*p+35)-3*b
*e*(a*e*(5+4*p)+c^2*(8*p^2+26*p+21)))*x^3*(b*x^4+c*x^2+a)^p*AppellF1(3/2,-
p,-p,5/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/
b^2/(5+4*p)/(7+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+
(-4*a*b+c^2)^(1/2)))^p)
```

3.400.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.75

$$\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx$$
$$= \frac{1}{35}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\left(35c^3\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+ex^2\left(35c^2\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+ex^2\left(21c\operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+5ex^2\operatorname{AppellF1}\left(\frac{7}{2},-p,-p\right.\right.\right.\right.$$

input
```
Integrate[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```

output
```
(x*(a + c*x^2 + b*x^4)^p*(35*c^3*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c
+ Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*(35*c^
2*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2
)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*(21*c*AppellF1[5/2, -p, -p, 7/2, (-2*
b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + 5*
e*x^2*AppellF1[7/2, -p, -p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b
*x^2)/(-c + Sqrt[-4*a*b + c^2])]))))/(35*((c - Sqrt[-4*a*b + c^2] + 2*b*x^
2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sq
rt[-4*a*b + c^2]))^p)
```

3.400.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1518, 2207, 1515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+ex^2\right)^3 \left(a+bx^4+cx^2\right)^p \, dx$$
$$\Big\downarrow \ 1518$$

$$\frac{\int (bx^4+cx^2+a)^p \left(ce^2(12pb+21b-5e-2ep)x^4-3e\left(ae^2-bc^2(4p+7)\right)x^2+bc^3(4p+7)\right)dx}{b(4p+7)}+$$
$$\frac{e^3x^3\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+7)}$$

$\big\downarrow$ 2207

$$\frac{\frac{\int \left(e\left(e^2\left(4p^2+16p+15\right)c^2+3b^2\left(16p^2+48p+35\right)c^2-3be\left(\left(8p^2+26p+21\right)c^2+ae(4p+5)\right)\right)x^2+c\left(a(2p+5)e^3-3ab(4p+7)e^2+b^2c^2\left(16p^2+48p+35\right)\right)\right)(bx^4}{b(4p+5)}}{b(4p+7)}$$
$$\frac{e^3x^3\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+7)}$$

$\big\downarrow$ 1515

$$\frac{\frac{\int \left(e\left(e^2\left(4p^2+16p+15\right)c^2+3b^2\left(16p^2+48p+35\right)c^2-3be\left(\left(8p^2+26p+21\right)c^2+ae(4p+5)\right)\right)x^2\left(bx^4+cx^2+a\right)^p+c\left(a(2p+5)e^3-3ab(4p+7)e^2+b^2c^2\left(16p^2+\right.\right.}{b(4p+5)}}{b(4p+7)}$$
$$\frac{e^3x^3\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+7)}$$

$\big\downarrow$ 2009

$$\frac{\frac{1}{3}ex^3\left(-3be\left(ae(4p+5)+c^2\left(8p^2+26p+21\right)\right)+3b^2c^2\left(16p^2+48p+35\right)+c^2e^2\left(4p^2+16p+15\right)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}}{}$$
$$\frac{e^3x^3\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+7)}$$

input
```
Int[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```

output
```
(e^3*x^3*(a + c*x^2 + b*x^4)^(1 + p))/(b*(7 + 4*p)) + ((c*e^2*(21*b - 5*e
+ 12*b*p - 2*e*p)*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c*(a*e^
3*(5 + 2*p) - 3*a*b*e^2*(7 + 4*p) + b^2*c^2*(35 + 48*p + 16*p^2))*x*(a + c
*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c
^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c - Sqrt[-4*a
*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (e*(c^2*e^2*(1
5 + 16*p + 4*p^2) + 3*b^2*c^2*(35 + 48*p + 16*p^2) - 3*b*e*(a*e*(5 + 4*p)
+ c^2*(21 + 26*p + 8*p^2)))*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p
, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b +
c^2])])/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c +
Sqrt[-4*a*b + c^2]))^p))/(b*(5 + 4*p)))/(b*(7 + 4*p))
```

3.400. $\int \left(c+ex^2\right)^3\left(a+cx^2+bx^4\right)^p\,dx$

3.400.3.1 Defintions of rubi rules used

rule 1515
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

3.400.4 Maple [F]

$$\int \left(e\,x^{2}+c\right)^{3} \left(b\,x^{4}+c\,x^{2}+a\right)^{p}dx$$

input
```
int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
```

output
```
int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
```

3.400.5 Fricas [F]

$$\int (c+ex^2)^3 (a+cx^2+bx^4)^p \, dx = \int (ex^2+c)^3 (bx^4+cx^2+a)^p \, dx$$

input
```
integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^3*x^6 + 3*c*e^2*x^4 + 3*c^2*e*x^2 + c^3)*(b*x^4 + c*x^2 + a)^p
, x)
```

3.400.6 Sympy [F(-1)]

Timed out.

$$\int (c+ex^2)^3 (a+cx^2+bx^4)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x**2+c)**3*(b*x**4+c*x**2+a)**p,x)
```

output
```
Timed out
```

3.400.7 Maxima [F]

$$\int (c+ex^2)^3 (a+cx^2+bx^4)^p \, dx = \int (ex^2+c)^3 (bx^4+cx^2+a)^p \, dx$$

input
```
integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

3.400.8 Giac [F]

$$\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^3 \left(bx^4+cx^2+a\right)^p \, dx$$

input
```
integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^2 + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

3.400.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)^3 \left(a+cx^2+bx^4\right)^p \, dx = \int \left(e\,x^2+c\right)^3 \left(b\,x^4+c\,x^2+a\right)^p \, dx$$

input
```
int((c + e*x^2)^3*(a + b*x^4 + c*x^2)^p,x)
```

output
```
int((c + e*x^2)^3*(a + b*x^4 + c*x^2)^p, x)
```

3.401 $\int (c+ex^2)^2 (a+cx^2+bx^4)^p \, dx$

3.401.1 Optimal result

Integrand size = 24, antiderivative size = 358

$$\int (c+ex^2)^2 (a+cx^2+bx^4)^p \, dx = \frac{e^2x(a+cx^2+bx^4)^{1+p}}{b(5+4p)}$$
$$-\frac{(ae^2-bc^2(5+4p))\,x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\right.}{b(5+4p)}$$
$$+\frac{ce(10b-3e+8bp-2ep)x^3\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p \operatorname{AppellF1}\left(\frac{3}{2},-p,\right.}{3b(5+4p)}$$

output
```
e^2*x*(b*x^4+c*x^2+a)^(p+1)/b/(5+4*p)-(a*e^2-b*c^2*(5+4*p))*x*(b*x^4+c*x^2
+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(
-4*a*b+c^2)^(1/2)))/b/(5+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2
*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)+1/3*c*e*(8*b*p-2*e*p+10*b-3*e)*x^3*(b*x^
4+c*x^2+a)^p*AppellF1(3/2,-p,-p,5/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x
^2/(c+(-4*a*b+c^2)^(1/2)))/b/(5+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p
)/((1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)
```

3.401.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.85

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx = \frac{1}{15}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\left(15c^2\,\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+ex^2\left(10c\,\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+3ex^2\,\text{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right)\right)$$

input
```
Integrate[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

output
```
(x*(a + c*x^2 + b*x^4)^p*(15*c^2*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c
+ Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*(10*c*
AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/
(-c + Sqrt[-4*a*b + c^2])] + 3*e*x^2*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^2)
/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])))/(15*((c
 - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4
*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

3.401.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1518, 25, 1515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+ex^2\right)^2 \left(a+bx^4+cx^2\right)^p \, dx$$

$$\Big\downarrow \; 1518$$

$$\frac{\int -\left(\left(-b(4p+5)c^2-e(8pb+10b-3e-2ep)x^2c+ae^2\right)\left(bx^4+cx^2+a\right)^p\right)dx}{b(4p+5)}+\frac{e^2x\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+5)}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^2x\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+5)}-\frac{\int \left(-b(4p+5)c^2-e(8pb+10b-3e-2ep)x^2c+ae^2\right)\left(bx^4+cx^2+a\right)^p dx}{b(4p+5)}$$

$$\Big\downarrow \text{1515}$$

$$\frac{e^2x\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+5)}-\frac{\int \left(ce(-8pb-10b+3e+2ep)x^2\left(bx^4+cx^2+a\right)^p+ae^2\left(1-\frac{bc^2(4p+5)}{ae^2}\right)\left(bx^4+cx^2+a\right)^p\right)dx}{b(4p+5)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2x\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+5)}-\frac{x\left(ae^2-bc^2(4p+5)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p \text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{c^2-4a}}\right.}{}$$

input
```
Int[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

output
```
(e^2*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) - (((a*e^2 - b*c^2*(5 +
4*p))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - S
qrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) -
(c*e*(10*b - 3*e + 8*b*p - 2*e*p)*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2,
-p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*
a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)
/(c + Sqrt[-4*a*b + c^2]))^p))/(b*(5 + 4*p))
```

3.401. $\int \left(c+ex^2\right)^2\left(a+cx^2+bx^4\right)^p\,dx$

3.401.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1515
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

rule 1518
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Simp[1/(c*(4*p + 2*q + 1))   Int[(a + b*x^2 + c*x^4)^p*Expand
ToSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*
p + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
 FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IGtQ[q, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.401.4 Maple [F]

$$\int \left(e\,x^{2}+c\right)^{2} \left(b\,x^{4}+c\,x^{2}+a\right)^{p}dx$$

input
```
int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)
```

output
```
int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)
```

3.401.5 Fricas [F]

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx=\int \left(ex^2+c\right)^2\left(bx^4+cx^2+a\right)^p dx$$

input
```
integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*c*e*x^2 + c^2)*(b*x^4 + c*x^2 + a)^p, x)
```

3.401.6 Sympy [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x**2+c)**2*(b*x**4+c*x**2+a)**p,x)
```

output
```
Timed out
```

3.401.7 Maxima [F]

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^2 \left(bx^4+cx^2+a\right)^p \, dx$$

input
```
integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

3.401.8 Giac [F]

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx = \int \left(ex^2+c\right)^2 \left(bx^4+cx^2+a\right)^p \, dx$$

input
```
integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^2 + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

3.401.9 Mupad [F(-1)]

Timed out.

$$\int (c+ex^2)^2 (a+cx^2+bx^4)^p \, dx = \int (e\,x^2+c)^2 (b\,x^4+c\,x^2+a)^p \, dx$$

input
```
int((c + e*x^2)^2*(a + b*x^4 + c*x^2)^p,x)
```

output
```
int((c + e*x^2)^2*(a + b*x^4 + c*x^2)^p, x)
```

3.402 $\int (c+ex^2)(a+cx^2+bx^4)^p\,dx$

3.402.1 Optimal result

Integrand size = 22, antiderivative size = 274

$$\int (c+ex^2)(a+cx^2+bx^4)^p\,dx = cx\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\,\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)+\frac{1}{3}ex^3\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\,\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)$$

output
```
c*x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2
)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)
/((1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)+1/3*e*x^3*(b*x^4+c*x^2+a)^p*Appell
F1(3/2,-p,-p,5/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^
(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c^2)
^(1/2)))^p)
```

3.402.2 Mathematica [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.85

$$\int \left(c+ex^2\right) \left(a+cx^2+bx^4\right)^p \, dx$$
$$=\frac{1}{3}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\left(3c\,\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+ex^2\,\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right)$$

input
```
Integrate[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

output
```
(x*(a + c*x^2 + b*x^4)^p*(3*c*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + S
qrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*AppellF1[
3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqr
t[-4*a*b + c^2])]))/(3*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*
b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))
^p)
```

3.402.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+ex^2\right) \left(a+bx^4+cx^2\right)^p \, dx$$
$$\Big\downarrow \text{1515}$$
$$\int \left(ex^2\left(a+bx^4+cx^2\right)^p+c\left(a+bx^4+cx^2\right)^p\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}ex^3\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}(a+bx^4+cx^2)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c-\sqrt{c^2-4ab}}\right.$$
$$cx\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}(a+bx^4+cx^2)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\right.$$

input
```
Int[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

output
```
(c*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt
[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c
- Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (e*
x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[
-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c
 - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

3.402.3.1 Defintions of rubi rules used

rule 1515
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.402.4 Maple [F]

$$\int \left(e\,x^2+c\right)\left(b\,x^4+c\,x^2+a\right)^p dx$$

input
```
int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

output
```
int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

3.402.5 Fricas [F]

$$\int (c+ex^2)(a+cx^2+bx^4)^p\,dx = \int (ex^2+c)(bx^4+cx^2+a)^p\,dx$$

input
```
integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

3.402.6 Sympy [F]

$$\int (c+ex^2)(a+cx^2+bx^4)^p\,dx = \int (c+ex^2)(a+bx^4+cx^2)^p\,dx$$

input
```
integrate((e*x**2+c)*(b*x**4+c*x**2+a)**p,x)
```

output
```
Integral((c + e*x**2)*(a + b*x**4 + c*x**2)**p, x)
```

3.402.7 Maxima [F]

$$\int (c+ex^2)(a+cx^2+bx^4)^p\,dx = \int (ex^2+c)(bx^4+cx^2+a)^p\,dx$$

input
```
integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

3.402.8 Giac [F]

$$\int \left(c+ex^2\right)\left(a+cx^2+bx^4\right)^p\,dx=\int \left(ex^2+c\right)\left(bx^4+cx^2+a\right)^p\,dx$$

input
```
integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x^2 + c)*(b*x^4 + c*x^2 + a)^p, x)
```

3.402.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+ex^2\right)\left(a+cx^2+bx^4\right)^p\,dx=\int \left(e\,x^2+c\right)\,\left(b\,x^4+c\,x^2+a\right)^p\,dx$$

input
```
int((c + e*x^2)*(a + b*x^4 + c*x^2)^p,x)
```

output
```
int((c + e*x^2)*(a + b*x^4 + c*x^2)^p, x)
```

3.403 $\int (a+cx^2+bx^4)^p \, dx$

3.403.1 Optimal result

Integrand size = 14, antiderivative size = 133

$$\int (a+cx^2+bx^4)^p \, dx = x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)$$

output
```
x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2))
,-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/(
(1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)
```

3.403.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$\int (a+cx^2+bx^4)^p \, dx = x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)$$

input
```
Integrate[(a + c*x^2 + b*x^4)^p,x]
```

output
```
(x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-
4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/(((c - Sqrt[-4*a*b +
c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b
*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

3.403.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1418, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{1418}$$

$$\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\int \left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^p\left(\frac{2bx^2}{c+\sqrt{c^2-4ab}}+1\right.$$

$$\Big\downarrow \text{333}$$

$$x\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p \text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\right.$$

input
```
Int[(a + c*x^2 + b*x^4)^p,x]
```

output
```
(x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-
4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c -
Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

3.403.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1418
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p]))   Int[(1
 + 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b,
c, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

3.403.4 Maple [F]

$$\int \left(b\,x^{4}+c\,x^{2}+a\right)^{p}dx$$

input
```
int((b*x^4+c*x^2+a)^p,x)
```

output
```
int((b*x^4+c*x^2+a)^p,x)
```

3.403.5 Fricas [F]

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(bx^4+cx^2+a\right)^p\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + c*x^2 + a)^p, x)
```

3.403.6 Sympy [F]

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(a+bx^4+cx^2\right)^p\,dx$$

input
```
integrate((b*x**4+c*x**2+a)**p,x)
```

output
```
Integral((a + b*x**4 + c*x**2)**p, x)
```

3.403.7 Maxima [F]

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(bx^4+cx^2+a\right)^p\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + c*x^2 + a)^p, x)
```

3.403.8 Giac [F]

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(bx^4+cx^2+a\right)^p\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + c*x^2 + a)^p, x)
```

3.403.9 Mupad [F(-1)]

Timed out.

$$\int (a + cx^2 + bx^4)^p \, dx = \int (b\,x^4 + c\,x^2 + a)^p \, dx$$

input
```
int((a + b*x^4 + c*x^2)^p,x)
```

output
```
int((a + b*x^4 + c*x^2)^p, x)
```

3.404 $\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$

3.404.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx = \text{Int}\left(\frac{(a+cx^2+bx^4)^p}{c+ex^2},x\right)$$

output
```
Unintegrable((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

3.404.2 Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx = \int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$$

input
```
Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]
```

output
```
Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]
```

3.404.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1570}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4+cx^2)^p}{c+ex^2}\,dx$$
$$\Big\downarrow \ 1570$$
$$\int \frac{(a+bx^4+cx^2)^p}{c+ex^2}dx$$

input
```
Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]
```

output
```
$Aborted
```

3.404.3.1 Defintions of rubi rules used

rule 1570
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Unintegrable[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ
[{a, b, c, d, e, p, q}, x]
```

3.404.4 Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(b\,x^4+c\,x^2+a)^p}{e\,x^2+c}dx$$

input
```
int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

output
```
int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

3.404.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx = \int \frac{(bx^4+cx^2+a)^p}{ex^2+c}\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="fricas")
```

output
```
integral((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

3.404.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+c*x**2+a)**p/(e*x**2+c),x)
```

output
```
Timed out
```

3.404.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx = \int \frac{(bx^4+cx^2+a)^p}{ex^2+c}\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

3.404.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx = \int \frac{(bx^4+cx^2+a)^p}{ex^2+c}\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c), x)
```

3.404.9 Mupad [N/A]

Not integrable

Time = 7.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx = \int \frac{(b\,x^4+c\,x^2+a)^p}{e\,x^2+c}\,dx$$

input
```
int((a + b*x^4 + c*x^2)^p/(c + e*x^2),x)
```

output
```
int((a + b*x^4 + c*x^2)^p/(c + e*x^2), x)
```

3.405 $\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$

3.405.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx = \text{Int}\left(\frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}, x\right)$$

output
```
Unintegrable((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

3.405.2 Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx = \int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

input
```
Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]
```

output
```
Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]
```

3.405.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1570}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4+cx^2)^p}{(c+ex^2)^2} \, dx$$

$$\Big\downarrow \text{1570}$$

$$\int \frac{(a+bx^4+cx^2)^p}{(c+ex^2)^2}dx$$

input
```
Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]
```

output
```
$Aborted
```

3.405.3.1 Defintions of rubi rules used

rule 1570
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Unintegrable[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ
[{a, b, c, d, e, p, q}, x]
```

3.405.4 Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(b\,x^4+c\,x^2+a)^p}{(e\,x^2+c)^2}dx$$

input
```
int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

output
```
int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

3.405.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx = \int \frac{(bx^4+cx^2+a)^p}{(ex^2+c)^2}\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4 + c*x^2 + a)^p/(e^2*x^4 + 2*c*e*x^2 + c^2), x)
```

3.405.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+c*x**2+a)**p/(e*x**2+c)**2,x)
```

output
```
Timed out
```

3.405.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx = \int \frac{(bx^4+cx^2+a)^p}{(ex^2+c)^2}\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)
```

3.405.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(bx^4+cx^2+a)^p}{(ex^2+c)^2}\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + c*x^2 + a)^p/(e*x^2 + c)^2, x)
```

3.405.9 Mupad [N/A]

Not integrable

Time = 8.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2}\,dx=\int \frac{(b\,x^4+c\,x^2+a)^p}{(e\,x^2+c)^2}\,dx$$

input
```
int((a + b*x^4 + c*x^2)^p/(c + e*x^2)^2,x)
```

output
```
int((a + b*x^4 + c*x^2)^p/(c + e*x^2)^2, x)
```

3.406 $\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx$

3.406.1 Optimal result

Integrand size = 24, antiderivative size = 446

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \frac{(ef-dg)\arctan\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{(ef-dg)\operatorname{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}}$$
$$+ \frac{\left(\sqrt{c}df+\sqrt{a}eg\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$- \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

output
```
1/2*(-d*g+e*f)*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-
c*d^4)^(1/2)-1/2*(-d*g+e*f)*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/
(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(1/2)-1/4*(-d*g+e*f)*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^
(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^
4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/e/(e^2*a^(1/2)+d^2*c
^(1/2))/(c*x^4+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*2^(1/2))*(e*g*a^(1/2)+d*f*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^
(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x
^4+a)^(1/2)
```

3.406.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.75 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.61

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \frac{-\frac{ig\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}+\frac{(ef-dg)\left(\sqrt[4]{c}de\sqrt{a+cx^4}\arctan\left(\frac{\sqrt{c}\left(d^2-e^2x^2\right)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)-\sqrt[4]{-1}\sqrt[4]{a}\sqrt{-cd^4-ae}\right.}{\sqrt[4]{c}d\sqrt{-cd^4-ae^4}}}{e\sqrt{a+cx^4}}$$

input
```
Integrate[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

output
```
(((-I)*g*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]
*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + ((e*f - d*g)*(c^(1/4)*d*e*Sqrt[a + c
*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4]])/Sqrt[-(c*d^4)
 - a*e^4]] - (-1)^(1/4)*a^(1/4)*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]
*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a
^(1/4)], -1]))/(c^(1/4)*d*Sqrt[-(c*d^4) - a*e^4]))/(e*Sqrt[a + c*x^4])
```

3.406.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2280, 27, 1577, 488, 219, 2227, 27, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\sqrt{a+cx^4}(d+ex)}dx$$

$$\Big\downarrow \text{ 2280}$$

$$\int \frac{(dg-ef)x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx+\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx-(ef-dg)\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \text{ 1577}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx-\frac{1}{2}(ef-dg)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2$$

$$\Big\downarrow \text{ 488}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx+\frac{1}{2}(ef-dg)\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx+\frac{(ef-dg)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

$$\Big\downarrow \text{ 2227}$$

$$\frac{\left(\sqrt{a}eg+\sqrt{c}df\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt{a}de(ef-dg)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{(ef-dg)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(\sqrt{a}eg+\sqrt{c}df\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{de(ef-dg)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{(ef-dg)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{de(ef-dg)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}eg+\sqrt{c}df\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}+\frac{(ef-dg)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

$$\Big\downarrow \text{2223}$$

$$\frac{de(ef-dg)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\text{arctanh}\left(\frac{x\sqrt{ae^4}\cdots}{de\sqrt{a\cdots}}\right)}{2de\sqrt{ae^4+cd^4}}\right.}{\sqrt{a}e^2+\sqrt{c}d^2}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}eg+\sqrt{c}df\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}+\frac{(ef-dg)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

input
```
Int[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

output
```
((e*f - d*g)*ArcTanh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a +
c*x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[c]*d*f + Sqrt[a]*e*g)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*
e^2)*Sqrt[a + c*x^4]) + (d*e*(e*f - d*g)*(((Sqrt[c]*d^2 + Sqrt[a]*e^2)*Arc
Tanh[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*d*e*Sqrt[c*d^4 + a
*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(
4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4
)*c^(1/4)*Sqrt[a + c*x^4])))/(Sqrt[c]*d^2 + Sqrt[a]*e^2)
```

3.406. $\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx$

3.406.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

3.406. $\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx$

rule 2280
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

3.406.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.56

method	result
default	$\frac{g\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{(-dg+ef)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}}{\sqrt{c}\,d}\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}\right.}{e^2}$
elliptic	$\frac{g\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{(dg-ef)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2}\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}\right.}{e^2}$

input
```
int((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
g/e/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),
I)+(-d*g+e*f)/e^2*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2+2
*a)/(c/e^4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)*e/d*(
1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(
1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*e^2/d^2,(-I
/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

3.406.5 Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.406.6 Sympy [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{f+gx}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((f + g*x)/(sqrt(a + c*x**4)*(d + e*x)), x)
```

3.406.7 Maxima [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{gx+f}{\sqrt{cx^4+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

3.406.8 Giac [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx=\int \frac{gx+f}{\sqrt{cx^4+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

3.406.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx=\int \frac{f+g\,x}{\sqrt{c\,x^4+a}\,\left(d+e\,x\right)}\,dx$$

input
```
int((f + g*x)/((a + c*x^4)^(1/2)*(d + e*x)),x)
```

output
```
int((f + g*x)/((a + c*x^4)^(1/2)*(d + e*x)), x)
```

3.407 $\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx$

3.407.1 Optimal result

Integrand size = 26, antiderivative size = 218

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \frac{(ef-dg)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cd^4-ae^4}\sqrt{-a+cx^4}}\right)}{2\sqrt{cd^4-ae^4}} + \frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}(ef-dg)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de\sqrt{-a+cx^4}}$$

output
```
1/2*(-d*g+e*f)*arctanh((-c*d^2*x^2+a*e^2)/(-a*e^4+c*d^4)^(1/2)/(c*x^4-a)^(
1/2))/(-a*e^4+c*d^4)^(1/2)+a^(1/4)*g*EllipticF(c^(1/4)*x/a^(1/4),I)*(1-c*x
^4/a)^(1/2)/c^(1/4)/e/(c*x^4-a)^(1/2)+a^(1/4)*(-d*g+e*f)*EllipticPi(c^(1/4
)*x/a^(1/4),e^2*a^(1/2)/d^2/c^(1/2),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/d/e/(c*x^
4-a)^(1/2)
```

3.407.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.11 (sec) , antiderivative size = 719, normalized size of antiderivative = 3.30

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx$$

$$= \frac{-\frac{ig\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}e} + \frac{if\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2\sqrt{-\frac{(1-i)\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{i\sqrt[4]{a}+\sqrt[4]{c}x}}\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+i\sqrt[4]{c}x\right)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2}}\,\Big(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

output
```
(((-I)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*
x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*e) + (I*f*(a^(1/4) - I*c^(1/4)*x)^2*Sqr
t[((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/4)*x)]*Sqrt[((1 + I)*
(a^(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1/4)*x)^2]*
((-(c^(1/4)*d) + a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1
/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] - (1 - I)*a^(1/4)*e*EllipticPi
[((1 - I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin[Sqrt[
((1 + I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]))/(a^(
1/4)*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e)) + (d*g*(a^(1/4)
 - I*c^(1/4)*x)^2*Sqrt[((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/
4)*x)]*Sqrt[((1 + I)*(a^(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/
4) - I*c^(1/4)*x)^2]*(I*(c^(1/4)*d - a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1
+ I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] + (1 + I)*
a^(1/4)*e*EllipticPi[((1 - I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1
/4)*e), ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(
1/4)*x)]], 2]))/(a^(1/4)*e*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/
4)*e)))/Sqrt[-a + c*x^4]
```

3.407.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2280, 27, 1577, 488, 219, 2229, 765, 762, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\sqrt{cx^4-a}(d+ex)}dx$$

$$\Big\downarrow \text{ 2280}$$

$$\int \frac{(dg-ef)x}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx+\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx-(ef-dg)\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx$$

$$\Big\downarrow \text{ 1577}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx-\frac{1}{2}(ef-dg)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx^2$$

$$\Big\downarrow \text{ 488}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx+\frac{1}{2}(ef-dg)\int \frac{1}{cd^4-ae^4-x^4}d\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}}$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx+\frac{(ef-dg)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}$$

$$\Big\downarrow \text{ 2229}$$

$$\frac{d(ef-dg)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx}{e}+\frac{g\int \frac{1}{\sqrt{cx^4-a}}dx}{e}+\frac{(ef-dg)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{d(ef-dg)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx}{e}+\frac{g\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{e\sqrt{cx^4-a}}+\frac{(ef-dg)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{d(ef-dg)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4-a}}dx}{e}+\frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{cx^4-a}}+\frac{(ef-dg)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}$$

$$\Big\downarrow \text{ 1543}$$

$$\frac{d\sqrt{1-\frac{cx^4}{a}}(ef-dg)\int \frac{1}{(d^2-e^2x^2)\sqrt{1-\frac{cx^4}{a}}}dx}{e\sqrt{cx^4-a}}+\frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{cx^4-a}}+\frac{(ef-dg)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}$$

$$\Big\downarrow \text{ 1542}$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(ef-dg)\,\text{EllipticPi}\left(\frac{\sqrt{a}e^2}{\sqrt{c}d^2},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de\sqrt{cx^4-a}}+\frac{\sqrt[4]{a}g\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{cx^4-a}}+\frac{(ef-dg)\text{arctanh}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}$$

input
```
Int[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

output
```
((e*f - d*g)*ArcTanh[(a*e^2 - c*d^2*x^2)/(Sqrt[c*d^4 - a*e^4]*Sqrt[-a + c*
x^4])])/(2*Sqrt[c*d^4 - a*e^4]) + (a^(1/4)*g*Sqrt[1 - (c*x^4)/a]*EllipticF
[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*Sqrt[-a + c*x^4]) + (a^(1/4)
*(e*f - d*g)*Sqrt[1 - (c*x^4)/a]*EllipticPi[(Sqrt[a]*e^2)/(Sqrt[c]*d^2), A
rcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e*Sqrt[-a + c*x^4])
```

3.407.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2229
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[B/e   Int[1/Sqrt[a + c*x^4], x], x] + Simp[(e*A - d*B)/
e   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e, A, B}
, x] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

rule 2280
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

3.407. $\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx$

3.407.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.13

method	result
default	$\frac{g\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}+\frac{(-dg+ef)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}-2a}{2\sqrt{\frac{c\,d^4}{e^4}-a}\,\sqrt{c\,x^4-a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}-a}}+\frac{e\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},-\frac{e^2\sqrt{a}}{d^2\sqrt{c}}\cdots\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4-a}}\right)}{e^2}$
elliptic	$\frac{g\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}-\frac{(dg-ef)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}-2a}{2\sqrt{\frac{c\,d^4}{e^4}-a}\,\sqrt{c\,x^4-a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}-a}}+\frac{e\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},-\frac{e^2\sqrt{a}}{d^2\sqrt{c}},\cdots\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4-a}}\right)}{e^2}$

input
```
int((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
g/e/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2
),I)+(-d*g+e*f)/e^2*(-1/2/(c/e^4*d^4-a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2
-2*a)/(c/e^4*d^4-a)^(1/2)/(c*x^4-a)^(1/2))+1/(-1/a^(1/2)*c^(1/2))^(1/2)*e/
d*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a
)^(1/2)*EllipticPi(x*(-1/a^(1/2)*c^(1/2))^(1/2),-e^2*a^(1/2)/d^2/c^(1/2),(
1/a^(1/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)))
```

3.407.5 Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.407.6 Sympy [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \int \frac{f+gx}{\sqrt{-a+cx^4}\,(d+ex)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x**4-a)**(1/2),x)
```

output
```
Integral((f + g*x)/(sqrt(-a + c*x**4)*(d + e*x)), x)
```

3.407.7 Maxima [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \int \frac{gx+f}{\sqrt{cx^4-a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

3.407.8 Giac [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \int \frac{gx+f}{\sqrt{cx^4-a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(sqrt(c*x^4 - a)*(e*x + d)), x)
```

3.407.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx=\int \frac{f+g\,x}{\sqrt{c\,x^4-a}\,\left(d+e\,x\right)}\,dx$$

input
```
int((f + g*x)/((c*x^4 - a)^(1/2)*(d + e*x)),x)
```

output
```
int((f + g*x)/((c*x^4 - a)^(1/2)*(d + e*x)), x)
```

3.408 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$

3.408.1 Optimal result

Integrand size = 40, antiderivative size = 65

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

output
```
1/3*arctanh((1+x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1
/2))*(-3+2*3^(1/2))^(1/2)
```

3.408.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\sqrt{9+6\sqrt{3}}\sqrt{-4+4\sqrt{3}x^2+x^4}}{2+\left(-2-2\sqrt{3}\right)x+\left(2+\sqrt{3}\right)x^2}\right)$$

input
```
Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x
^4]),x]
```

output
```
(Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[9 + 6*Sqrt[3]]*Sqrt[-4 + 4*Sqrt[3]*x^2
 + x^4])/(2 + (-2 - 2*Sqrt[3])*x + (2 + Sqrt[3])*x^2)])/3
```

3.408.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2278, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^4+4\sqrt{3}x^2-4}}dx$$

$$\Big\downarrow \text{2278}$$

$$-4\left(2-\sqrt{3}\right)\int \frac{1}{\frac{4\left(x-\sqrt{3}+1\right)^4}{x^4+4\sqrt{3}x^2-4}+12\left(3-2\sqrt{3}\right)}d\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{x^4+4\sqrt{3}x^2-4}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}}$$

input
```
Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x
]
```

output
```
((2 - Sqrt[3])*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[
-4 + 4*Sqrt[3]*x^2 + x^4])])/Sqrt[3*(-3 + 2*Sqrt[3])]
```

3.408.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2278
```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Simp[(-A^2)*((B*d + A*e)/e)   Subst[Int[1/(6*A^3*
B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /
; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e
^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] &&
EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

3.408.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.54 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.03

method	result
elliptic	$\frac{\sqrt{1-\left(\frac{\sqrt{3}}{2}-1\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\,F\left(x\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right), i\sqrt{1+4\sqrt{3}\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{-4+x^4+4x^2\sqrt{3}}} - 2\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{4\sqrt{3}\left(-1-\sqrt{3}\right)^2-8+4x^2\sqrt{3}+2x\cdots}{2\sqrt{\left(-1-\sqrt{3}\right)^4+4\sqrt{3}\left(-1-\sqrt{3}\right)^2-4\cdots}}\cdots\right.}{2\sqrt{\left(-1-\sqrt{3}\right)^4+4\sqrt{3}\left(-1-\sqrt{3}\cdots\right.}}\right.$

input
```
int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/(1/2*I*3^(1/2)-1/2*I)*(1-(1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),I
*(1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*3^
(1/2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*x^
2*3^(1/2)+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4
)^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1/2))-1/(1/2*3^(1/2)-1)^(1/2)/(-1-3^(1/2))
*(1-(1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*x^2
*3^(1/2))^(1/2)*EllipticPi((1/2*3^(1/2)-1)^(1/2)*x,1/(1/2*3^(1/2)-1)/(-1-3
^(1/2))^2,(1+1/2*3^(1/2))^(1/2)/(1/2*3^(1/2)-1)^(1/2)))
```

3.408.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(47) = 94.

Time = 0.40 (sec) , antiderivative size = 323, normalized size of antiderivative = 4.97

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$$
$$= \frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37\,x^{12}-204\,x^{11}+804\,x^{10}-2408\,x^9+3708\,x^8-5472\,x^7+6432\,x^6+10944\,x^5+1}{}\right.$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x, algo
rithm="fricas")
```

output
```
1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 +
3708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864
*x^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 10
08*x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320
*x^7 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256)
 - 1152*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqr
t(3)*(7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5
 - 3696*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 +
 12*x^11 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*
x^4 - 320*x^3 + 768*x^2 - 384*x + 64))
```

3.408.6 Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}\,dx$$

input
```
integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*x**2*3**(1/2))**(1/2),x
)
```

output
```
Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
 4)), x)
```

3.408.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) +
1)), x)
```

3.408.8 Giac [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) +
1)), x)
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx=\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^4+4\sqrt{3}\,x^2-4}}\,dx$$

input
```
int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
x)
```

output
```
int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
 x)
```

3.409 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$

3.409.1 Optimal result

Integrand size = 40, antiderivative size = 63

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)$$

output
```
-1/3*arctan((1+x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-4+x^4-4*x^2*3^(1/2))^(1/
2))*(3+2*3^(1/2))^(1/2)
```

3.409.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\sqrt{-9+6\sqrt{3}}\sqrt{-4-4\sqrt{3}x^2+x^4}}{-2+\left(2-2\sqrt{3}\right)x+\left(-2+\sqrt{3}\right)x^2}\right)$$

input
```
Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x
^4]),x]
```

output
```
-1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-9 + 6*Sqrt[3]]*Sqrt[-4 - 4*Sqrt[3]
*x^2 + x^4])/(-2 + (2 - 2*Sqrt[3])*x + (-2 + Sqrt[3])*x^2)])
```

3.409.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2278, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}dx$$

$$\Big\downarrow \text{ 2278}$$

$$-4\left(2+\sqrt{3}\right)\int \frac{1}{\frac{4\left(x+\sqrt{3}+1\right)^4}{x^4-4\sqrt{3}x^2-4}+12\left(3+2\sqrt{3}\right)}d\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{x^4-4\sqrt{3}x^2-4}}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x
]
```

output
```
-(((2 + Sqrt[3])*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3])]*Sqrt[
-4 - 4*Sqrt[3]*x^2 + x^4])])/Sqrt[3*(3 + 2*Sqrt[3])])
```

3.409.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2278
```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Simp[(-A^2)*((B*d + A*e)/e)   Subst[Int[1/(6*A^3*
B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /
; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e
^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] &&
EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

3.409.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.42 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.94

method	result
elliptic	$\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{\sqrt{3}}{2}+1\right)x^2}\,F\left(x\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right),i\sqrt{1-4\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-4+x^4-4x^2\sqrt{3}}}+2\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{-4\sqrt{3}\left(\sqrt{3}-1\right)^2-8-4x^2\sqrt{\ }}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}-1\right)^2-}}\right.}{2\sqrt{\left(\sqrt{3}-1\right)^4-4\sqrt{3}\left(\sqrt{3}\right.}}\right.$

input
```
int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)
*x^2)^(1/2)/(-4+x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2))
,I*(1-4*3^(1/2)*(-1/2*3^(1/2)+1))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*
3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*x
^2*3^(1/2)+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^
(1/2)/(-4+x^4-4*x^2*3^(1/2))^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(
1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*x^2)^(1/2)/(-4+x^4-4*x^2
*3^(1/2))^(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))/(3^
(1/2)-1)^2,(-1/2*3^(1/2)+1)^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

3.409.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(45) = 90$.

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$$
$$=\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9x^4-30x^3+18x^2-2\sqrt{3}(2x^4-10x^3+3x^2-10x+2)+24\right)\sqrt{x^4-4\sqrt{3}x}}{11x^6-42x^5+66x^4-176x^3-132x^2-168x-88}\right.$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2),x, algo
rithm="fricas")
```

output
```
1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^
4 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*
sqrt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

3.409.6 Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

input
```
integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*x**2*3**(1/2))**(1/2),x
)
```

output
```
Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
 4)), x)
```

3.409.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\,\sqrt{3}x^2-4}(x-\sqrt{3}+1)}\,dx$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) +
1)), x)
```

3.409.8 Giac [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\,\sqrt{3}x^2-4}(x-\sqrt{3}+1)}\,dx$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) +
1)), x)
```

3.409.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\,\sqrt{3}\,x^2-4}\,\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
x)
```

output
```
int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
 x)
```

3.410 $\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx$

3.410.1 Optimal result

Integrand size = 46, antiderivative size = 72

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\left(1-\sqrt{3}+2x\right)^2}{2\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-1+4\sqrt{3}x^2+4x^4}}\right)$$

output
```
1/3*arctanh(1/2*(1+2*x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-1+4*x^4+4*x^2*3^(
1/2))^(1/2))*(-3+2*3^(1/2))^(1/2)
```

3.410.2 Mathematica [A] (verified)

Time = 8.82 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx = \frac{1}{3}\sqrt{-3+2\sqrt{3}}\text{arctanh}\left(\frac{\sqrt{9+6\sqrt{3}}\sqrt{-1+4\sqrt{3}x^2+4x^4}}{1+\left(-2-2\sqrt{3}\right)x+\left(4+2\sqrt{3}\right)x^2}\right)$$

input
```
Integrate[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2
 + 4*x^4]),x]
```

output
```
(Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[9 + 6*Sqrt[3]]*Sqrt[-1 + 4*Sqrt[3]*x^2
 + 4*x^4])/(1 + (-2 - 2*Sqrt[3])*x + (4 + 2*Sqrt[3])*x^2)])/3
```

3.410.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2278, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-\sqrt{3}+1}{\left(2x+\sqrt{3}+1\right)\sqrt{4x^4+4\sqrt{3}x^2-1}}dx$$

$$\Big\downarrow \text{2278}$$

$$-4\left(2-\sqrt{3}\right)\int \frac{1}{\frac{2\left(2x-\sqrt{3}+1\right)^4}{4x^4+4\sqrt{3}x^2-1}+24\left(3-2\sqrt{3}\right)}d\frac{\left(2x-\sqrt{3}+1\right)^2}{\sqrt{4x^4+4\sqrt{3}x^2-1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(2x-\sqrt{3}+1\right)^2}{2\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{4x^4+4\sqrt{3}x^2-1}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}}$$

input
```
Int[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x
^4]),x]
```

output
```
((2 - Sqrt[3])*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3*(-3 + 2*Sqrt[3])]*S
qrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4])])/Sqrt[3*(-3 + 2*Sqrt[3])]
```

3.410.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2278
```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Simp[(-A^2)*((B*d + A*e)/e)   Subst[Int[1/(6*A^3*
B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /
; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e
^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] &&
EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

3.410.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.38 (sec) , antiderivative size = 336, normalized size of antiderivative = 4.67

method	result
elliptic	$\frac{\sqrt{1-\left(2\sqrt{3}-4\right)x^2}\,\sqrt{1-\left(2\sqrt{3}+4\right)x^2}\,F\left(x\left(i\sqrt{3}-i\right),i\sqrt{1+\sqrt{3}\left(2\sqrt{3}+4\right)}\right)}{\left(i\sqrt{3}-i\right)\sqrt{-1+4x^4+4x^2\sqrt{3}}}-\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{4\sqrt{3}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^2-2+4x^2\sqrt{3}+\cdots}{2\sqrt{4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^4+4\sqrt{3}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^2\cdots}}\right)}{2\sqrt{4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^4+4\sqrt{3}\left(-\frac{1}{2}-\cdots}}\right.$

input
```
int((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/(I*3^(1/2)-I)*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(2*3^(1/2)+4)*x^2)^(1/2)/(-
1+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I*3^(1/2)-I),I*(1+3^(1/2)*(2*3^(
1/2)+4))^(1/2))-3^(1/2)*(-1/2/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2*
3^(1/2))^2-1)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1/2-1/2*3^(1/2))^2-2+4*x^2*3^
(1/2)+8*x^2*(-1/2-1/2*3^(1/2))^2)/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-
1/2*3^(1/2))^2-1)^(1/2)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2))-1/(2*3^(1/2)-4)^(1
/2)/(-1/2-1/2*3^(1/2))*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(2*3^(1/2)+4)*x^2)^(
1/2)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticPi((2*3^(1/2)-4)^(1/2)*x,1/(2*
3^(1/2)-4)/(-1/2-1/2*3^(1/2))^2,(2*3^(1/2)+4)^(1/2)/(2*3^(1/2)-4)^(1/2)))
```

3.410.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(52) = 104$.

Time = 0.39 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.56

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx$$
$$=\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{2368\,x^{12}-6528\,x^{11}+12864\,x^{10}-19264\,x^{9}+14832\,x^{8}-10944\,x^{7}+6432\,x^{6}+547}{}\right.$$

input
```
integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2),x
, algorithm="fricas")
```

output
```
1/12*sqrt(2*sqrt(3) - 3)*log(-(2368*x^12 - 6528*x^11 + 12864*x^10 - 19264*
x^9 + 14832*x^8 - 10944*x^7 + 6432*x^6 + 5472*x^5 + 3708*x^4 + 2408*x^3 +
804*x^2 + (1728*x^10 - 4800*x^9 + 8208*x^8 - 8928*x^7 + 6048*x^6 - 3024*x^
5 - 504*x^4 - 504*x^3 - 324*x^2 + 2*sqrt(3)*(496*x^10 - 1408*x^9 + 2304*x^
8 - 2640*x^7 + 1848*x^6 - 504*x^5 + 336*x^4 + 204*x^3 + 63*x^2 + 26*x + 4)
 - 72*x - 15)*sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*sqrt(2*sqrt(3) - 3) + 3*sqrt
(3)*(448*x^12 - 1280*x^11 + 2560*x^10 - 3200*x^9 + 3696*x^8 - 1920*x^7 - 9
60*x^5 - 924*x^4 - 400*x^3 - 160*x^2 - 40*x - 7) + 204*x + 37)/(64*x^12 +
384*x^11 + 768*x^10 + 320*x^9 - 720*x^8 - 576*x^7 + 384*x^6 + 288*x^5 - 18
0*x^4 - 40*x^3 + 48*x^2 - 12*x + 1))
```

3.410.6 Sympy [F]

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2x-\sqrt{3}+1}{\left(2x+1+\sqrt{3}\right)\sqrt{4x^4+4\sqrt{3}x^2-1}}\,dx$$

input
```
integrate((1+2*x-3**(1/2))/(1+2*x+3**(1/2))/(-1+4*x**4+4*x**2*3**(1/2))**(
1/2),x)
```

output
```
Integral((2*x - sqrt(3) + 1)/((2*x + 1 + sqrt(3))*sqrt(4*x**4 + 4*sqrt(3)*
x**2 - 1)), x)
```

3.410.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^4+4\,\sqrt{3}x^2-1}\left(2\,x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt
(3) + 1)), x)
```

3.410.8 Giac [F]

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^4+4\,\sqrt{3}x^2-1}\left(2\,x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2),x
, algorithm="giac")
```

output
```
integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt
(3) + 1)), x)
```

3.410.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2\,x-\sqrt{3}+1}{\sqrt{4\,x^4+4\,\sqrt{3}\,x^2-1}\,\left(2\,x+\sqrt{3}+1\right)}\,dx$$

input
```
int((2*x - 3^(1/2) + 1)/((4*3^(1/2)*x^2 + 4*x^4 - 1)^(1/2)*(2*x + 3^(1/2)
+ 1)),x)
```

output
```
int((2*x - 3^(1/2) + 1)/((4*3^(1/2)*x^2 + 4*x^4 - 1)^(1/2)*(2*x + 3^(1/2)
+ 1)), x)
```

3.411 $\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx$

3.411.1 Optimal result

Integrand size = 46, antiderivative size = 70

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\left(1+\sqrt{3}+2x\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-1-4\sqrt{3}x^2+4x^4}}\right)$$

output
```
-1/3*arctan(1/2*(1+2*x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-1+4*x^4-4*x^2*3^(1
/2))^(1/2))*(3+2*3^(1/2))^(1/2)
```

3.411.2 Mathematica [A] (verified)

Time = 8.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx = -\frac{1}{3}\sqrt{3+2\sqrt{3}}\arctan\left(\frac{\sqrt{-9+6\sqrt{3}}\sqrt{-1-4\sqrt{3}x^2+4x^4}}{-1+\left(2-2\sqrt{3}\right)x+\left(-4+2\sqrt{3}\right)x^2}\right)$$

input
```
Integrate[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2
 + 4*x^4]),x]
```

output
```
-1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-9 + 6*Sqrt[3]]*Sqrt[-1 - 4*Sqrt[3]
*x^2 + 4*x^4])/(-1 + (2 - 2*Sqrt[3])*x + (-4 + 2*Sqrt[3])*x^2)])
```

3.411.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2278, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+\sqrt{3}+1}{\left(2x-\sqrt{3}+1\right)\sqrt{4x^4-4\sqrt{3}x^2-1}}dx$$

$$\Big\downarrow \text{2278}$$

$$-4\left(2+\sqrt{3}\right)\int \frac{1}{\frac{2\left(2x+\sqrt{3}+1\right)^4}{4x^4-4\sqrt{3}x^2-1}+24\left(3+2\sqrt{3}\right)}d\frac{\left(2x+\sqrt{3}+1\right)^2}{\sqrt{4x^4-4\sqrt{3}x^2-1}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(2x+\sqrt{3}+1\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{4x^4-4\sqrt{3}x^2-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x
^4]),x]
```

output
```
-(((2 + Sqrt[3])*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3*(3 + 2*Sqrt[3])]*S
qrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4])])/Sqrt[3*(3 + 2*Sqrt[3])])
```

3.411.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2278
```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Simp[(-A^2)*((B*d + A*e)/e)   Subst[Int[1/(6*A^3*
B*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /
; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e
^4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] &&
EqQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

3.411.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.35 (sec) , antiderivative size = 336, normalized size of antiderivative = 4.80

method	result
elliptic	$\frac{\sqrt{1-\left(-2\sqrt{3}-4\right)x^{2}}\,\sqrt{1-\left(-2\sqrt{3}+4\right)x^{2}}\,F\left(x\left(i+i\sqrt{3}\right),i\sqrt{1-\sqrt{3}\left(-2\sqrt{3}+4\right)}\right)}{\left(i+i\sqrt{3}\right)\sqrt{-1+4x^{4}-4x^{2}\sqrt{3}}}+\sqrt{3}\left(-\frac{\operatorname{arctanh}\left(\frac{-4\sqrt{3}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^{2}-2-4x^{2}}{2\sqrt{4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^{4}-4\sqrt{3}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right.}}\right.}{2\sqrt{4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)^{4}-4\sqrt{3}\left(\right.}}\right.$

input
```
int((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/(I+I*3^(1/2))*(1-(-2*3^(1/2)-4)*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(1/2)/
(-1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I+I*3^(1/2)),I*(1-3^(1/2)*(-2*
3^(1/2)+4))^(1/2))+3^(1/2)*(-1/2/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(
1/2)-1/2)^2-1)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(1/2*3^(1/2)-1/2)^2-2-4*x^2*3
^(1/2)+8*x^2*(1/2*3^(1/2)-1/2)^2)/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^
(1/2)-1/2)^2-1)^(1/2)/(-1+4*x^4-4*x^2*3^(1/2))^(1/2))-1/(-2*3^(1/2)-4)^(1/
2)/(1/2*3^(1/2)-1/2)*(1-(-2*3^(1/2)-4)*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(
1/2)/(-1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticPi((-2*3^(1/2)-4)^(1/2)*x,1/(-
2*3^(1/2)-4)/(1/2*3^(1/2)-1/2)^2,(-2*3^(1/2)+4)^(1/2)/(-2*3^(1/2)-4)^(1/2)
))
```

3.411. $\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^{2}+4x^{4}}}\,dx$

3.411.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(50) = 100.

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.63

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx$$
$$=\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(36\,x^4-60\,x^3+18\,x^2-\sqrt{3}(16\,x^4-40\,x^3+6\,x^2-10\,x+1)+6\right)\sqrt{4\,x^4-4\,\sqrt{3}}}{88\,x^6-168\,x^5+132\,x^4-176\,x^3-66\,x^2-42\,x-11}\right.$$

input
```
integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2),x
, algorithm="fricas")
```

output
```
1/6*sqrt(2*sqrt(3) + 3)*arctan(-(36*x^4 - 60*x^3 + 18*x^2 - sqrt(3)*(16*x^
4 - 40*x^3 + 6*x^2 - 10*x + 1) + 6)*sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*sqrt(2
*sqrt(3) + 3)/(88*x^6 - 168*x^5 + 132*x^4 - 176*x^3 - 66*x^2 - 42*x - 11))
```

3.411.6 Sympy [F]

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx=\int \frac{2x+1+\sqrt{3}}{\left(2x-\sqrt{3}+1\right)\sqrt{4x^4-4\sqrt{3}x^2-1}}\,dx$$

input
```
integrate((1+2*x+3**(1/2))/(1+2*x-3**(1/2))/(-1+4*x**4-4*x**2*3**(1/2))**(
1/2),x)
```

output
```
Integral((2*x + 1 + sqrt(3))/((2*x - sqrt(3) + 1)*sqrt(4*x**4 - 4*sqrt(3)*
x**2 - 1)), x)
```

3.411.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx = \int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}x^2-1}\left(2\,x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt
(3) + 1)), x)
```

3.411.8 Giac [F]

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx = \int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}x^2-1}\left(2\,x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2),x
, algorithm="giac")
```

output
```
integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt
(3) + 1)), x)
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}x^2+4x^4}}\,dx = \int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}\,x^2-1}\,\left(2\,x-\sqrt{3}+1\right)}\,dx$$

input
```
int((2*x + 3^(1/2) + 1)/((4*x^4 - 4*3^(1/2)*x^2 - 1)^(1/2)*(2*x - 3^(1/2)
+ 1)),x)
```

output
```
int((2*x + 3^(1/2) + 1)/((4*x^4 - 4*3^(1/2)*x^2 - 1)^(1/2)*(2*x - 3^(1/2)
+ 1)), x)
```

3.412 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$

3.412.1 Optimal result

Integrand size = 29, antiderivative size = 560

$$\begin{aligned}\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx &= \frac{(ef-dg)\arctan\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2(bd^2+ae^2)}} - \frac{(ef-dg)\text{arctanh}\left(\frac{bd^2+2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2+ae^4}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2+ae^4}} \\ &+ \frac{(\sqrt{c}df+\sqrt{a}eg)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}} \\ &- \frac{(\sqrt{c}d^2-\sqrt{a}e^2)(ef-dg)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}}\end{aligned}$$

output
```
1/2*(-d*g+e*f)*arctan(x*(-a*e^4-b*d^2*e^2-c*d^4)^(1/2)/d/e/(c*x^4+b*x^2+a)
^(1/2))/(-a*e^4-b*d^2*e^2-c*d^4)^(1/2)-1/2*(-d*g+e*f)*arctanh(1/2*(b*d^2+2
*a*e^2+(b*e^2+2*c*d^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2+a)^
(1/2))/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)-1/4*(-d*g+e*f)*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^
(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)
+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1
/4)/d/e/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(
c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(e*g*a^(1/
2)+d*f*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2
))^2)^(1/2)/a^(1/4)/c^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+b*x^2+a)^(1/2
)
```

3.412.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.74 (sec) , antiderivative size = 3652, normalized size of antiderivative = 6.52

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^
2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(Sqr
t[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*e*Sqrt[a + b*x^2 + c*x^4]) + (2*(
Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*
c]/c]/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)^2*Sqr
t[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/
Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c)
+ Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2
 - 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] -
Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*
a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b
 + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + 2*x)
)/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(
Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - Sq
rt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2
 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt
[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b +
Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x)...
```

3.412.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2279, 27, 1576, 1154, 219, 2226, 27, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{2279}$$

$$\int \frac{(dg-ef)x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx+\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx-(ef-dg)\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \text{1576}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx-\frac{1}{2}(ef-dg)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1154}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx+(ef-dg)\int \frac{1}{4(cd^4+be^2d^2+ae^4)-x^4}d\left(-\frac{bd^2+2ae^2+(2cd^2+be^2)x^2}{\sqrt{cx^4+bx^2+a}}\right)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx-\frac{(ef-dg)\text{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \text{2226}$$

$$\frac{(\sqrt{a}eg+\sqrt{c}df)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt{a}de(ef-dg)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}-\frac{(ef-dg)\text{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(\sqrt{a}eg+\sqrt{c}df)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{de(ef-dg)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}-\frac{(ef-dg)\text{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{de(ef-dg)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)(\sqrt{a}eg+\sqrt{c}df)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}-\frac{(ef-dg)\text{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \text{2222}$$

3.412. $\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$

$$\frac{de(ef-dg)\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{(\sqrt{a}e^2+\sqrt{c}d^2)\text{arcta}}{2de\sqrt{ae^4}}\right.}{\sqrt{a}e^2+\sqrt{c}d^2}$$

$$\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)(\sqrt{a}eg+\sqrt{c}df)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\,(\sqrt{a}e^2+\sqrt{c}d^2)}-\frac{(ef-dg)\text{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

input
```
Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*((e*f - d*g)*ArcTanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqr
t[c*d^4 + b*d^2*e^2 + a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c*d^4 + b*d^2
*e^2 + a*e^4] + ((Sqrt[c]*d*f + Sqrt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^
2 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (d*e*(e*f - d*g)*(((Sqrt[c]*d^
2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]*x)/(d*e*Sqrt[a +
 b*x^2 + c*x^4])])/(2*d*e*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((Sqrt[a]/d^2
 - Sqrt[c]/e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt
[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])
/(4*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/(Sqrt[c]*d^2 + Sqrt[a]*e^2)
```

3.412.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2279
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

3.412. $\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$

3.412.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.78

method	result
default	$\frac{g\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}+\frac{(-dg+ef)\left(-\frac{\mathrm{arcta}}{}\right.}{}$
elliptic	$\frac{g\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}-\frac{(dg-ef)\left(-\frac{\mathrm{arctanh}}{}\right.}{}$

input
```
int((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*g/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(-d*g+e*f)/e^2*(-1/2/(c/e^4*d^4+b/e^
2*d^2+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2+b/e^2*d^2+b*x^2+2*a)/(c/e^4*d^
4+b/e^2*d^2+a)^(1/2)/(c*x^4+b*x^2+a)^(1/2))+2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*e/d*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1+1/2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c+b^2)^(1/2))*a*e^2/d^2,(-
1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)))
```

3.412.5 Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.412.6 Sympy [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((f + g*x)/((d + e*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

3.412.7 Maxima [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{gx+f}{\sqrt{cx^4+bx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

3.412.8 Giac [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{gx+f}{\sqrt{cx^4+bx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

3.412.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{f+g\,x}{(d+e\,x)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((f + g*x)/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((f + g*x)/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.413 $\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$

3.413.1 Optimal result

Integrand size = 31, antiderivative size = 527

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{(ef-dg)\operatorname{arctanh}\left(\frac{bd^2-2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2-ae^4}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2-ae^4}}$$
$$+\frac{\sqrt{b+\sqrt{b^2+4ac}}\,g\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}}$$
$$+\frac{\sqrt{-b+\sqrt{b^2+4ac}}(ef-dg)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticPi}\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2},\arcsin\left(\frac{\sqrt{2}\cdots}{\sqrt{-b+\cdots}}\right.\right.}{\sqrt{2}\sqrt{c}de\sqrt{-a+bx^2+cx^4}}$$

output
```
-1/2*(-d*g+e*f)*arctanh(1/2*(b*d^2-2*a*e^2+(b*e^2+2*c*d^2)*x^2)/(-a*e^4+b*
d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2-a)^(1/2))/(-a*e^4+b*d^2*e^2+c*d^4)^(1/2)
+1/2*g*(1/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^
2)^(1/2)))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2)/(
1+2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/(b-(4*a*c+b^2
)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b+(4*a*c+b^2)^(1/2))^(
1/2)/e*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*a*c+b^2)^(1
/2)))/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)+1/2*(-d*g+e*f)*EllipticPi(x
*2^(1/2)*c^(1/2)/(-b+(4*a*c+b^2)^(1/2))^(1/2),-1/2*e^2*(b-(4*a*c+b^2)^(1/2
))/c/d^2,((b-(4*a*c+b^2)^(1/2))/(b+(4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(
b-(4*a*c+b^2)^(1/2)))^(1/2)*(-b+(4*a*c+b^2)^(1/2))^(1/2)*(1+2*c*x^2/(b+(4*
a*c+b^2)^(1/2)))^(1/2)/d/e*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)
```

3.413.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.68 (sec) , antiderivative size = 3658, normalized size of antiderivative = 6.94

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-
b + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^
2 + 4*a*c]))]*x], (-b - Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c])])/(Sqr
t[2]*Sqrt[-(c/(-b - Sqrt[b^2 + 4*a*c]))]*e*Sqrt[-a + b*x^2 + c*x^4]) + (2*
(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a
*c]/c]/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x)^2*Sq
rt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]
/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c)
 + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqr
t[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^
2 + 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] -
 Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4
*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-
b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + 2*x
))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*
(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - S
qrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^
2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqr
t[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b +
 Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x...
```

3.413.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2279, 27, 1576, 1154, 219, 2228, 1417, 320, 1544, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}dx$$

$$\Big\downarrow \; 2279$$

$$\int \frac{(dg-ef)x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx+\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx-(ef-dg)\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx$$

$$\Big\downarrow \; 1576$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx-\frac{1}{2}(ef-dg)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx^2$$

$$\Big\downarrow \; 1154$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx+(ef-dg)\int \frac{1}{4(cd^4+be^2d^2-ae^4)-x^4}d\left(-\frac{bd^2-2ae^2+(2cd^2+be^2)x^2}{\sqrt{cx^4+bx^2-a}}\right)$$

$$\Big\downarrow \; 219$$

$$\int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx-\frac{(ef-dg)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \; 2228$$

$$\frac{d(ef-dg)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx}{e}+\frac{g\int \frac{1}{\sqrt{cx^4+bx^2-a}}dx}{e}-\frac{(ef-dg)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \; 1417$$

$$\frac{g\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}}dx}{e\sqrt{-a+bx^2+cx^4}}+\frac{d(ef-dg)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx}{e}-\frac{(ef-dg)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \; 320$$

3.413. $\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}dx$

$$\frac{d(ef-dg)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2-a}}dx}{e}+\frac{g\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}-\frac{(ef-dg)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \text{1544}$$

$$\frac{d\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}(ef-dg)\int \frac{1}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2+4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2+4ac}}+1}(d^2-e^2x^2)}dx}{e\sqrt{-a+bx^2+cx^4}}+\frac{g\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}-\frac{(ef-dg)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}(ef-dg)\text{EllipticPi}\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2},\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2+4ac}-b}}\right),\frac{b-}{b+}\right.}{\sqrt{2}\sqrt{c}de\sqrt{-a+bx^2+cx^4}}$$

$$\frac{g\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}e\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}-\frac{(ef-dg)\text{arctanh}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

input
```
Int[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

3.413. $\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$

output
```
-1/2*((e*f - d*g)*ArcTanh[(b*d^2 - 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqr
t[c*d^4 + b*d^2*e^2 - a*e^4]*Sqrt[-a + b*x^2 + c*x^4])])/Sqrt[c*d^4 + b*d^
2*e^2 - a*e^4] + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*g*(1 + (2*c*x^2)/(b - Sqrt[b
^2 + 4*a*c]))*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a
*c]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*e
*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2
 + 4*a*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*(e*
f - d*g)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
 + Sqrt[b^2 + 4*a*c])]*EllipticPi[-1/2*((b - Sqrt[b^2 + 4*a*c])*e^2)/(c*d^
2), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (b - Sqrt[b^
2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*e*Sqrt[-a + b*x^2
 + c*x^4])
```

3.413.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

rule 1544
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/((d + e*x^2)*S
qrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& NegQ[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2228
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[B/e   Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Simp[(e*A - d*B)/e   Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
 x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a
]
```

rule 2279
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

3.413. $\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}} \, dx$

3.413.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.83

method	result
default	$\frac{g\sqrt{4+\frac{2(-b+\sqrt{4ac+b^2})x^2}{a}}\sqrt{4-\frac{2(b+\sqrt{4ac+b^2})x^2}{a}}\,F\left(\frac{x\sqrt{-\frac{2(-b+\sqrt{4ac+b^2})}{a}}}{2},\frac{\sqrt{-4-\frac{2b(b+\sqrt{4ac+b^2})}{ac}}}{2}\right)}{2e\sqrt{-\frac{2(-b+\sqrt{4ac+b^2})}{a}}\sqrt{cx^4+bx^2-a}}+\frac{(-dg+ef)\left(-\frac{\operatorname{arctanh}\left(\frac{}{2\sqrt{g}}\right)}{2}\right.}{}$
elliptic	$\frac{g\sqrt{4+\frac{2(-b+\sqrt{4ac+b^2})x^2}{a}}\sqrt{4-\frac{2(b+\sqrt{4ac+b^2})x^2}{a}}\,F\left(\frac{x\sqrt{-\frac{2(-b+\sqrt{4ac+b^2})}{a}}}{2},\frac{\sqrt{-4-\frac{2b(b+\sqrt{4ac+b^2})}{ac}}}{2}\right)}{2e\sqrt{-\frac{2(-b+\sqrt{4ac+b^2})}{a}}\sqrt{cx^4+bx^2-a}}-\frac{(dg-ef)\left(-\frac{\operatorname{arctanh}\left(\frac{2c}{2\sqrt{\frac{cd}{e^4}}}\right)}{2\sqrt{}}\right.}{}$

input
```
int((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*g/e/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*E
llipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b
^2)^(1/2))/a/c)^(1/2))+(-d*g+e*f)/e^2*(-1/2/(c/e^4*d^4+b/e^2*d^2-a)^(1/2)*
arctanh(1/2*(2*c*x^2/e^2*d^2+b/e^2*d^2+b*x^2-2*a)/(c/e^4*d^4+b/e^2*d^2-a)^
(1/2)/(c*x^4+b*x^2-a)^(1/2))+1/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*e/d*(
1+1/2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1-1/2*(b+(4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)
^(1/2)*x,-2/(-b+(4*a*c+b^2)^(1/2))*a*e^2/d^2,1/2*2^(1/2)*((b+(4*a*c+b^2)^(
1/2))/a)^(1/2)/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)))
```

3.413.5 Fricas [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{gx+f}{\sqrt{cx^4+bx^2-a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 - a)*(g*x + f)/(c*e*x^5 + c*d*x^4 + b*e*x^3 +
b*d*x^2 - a*e*x - a*d), x)
```

3.413.6 Sympy [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral((f + g*x)/((d + e*x)*sqrt(-a + b*x**2 + c*x**4)), x)
```

3.413.7 Maxima [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{gx+f}{\sqrt{cx^4+bx^2-a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```

3.413.8 Giac [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{gx+f}{\sqrt{cx^4+bx^2-a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(e*x + d)), x)
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{f+g\,x}{(d+e\,x)\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

input
```
int((f + g*x)/((d + e*x)*(b*x^2 - a + c*x^4)^(1/2)),x)
```

output
```
int((f + g*x)/((d + e*x)*(b*x^2 - a + c*x^4)^(1/2)), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```